# ICT 4203

# Computer Graphics and Animation

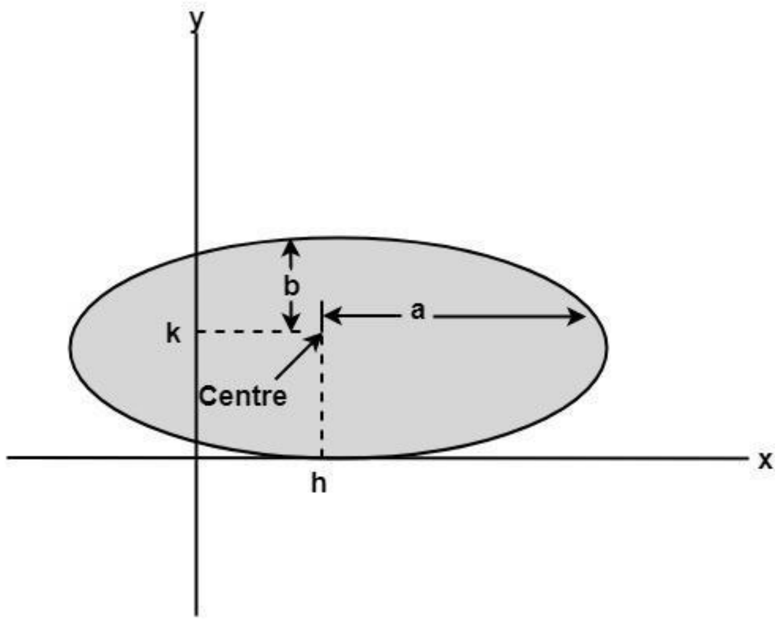
## Lecture 06

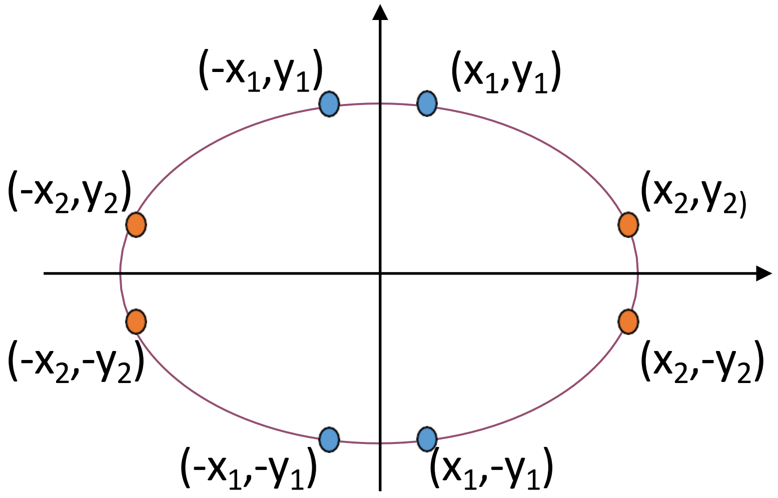
Scan-Conversion: Ellipse

Md. Mahmudur Rahman
Lecturer
Institute of Information Technology

# Scan Converting a Ellipse

- The ellipse is also a symmetric figure like a circle but is four-way symmetry rather than eight-way.




- There are two methods of defining an Ellipse:
  - Polynomial Method
  - Trigonometric Method

# Polynomial Method

- The ellipse has a major and minor axis. If a and b are major and minor axis respectively. The centre of ellipse is (h, k). The value of x will be incremented from h to a and value of y will be calculated using the following formula:

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1 \qquad y = b\sqrt{1-\frac{(x-h)^2}{a^2}}+k$$

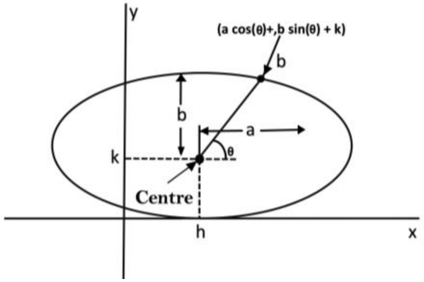
**Drawback of Polynomial Method:**

- It requires squaring of values. So floating point calculation is required.
- Routines developed for such calculations are very complex and slow.

# Trignometric Method

- The following equation defines an ellipse trigonometrically:
  - $x = a \cos\theta + h$ and $y = b \sin\theta + k$.

    where (x, y) = the current coordinates.
  - a = length of major axis.
  - b = length of minor axis.
  - $\theta$= current angle.
  - (h, k) = ellipse center.



