1. Linear Applications 1

Exercise 1 *Let $f:\mathbb{R}^2 \longrightarrow \mathbb{R}^3$ be the linear application defined by $f(x,y) = (x+y, x+y, x+y)$.*

a) Compute the matrix of f referred to the standard basis $\mathscr{B}$ and $\mathscr{C}$ of $\mathbb{R}^2$ and $\mathbb{R}^3$, respectively.

b) Find out the matrix of f when we change to the basis $\mathscr{B} = \{(1,1),(-1,1)\}$ in $\mathbb{R}^2$, and $\mathscr{C} = \{(1,1,1),(0,1,1),(0,0,2)\}$ in $\mathbb{R}^3$.

c) Find out the image of the vector $\vec{v} = (1,2)$.

Exercise 2 *Considering $f:\mathbb{R}^2 \longrightarrow \mathbb{R}^2$ given by its matrix por $A = \begin{pmatrix} 1 & 1 \\ 1 & -2 \end{pmatrix}$. Obtain the component functions that define f.*

Exercise 3 *Considering $f:\mathbb{R}^3 \longrightarrow \mathbb{R}^4$ defined by*

$$f(1,1,0) = (3,2,0,1), \quad f(2,3,1) = (1,-2,1,1) \quad y \quad f(0,-2,1) = (4,0,1,2).$$

Give its matrix referred to the standard basis in both spaces, and compute the image of the vector $\vec{v} = (1,2,3)$.

Exercise 4 *Let $f:\mathbb{R}_3[x] \longrightarrow \mathscr{M}_2(\mathbb{R})$ be the linear application defined by*

$$f(a_3x^3 + a_2x^2 + a_1x + a_0) = \begin{pmatrix} 3a_3 - a_2 & a_1 \\ 2a_2 + a_0 & a_2 \end{pmatrix}.$$

Obtain the matrix of f referred to the basis:

$$B = \{1, 1+x, 1+x+x^2, 1+x+x^2+x^3\}$$

and

$$C = \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ -1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix} \right\}.$$

Compute the image of the polynomial $p = 1 - x^3$.
