Morphological Image Processing

ICT4201: DIP

Part 2: Grey Scale Image

Grey Scale Morphology

- In grey scale images we have pixel values (0-255), but in binary we binary we have only (0-1).
- For grey scale images we also have basic operations, i.g. dilation, erosion, opening and closing and using these operations to develop several basic grey scale morphological algorithms.
- For grey scale digital images, input function or image represented as $f(x, y)$ and structuring element SE as $b(x, y)$, where both the functions are discrete
- Here we assume that if Z denotes the real set of integers, then (x, y) are integers from the domain Z×Z and 'f' and 'b' are functions that assign grey level values to distinct pair of coordinates (x, y).
    - If grey level values are also integers, Z → R

Dilation with Greyscale Image

- Greyscale dilation of '*f*' and '*b*' denotes $f \oplus b$ is defined as:
  - $(f \oplus b)(s,t) = \max\{f(s-x, t-y) + b(x,y) \mid (s-x),(t-y) \in D_f; (x,y) \in D_b\}$ ……..(1)
  - where D_f and D_b are the domains of '*f*' and '*b*', respectively.
- The condition (in terms of set theory) that $(s-x)$ and $(t-y)$ have to be in the domain of '*f*' and (x,y) have to be in that of '*b*'
  - Analogous to the conditions in binary destination of dilation, where two sets have to overlap by at least one element or have some common element between input image and SE.
  - Eq(1) is similar to the 2D convolution, with the max operation replacing the sums of convolution and addition replacing the products of convolution.
- If we write eq (1) for single variable it can be expressed as:
  - $f \oplus b = \max\{f(s-x) + b(x) \mid (s-x) \in D_f; (x) \in D_b\}$
  - For convolution, $f(-x)$ is simply $f(x)$, mirrored with respect to the x axis, i.e. the function $f(s-x)$ moves to the right for positive values and to the left for the negative values

Dilation with Greyscale Image

- The general effects of performing dilation on a greyscale image is two-fold:
  - If all the values of the structured element are positive, the output image tends to be brighter than the input
  - Dark details either reduced or eliminated, depending on how their values and shapes relate to the structuring element used for dilation.

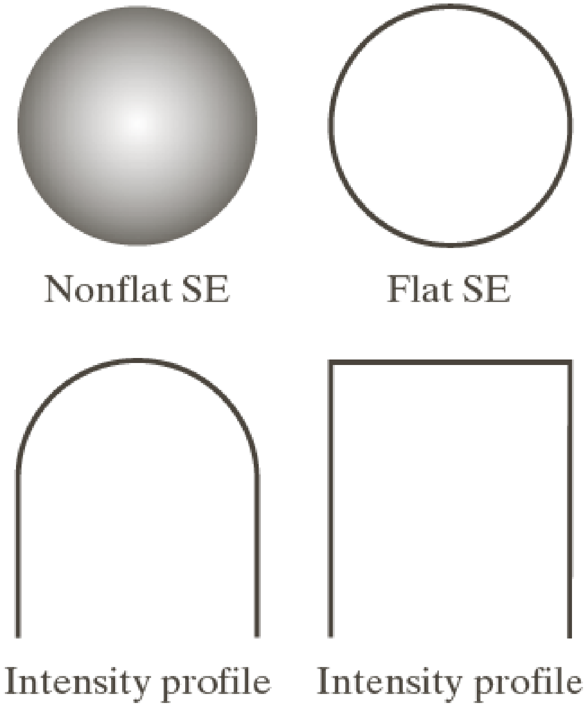
Erosion with Greyscale Image

- Greyscale erosion of '*f*' and '*b*' denotes $f \ominus b$ is defined as:
  - $(f \ominus b)(s,t) = \min\{f(s+x, t+y) - b(x,y) \big| (s+x),(t+y) \in D_f; (x,y) \in D_b\}$ …….(2)
  - where D_f and D_b are the domains of '*f*' and '*b*', respectively.
- The condition (in terms of set theory) that $(s-x)$ and $(t-y)$ have to be in the domain of '*f*' and (x,y) have to be in that of '*b*'
  - Analogous to the conditions in binary destination of erosion, where the SE has to be completely contained by the set being eroded.
  - Eq(2) is similar to the 2D correlation, with the min operation replacing the sums of correlation and subtraction replacing the products of correlation.
- If we write eq (1) for single variable it can be expressed as:
  - $f \ominus b = \min\{f(s+x) - b(x) \big| (s+x) \in D_f; (x) \in D_b\}$
  - For correlation, the function $f(s+x)$ moves to the left for positive values and to the right for the negative values
  - *b* completely contains within *f*

