Fundamentals of DIP

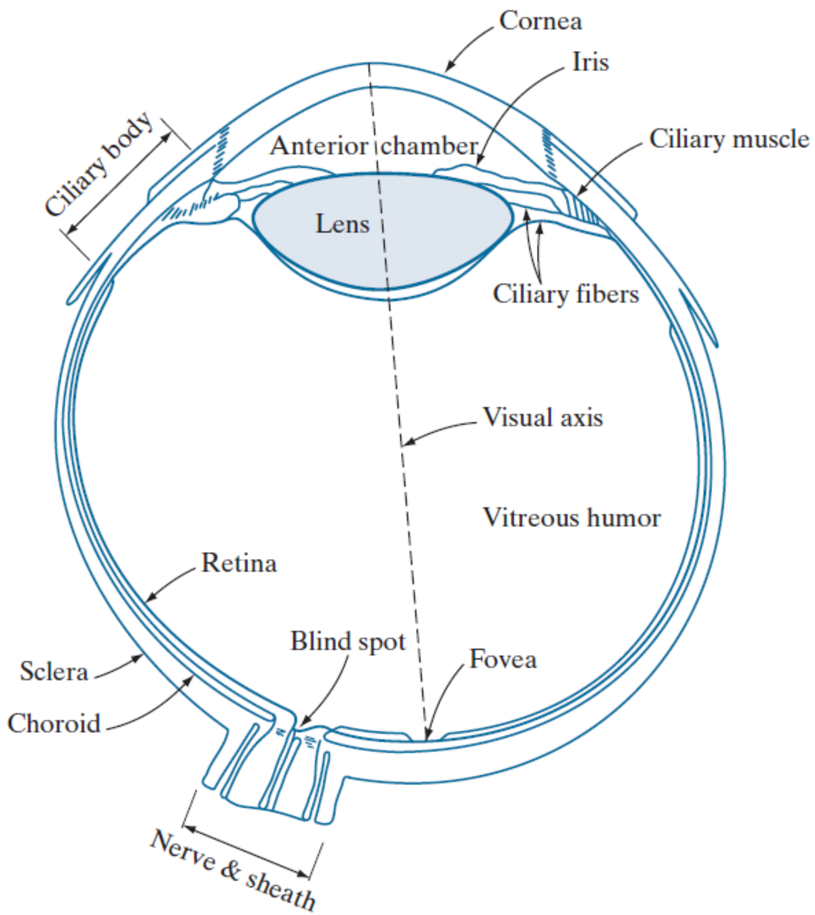
ICT4201: DIP

ELEMENTS OF VISUAL PERCEPTION

- Humans can choose different techniques for image processing using intuition and analysis.
- To understand image processing properly we have to learn the techniques used by human eye to form and perceive images, i.e. how human can see using eyes.
- We are interested in learning the physical limitations of human vision in terms of factors that also are used in our work with digital images. Factors such as how human and electronic imaging devices compare in terms of resolution and ability to adapt to changes in illumination are not only interesting, they are also important from a practical point of view.

STRUCTURE OF THE HUMAN EYE

- The eye is nearly a sphere (with a diameter of about 20 mm)
  - The *cornea* and *sclera* outer cover
  - The *choroid*
    - *Lens and ciliary muscle*
  - The *retina*
    - When the eye is focused, light from an object is imaged on the retina.
    - There are two types of receptors: *cones* and *rods*.
    - Cones:
      - Inside *fovea* and are between 6 and 7 million
      - Highly color sensitive
      - Muscles makes images formed in fovea
      - Vision is called *photopic* or *bright-light* vision.



STRUCTURE OF THE HUMAN EYE

- The number of rods is much larger: Some 75 to 150 million are distributed over the retina.
- The larger area of distribution, and the fact that several rods are connected to a single nerve ending, reduces the amount of detail discernible by these receptors.
- Rods capture an overall image of the field of view. They are not involved in color vision, and are sensitive to low levels of illumination.
  - For example, objects that appear brightly colored in daylight appear as colorless forms in moonlight because only the rods are stimulated.
  - This phenomenon is known as scotopic or dim-light vision.

FIGURE 2.2 Distribution of rods and cones in the retina.

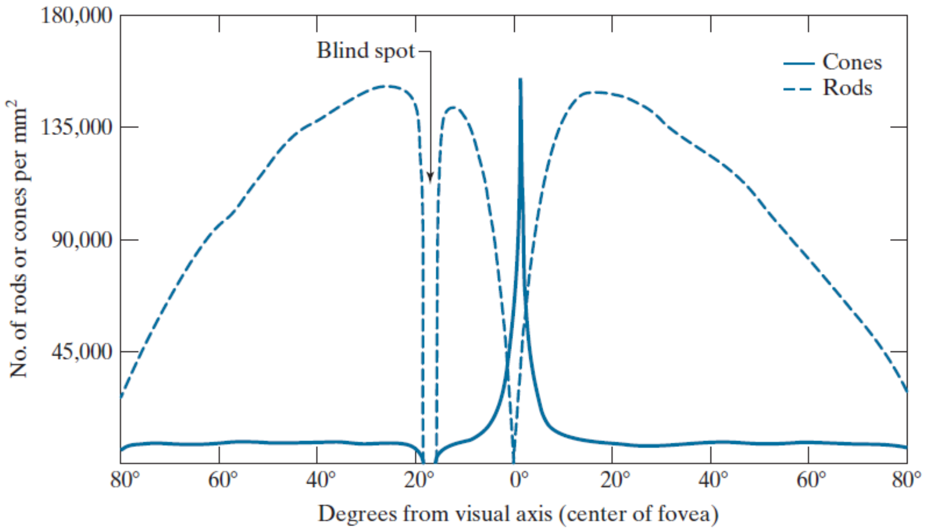
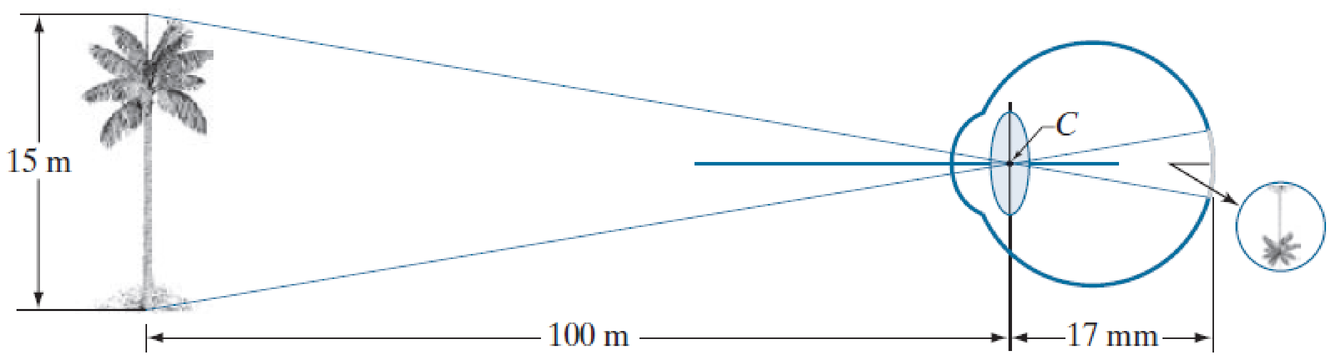


IMAGE FORMATION IN THE EYE

FIGURE 2.3 Graphical representation of the eye looking at a palm tree. Point C is the focal center of the lens.



- In an ordinary photographic camera, the lens has a fixed focal length. Focusing at various distances is achieved by varying the distance between the lens and the imaging plane, where the film (or imaging chip in the case of a digital camera) is located.
- In the human eye, the converse is true; the distance between the center of the lens and the imaging sensor (the retina) is fixed, and the focal length needed to achieve proper focus is obtained by varying the shape of the lens.
- The range of focal lengths is approximately 14 mm to 17 mm, In the fig. $15/100 = h/17$ or $h = 2.5$ mm, is the height of the retinal image.

BRIGHTNESS ADAPTATION AND DISCRIMINATION

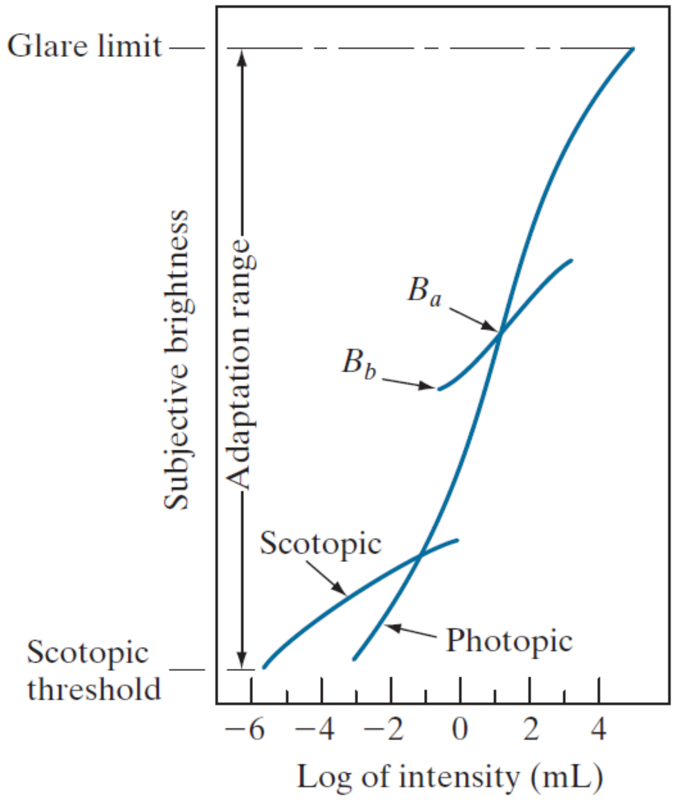
- Because digital images are displayed as sets of discrete intensities, the eye's ability to discriminate between different intensity levels is an important consideration in presenting image processing results.