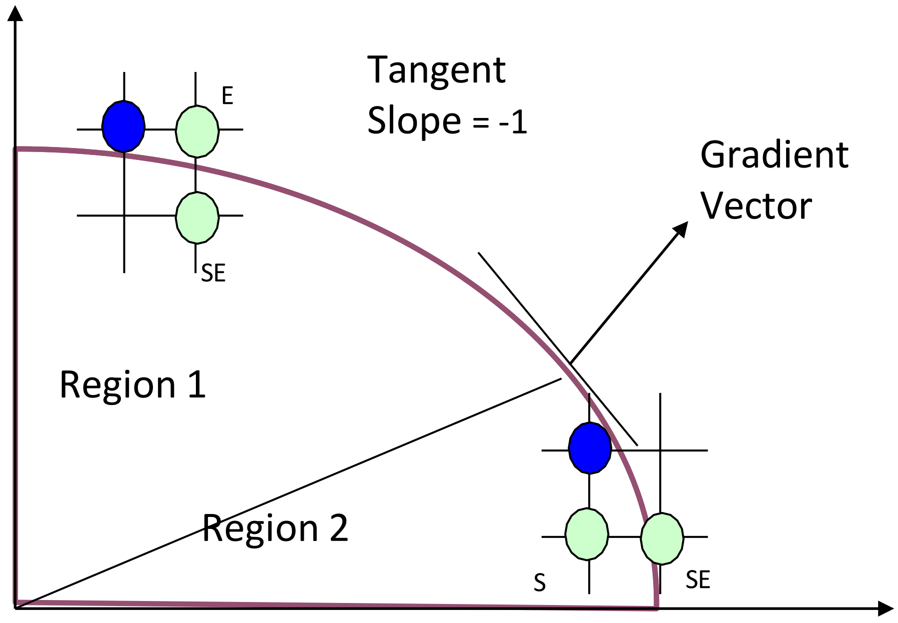

- In this method, the value of $\theta$ is varied from 0 to $\pi/2$ radians. The remaining points are found by symmetry.

**Drawback:**

- This is an inefficient method.
- It is not an interactive method for generating ellipse.
- The table is required to see the trigonometric value.
- Memory is required to store the value of $\theta$.

# Midpoint Algorithm

- Implicit equation is: $F(x,y) = b^2x^2 + a^2y^2 - a^2b^2 = 0$
- We have only 4-way symmetry
- There exists two regions:
  - In **Region 1** dx > dy
    - Increase x at each step
    - y may decrease
  - In **Region 2** dx < dy
    - Decrease y at each step
    - x may increase



# Pseudo Code

```
x=0, y=b;
fx=0, fy=a²b
p=b²-a²b+1/4(a²)
while (fx<fy) {
   setPixel (x, y);
   x++;
   fx=fx+b²;
   if(p<0)
          p=p+fx+b²;
   else{
          y--;
          fy=fy-a²;
          p=p+fx-fy+b²;
   }
}
setPixel(x,y);
```

```
p=b²(x+0.5)²+a²(y-1)²-a²b²
while (y>0) {
   y--;
   fy=fy-a²;
   if(p>=0)
          p=p-fy+a²;
   else{
          x++;
          fx=fx+b²;
          p=p+fx-fy+a²;
   }
setPixel(x,y);
}
```

# Example

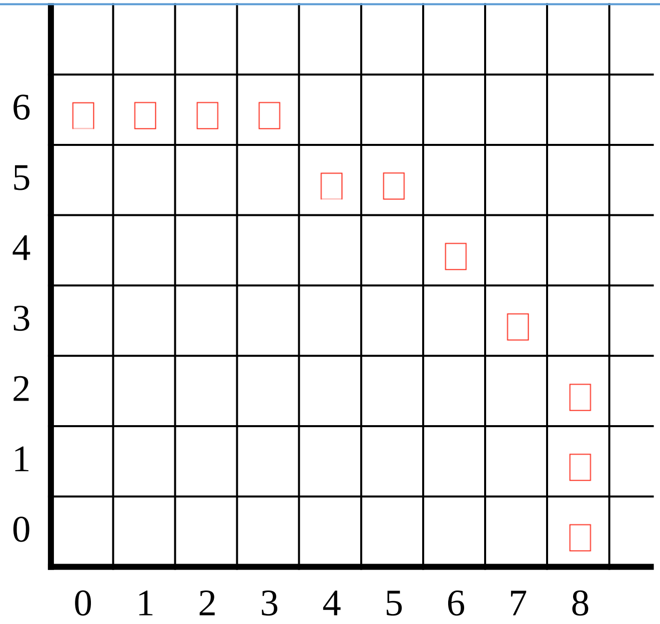
- Draw an ellipse with *a*= 8, *b* = 6 using Midpoint Ellipse Algorithm.
- **Solution:**
- **Region 1: initial (0,6)**