Gray-Scale Morphology

$f(x, y)$: gray-scale image

$b(x, y)$: structuring element



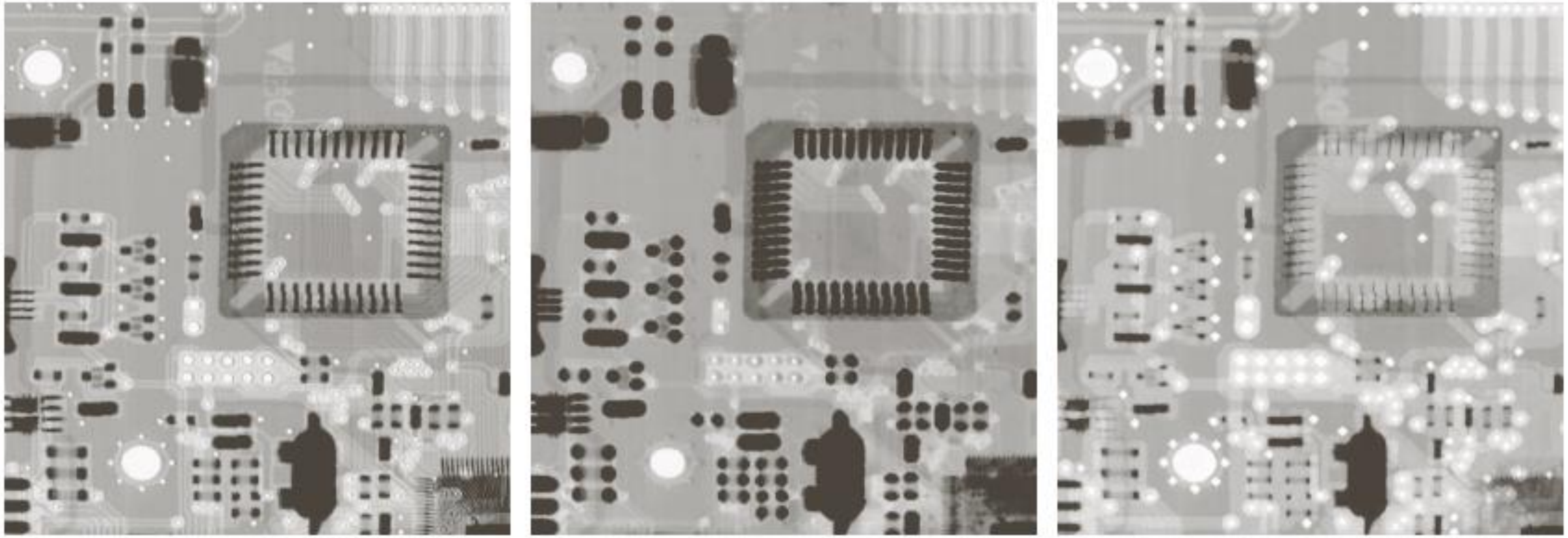
a b
c d

FIGURE 9.34 Nonflat and flat structuring elements, and corresponding horizontal intensity profiles through their center. All examples in this section are based on flat SEs.

Gray-Scale Morphology: Erosion and Dilation by Flat Structuring

$$\left[f \ominus b \right](x, y) = \min_{(s,t)\in b} \left\{ f(x+s, y+t) \right\}$$

$$\left[f \oplus b \right](x, y) = \max_{(s,t)\in b} \left\{ f(x-s, y-t) \right\}$$



a b c

FIGURE 9.35 (a) A gray-scale X-ray image of size 448 × 425 pixels. (b) Erosion using a flat disk SE with a radius of two pixels. (c) Dilation using the same SE. (Original image courtesy of Lixi, Inc.)

Properties of Gray-scale Opening

$$(a) \quad f \circ b \dashv f$$

$$(b) \quad \text{if } f_1 \dashv f_2, \text{ then } \left(f_1 \circ b \right) \dashv \left(f_2 \circ b \right)$$

$$(c) \quad \left(f \circ b \right) \circ b = f \circ b$$

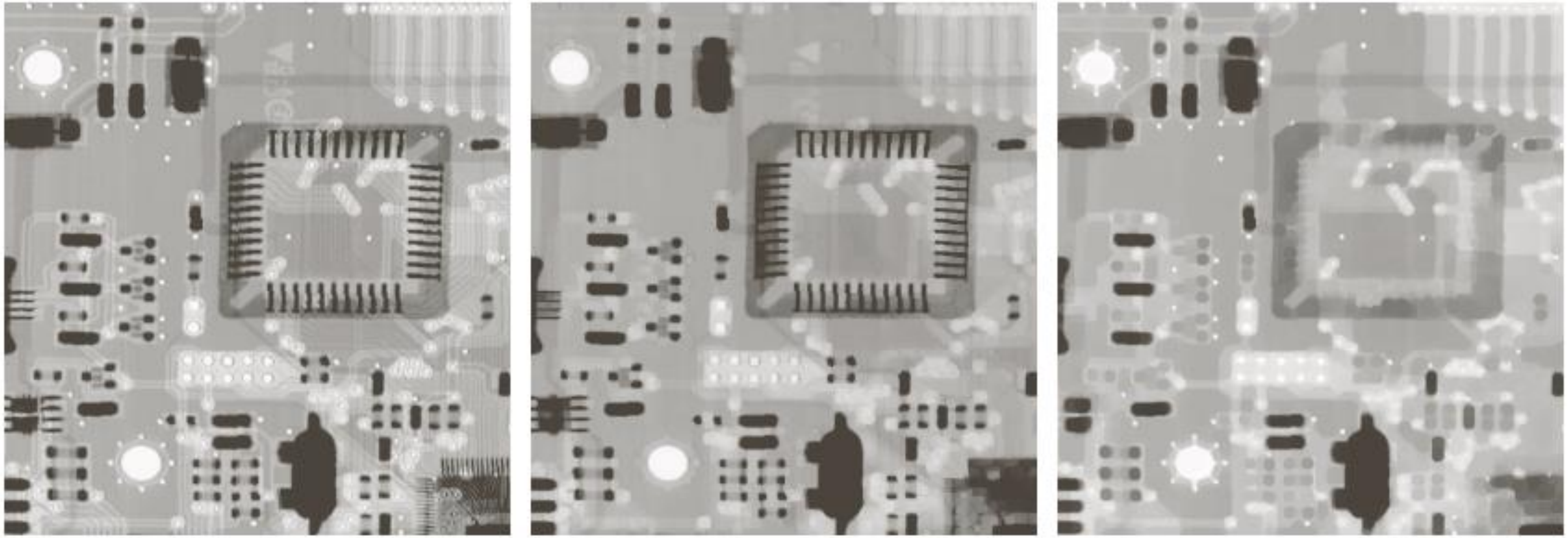
where $e \dashv r$ denotes e is a subset of r and also $e(x, y) \leq r(x, y)$.

