

Frequency-Response Masking FIR Filters

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Introduction

Introduction to FRM Filters
Filters Preserving Phase

Frequency Response Masking

Narrow Band Filter Design
Arbitrary Bandwidth Filter Design

Parameter Optimization

Ripples of F
Optimizing F and M
Further Optimization

Examples

Conclusion

Frequency-Response Masking Filters

- ▶ frequency-response masking filters are a technique to design sharp low-pass, high-pass, bandpass and bandstop filters with arbitrary passband bandwidth
- ▶ furthermore linear phase FIR filters are generated, which have advantages such as guaranteed stability and are free of phase distortion
- ▶ however, the problem with FIR filters is the high complexity for sharp filters

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Advantages

- ▶ with the frequency-response masking technique the resulting filter has very sparse coefficients
- ▶ since only a very small fraction of its coefficient values are nonzero, its complexity is very much lower than the infinite wordlength minimax optimum filter
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Linear Phase FIR Filters

- ▶ in linear phase FIR filters phase is a linear function of frequency
- ▶ they have a symmetric impulse response
- ▶ the phase delay ($-\frac{phase}{\omega}$) is $\frac{N-1}{2}$ at every frequency
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Zero Phase Filters

- ▶ are a special case of linear phase filters, where the phase delay is zero
- ▶ impulse response of a zero phase filter is even about time 0:

$$h(n) = h(-n)$$

therefore this filter cannot be causal

- ▶ a real, even impulse response corresponds to a real, even frequency response
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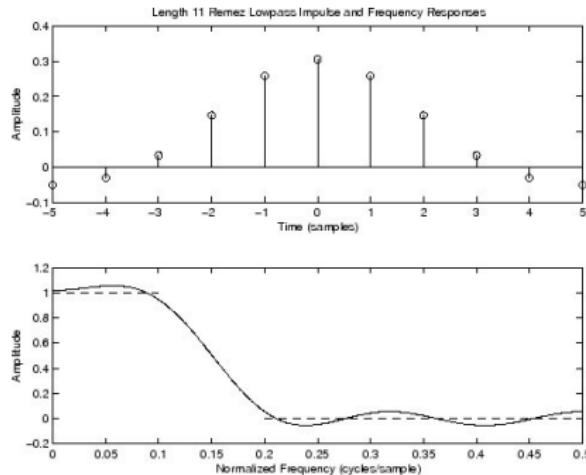
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Zero Phase Filters

Impulse and frequency response of a length 11 zero-phase FIR lowpass filter:



Symmetric Linear Phase Filters

- ▶ are derived from a delayed zero-phase filter
- ▶ are causal and symmetric about the midpoint:

$$h(n) = h(N - 1 - n), n = 0, 1, \dots, N - 1$$

- ▶ H_{ZP} is a zero-phase filter, N is odd:

$$h_{ZP}(n) = h\left(n - \frac{N-1}{2}\right), n = 0, 1, \dots, N - 1$$

$$H(z) = z^{-\frac{N-1}{2}} H_{ZP}(z)$$

$$H(e^{j\omega T}) = e^{-j\omega \frac{N-1}{2}T} H_{ZP}(e^{j\omega T})$$

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Basic Principle

The basic principle of frequency masking is the following:

- ▶ in a linear phase model filter each delay is replaced by M delays
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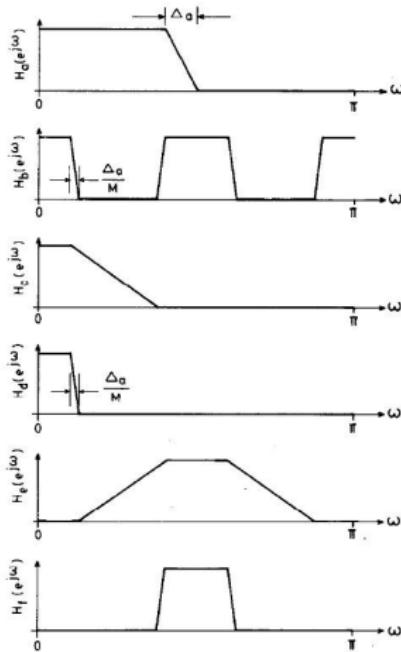
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Narrow Band Filter Design

Frequency Masking Principle:



