Räkneregler för sannolikheter:

$S \Rightarrow$ "sample space" = utfallsrum

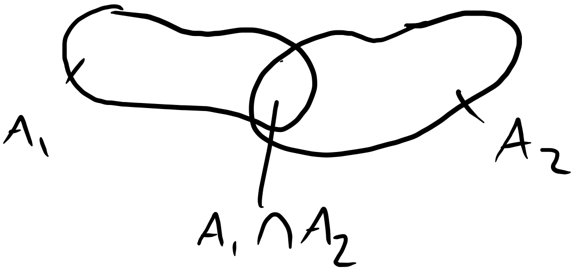
Axiom $\boxed{P[S] = 1}$

$P[A] \geq 0$ för alla A

$P[\emptyset] = 0$

$P[A'] = 1 - P[A]$

$$P[A_1 \cup A_2] = P[A_1] + P[A_2] - P[A_1 \cap A_2]$$



90% $p = 0{,}9$

Om A_1 = primära motorn fungerar

A_2 = sekundära motorn fungerar

$$P[A_1 \cap A_2] = 0.9 \cdot 0.9 = 0{,}81$$

$$P[A_1 \cup A_2] = 0{,}9 + 0.9 - 0{,}81 = 0{,}99$$

Låt $A_1, A_2, \ldots$ vara ömsesidigt uteslutande händelser

$$P[A_1 \cup A_2 \cup \ldots] = P[A_1] + P[A_2] + \ldots$$

<u>Oberoende händelser</u>

A_1 och A_2 är oberoende omm

$$P[A_1 \cap A_2] = P[A_1]\, P[A_2]$$

<u>Beroende händelser</u>

$$P[A_2 \mid A_1] = \frac{P[A_1 \cap A_2]}{P[A_1]}$$

Oberoende händelser (alt):

$$P[A_2 | A_1] = P[A_2] \quad (P[A_1] \neq 0)$$

$$P[A_1 | A_2] = P[A_1] \quad (P[A_2] \neq 0)$$

multiplikationsregeln:

$$P[A_1 \cap A_2] = P[A_2 | A_1] P[A_1]$$

Bayes Teorem

Låt $A_1, A_2, A_3, \ldots, A_n$ vara en samling ömsesidigt uteslutande händelser vars union är S. Låt B vara en händelse $P[B] \neq 0$. Då gäller för varje händelse A_j, $j = 1, 2, 3, \ldots, n$:

$$P[A_j | B] = \frac{P[B | A_j] P[A_j]}{\sum_{i=1}^{n} P[B | A_i] P[A_i]}$$

A chemist analyses water near a factory.
Past experience shows that 38% of samples contain toxic levels of lead or mercury.
In addition 32% contain toxic levels of lead and 16% contain toxic levels of mercury.

Q: What is the probability that a random sample contains toxic levels of lead only?

A_1 : sample contains toxic levels of lead

A_2 : sample contains toxic levels of mercury

Given: $P[A_1] = .32$, $P[A_2] = .16$

$P[A_1 \cup A_2] = .38$

A_1 : sample contains toxic levels of lead

A_2 : sample contains toxic levels of mercury

Given: $P[A_1] = .32$, $P[A_2] = .16$

$P[A_1 \cup A_2] = .38$

$$P[A_1 \cup A_2] = P[A_1] + P[A_2] - P[A_1 \cap A_2]$$

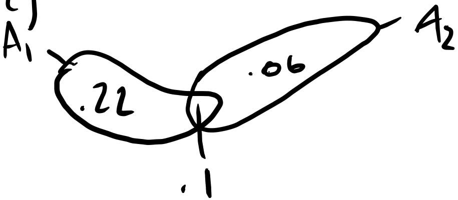
$$.38 = .32 + .16 - P[A_1 \cap A_2]$$

$$P[A_1 \cap A_2] = .1$$

a)

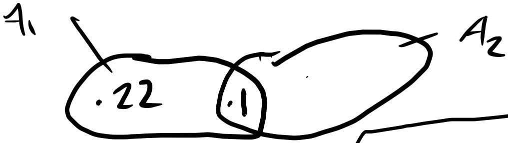
$P[A_1 \cap A_2] = .1$

c)