Properties of Gray-scale Closing

$$(a)\quad f \dashv f \bullet b$$

$$(b)\quad \text{if } f_1 \dashv f_2,\ \text{then } \left(f_1 \bullet b \right) \dashv \left(f_2 \bullet b \right)$$

$$(c)\quad \left(f \bullet b \right) \bullet b = f \bullet b$$



a b c

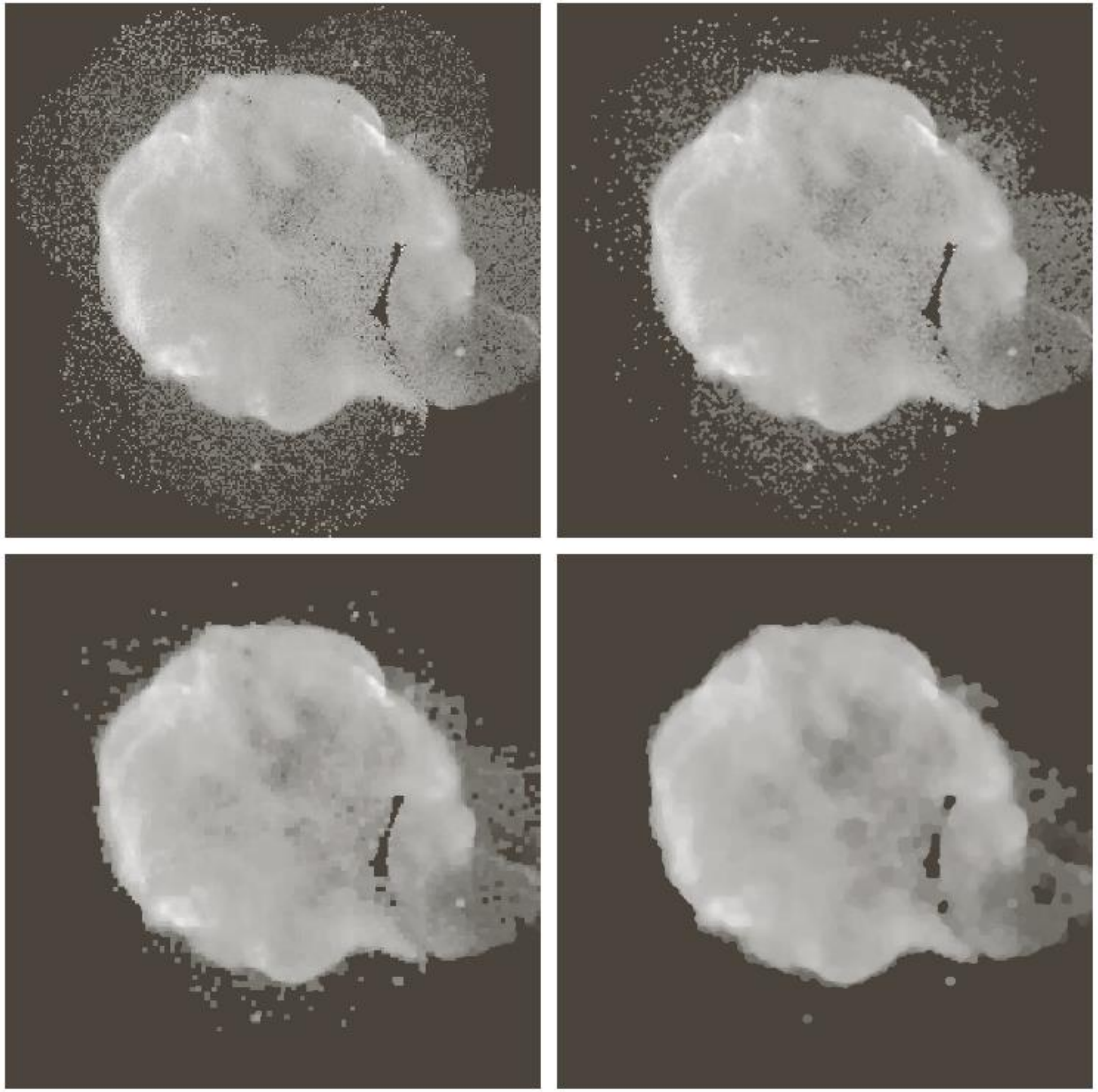
FIGURE 9.37 (a) A gray-scale X-ray image of size 448 × 425 pixels. (b) Opening using a disk SE with a radius of 3 pixels. (c) Closing using an SE of radius 5.

Morphological Smoothing

- Opening suppresses bright details smaller than the specified SE, and closing suppresses dark details.

- Opening and closing are used often in combination as morphological filters for image smoothing and noise removal.

Morphological Smoothing



a b
c d

FIGURE 9.38 (a) 566 × 566 image of the Cygnus Loop supernova, taken in the X-ray band by NASA's Hubble Telescope. (b)–(d) Results of performing opening and closing sequences on the original image with disk structuring elements of radii, 1, 3, and 5, respectively. (Original image courtesy of NASA.)