- The range of light intensity levels
  - the human visual system can adapt is enormous—on the order of 10^{10}— from the scotopic threshold to the glare limit.
  - Measured by the energy of light source.

- Subjective brightness is a logarithmic function of the light intensity incident on the eye.

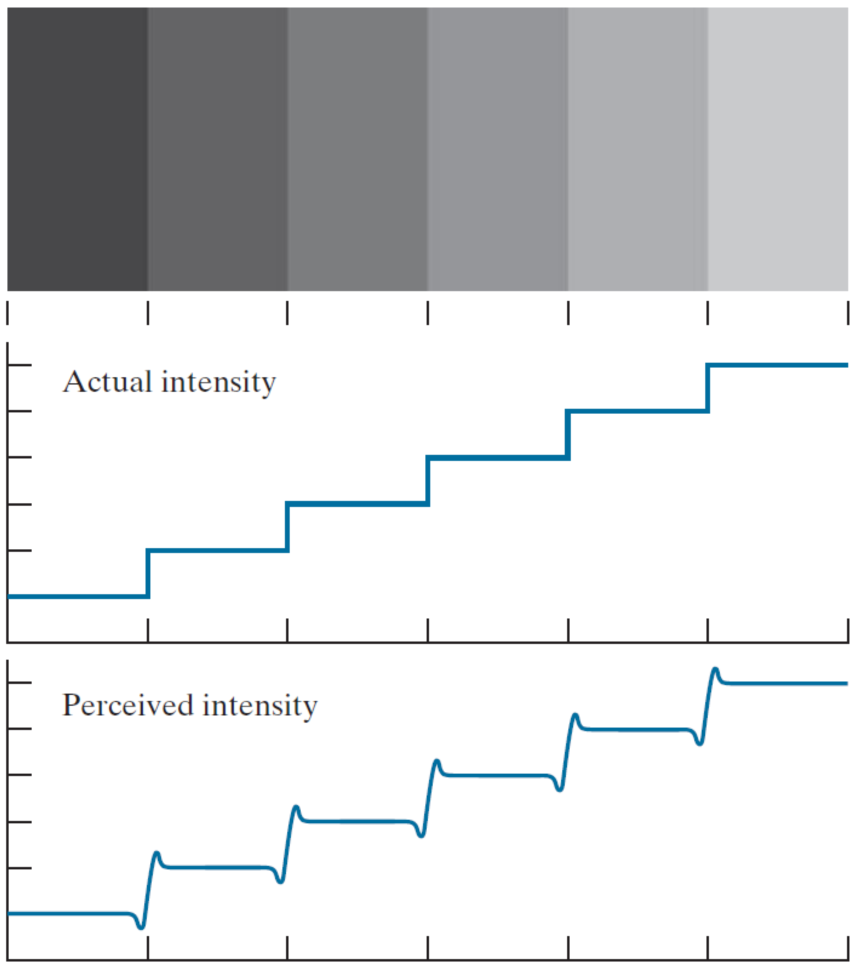
- The total range that can be perceived simultaneously is much smaller.

- Adaptation is based on brightness level.



MACH BAND EFFECT

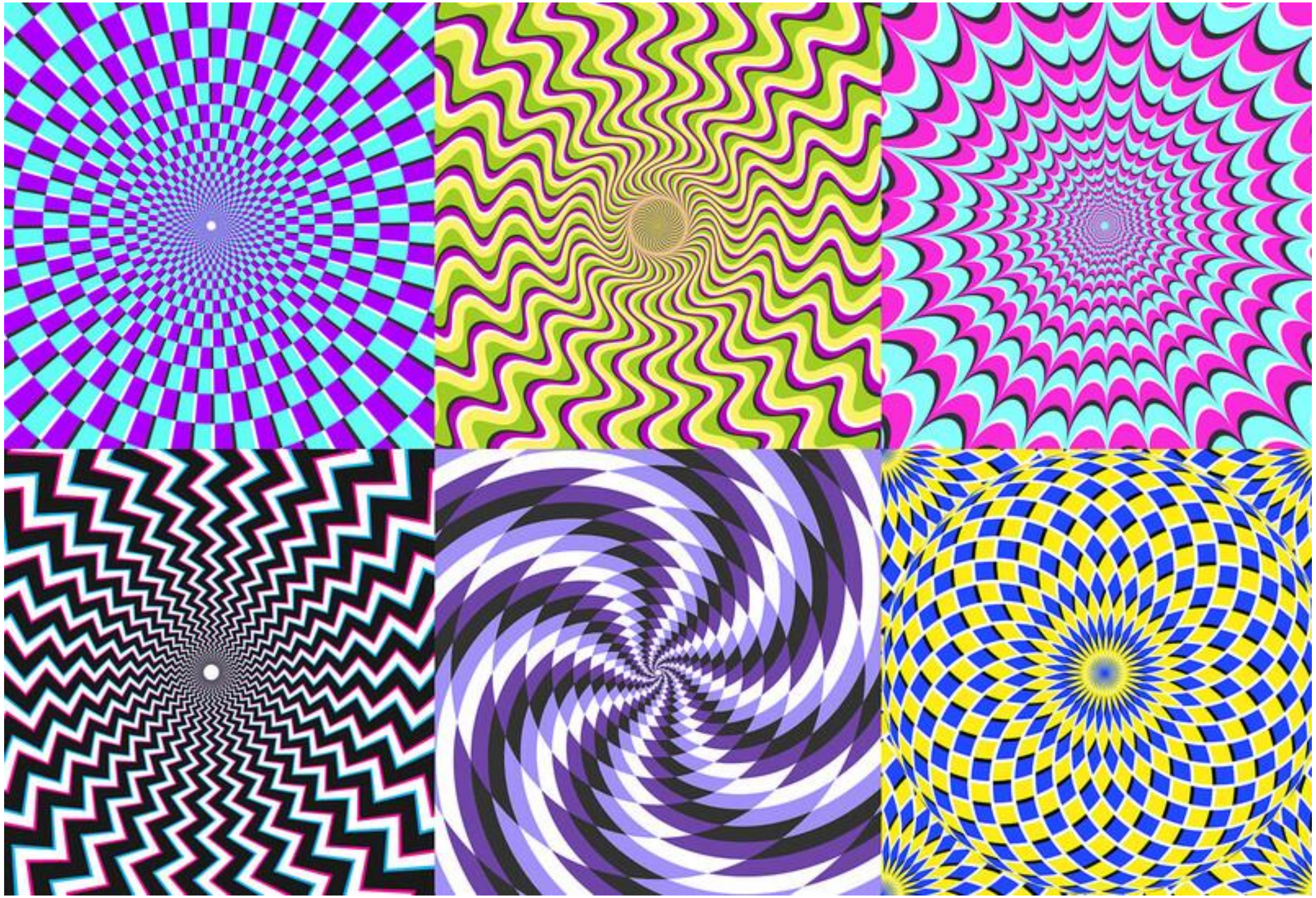
- The visual system tends to undershoot or overshoot around the boundary of regions of different intensities
- Although the intensity of the stripes is constant, we actually perceive a brightness pattern that is strongly scalloped, especially near the boundaries.
- These seemingly scalloped bands are called Mach bands after Ernst Mach, who first described the phenomenon in 1865.
- In neurobiology, lateral inhibition is the capacity of an excited neuron to reduce the activity of its neighbors.



SIMULTANEOUS CONTRAST

- A region's perceived brightness does not depend only on its intensity, as Fig. demonstrates.
- All the center squares have exactly the same intensity, but each appears to the eye to become darker as the background gets lighter.
- A more familiar example is a piece of paper that looks white when lying on a desk, but can appear totally black when used to shield the eyes while looking directly at a bright sky.