| i | $X_i$ | $y_i$ | $p_i$ | $p_{i+1}$ | $x_{i+1}$ | $y_{i+1}$ | $F_{x+1}$ | $F_{y+1}$ |
|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 6 | -332 | -260 | 1 | 6 | 36 | 384 |
| 1 | 1 | 6 | -260 | -152 | 2 | 6 | 72 | 384 |
| 2 | 2 | 6 | -152 | -8 | 3 | 6 | 108 | 384 |
| 3 | 3 | 6 | -8 | 172 | 4 | 6 | 144 | 384 |
| 4 | 4 | 6 | 172 | 68 | 5 | 5 | 180 | 320 |
| 5 | 5 | 5 | 68 | 64 | 6 | 4 | 216 | 256 |
| 6 | 6 | 4 | 64 | 160 | 7 | 3 | 252 | 192 |
| 7 | 7 | 3 | | | | | | |

# Continue...

- **Region 2: initial (7,3)**

| $i$ | $x_i$ | $y_i$ | $p_i$ | $p_{i+1}$ | $x_{i+1}$ | $y_{i+1}$ | $f_x$ | $f_y$ |
|---|---|---|---|---|---|---|---|---|
| 8 | 7 | 3 | -23 | 201 | 8 | 2 | 288 | 128 |
| 9 | 8 | 2 | 201 | 201 | 8 | 1 | 288 | 64 |
| 10 | 8 | 1 | 201 | 265 | 8 | 0 | 288 | 0 |
| 11 | 8 | 0 | | | | | | |

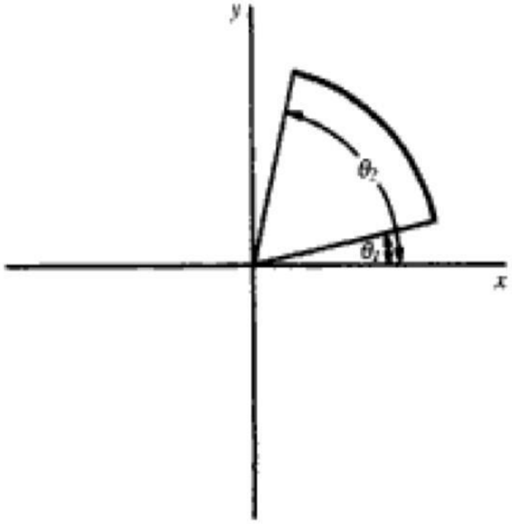


# Exercise

- Generate all points of an ellipse with $a$= 6, $b$ = 8 using Midpoint Ellipse Algorithm.
- Generate all points of an ellipse with $a$= 14, $b$ = 10 using Midpoint Ellipse Algorithm center at (15, 10).

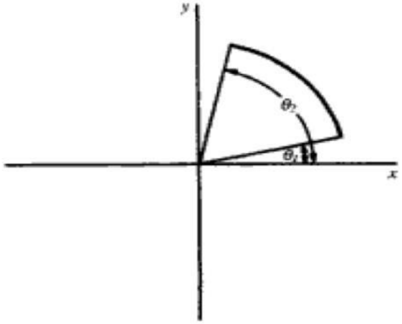
# Scan-Converting ARCS

- An Arc can be generated using either the polynomial or the trigonometric method.
- When the trigonometric method is used, the starting value is set to $\theta_1$ and the ending value is set to $\theta_2$.
- The rest of the steps are similar to those used when scan-converting a circle, except that symmetry is not used.



# Scan-Converting Sectors

- A sector is scan-converted by using any of the methods of scan-converting an arc, and then scan-converting two lines from the center of the arc to the endpoints of the arc.
- For example, assume that a sector whose center is at point (h,k) is to be scan-converted.
- First scan-convert an arc from $\theta_1$ to $\theta_2$.
- Next, a line to be scan-converted from (h, k) to ($r\cos\theta_1+h$, $r\sin\theta_1+k$).
- A second line to be scan-converted from (h, k) to ($r\cos\theta_2+h$, $r\sin\theta_2+k$).



# Scan-Converting a Rectangle

- A rectangle whose sides are parallel to the co-ordinates axes may be constructed if the locations of two vertices are known. The remaining corner points are then derived.
- Once the vertices are known, the four sets of co-ordinates are sent to line routine and the rectangle is scan-converted.
- In the case of the rectangle shown, lines would be drawn as follows:
  - Line: $(x_1, y_1)$ to $(x_1, y_2)$
  - Line: $(x_1, y_2)$ to $(x_2, y_2)$
  - Line: $(x_2, y_2)$ to $(x_2, y_1)$
  - Line: $(x_2, y_1)$ to $(x_1, y_1)$