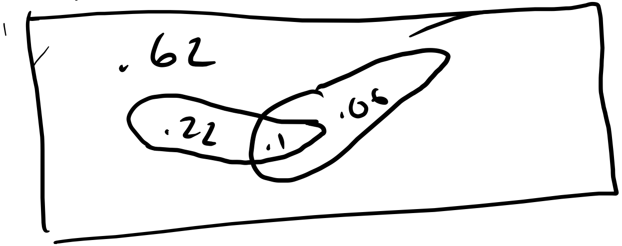
$P[A_1' \cap A_2] = .06$

b)

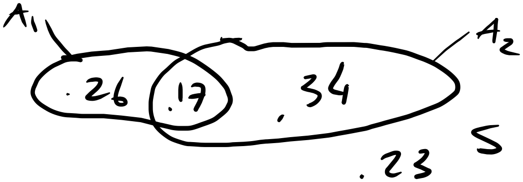


$\boxed{P[A_1 \cap A_2'] = .22}$

d)



$P[A_1' \cap A_2'] = .62$



$P[A_2 \mid A_1]$ ← den vi söker

A_1 : protein present

A_2 : child is male

$P[A_1] = .43$

$P[A_2] = .51$

$P[A_1 \cap A_2] = .17$

$$\frac{.17}{.43} = .395$$

$$P[A_2 \mid A_1] = \frac{P[A_1 \cap A_2]}{P[A_1]}$$

A_1: spader dras

A_2: honorcard (10, J, Q, K, A)

$P[A_1] = \frac{13}{52}$ $P[A_1 \cap A_2] = \frac{5}{52}$

$P[A_2] = \frac{20}{52}$

Är A_1 och A_2 oberoende?

$P[A_1]P[A_2] = (13/52)(20/52) = 5/52$

A_1 : main operable

A_2 : first backup operable

A_3 : second backup operable

$$P[A_1] = P[A_2] = P[A_3] = .9$$

oberoende

$$P[A_1 \cap A_2 \cap A_3] = P[A_1]P[A_2]P[A_3]$$

$$= (.9)^3 = .729$$

För bilolyckor på en viss väg gäller:

E : fortkörning (någon förare körde för fort)

A : rattfylla (någon — " — var full)

$P[E] = .4$ $\qquad$ $P[E'] = 1 - .4 = .6$

$P[A] = .3$ $\qquad$ $P[A'] = 1 - .3 = .7$

$P[E \mid A] = .6$

$P[E \mid A'] = .1$

$$P[A \mid E] = \frac{P[E \cap A]}{P[E]}$$

$$P[A \mid E]\, P[E] = P[E \cap A]$$

$$E = (E \cap A) \cup (E \cap A')$$

$$E = (E \cap A) \cup (E \cap A')$$

$$P[E] = P[E \cap A] + P[E \cap A']$$

$$P[E \cap A'] = P[E \mid A']\,P[A']$$

$$P[A \mid E] = \frac{P[E \cap A]}{P[E]}$$

$$= \frac{P[E \mid A]\ P[A]}{P[E \mid A]P[A] + P[E \mid A']P[A']}$$

$$= \frac{.6 \cdot .3}{(.6)(.3) + (.1)\cdot(.7)} = .72$$

Blodtyper i USA:

typ A : 41%

typ B : 9%

typ AB : 4%

typ O : 46%

typ O men klassad som A : 4%

typ A och korrekt : 88%

typ B klassad som A : 4%

typ AB klassad som A : 10%

Antag en individ klassad som typ A.

Vad är sannolikheten att individen är typ A?

A_1 : blodtypen är A

A_2 : blodtypen är B

A_3 : — " — AB

A_4 : — " — O

B : klassad som typ A

$$P[A_1 \mid B]$$

$$\begin{cases} P[A_1] = .41 \\ P[A_2] = .09 \\ P[A_3] = .04 \\ P[A_4] = .46 \end{cases}$$

$P[B|A_1] = .88$

$P[B|A_2] = .04$

$P[B|A_3] = .10$

$P[B|A_4] = .04$

$$P[A_1|B] = \frac{P[B|A_1]\cdot P[A_1]}{P[B|A_1]P[A_1] + P[B|A_2]P[A_2] + P[B|A_3]P[A_3] + P[B|A_4]P[A_4}$$

$$= \frac{(.88)(.41)}{(.88)(.41) + (.04)(.09) + (.10)(.04) + (.04)(.46)}$$

$$= .93$$