Optical Illusion

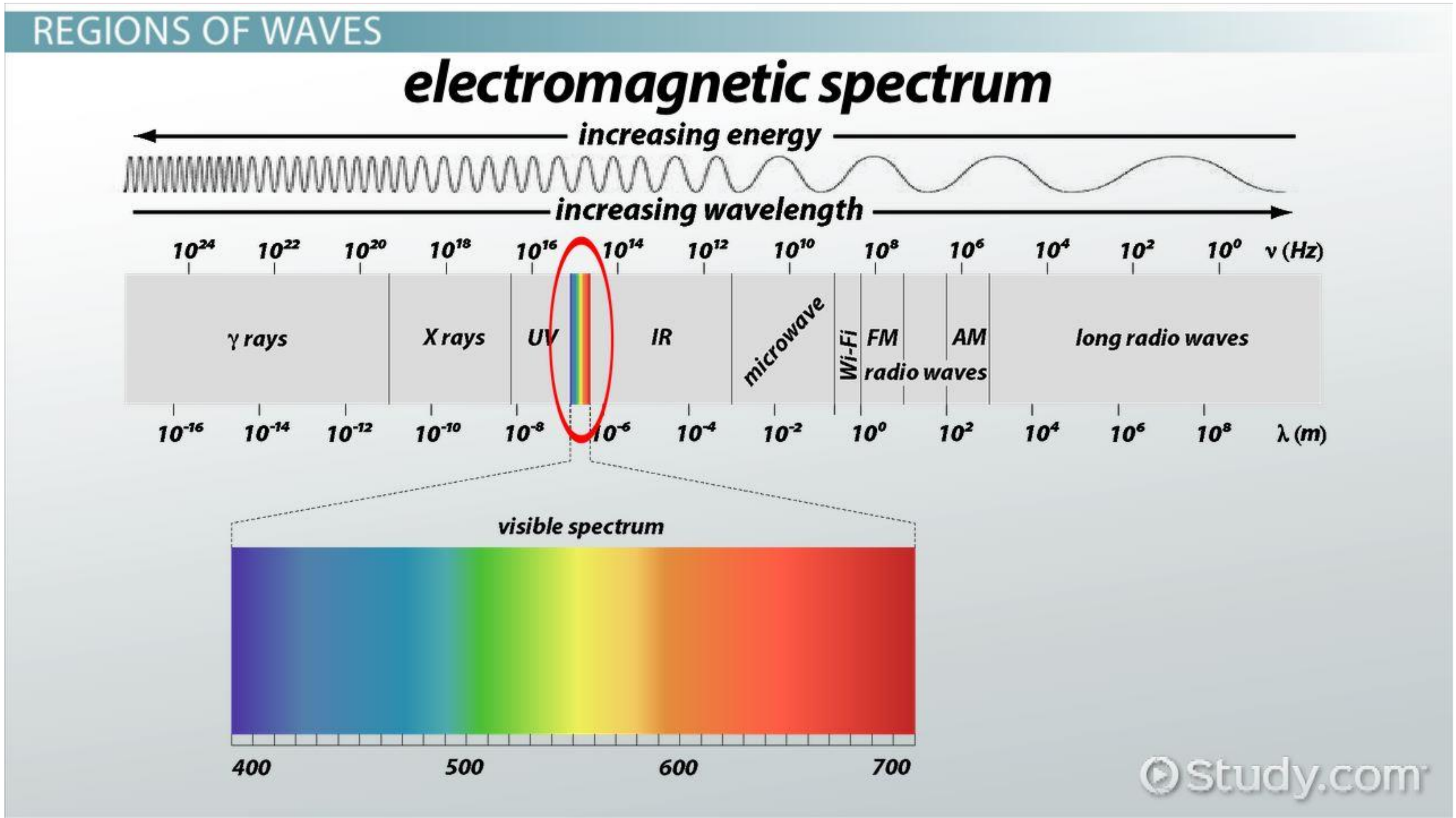


Human Visual Perception

- Nonlinear
- Small range of discrimination
- Very subjective

LIGHT AND THE ELECTROMAGNETIC SPECTRUM

In 1666, Sir Isaac Newton discovered that when a beam of sunlight passes through a glass prism, the emerging beam of light is not white but consists instead of a continuous spectrum of colors ranging from violet at one end to red at the other.



LIGHT AND THE ELECTROMAGNETIC SPECTRUM

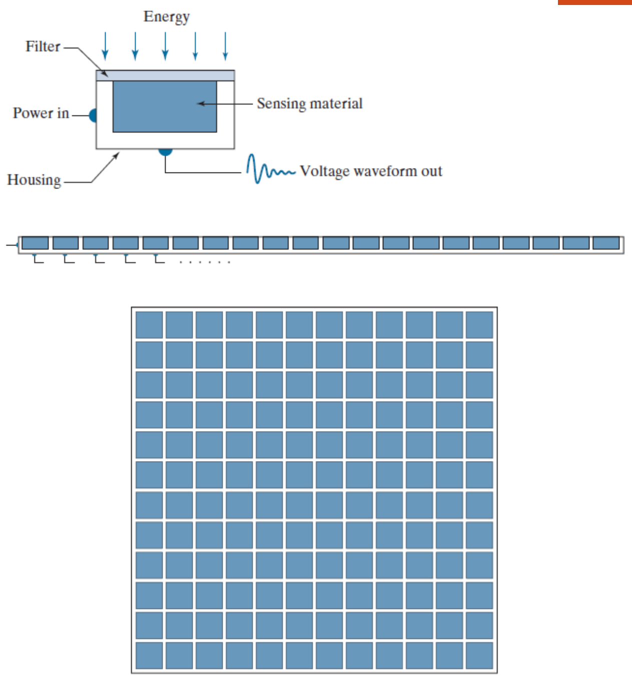
- The colors perceived in an object are determined by the nature of the light *reflected* by the object.
  - Light that is void of color is called *monochromatic* (or *achromatic*) light. The only attribute of monochromatic light is its intensity, vary from black to grays and finally to white. (grayscale/gray level)
  - *Chromatic* (color) light spans the electromagnetic energy spectrum from approximately 0.43 to 0.79 mm
  - *Radiance* is the total amount of energy that flows from the light source, and it is usually measured in watts (W).
  - *Luminance*, measured in lumens (lm), gives a measure of the amount of energy an observer *perceives* from a light source.
  - *Brightness* is a subjective descriptor of light perception that is practically impossible to measure. It embodies the achromatic notion of intensity and is one of the key factors in describing color sensation.

IMAGE SENSING AND ACQUISITION

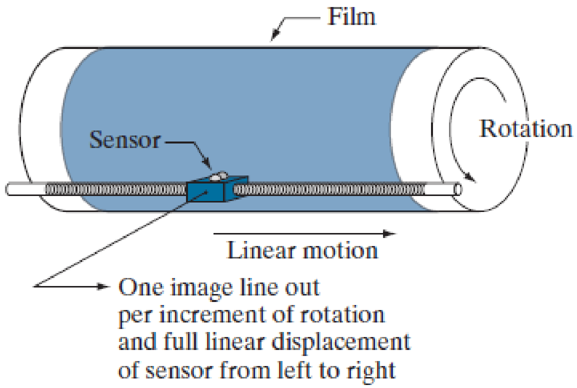
- Most of the images in which we are interested are generated by the combination of an “illumination” source and the reflection or absorption of energy from that source by the elements of the “scene” being imaged.
- Sensor: Incoming energy → a voltage by combination of input electrical power and sensor material.
- (photon → electron → voltage)
- Types of sensor: –
  - Mechanical Sensor
    - Single sensor -- photodiode
    - Line sensor -- The "modulation" of the light intensity causes corresponding variations in the sensor voltage, which are ultimately converted to image intensity levels by digitization.
  - Array sensor

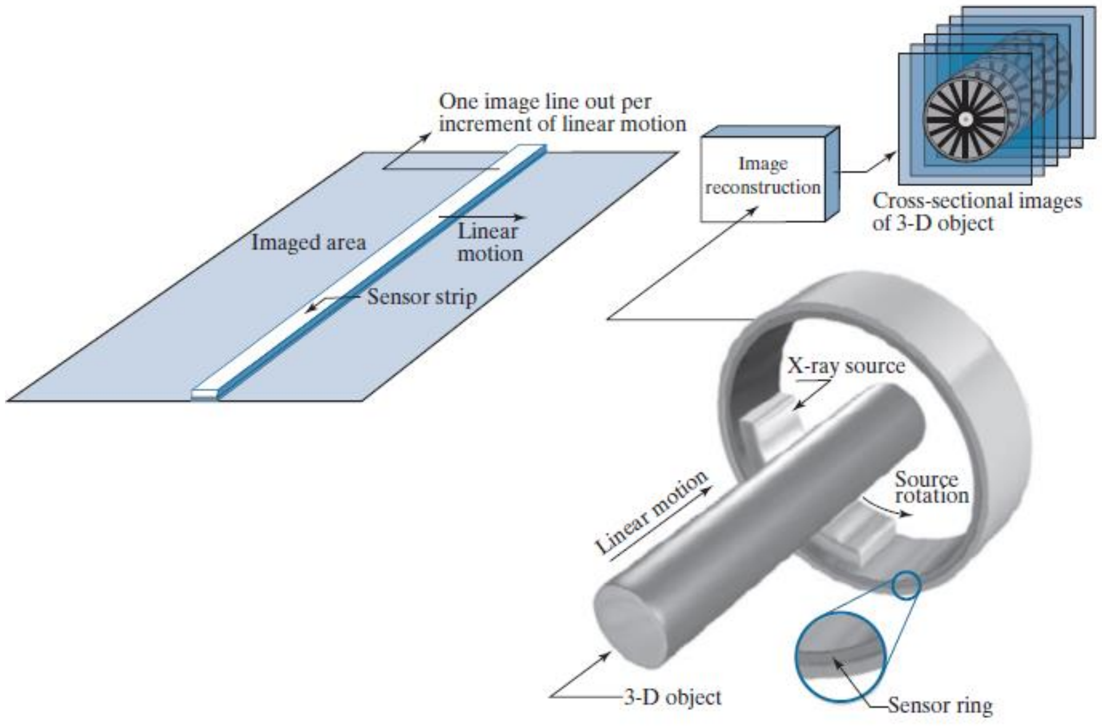
a
b
c

FIGURE 2.12
(a) Single sensing element.
(b) Line sensor.
(c) Array sensor.



SINGLE SENSOR AND LINE SENOR

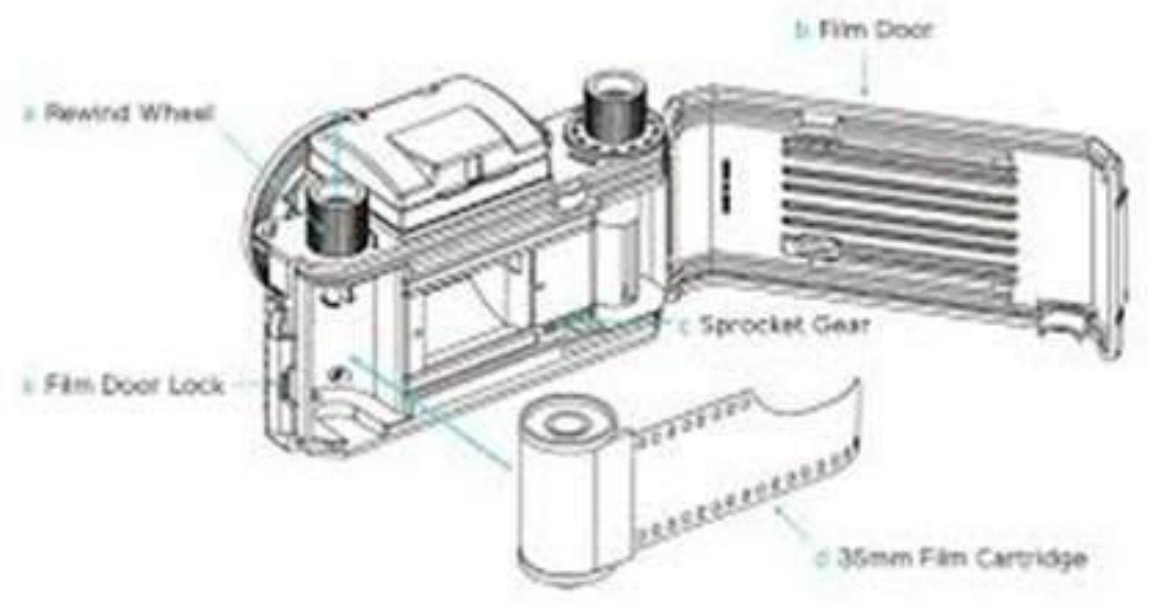




This arrangement is used in most flat bed scanners. Sensing devices with 4000 or more in-line sensors are possible.
Used in airborne imaging applications

Sensor strips in a ring configuration are used in medical and industrial imaging to obtain cross-sectional ("slice") images of 3-D objects.
Needs computerized digital processing for image construction, expensive to use.