- ▶ low-pass filter $H_a(z)$ with transition width Δ_a (model filter)
 - ▶ replacing each delay by M delays: $H_b(z) = H_a(z^M)$
 - ▶ masking filter $H_c(z)$
 - ▶ resulting frequency response:

$$H_d(e^{j\omega}) = H_b(e^{j\omega})H_c(e^{j\omega})$$
with transition width Δ_a/M
 - ▶ masking filter $H_e(z)$
 - ▶ resulting frequency response: $H_f(e^{j\omega}) = H_b(e^{j\omega})H_e(e^{j\omega})$

Narrow Band Filter Design

- ▶ This describes a method of deriving sharp filters (Δ_a/M) from filters with much wider transition band (Δ_a)
- ▶ Advantages: only a few coefficients in the model filter are nonzero, so the complexity is very low
- ▶ Problem: only suitable for narrow-band filters, because the passband bandwidth is reduced by the same factor

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Arbitrary Bandwidth Design

Consider a filter F_c complementary to the masking filter F_a :

- ▶ z-transform of the symmetric linear phase filter F_a :

$$F_a(z) = z^{-\frac{N-1}{2}} F_{a,ZP}(z)$$

where $F_{a,ZP}(z)$ is a zero-phase filter and N is odd

- ▶ the complementary filter F_c :

$$F_c(z) = z^{-\frac{N-1}{2}} (1 - F_{a,ZP}(z))$$

- ▶ this results in

$$F_c(z) = z^{-\frac{N-1}{2}} - F_a(z)$$

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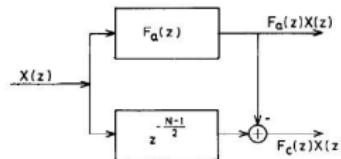
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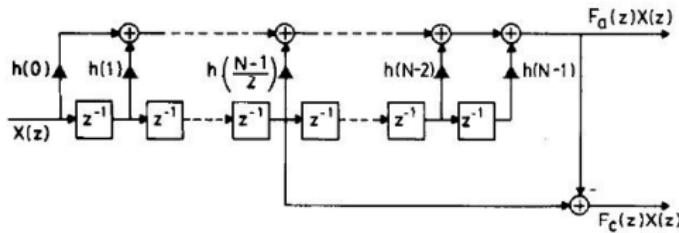
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Complementary Filter Pair

F_c can be implemented by subtracting the output of F_a from a delayed version of the input:

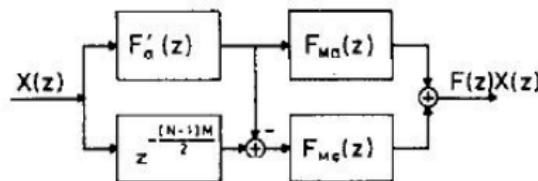


without extra delays:



Masking Filters

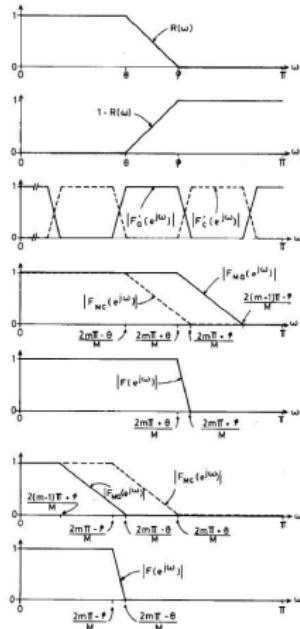
If two masking filters, F_{Ma} and F_{Mc} for F_a and F_c , are used, it's possible to design wide-band sharp filters:



$$F(z) = F_a(z^M)F_{Ma}(z) + \left(z^{-\frac{N-1}{2}} - F_a(z^M)\right)F_{Mc}(z)$$

Arbitrary Bandwidth Filter Design

Arbitrary Bandwidth Masking Principle:



- ▶ model filter F_a , cutoff frequencies θ and ϕ
- ▶ complementary filter F_c
- ▶ replacing
 - each delay of F_a and F_c by M delays to get periodic, complementary model filters
- ▶ masking filters F_{Ma} and F_{Mc}
- ▶ resulting frequency response $F(e^{j\omega})$ with band edges ω_P and ω_S
- ▶ other masking filters F_{Ma} and F_{Mc}
- ▶ resulting frequency response $F(e^{j\omega})$

Arbitrary Bandwidth Filter Design

One can distinguish two cases:

- ▶ Case1: the frequency response of F near the transition band is determined mainly by F_a , pass- and stopband is defined by

$$\omega_P = \frac{2m\pi + \theta}{M}, \omega_S = \frac{2m\pi + \phi}{M}$$

- ▶ Case2: mainly determined by F_c , then pass- and stopband is defined by

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Synthesis Problem

In a synthesis problem the following has to be considered:

