OyG

Ondas y Electromagnetismo - Cristina y Jaime

30% → Pruebas portemas → 1. Electroestática y corriente eléctrica ≥ 4
→ 2. Magnetostática y campos electromagnéticos variable con el tiempo. ≥ 4

7% → Actitud y ejercicios en clase ≥ 4

13% → Laboratorio ≥ 4

50% → Examen Final ≥ 4

≥ 5

Chapter 1. Electrostatics

$q_e = 1'6 \cdot 10^{-19} C$ Ley de Coulomb: $\boxed{\vec{F} = -k \frac{q_1 \cdot q_2}{r_{12}^2} \vec{V_r}}$ $\vec{V_r} = \frac{\vec{r_{12}}}{|\vec{r_{12}}|_2}$ $|\vec{V_r}| = 1$

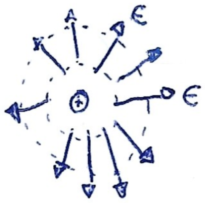
$k = 8.99 \cdot 10^9 \frac{N \cdot m^2}{C^2} = \frac{1}{4\pi \varepsilon_0}$; $\varepsilon_0 = 8'85 \cdot 10^{-12} \frac{C^2}{m^2 N}$

$k = 10^{-7} c^2 \frac{N \cdot s^2}{C^2}$; c: Velocidad de la luz

$\frac{N}{C} = \frac{V}{m}$

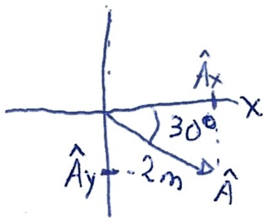
Electric Field of a point

$\boxed{\vec{E} = k \frac{q}{r^2} \vec{V_r}}$



$\boxed{\vec{F} = q \cdot \vec{E}}$

Descompose a vector



$\hat{A}x = \text{Module} \cdot \cos 30° \hat{i} = \sqrt{3} \hat{i}$ m

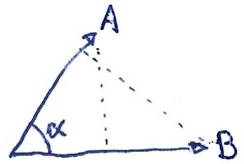
$\hat{A}y = \text{Module} \cdot \text{sen } 30° \hat{j} = -1 \hat{j}$ m

$|\vec{A}| = \sqrt[2]{A_x^2 + A_y^2 + A_z^2}$

$|\vec{B}| = \sqrt[2]{A_x^2 + A_y^2 + A_z^2}$

Scalar product

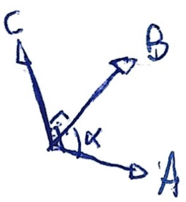
$\vec{A} \cdot \vec{B} = A_x \cdot B_x + A_y B_y + A_z B_z$ ES LA $\text{Proj}^{\perp}_{\vec{B}}(\vec{A}) = \text{Proj}^{\perp}_{\vec{A}}(\vec{B})$



$\boxed{\cos \alpha = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}| \cdot |\vec{B}|}}$

Vector Product

$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$



$|\vec{A} \times \vec{B}| = |\vec{A}| |\vec{B}| \text{ sen } \alpha$

Regla de la mano derecha: Índice $\vec{A}$
Corazón $\vec{B}$

Aceleration

$$\boxed{\begin{gathered}\sum \vec{F} = m\vec{a} \\ \vec{F} = q\vec{E}\end{gathered}}$$

Electric dipole

From $\ominus$ to $\oplus$

$\ominus \xrightarrow{\vec{L}} \oplus$

$-q \qquad +q$

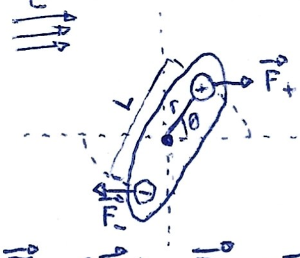
$$\boxed{\vec{P} = |q|\vec{L}}$$

↳ Dipole momentum

Torque = τ

$\sum \tau = 0 \rightarrow$ No rotation

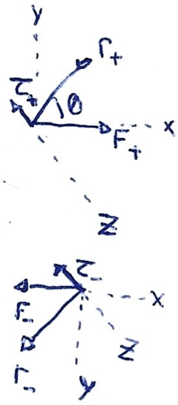
$$\boxed{\begin{gathered}\vec{\tau} = \vec{r} \times \vec{F} \\ = \vec{P} \times \vec{E}\end{gathered}}$$