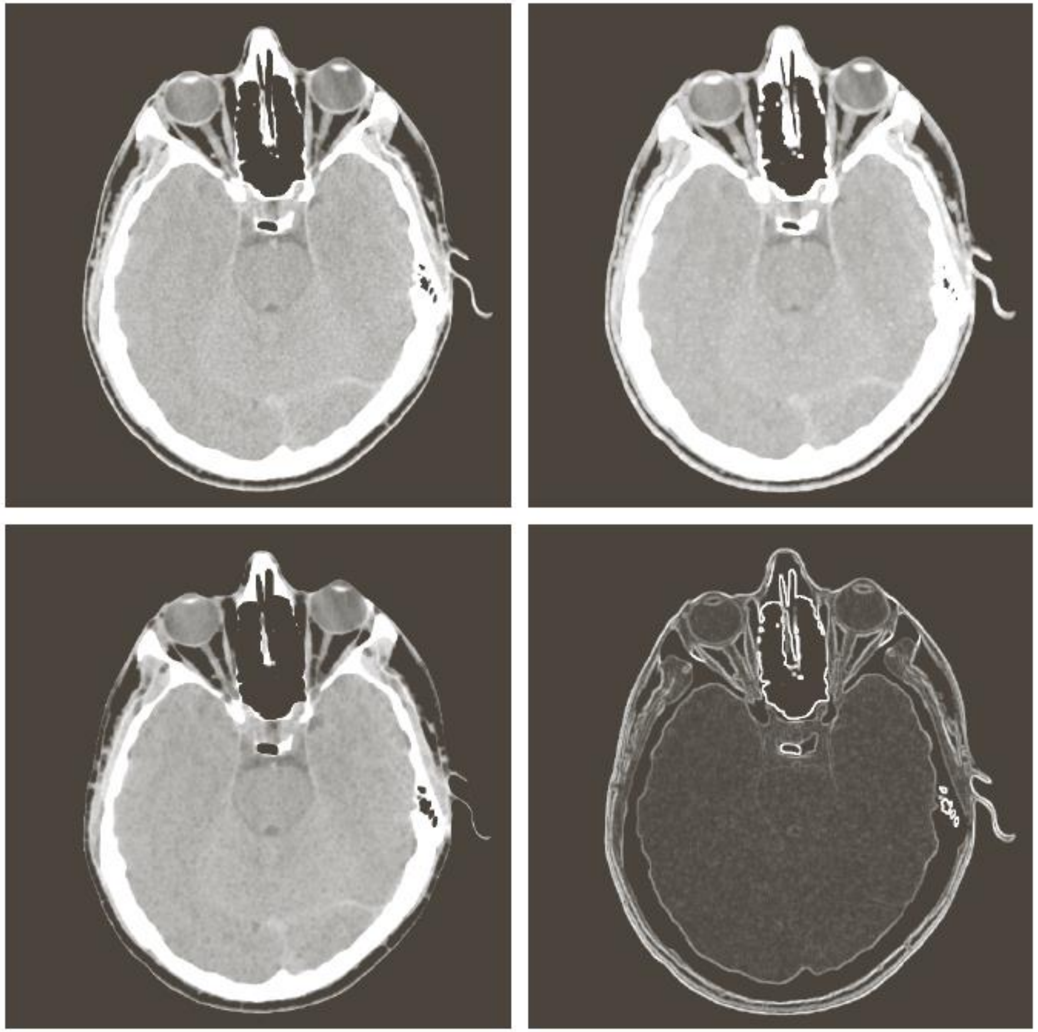
Morphological Gradient

- Dilation and erosion can be used in combination with image subtraction to obtain the morphological gradient of an image, denoted by g,

$$g = (f \oplus b) - (f \ominus b)$$

- The edges are enhanced and the contribution of the homogeneous areas are suppressed, thus producing a “derivative-like” (gradient) effect.

Morphological Gradient



a	b
c	d

FIGURE 9.39 (a) 512 × 512 image of a head CT scan. (b) Dilation. (c) Erosion. (d) Morphological gradient, computed as the difference between (b) and (c). (Original image courtesy of Dr. David R. Pickens, Vanderbilt University.)

Top-hat and Bottom-hat Transformations

- The top-hat transformation of a grayscale image f is defined as f minus its opening:

$$T_{hat}(f) = f - (f \circ b)$$

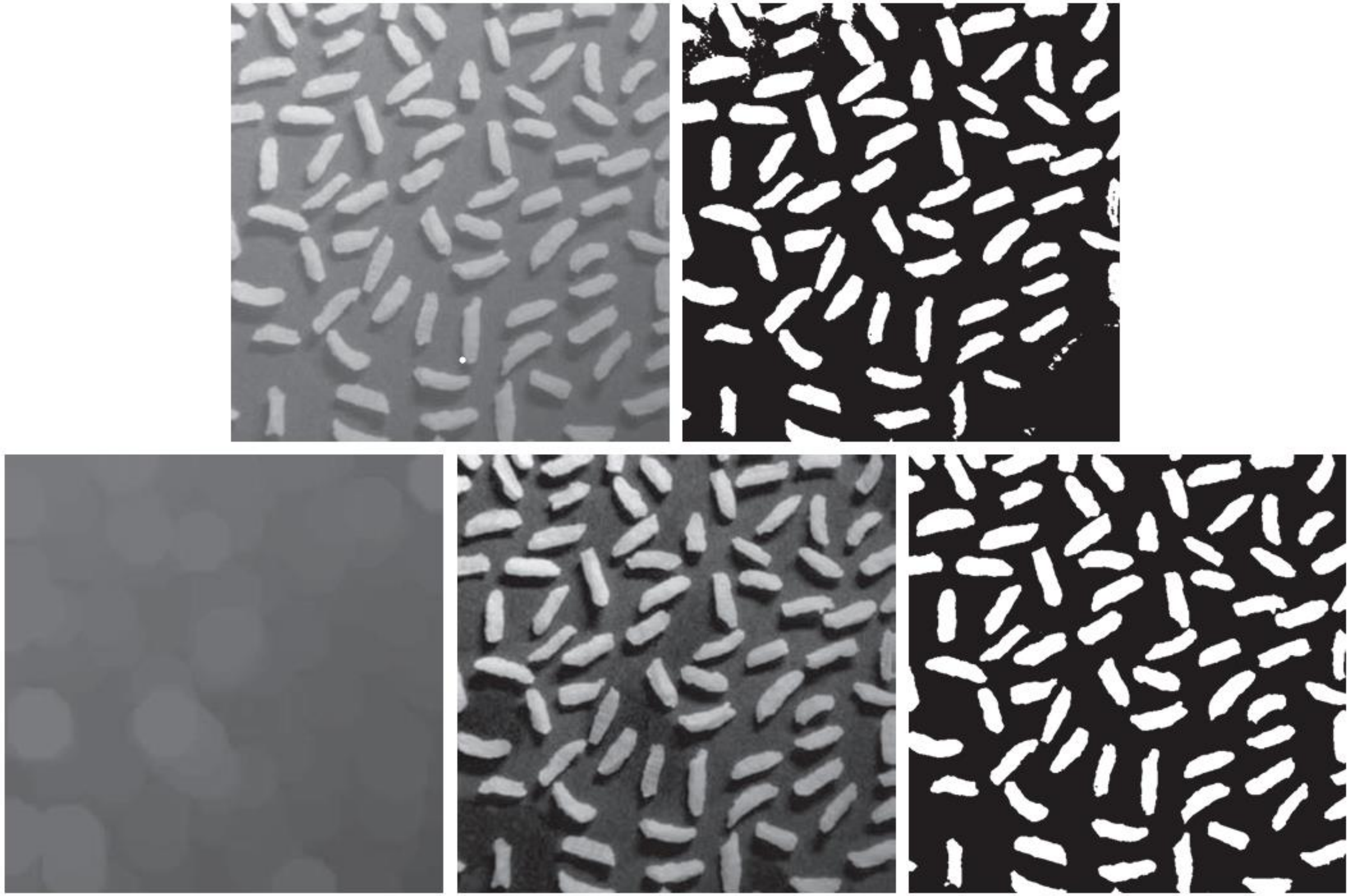
- The bottom-hat transformation of a grayscale image f is defined as its closing minus f:

$$B_{hat}(f) = (f \bullet b) - f$$

Top-hat and Bottom-hat Transformations

- One of the principal applications of these transformations is in removing objects from an image by using structuring element in the opening or closing operation

Example of Using Top-hat Transformation in Segmentation



a b
c d e

FIGURE 9.40 Using the top-hat transformation for *shading correction*. (a) Original image of size 600 × 600 pixels. (b) Thresholded image. (c) Image opened using a disk SE of radius 40. (d) Top-hat transformation (the image minus its opening). (e) Thresholded top-hat image.

Granulometry

- Granulometry deals with determining the size of distribution of particles in an image.

- Opening operations of a particular size should have the most effect on regions of the input image that contain particles of similar size.

- For each opening, the sum (**surface area**) of the pixel values in the opening is computed.