- ▶ ω_P and ω_S are given and m, M, θ, ϕ must be determined
- ▶ M should be chosen that the overall complexity of the filter is minimized
- ▶ this leads to an optimization problem:
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Ripples of F

In this section the ripples of the overall filter $F(e^{j\omega})$ are analyzed:

- ▶ let $G(\omega)$ be the desired value and $\delta(\omega)$ the deviation from this value for each filter
- ▶ for $F(e^{j\omega})$ this leads to:

$$G(\omega) + \delta(\omega) = (G_{Ma}(\omega) + \delta_{Ma}(\omega))(G_a(\omega) + \delta_a(\omega)) \\ + (G_{Mc}(\omega) + \delta_{Mc}(\omega))(1 - G_a(\omega) - \delta_a(\omega))$$

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Ripples, Fr. Range 1 + 2

- ▶ Frequency Range 1: $G_{Ma}(\omega) = G_{Mc}(\omega) = 1$ (passband)

$$G(\omega) = 1$$

$$G_a(\omega) = 1, \delta(\omega) \approx \delta_{Ma}(\omega)$$

$$G_a(\omega) = 0, \delta(\omega) \approx \delta_{Mc}(\omega)$$

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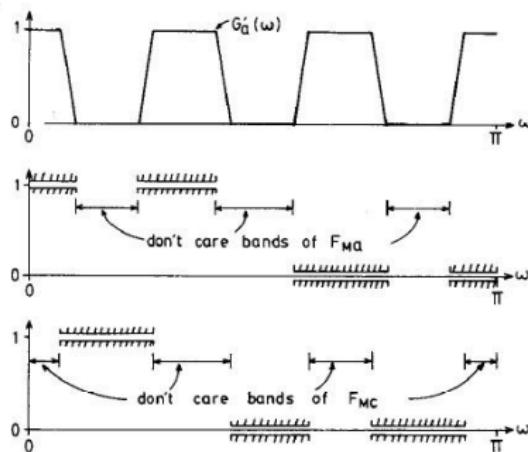
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Don't Care Bands

Therefore F_{Ma} and F_{Mc} could be interpreted as low-pass filters with don't care bands within their pass- and stopbands:



These don't care bands help to reduce the complexity of the masking filters.

Ripples, Fr. Range 3

Frequency Range 3: $G_{Ma}(\omega) \neq G_{Mc}(\omega)$, transition band

- ▶ here $\delta(\omega)$ is a function of $\delta_a(\omega)$, $\delta_{Ma}(\omega)$ and $\delta_{Mc}(\omega)$
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Optimization of F

F_a has to be designed to compensate for $\delta_{Ma}(\omega)$ and $\delta_{Mc}(\omega)$

- ▶ a linear equation relating $\delta(\omega)$ and F_a must be obtained:

$$\begin{aligned}\delta(\omega) = & F_{a,ZP}(M\omega)(G_{Ma}(\omega) + \delta_{Ma}(\omega) - G_{Mc}(\omega) - \delta_{Mc}(\omega)) \\ & + G_{Mc}(\omega) + \delta_{Mc}(\omega) - G(\omega)\end{aligned}$$

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- ▶ a good choice of M can be obtained by estimating the filter complexity for each M (nonzero multipliers) and then selecting the M which corresponds to the lowest estimate
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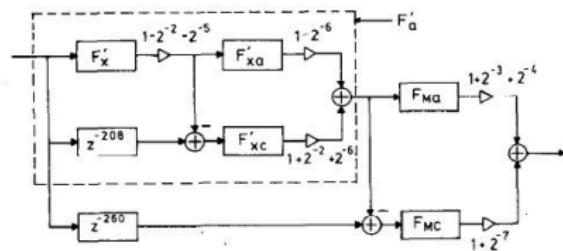
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Multistage Frequency Response Masking Design

The model and masking filters may again be synthesized using the frequency response masking technique, producing a multistage frequency response masking design:



Optimizing this technique is again subject of many more recent papers.