IMAGE FORMATION ON ANALOG CAMERAS

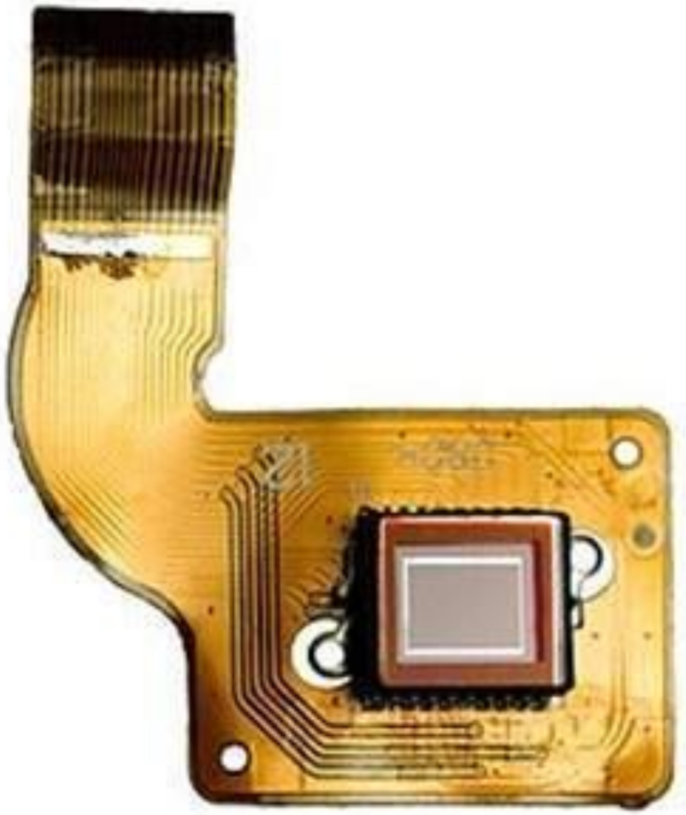


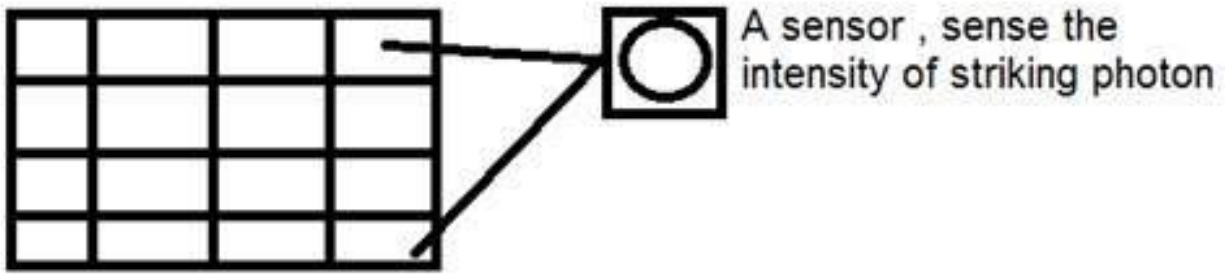
35mm strip

Photons (light particles) + silver halide → silver → image negative.

IMAGE FORMATION ON DIGITAL CAMERAS

In the digital camera , a CCD array of sensors is used for the image formation.

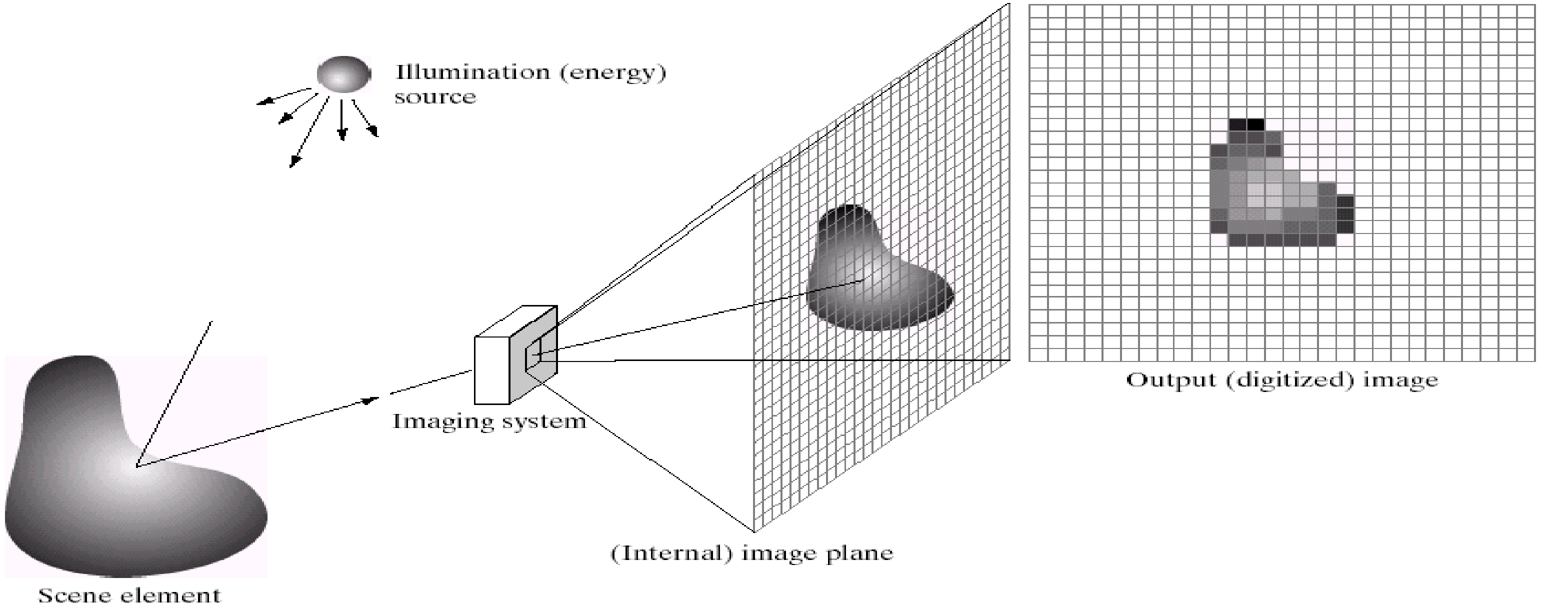




- CCD stands for charge-coupled device.
- It is an image sensor, and like other sensors it senses the values and converts them into an electric signal.
- In case of CCD it senses the image and convert it into electric signal.

IMAGE ACQUISITION USING SENSOR ARRAYS

- Process of acquiring an image
- Converting simple image into digitized image
- Involves some preprocessing like scaling etc also

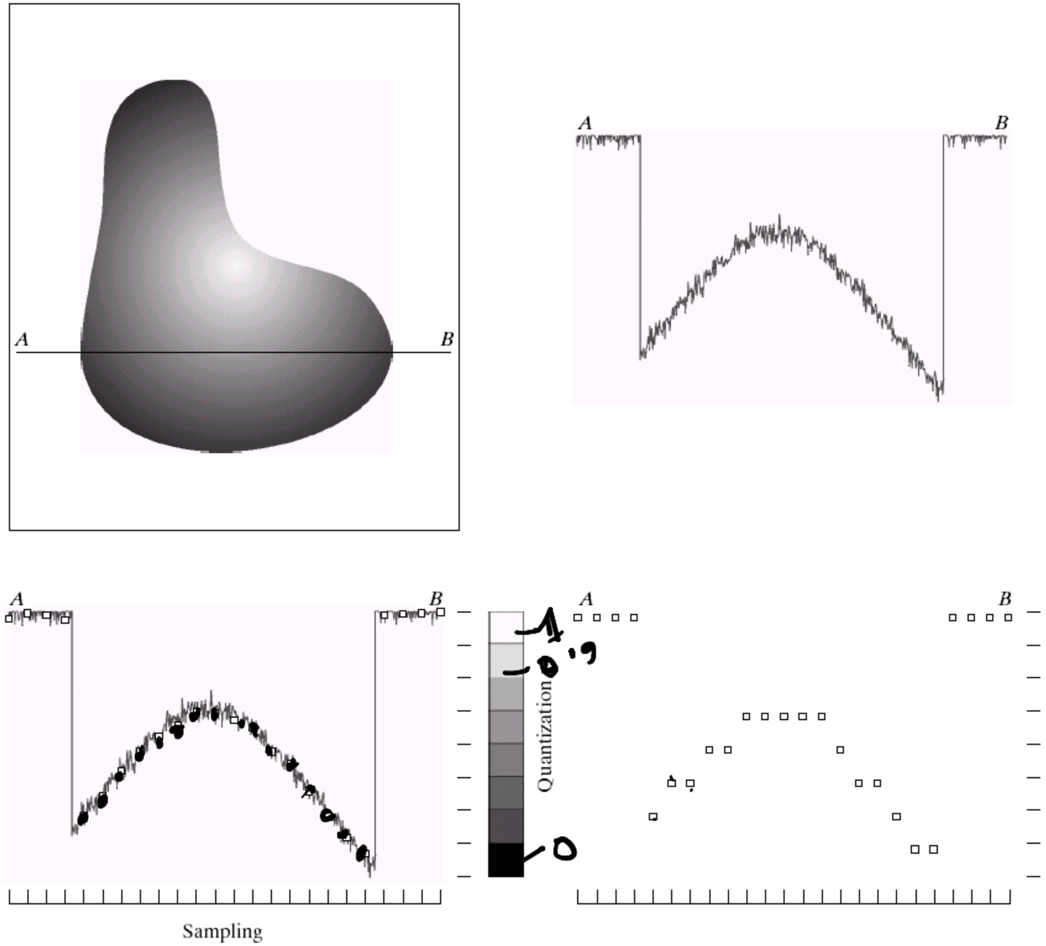


FUNCTIONAL REPRESENTATION OF IMAGES

- Two-D function $f(x, y)$, where (x, y) pixel position. Positive and bounded.
- $f(x, y)$, is characterized by two components:
  - Amount of source illumination incident on the source being viewed.
  - Amount of illumination reflected by the objects in the scene.
- Written as $f(x, y) = i(x, y)r(x, y)$, $i(x, y) < \infty$ illumination from light source, $r(x, y) \leq 1$, reflectance (bounded between 0 and 1) based on material properties,
  - e.g $r(x, y)$=0.01 for black velvet, $r(x, y) = 0.93$ for snow.
- Intensity of monochrome image $f(x, y)$ is synonymous with *grey levels*. By convention grey level are from 0 to L-1.
  - 0 - total absorption
  - 1 - total reflectance