Example

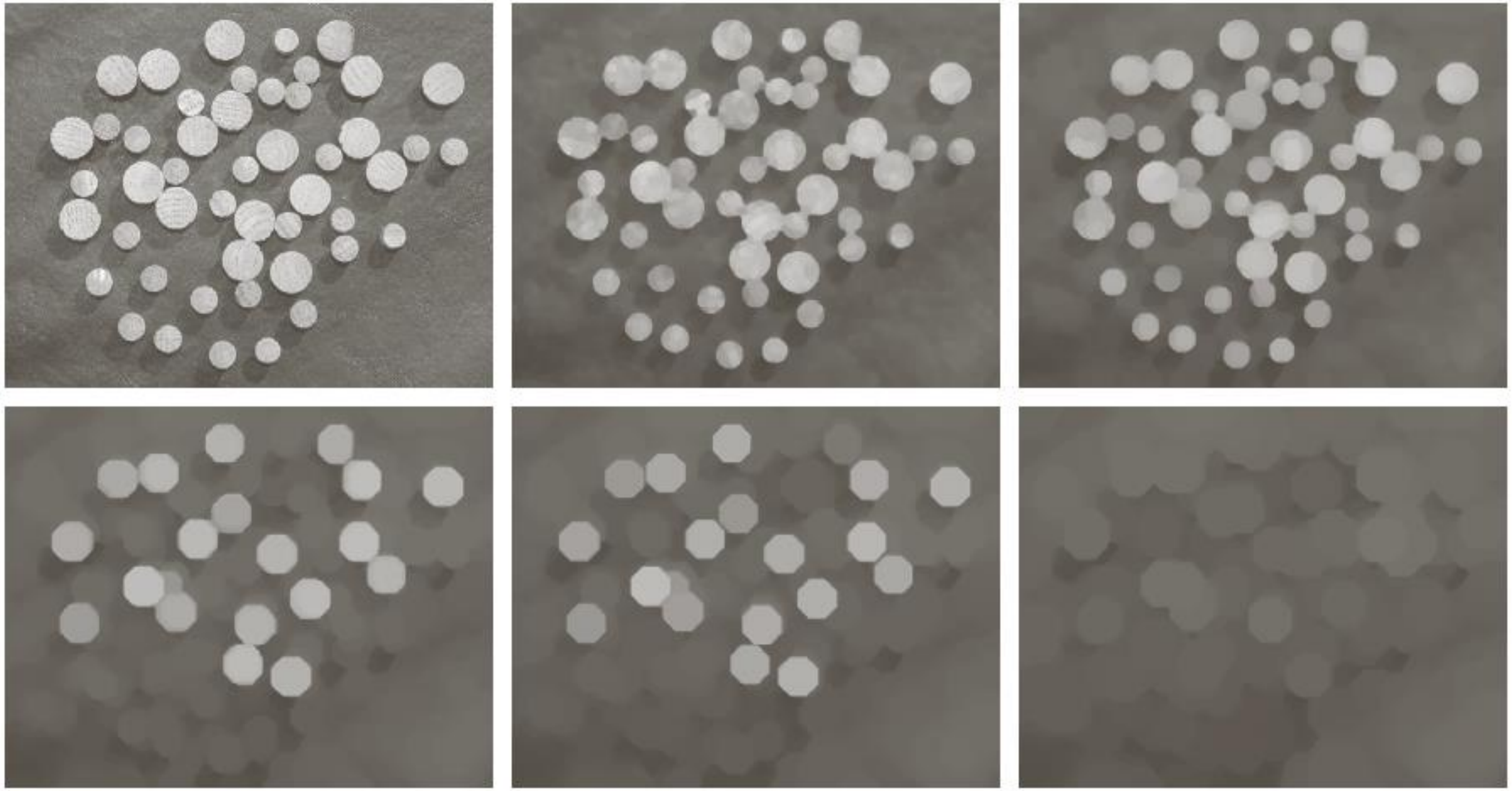


FIGURE 9.41 (a) 531 × 675 image of wood dowels. (b) Smoothed image. (c)–(f) Openings of (b) with disks of radii equal to 10, 20, 25, and 30 pixels, respectively. (Original image courtesy of Dr. Steve Eddins, The MathWorks, Inc.)

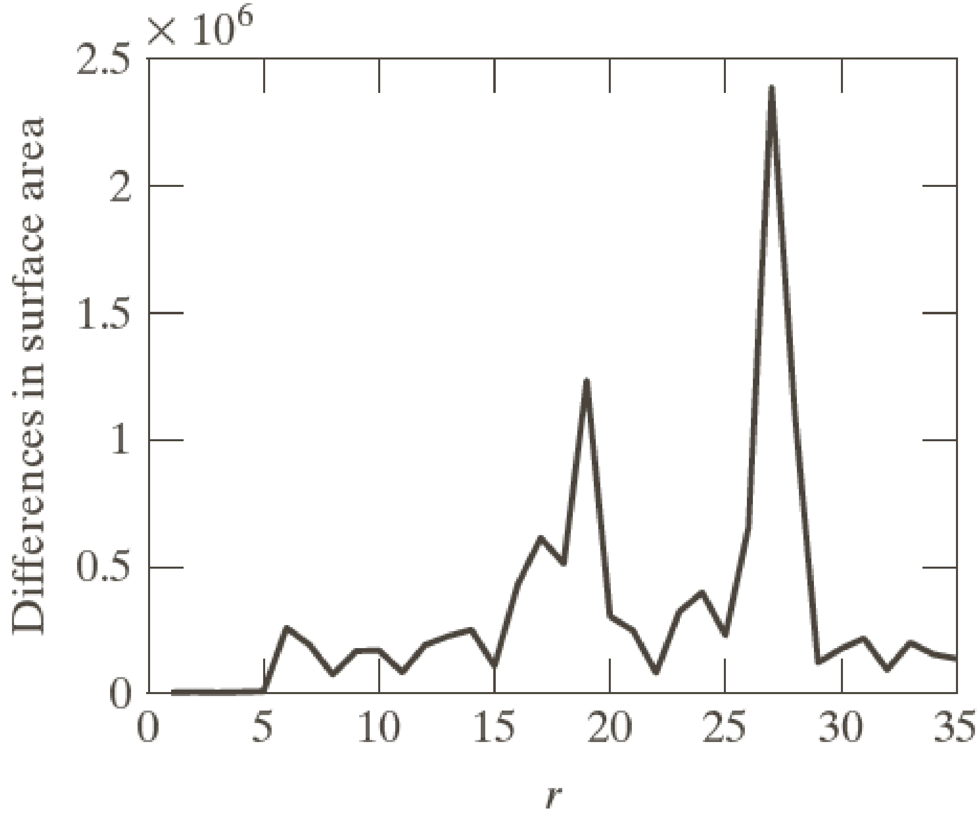
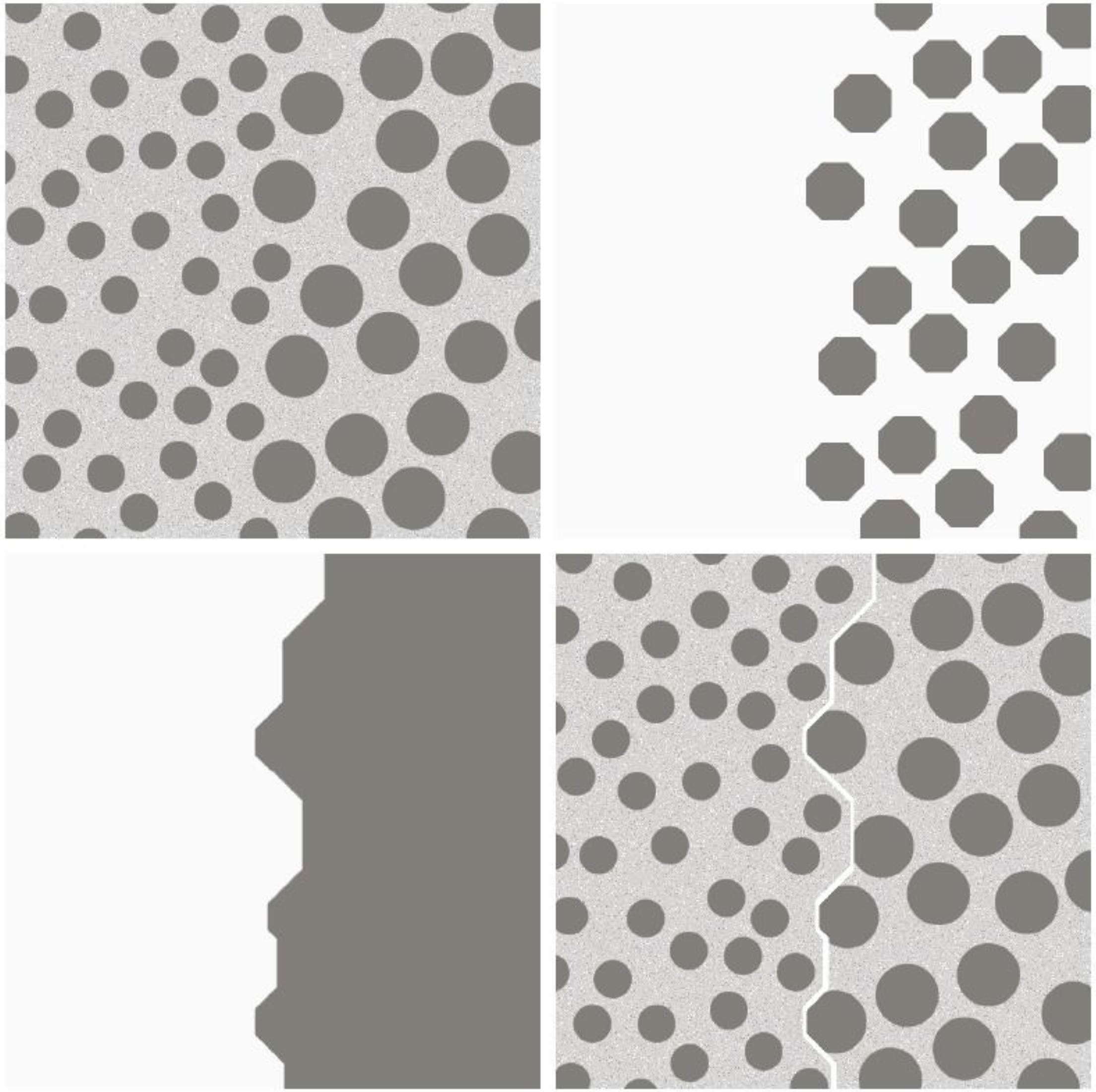