$|\vec{F}_+| = |\vec{F}_-|$

$\sum \vec{F} = 0$

$\vec{F}_+ = q\vec{E}\,\hat{i} \qquad \vec{F}_- = q\vec{E}\,\hat{i}$



$|\vec{\tau}_+| = |\vec{r}_+||\vec{F}_+|\sin\theta$

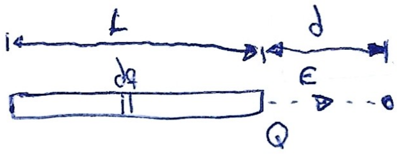
$= \frac{L}{2} qE\sin\theta$

$= |\tau_-|$

$\sum \tau = \cancel{2}\left(\frac{L}{\cancel{2}} \cdot qE\sin\theta\right) =$

$= \underbrace{qL}_{|\vec{P}|}E\sin\theta = |\vec{P}||\vec{E}|\sin\theta$

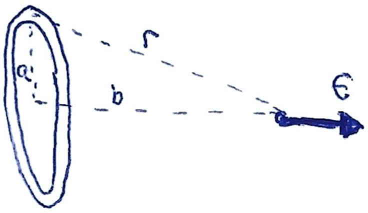
Electric field of a line



$$\boxed{\begin{gathered}E = k\frac{Q}{d(d+L)}\hat{d} \\ \lambda = \frac{Q}{L}\ (C/m)\end{gathered}}$$

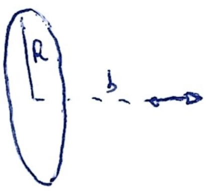
$$\boxed{\varepsilon_0 = 8'85 \cdot 10^{-12} \frac{C^2}{Nm^2}}$$

Electric field of a ring



$$\boxed{\begin{gathered}E = k\frac{Qb}{(a^2+b^2)^{3/2}}\hat{b} \\ \lambda = \frac{Q}{2\pi a}\ (C/m)\end{gathered}}$$

Electric field of a disk



$$\boxed{\begin{gathered}E = 2\lambda\pi\sigma\left[1 - \frac{b}{(b^2+R^2)^{1/2}}\right] \\ \sigma = \frac{Q}{\pi R^2}\ (C/m^2)\end{gathered}}$$

$$\boxed{\lambda = \frac{1}{4\pi\varepsilon_0}}$$

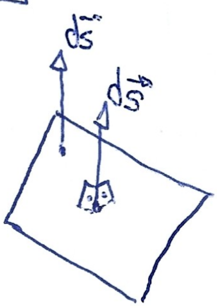
Gauss theorem:

$$\phi = \oint_S \vec{E} \cdot d\vec{S} = \frac{q_{enclosed}}{\varepsilon_0}$$

Closed surface

Electric Flux:

$$d\phi = \vec{E} \cdot d\vec{S} = E \cdot dS \cdot \cos\theta \quad \left[\frac{N}{C}m^2\right]$$



$q_{enc} = \lambda \cdot L\ (C)$

$q_{enc} = \sigma \cdot S\ (C)$

$q_{enc} = \rho \cdot V\ (C)$

Experimental measurements: General Concepts.

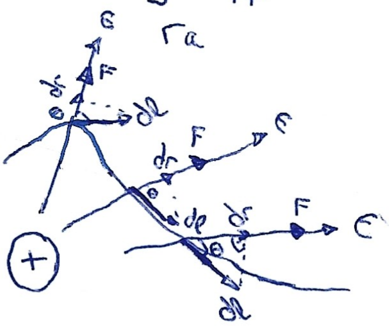
- Direct measu...: Calculadas con instrumentos
- Indirect meas...: Calculadas por fórmulas
- Uncertainty
- Unit

$$X = X + \overbrace{\delta X}^{\text{Uncertainty}}$$

$$t = t + \delta t \qquad V = \frac{X}{t}$$

Trabajo

$$W_{a\to b} = -\int_{r_a}^{r_b} \vec{F}_{qq_0}\cdot\vec{dl} = -\int_{r_a}^{r_b} F_{qq_0}\cdot dl\cdot\cos\theta = -\int_{r_a}^{r_b} \overbrace{\frac{qq_0}{4\pi\varepsilon_0\cdot r^2}}^{k\cdot\frac{qq_0}{r^2}}\ \overbrace{\cos\theta\, dl}^{dr} = -kqq_0\int_{r_a}^{r_b}\frac{1}{r^2}\,dr =$$



$$W_{a\to b} = -kqq_0\left(\frac{1}{r_B}-\frac{1}{r_A}\right) = -\left(E_{p_F}-E_{p_0}\right)\ J = N\cdot m$$