IMAGE SAMPLING AND QUANTIZATION

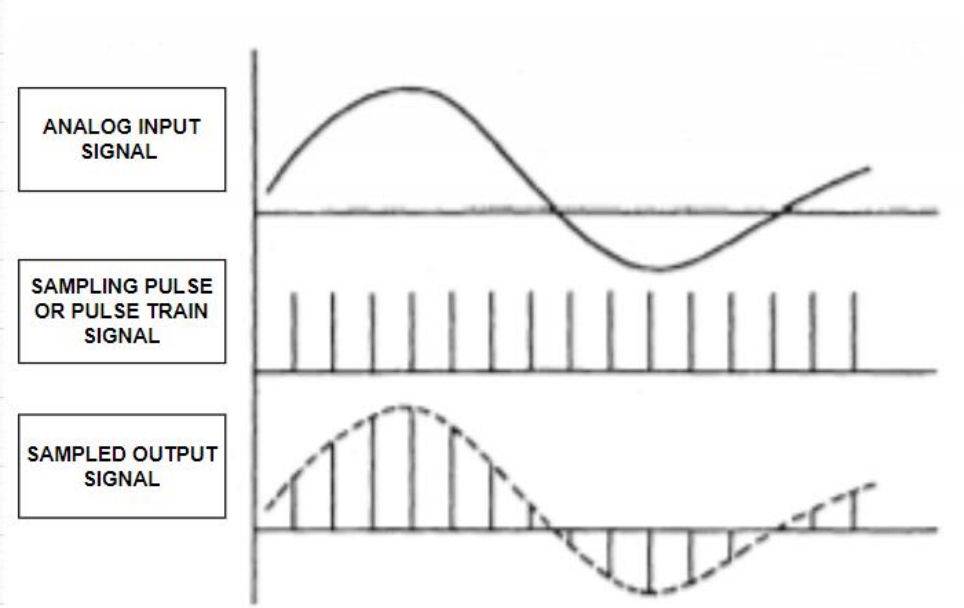
- Digitizing the coordinate value is called Sampling
- Digitizing the amplitude value is called quantization.



a b
c d

FIGURE 2.16 Generating a digital image. (a) Continuous image. (b) A scan line from A to B in the continuous image, used to illustrate the concepts of sampling and quantization. (c) Sampling and quantization. (d) Digital scan line.

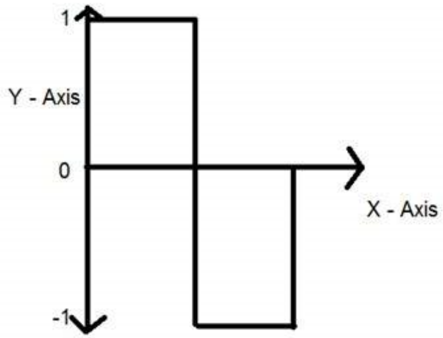
SAMPLING



- Sampling as its name suggests can be defined as take samples. Take samples of a digital signal over x axis. Sampling is done on an independent variable, x.
- To do the sampling we have to take a particular time period, $f = 1/T$, f is the sampling frequency.
- For any cont. signal $f(t)$, at first we have to multiply it with a pulse train or railing function, $r(t)$, to get the sampled signal as $f(n) = f(t) \times r(t)$. This signal is the approximation of the original signal. Original signal can be reconstructed using low pass filters.
- Sampling frequency decides the distance between samples and represents the unit pixel size.
- Shannon-Nyquest Theorem: $f \geq 2 \times f_{max}$, where f_{max} is the maximum frequency in image.

QUANTIZATION

- Quantization as its name suggest can be defined as dividing into quanta (partitions). Quantization is done on dependent variable. It is opposite to sampling.
- In case of this mathematical equation y = sin(x)
- Quantization is done on the Y variable. It is done on the y axis. The conversion of y axis infinite values to 1, 0, -1 (or any other level) is known as Quantization.
- These are the two basics steps that are involved while converting an analog signal to a digital signal.
- The quantization of a signal has been shown in the figure below.



Quantization

- It is quantizing the amplitude values. No. of quantization value is high enough for human perception of the high details in the image.
- Most of the DIP devices uses quantization levels, $k = 2^b$, where b is the no. of bits representing the grey level values of the image.
  - For 8-bit images, $k = 2^8 = 256$ levels
  - For 16-bit images, $k = 2^{16} = 65{,}536$ levels

OUTPUT OF SAMPLING AND QUANTIZATION

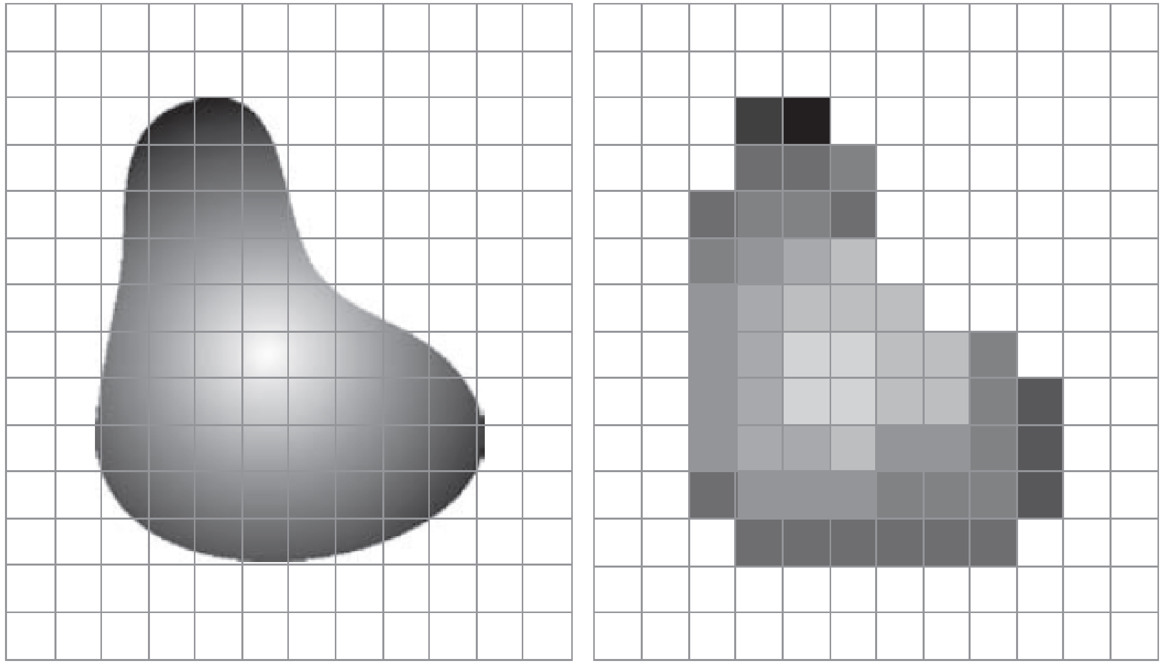


FIGURE 2.17
(a) Continuous image projected onto a sensor array. (b) Result of image sampling and quantization.

Quantization and Sampling

- Method of Sampling is determined by the sensor arrangement used acquire the image, such as single, line or array sensor.
- Quantization of sensor outputs complete the process of generating a digital image.
- The quality of digital image is determine to a large degree the number of samples or **Sampling** and discrete grey levels or **Quantization** used in this process.
- Finally we can say quality of an image in any device depends on the sample rate and grey levels used in digitizing the image.

Types of Sampling

- There are two types of sampling
    1. Down sampling: Images are scaled down by half by reducing the sample rate.
    2. Up Sampling: Images are scaled up using replication or interpolation.