FIGURE 9.42 Differences in surface area as a function of SE disk radius, r. The two peaks are indicative of two dominant particle sizes in the image.

Textual Segmentation

- Segmentation: the process of subdividing an image into regions.



a b
c d

FIGURE 9.43 Textural segmentation. (a) A 600 × 600 image consisting of two types of blobs. (b) Image with small blobs removed by closing (a). (c) Image with light patches between large blobs removed by opening (b). (d) Original image with boundary between the two regions in (c) superimposed. The boundary was obtained using a morphological gradient operation.

Gray-Scale Morphological Reconstruction (1)

- Let f and g denote the marker and mask image with the same size, respectively and f ≤ g.

The geodesic dilation of size 1 of f with respect to g is defined as

$$D_g^{(1)}(f) = \left(f \oplus g \right) \wedge g$$

where $\wedge$ denotes the point-wise minimum operator.

The geodesic dilation of size n of f with respect to g is defined as

$$D_g^{(n)}(f) = D_g^{(1)}\left[D_g^{(n-1)}(f) \right] \text{ with } D_g^{(0)}(f) = f$$

Gray-Scale Morphological Reconstruction (2)

- The geodesic erosion of size 1 of f with respect to g is defined as

$$E_g^{(1)}(f) = \left(f \ominus g \right) \vee g$$

where $\vee$ denotes the point-wise maximum operator.

The geodesic erosion of size n of f with respect to g is defined as

$$E_g^{(n)}(f) = E_g^{(1)}\left[E_g^{(n-1)}(f) \right] \text{ with } E_g^{(0)}(f) = f$$

Gray-Scale Morphological Reconstruction (3)

- The morphological reconstruction by dilation of a gray-scale mask image g by a gray-scale marker image f, is defined as the geodesic dilation of f with respect to g, iterated until stability is reached, that is,

