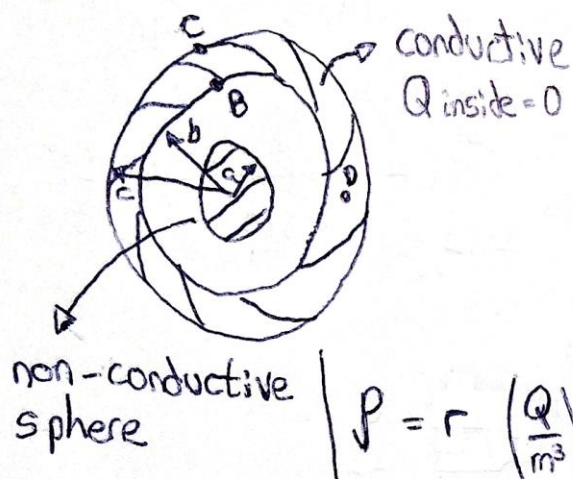


# Task 1.1:



$$Q_a = \int_V \rho dV = \int_0^a r \cdot 4\pi r^2 = \pi \left[ r^4 \right]_0^a = \pi a^4$$

$$V = \frac{4}{3} \pi r^3$$

$$dV = 4\pi r^2$$

$$\begin{cases} 1. \quad 0 = Q_B + Q_C \\ \quad 0 = Q_D = Q_A + Q_B \end{cases} \quad \begin{cases} Q_A = Q_C = \pi a^4 \\ Q_A = -Q_B = -\pi a^4 \end{cases}$$

$$\sigma_C = \frac{Q_C}{S_C} = \frac{\pi a^4}{4\pi c^2} = \frac{a^4}{4c^2}$$

$$\sigma_B = \frac{Q_B}{S_B} = \frac{-\pi a^4}{4\pi b^2} = \frac{-a^4}{4b^2}$$

$$2. \quad r < a$$

$$\phi = \oint_V \vec{E} \cdot d\vec{S} = \epsilon S = \epsilon 4\pi r^2 = \frac{Q_{enc}}{\epsilon_0} \rightarrow \epsilon = \frac{\pi r^4 \epsilon_0}{4\pi r^2 \epsilon_0} \Rightarrow \vec{E} = \frac{r^2}{4\epsilon_0} \left( \frac{N}{C} \right)$$

$$Q_{enc} = \int_V \rho dV = \int_0^r r 4\pi r^2 = \pi r^4$$

$$a < r < b$$

$$\phi = \epsilon S = \epsilon 4\pi r^2 = \frac{Q_a}{\epsilon_0} \rightarrow \epsilon = \frac{\pi a^4}{4\pi r^2 \epsilon_0} \Rightarrow \vec{E} = \frac{a^4}{4r^2 \epsilon_0} \left( \frac{N}{C} \right)$$

$$b < r < c$$

$$\phi = 0 \rightarrow \vec{E} = 0$$

$$Q_{enc} = 0$$

$$c < r$$

$$\phi = \epsilon S = \epsilon 4\pi r^2 = \frac{Q_a + 0}{\epsilon_0} \rightarrow \epsilon = \frac{\pi a^4}{4\pi r^2} \Rightarrow \vec{E} = \frac{a^4}{4r^2 \epsilon_0} \left( \frac{N}{C} \right)$$



3. A)  $\Delta V = V_{R1} - V_{\infty} = - \int_{\infty}^{R1=a/2} \vec{E} d\vec{r} = - \int_{\infty}^c \frac{a^4}{4r^2 \epsilon_0} dr - \int_c^b 0 dr - \int_b^a \frac{a^4}{4r^2 \epsilon_0} dr - \int_a^{a/2} \frac{r^2}{4\epsilon_0} dr =$

$$= \frac{a^4}{4\epsilon_0} \left( \left[ \frac{1}{r} \right]_{\infty}^c + \left[ \frac{1}{r} \right]_b^a \right) - \frac{1}{12\epsilon_0} \left[ r^3 \right]_a^{a/2} = \frac{a^3 (ab+cb-ac)}{4\epsilon_0 bc} + \frac{7a^3}{96\epsilon_0} =$$

$$= \frac{a^3}{4\epsilon_0} \left( \frac{ab+cb-ac}{bc} + \frac{7}{24} \right) \text{ (V)}$$

$R_1 = a/2$   
 $R_2 = 2a$   
 $R_3 = 2b$   
 $R_4 = 2c$

B)  $\Delta V = V_{R1} - V_{R4} = - \int_{\infty}^{R1=a/2} \vec{E} d\vec{r} - \int_{\infty}^{R4=2c} \vec{E} d\vec{r} = \frac{a^3}{4\epsilon_0} \left( \frac{ab+cb-ac}{bc} + \frac{7}{24} \right) - \int_{\infty}^{2c} \frac{a^4}{4r^2 \epsilon_0} dr =$

$$= \frac{a^3}{4\epsilon_0} \left( \frac{ab+cb-ac}{bc} + \frac{7}{24} + \frac{a}{2c} \right) = \frac{a^3}{4\epsilon_0} \cdot \frac{36ab+31bc-24ac}{24bc} \text{ (V)}$$

C)  $\Delta V = V_{R1} - V_{R3} = - \int_{\infty}^{R1} \vec{E} d\vec{r} - \int_{\infty}^{R3} \vec{E} d\vec{r} = \frac{a^3}{4\epsilon_0} \left( \frac{ab+cb-ac}{bc} + \frac{7}{24} \right) + \frac{a^4}{4c\epsilon_0} =$

$$= \frac{a^3}{4\epsilon_0} \cdot \frac{48ab+31bc-24ac}{24bc} \text{ (V)}$$

D)  $\Delta V = V_{R1} - V_{R2} = - \int_{\infty}^{R1} \vec{E} d\vec{r} - \int_{\infty}^{R2} \vec{E} d\vec{r} = - \int_{\infty}^{R1} \vec{E} d\vec{r} + \frac{a^4}{4\epsilon_0} \left( \left[ \frac{1}{r} \right]_{\infty}^c + \left[ \frac{1}{r} \right]_{b}^{2a} \right) =$

$$= \frac{a^3}{4\epsilon_0} \left( \frac{24ab+31bc-24ac}{24bc} + \frac{2ab+bc-2ac}{2bc} \right) = \frac{a^3}{4\epsilon_0} \cdot \frac{48ab+43bc-48ac}{24bc} \text{ (V)}$$

because from c to R3  $\Delta V = 0$