Powers-of-Two Design Technique

- ▶ the complexity of the filter may be further reduced by constraining all the coefficient values to be a sum or difference of two powers-of-two using the powers-of-two design technique
- ▶ in this case, the multiplication can be performed just by using shifts and adds

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Single-Stage Design

- ▶ The single-stage FRM low-pass filter, using the powers-of-two design technique, should meet the following specifications:
- ▶ bandedges at 0.3 and 0.305 sampling frequencies, maximum passband deviation is 0.1 dB and minimum stopband attenuation is -40 dB
- ▶ this filter requires 202 shift-add operations per sampling interval, whereas the infinite precision minimax optimum design requires 383 multiply and 382 add operations

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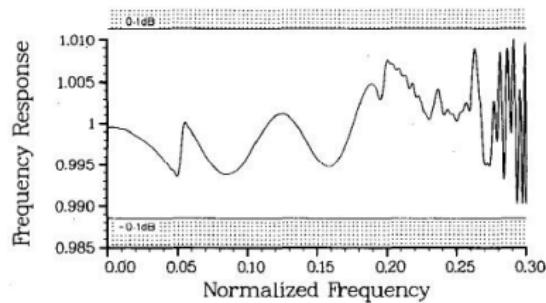
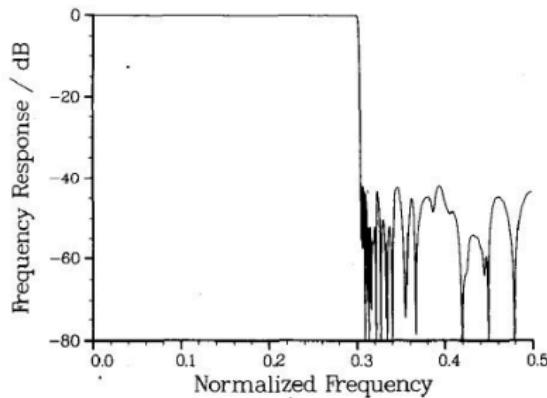
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Single-Stage Design Frequency Response

Frequency Response of the single-stage FRM low-pass filter:



Multi-Stage Design

- ▶ Now a multi-stage FRM low-pass filter with the following specifications should be designed:
- ▶ bandedges at 0.2 and 0.2001 sampling frequencies, maximum passband deviation is 0.05 dB and minimum stopband attenuation is -50 dB
- ▶ a five stage design was used with $M1 = M2 = M3 = M4 = 4$ and $M5 = 3$
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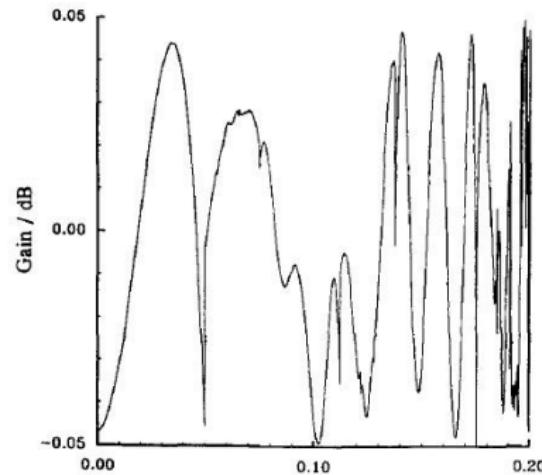
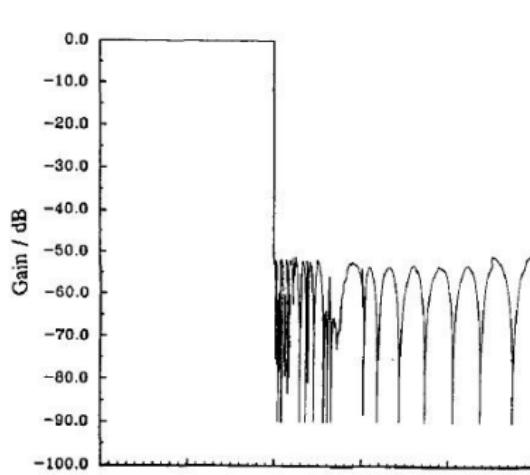
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References

- ▶ Yong Ching Lim; *Frequency-Response Masking Approach for the Synthesis of Sharp Linear Phase Digital Filters*; 1986, IEEE transactions on circuits and systems
- ▶ Julius O. Smith; *Introduction to Digital Filters*; 2006, Center for Computer Research in Music and Acoustics (CCRMA), Stanford University
- ▶ Yong Ching Lim and Yong Lian; *The Optimum Design of One- and Two-Dimensional FIR Filters Using the Frequency Response Masking Technique*; 1986, IEEE transactions on circuits and systems

Questions

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