$$R_g^D(f) = D_g^{(k)}\left(f\right)$$

$$\text{with k such that } D_g^{(k)}\left(f\right) = D_g^{(k+1)}\left(f\right)$$

The morphological reconstruction by erosion of g by f is defined as

$$R_g^E(f) = E_g^{(k)}\left(f\right)$$

$$\text{with k such that } E_g^{(k)}\left(f\right) = E_g^{(k+1)}\left(f\right)$$

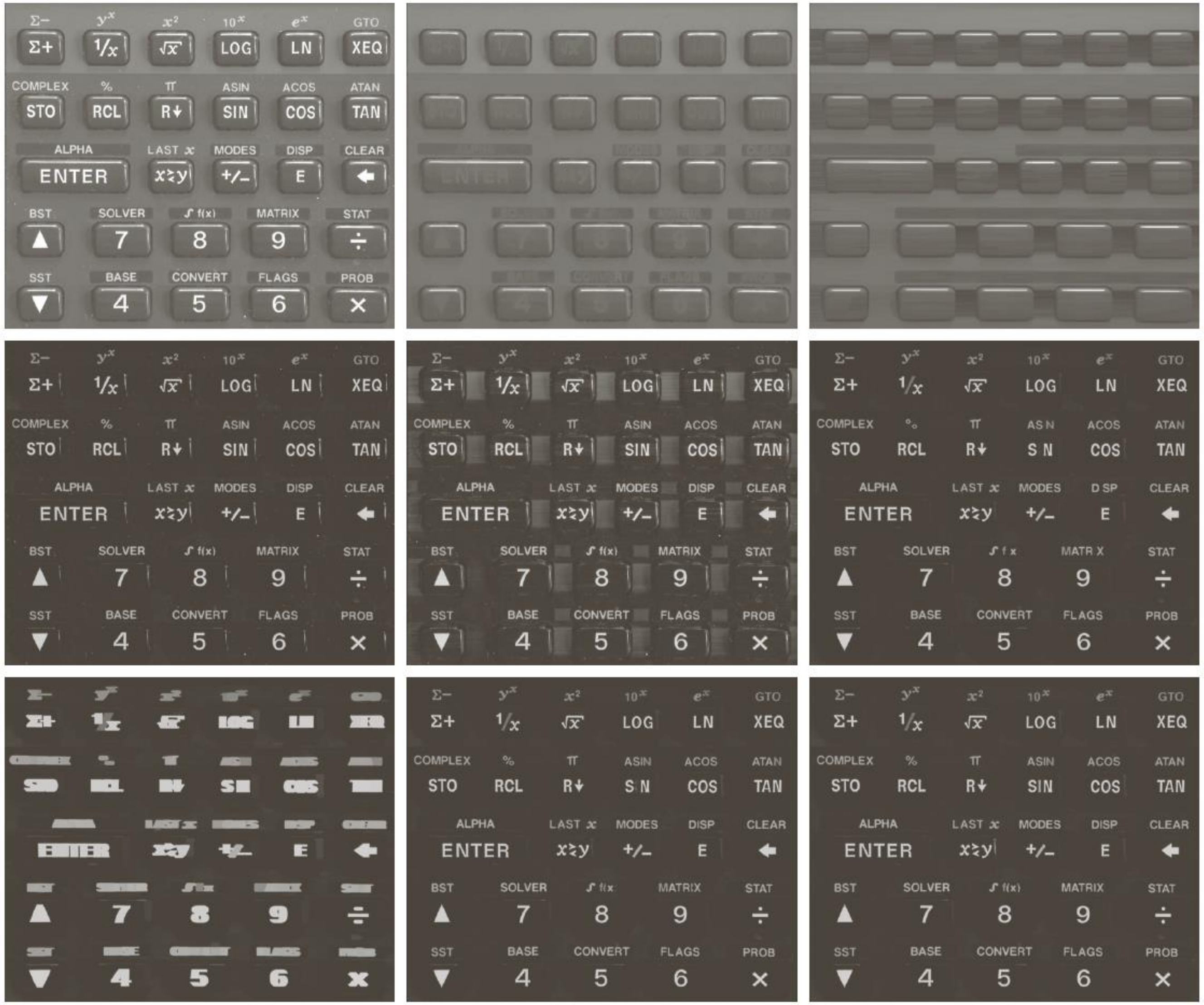
Gray-Scale Morphological Reconstruction (4)

- The opening by reconstruction of size n of an image f is defined as the reconstruction by dilation of f from the erosion of size n of f; that is,

$$O_R^{(n)}(f) = R_f^D\left[f \ominus nb\right]$$

The closing by reconstruction of size n of an image f is defined as the reconstruction by erosion of f from the dilation of size n of f; that is,

$$C_R^{(n)}(f) = R_f^E\left[f \oplus nb\right]$$



a b c
d e f
g h i

FIGURE 9.44 (a) Original image of size 1134 × 1360 pixels. (b) Opening by reconstruction of (a) using a horizontal line 71 pixels long in the erosion. (c) Opening of (a) using the same line. (d) Top-hat by reconstruction. (e) Top-hat. (f) Opening by reconstruction of (d) using a horizontal line 11 pixels long. (g) Dilation of (f) using a horizontal line 21 pixels long. (h) Minimum of (d) and (g). (i) Final reconstruction result. (Images courtesy of Dr. Steve Eddins, The MathWorks, Inc.)

Steps in the Example

1. Opening by reconstruction of the original image using a horizontal line of size 1x71 pixels in the erosion operation

$$O_R^{(n)}(f) = R_f^D\left[f \ominus nb \right]$$

2. Subtract the opening by reconstruction from original image

$$f' = f - O_R^{(n)}(f)$$

3. Opening by reconstruction of the f’ using a vertical line of size 11x1 pixels

$$f1 = O_R^{(n)}(f') = R_f^D\left[f' \ominus nb' \right]$$

4. Dilate f2 with a line SE of size 1x21, get f2.

Steps in the Example

5. Calculate the minimum between the dilated image f2 and and f', get f3.

6. By using f3 as a marker and the dilated image f2 as the mask,

$$R_{f2}^{D}(f3) = D_{f2}^{(k)}\left(f3\right)$$

$$\text{with k such that } D_{f2}^{(k)}\left(f3\right) = D_{f2}^{(k+1)}\left(f3\right)$$