- Example of Down/Sub Sampling:
    - Images are cut into half of its size. Consider the following image I of 4×4, which would be reduced in 2×2 size using
    - Single pixel selection
    - Statistical method/ Mean method

$$I = \begin{array}{|c|c|c|c|} \hline 3 & 3 & 3 & 3 \\ \hline 9 & 9 & 9 & 9 \\ \hline 3 & 3 & 3 & 3 \\ \hline 9 & 9 & 9 & 9 \\ \hline \end{array}$$

$$F = \begin{bmatrix} 3 & 3 \\ 3 & 3 \end{bmatrix}$$

Single pixel selection

Statistical Method

$$F' = \begin{bmatrix} 6 & 6 \\ 6 & 6 \end{bmatrix}$$

$$\begin{pmatrix} 3 & 3 \\ 9 & 9 \end{pmatrix} \rightarrow \frac{3 + 3 + 9 = 9}{4} = \frac{24}{4} = 6$$

Up Sampling using Replication

- Replication: We can increase the size of image by doubling its pixels, i.e. 2×2 image can be converted to 4×4. Let input image, $F = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$
  - Zero hold process: inserting zeros in the neighbouring pixels.
  - Repeating neighbouring pixels

$$I = \begin{bmatrix} 2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad I' = \begin{bmatrix} 2 & 2 & 1 & 1 \\ 2 & 2 & 1 & 1 \\ 1 & 1 & 3 & 3 \\ 1 & 1 & 3 & 3 \end{bmatrix}$$

Up Sampling using Interpolation

- Let input image, $F = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$, we can up sampling it into 4×4 image using Linear interpolation.
- Step 1: Zero hold process, $I =$

2	0	1	0
0	0	0	0
1	0	3	0
0	0	0	0

- Step 2: Row-wise interpolation:

$I =$

2	1.5	1	0.5
0	0	0	0
1	2	3	1.5
0	0	0	0

$$\frac{2+1}{2} = 1.5 \quad \frac{1+0}{2} = 0.5$$

$$\frac{1+3}{2} = 2 \quad \frac{3+0}{2} = 1.5$$

- Step 3: Column-wise Interpolation

$I =$

2	1.5	1	0.5
1.5	1.75	2	1
1	2	3	1.5
0.5	1	1.5	0.75

$$\frac{2+1}{2} = 1.5 \quad \frac{1.5+2}{2} = 1.75 \quad \frac{1+3}{2} = 2 \quad \frac{0.5+1.5}{2} = 1$$

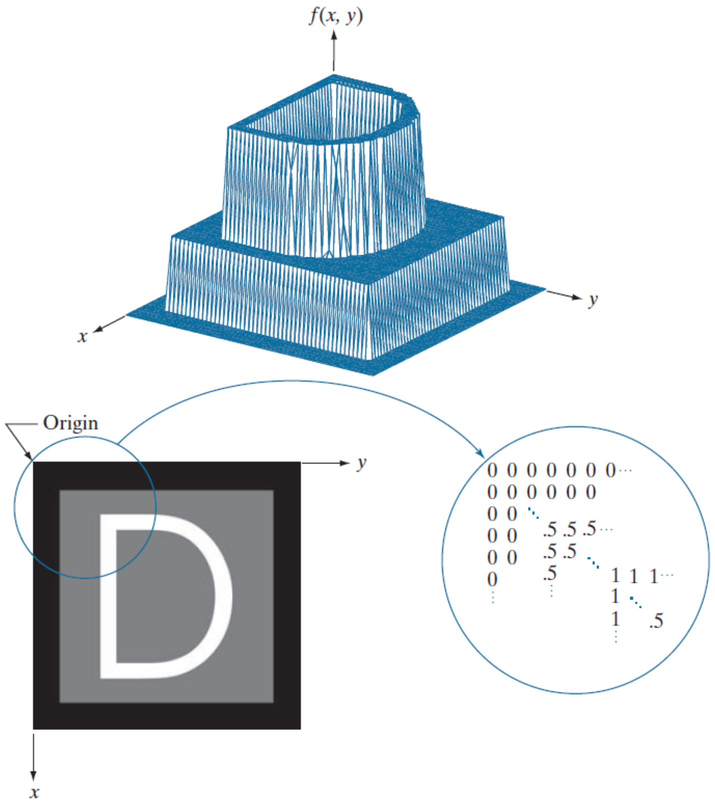
$$\frac{2+1}{2} = 1.5 \quad \frac{2+0}{2} = 1 \quad \frac{3+0}{2} = 1.5 \quad \frac{1.5+0}{2} = 0.75$$

REPRESENTING DIGITAL IMAGES

- Let $f(s, t)$ represent a *continuous* image function of two continuous variables, s and t, using sampling and quantization we convert it into digital image, $f(x, y)$, containing M rows and N columns.
- The spatial coordinates are integer values such as: $x = 0, 1, 2, \ldots, M-1$ and $y = 0, 1, 2, \ldots, N-1$.
- Figure 2.18(a) is a plot of the function, with two axes determining spatial location and the third axis being the values of f as a function of x and y.
- Fig. 2.18(b) is more common, and it shows $f(x, y)$ as it would appear on a computer display or photograph.
- Fig. 2.18(c) shows, the third representation is an array (matrix) composed of the numerical values of $f(x, y)$. This is the representation used for computer processing.

a
b c

FIGURE 2.18
(a) Image plotted as a surface. (b) Image displayed as a visual intensity array. (c) Image shown as a 2-D numerical array. (The numbers 0, .5, and 1 represent black, gray, and white, respectively.)



REPRESENTING DIGITAL IMAGES

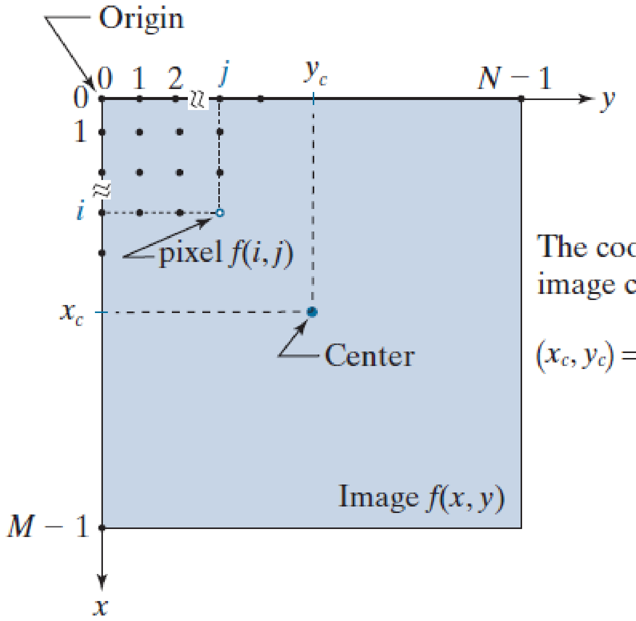
- We write the representation of an $M \times N$ numerical array as

$$f(x,y)=\begin{bmatrix} f(0,0) & f(0,1) & \cdots & f(0,N-1) \\ f(1,0) & f(1,1) & \cdots & f(1,N-1) \\ \vdots & \vdots & & \vdots \\ f(M-1,0) & f(M-1,1) & \cdots & f(M-1,N-1) \end{bmatrix}$$

- We can also represent a digital image in a traditional matrix form, where $a_{ij} = f(i, j)$.

$$\mathbf{A}=\begin{bmatrix} a_{0,0} & a_{0,1} & \cdots & a_{0,N-1} \\ a_{1,0} & a_{1,1} & \cdots & a_{1,N-1} \\ \vdots & \vdots & & \vdots \\ a_{M-1,0} & a_{M-1,1} & \cdots & a_{M-1,N-1} \end{bmatrix}$$

- The number of intensity levels, L, being an integer power of two; that is $L = 2^k$,where k is an integer. We assume that the discrete levels are equally spaced and that they are integers in the range $[0, L-1]$.



The coordinates of the image center are

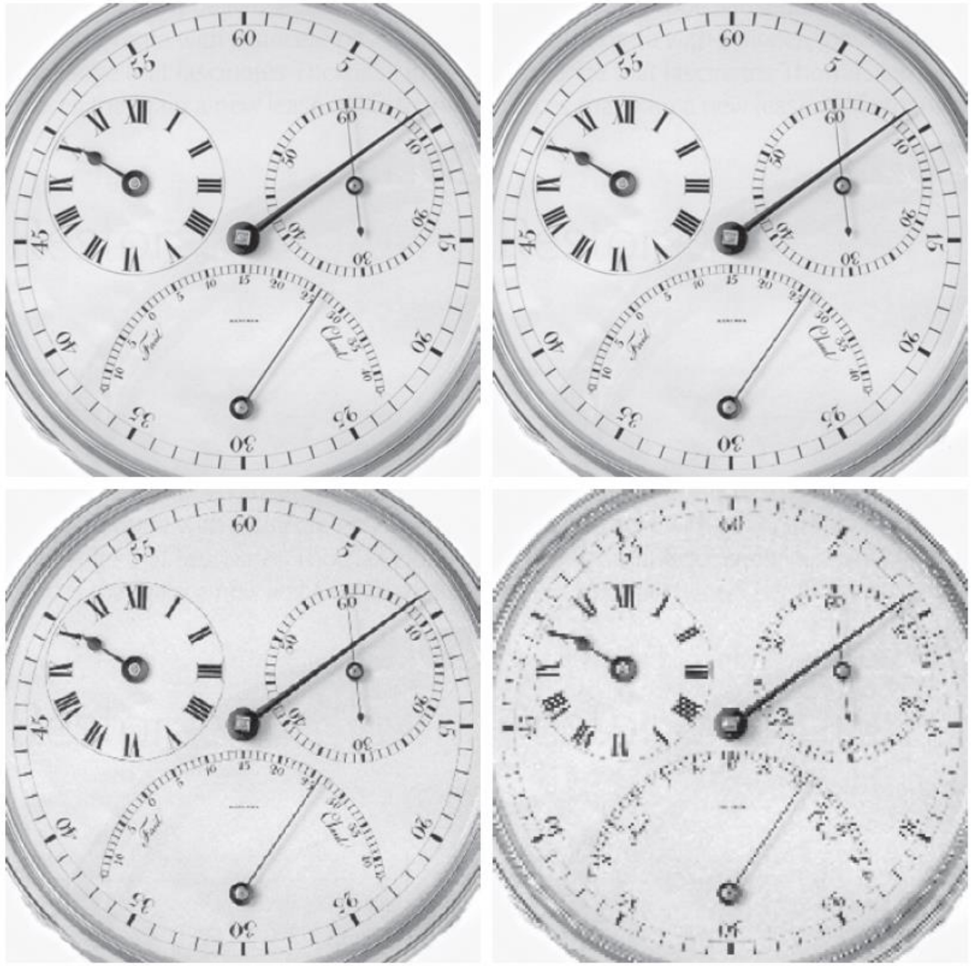
$$(x_c, y_c) = \left(\text{floor}\left(\frac{M}{2}\right), \text{floor}\left(\frac{N}{2}\right)\right)$$

SPATIAL RESOLUTION

- *Spatial resolution* is a measure of the smallest perceptible details in an image.
  - Quantitatively, spatial resolution can be stated in several ways, with *line pairs per unit distance*, and *dots (pixels) per unit distance* being common measures
  - Depends on sampling rate, i.e. the number of pixels per unit distance.

a b
c d

FIGURE 2.23 Effects of reducing spatial resolution. The images shown are at: (a) 930 dpi, (b) 300 dpi, (c) 150 dpi, and (d) 72 dpi.

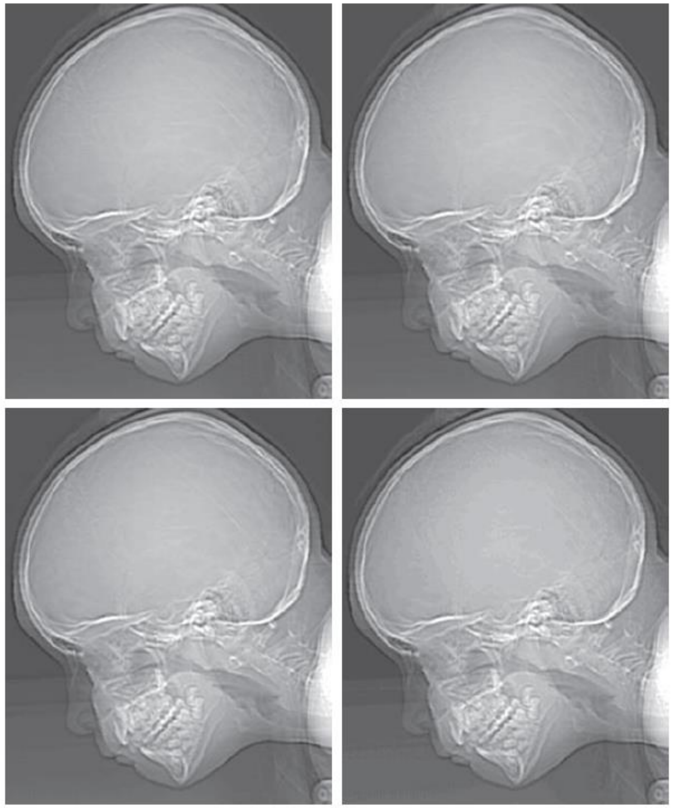


INTENSITY RESOLUTION

- *Intensity resolution* similarly refers to the smallest *discernible* change in intensity level.
  - Based on hardware considerations, the number of intensity levels usually is an integer power of two as mentioned earlier., commonly 8 bits, 16 bits or 32 bits (very rare)
  - Measured by Quantization rate.

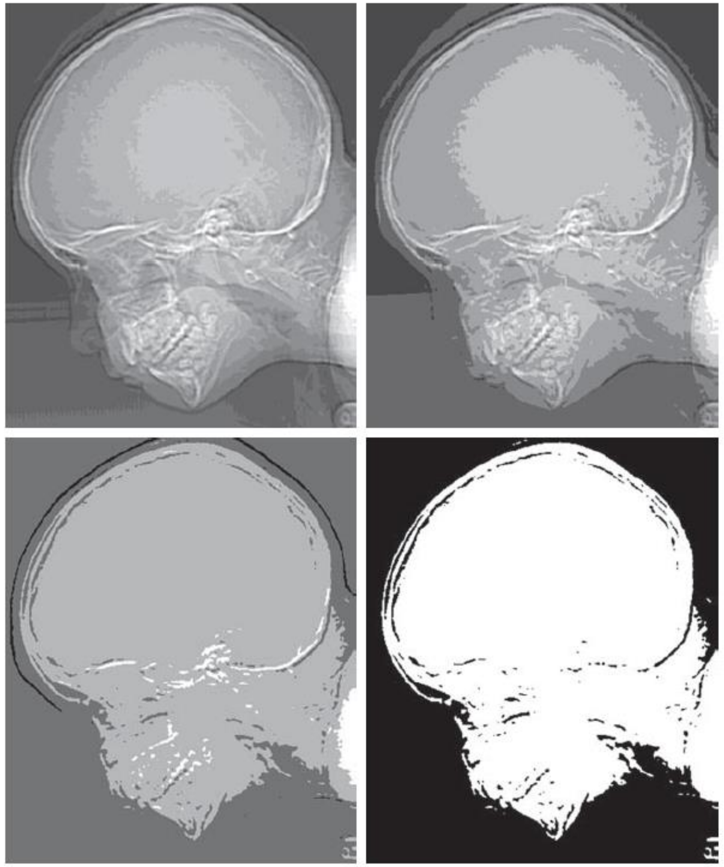
a b
c d

FIGURE 2.24
(a) 774 × 640, 256-level image. (b)-(d) Image displayed in 128, 64, and 32 intensity levels, while keeping the spatial resolution constant. (Original image courtesy of the Dr. David R. Pickens, Department of Radiology & Radiological Sciences, Vanderbilt University Medical Center.)



e f
g h

FIGURE 2.24
(*Continued*) (e)-(h) Image displayed in 16, 8, 4, and 2 intensity levels.



SPATIAL AND INTENSITY RESOLUTION

- Both are digitization dependent
  - Spatial resolution depends on the number of Samples (N)
  - Intensity resolution depends on the number of bits (K)
- Different artifacts
  - Too low spatial resolution creates jagged lines
  - Too low intensity resolution creates false contouring
- Sensitivity
  - Spatial resolution is more sensitive to shape variation
  - Intensity resolution is more sensitive to the lighting variation.
- The experiment by Huang [1965] consisted of a set of subjective tests. Images similar to those shown in Fig. 2.25 were used

a b c

FIGURE 2.25 (a) Image with a low level of detail. (b) Image with a medium level of detail. (c) Image with a relatively large amount of detail. (Image (b) courtesy of the Massachusetts Institute of Technology.)

- The woman's face represents an image with relatively little detail; only lighting information.
- The picture of the cameraman contains an intermediate amount of detail; less lighting information but more geometric information.
- The crowd picture contains, by comparison, a large amount of detail, Most geometric and least lighting information.

SPATIAL AND INTENSITY RESOLUTION

- Results were summarized in the form of so-called *iso-preference curves* in the *Nk*-plane.
  - Each point in the *Nk*-plane represents an image having values of *N* and *k* equal to the coordinates of that point.
  - Change the *N* and *k* values and compare the quality of the images.
  - Each curves represents the image with same quality judged by the observers.
- The graph shows
  - Observe that iso-preference curves tend to become more vertical as the detail in the image increases. This result suggests that for images with a large amount of detail only a few intensity levels may be needed, e.g. the crowd has nearly vertical curve.
  - The other two image categories remained the same in some intervals in which the number of samples was increased, but the number of intensity levels actually decreased. The most likely reason for this result is that a decrease in *k* tends to increase the apparent contrast, a visual effect often perceived as improved image quality.
  - Images with shapes are sensitive to the intensity resolution but less sensitive to spatial resolution.

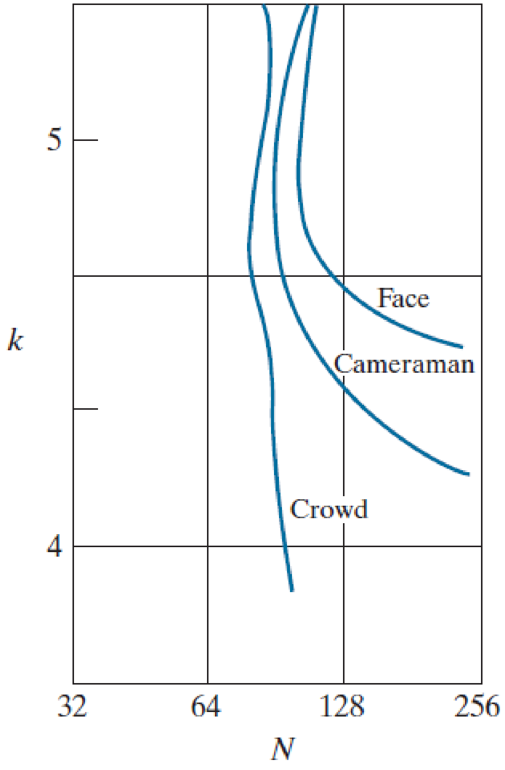


IMAGE INTERPOLATION

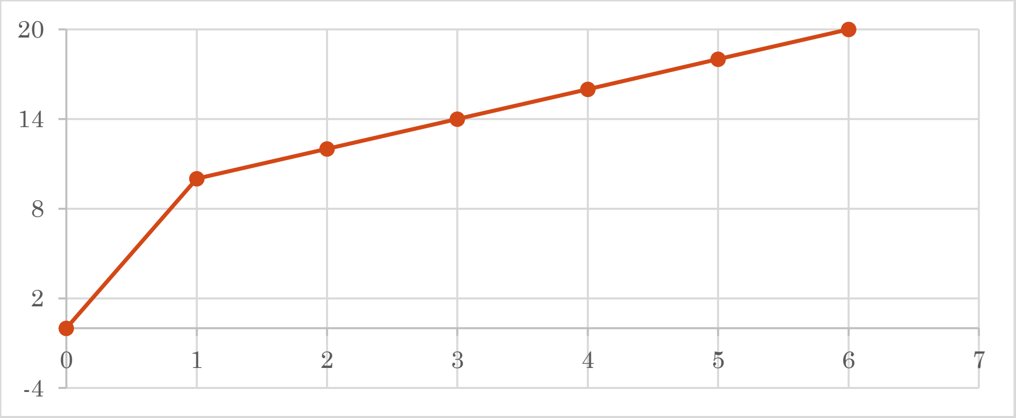
- A basic tool used extensively in lots of tasks such as Zooming, shrinking, rotating, geometric correction – Reconstruction.
  - image resizing (shrinking and zooming), which are basically image resampling methods.
- *Interpolation* is the process of using known data to estimate values at unknown locations.
- Example: Suppose that an image of size 500 × 500 pixels has to be enlarged 1.5 times to 750 × 750 pixels.
  - create an imaginary 750 × 750 grid with the same pixel spacing as the original image
  - then shrink it so that it exactly overlays the original image
  - the pixel spacing in the shrunken 750 × 750 grid will be less than the pixel spacing in the original image
  - To assign an intensity value to any point in the overlay, we look for its closest pixel in the underlying original image and assign the intensity of that pixel to the new pixel in the 750 × 750 grid.
  - When intensities have been assigned to all the points in the overlay grid, we expand it back to the specified size to obtain the resized image.
- This method is called *nearest neighbor interpolation* because it assigns to each new location the intensity of its nearest neighbor in the original image

IMAGE INTERPOLATION-- Example

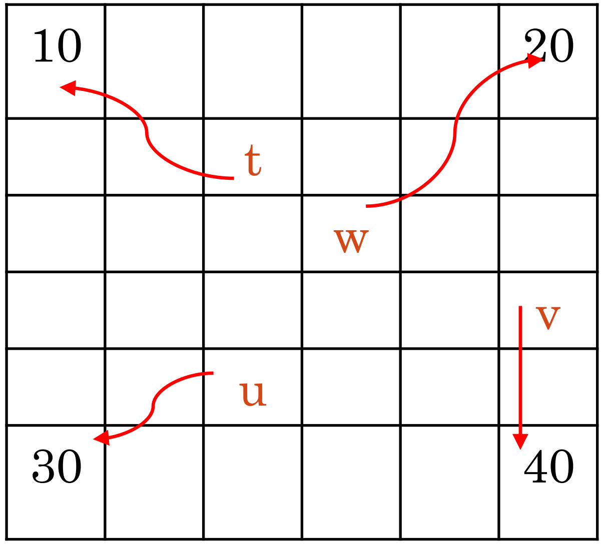
1. Let us expand a 2×2 image by factor 5.
   - If we expand then we get 6×6 grid as shown below:
   - We have to fil the rest of the 32 values closest to the original 4 values.
   - Find the nearest value for each blank space, which is called nearest neighbour interpolation.
     - t= 10, w = 20, u= 30 and v=10

2. Find the values of $f(3)$ using linear interpolation, given that $f(1)$ =10 and $f(6)$= 20.
   - Given that
   - $x_0 = 1$, y_0= 10
   - $x_1 = 6$, y_1= 20
   - $x = 3$, $y = y_0 + \frac{y_1 - y_0}{x_1 - x_0}(x - x_0) = 10 + \frac{20-10}{6-1}(3-1)$

$$= 10 + \frac{10}{5}(2) = 14$$



10	20
30	40



10	10	10	20	20	20
10	10	10	20	20	20
10	10	10	20	20	20
30	30	30	40	40	40
30	30	30	40	40	40
30	30	30	40	40	40

OTHER TECHNIQUES OF IMAGE INTERPOLATION

- N.N. interpolation is simple but, it has the tendency to produce undesirable artifacts, such as severe distortion of straight edges.
- A more suitable approach is *bilinear interpolation*, in which we use the four nearest neighbors to estimate the intensity at a given location.
  - Let (x, y) denote the coordinates of the location to which we want to assign an intensity value
  - Let $v(x, y)$ denote that intensity value. For bilinear interpolation, the assigned value is obtained using the equation $v(x, y) = ax + by + cxy + d$
  - where the four coefficients are determined from the four equations in four unknowns that can be written using the *four* nearest neighbors of point (x, y).
- The next level of complexity is *bicubic interpolation*, which involves the sixteen nearest neighbors of a point. The intensity value assigned to point (x, y) is obtained using the equation

$$v(x, y) = \sum_{i=0}^{3} \sum_{j=0}^{3} a_{ij} x^i y^j$$

  - Used in commercial applications such as Adobe Photoshop and Corel Photopaint.

NEIGHBOURHOOD OF PIXELS

- Images can be represented as

(x-1, y+1)	(x, y+1)	(x+1, y+1)
(x-1, y)	(x, y)	(x+1, y)
(x-1, y-1)	(x-1, y-1)	(x+1, y-1)

  - $N_4(P)$ 4-neighbours: the set of horizontal and vertical neighbours

	(x, y+1)	
(x-1, y)	(x, y)	(x+1, y)
	(x-1, y-1)	

  - $N_D(P)$ diagonal neighbours: the set of 4 diagonal neighbours

(x-1, y+1)		(x+1, y+1)
	(x, y)	
(x-1, y-1)		(x+1, y-1)

  - $N_8(P)$ 8-neighbours : All 4-neighbours, $N_8(P)= N_4(P) + N_D(P)$

(x-1, y+1)	(x, y+1)	(x+1, y+1)
(x-1, y)	(x, y)	(x+1, y)
(x-1, y-1)	(x-1, y-1)	(x+1, y-1)

ADJACENCY AND CONNECTIVITY

- *Adjacency*- Two pixels p and q are adjacent if q is in N(p) where N(p) is the neighborhood of p and they have closely related or same pixel values. Three common definitions of neighborhood are

1. 4-adjacency: If p=(x,y), values are similar, but q is

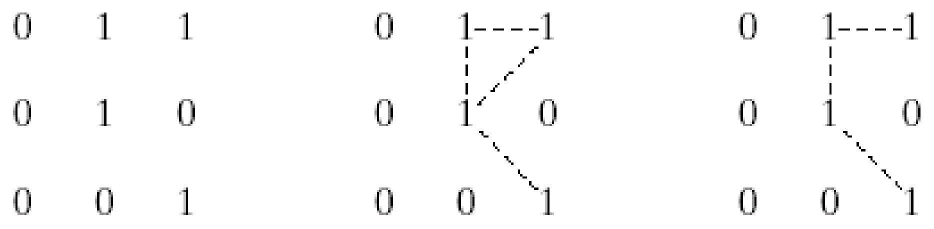
   either (x-1,y),(x+1,y),(x,y-1),(x,y+1)

1. 8-adjacency: It is possible for q to be one of the diagonal points (x-1,y-1),(x-1,y+1),(x+1,y-1),(x+1,x+1).

2. m-adjacency: Either q is 4-adjacent to p, or q is a diagonal point and the intersection of the four neighborhood of p and that of q have no similar pixel values.

ADJACENCY EXAMPLES

- 4-adjacency and 8- adjacency and not connected pixels
- $I = \begin{matrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{matrix}$ where v = {1} $\qquad I = \begin{matrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{matrix}$
- $I = \begin{matrix} 54 & 10 & 100 & 8 \\ 81 & 150 & 2 & 34 \\ 201 & 200 & 3 & 45 \\ 7 & 70 & 147 & 56 \end{matrix}$ where v={1,2,3, ……..,10} $\qquad I = \begin{matrix} 54 & 10 & 100 & 8 \\ 81 & 150 & 2 & 34 \\ 201 & 200 & 3 & 45 \\ 7 & 70 & 147 & 56 \end{matrix}$



For m-adjacency 4- adjacency should be applied first.

a b c

FIGURE 2.26 (a) Arrangement of pixels; (b) pixels that are 8-adjacent (shown dashed) to the center pixel; (c) *m*-adjacency.

DISTANCE MEASUREMENTS OF PIXELS

- For pixels p and q with coordinates (x, y) and (s,t); then
  - *Euclidean distance* between p and q is defined as $D_e(p,q) = [(x-s)^2 + (y-t)^2]^{1/2}$
  - The D_4 *distance*, (called the *city-block distance*) between p and q is defined as

$$D_e(p,q) = |(x-s)| + |(y-t)|$$

  - For example, the pixels with $D4$ distance ≤ 2 from (x, y) (the center point) form the following contours of constant distance:
  - The pixels with $D_4 = 1$ are the 4-neighbors of (x, y).

$$\begin{array}{ccccc} & & 2 & & \\ & 2 & 1 & 2 & \\ 2 & 1 & 0 & 1 & 2 \\ & 2 & 1 & 2 & \\ & & 2 & & \end{array}$$

- The D_8 *distance* (called the *chessboard distance*) between p and q is defined as

$$D_8(p,q) = \max(|(x-s)|, |(y-t)|)$$

- In this case, the pixels with D_8 distance from (x, y) less than or equal to some value d form a square centered at (x, y). For example, the pixels with D_8 distance ≤ 2 form the following contours of constant distance:

$$\begin{array}{ccccc} 2 & 2 & 2 & 2 & 2 \\ 2 & 1 & 1 & 1 & 2 \\ 2 & 1 & 0 & 1 & 2 \\ 2 & 1 & 1 & 1 & 2 \\ 2 & 2 & 2 & 2 & 2 \end{array}$$

DISTANCE MEASUREMENTS OF PIXELS

- the D_m distance between two points is defined as the shortest m-path between the points. In this case, the distance between two pixels will depend on the values of the pixels along the path, as well as the values of their neighbors.
  - For instance, consider the following arrangement of pixels and assume that p, p_2 , and p_4 have a value of 1, and that p_1 and p_3 can be 0 or 1:

	p_3	p_4
p_1	p_2	
p		

  - Suppose that we consider adjacency of pixels valued 1 (i.e.,$V = \{1\}$). If p_1 and p_3 are 0, the length of the shortest m-path (the D_m distance) between p and p_4 is 2. If p_1 is 1, then p_2 and p will no longer be m-adjacent and the length of the shortest m-path becomes 3 (the path goes through the points $pp_1p_2\,p_4$).
  - Similar comments apply if p_3 is 1 (and p_1 is 0); in this case, the length of the shortest m-path also is 3. Finally, if both p_1 and p_3 are 1, the length of the shortest m-path between p and p_4 is 4. In this case, the path goes through the sequence of points $p\ p_1\ p_2\,p_3\ p_4$.

Thank You

Practice more!