$$E_p = q_0\cdot V = k\frac{q\cdot q_0}{r}$$

$$W_{a\to b} = -q_0\left(V_F - V_0\right)\ J$$

$$\Delta E_p = -W_{a\to b}$$

$$V = k\frac{q}{r}\ \left(\frac{J}{C}\right)$$

$$\overbrace{1eV}^{J} = \overbrace{1V}^{\overset{J/C}{V}}\cdot\overbrace{1{,}6\cdot10^{-19}C}^{C}$$

$$\Delta V = -\frac{W_{a\to b}}{q_0}$$

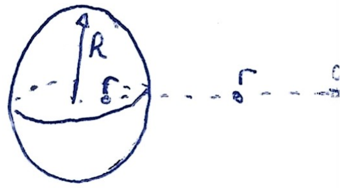
$$E_c = \frac{1}{2}mv^2$$

$$\vec{E}\ (N/C) \qquad (\Delta)V = -\int_A^B \vec{E}\,\vec{dl}$$

$\boxed{\text{Insulating}}$: Charges are distributed homogeneously along the volume/surface

$\boxed{\text{Conductive}}$: After electrostatic equilibrium, charges will move to the surface(s).

Potential Inside and outside a spherical shell of Charge Q $\qquad V = \vec{E}\cdot\vec{r}$



Hollow Sphere

• Outside $(r>R)$: $\vec{E} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{r^2}\,\hat{u}_r$

$$\Delta V = V_r - \underset{\substack{\downarrow \\ (V=0 \text{ if } r=\infty)}}{V_{ref}} = -\int_{ref}^{r}\vec{E}\,d\vec{r} = -\int_{\infty}^{r} E\,dr = \int_{\infty}^{r} k\cdot\frac{Q}{r^2}\,dr = \frac{Q}{4\pi\varepsilon_0 r}$$

• Inside $(r<R)$ $\vec{E}=0 \rightarrow Q_{enc}=0$

$$\Delta V = V_r - \underset{\substack{\downarrow \\ V=0 \text{ if } r=\infty}}{V_{ref}} = -\int_{\infty}^{r}\vec{E}\,d\vec{r} = \underbrace{-\int_{\infty}^{R} k\cdot\frac{Q}{r^2}\,dr}_{\text{Fuera esfera}}\ \underbrace{-\int_{R}^{r} 0\,dr}_{\text{Dentro}} = \frac{Q}{4\pi\varepsilon_0\cdot R}$$

Full Sphere

• Outside $(r>R)$: $\vec{E} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{r^2}\,\hat{u}_r$

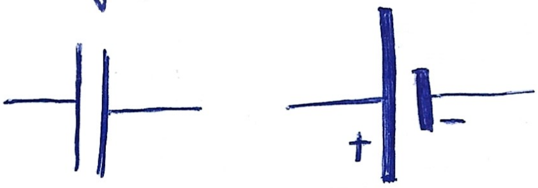
$$\Delta V = V_r - \underset{0}{\cancel{V_{ref}}} = -\int_{\infty}^{r}\vec{E}\,d\vec{r} = \frac{Q}{4\pi\varepsilon_0 r}$$

• Inside $(r<R)$: $\vec{E} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Qr}{R^3}\,\hat{u}_r$

$$\Delta V = V_r - \underset{0}{\cancel{V_{ref}}} = -\int_{\infty}^{r}\vec{E}\,d\vec{r} = -\int_{\infty}^{R}\frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{r^2}\,dr - \int_{R}^{r}\frac{1}{4\pi\varepsilon_0}\cdot\frac{Qr}{R^3}\,dr = \left[\frac{Q}{4\pi\varepsilon_0 R} - \frac{Q}{4\pi\varepsilon_0 R^3}\cdot\frac{r^2}{2}\right]_R^r =$$

$$= \frac{Q(3R^2 - r^2)}{8\pi\varepsilon_0 R^3}$$

Capacitors and capacitance $\rightarrow$ $C = \dfrac{Q}{V_{ab}}$ (F)



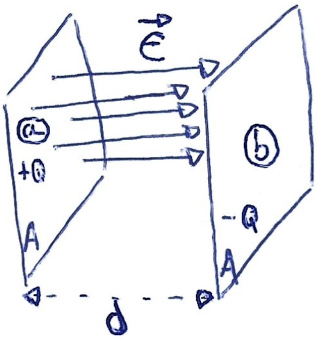
C → coulomb

$$\text{Farads} = 1F = \frac{1C}{1V} = \frac{1C^2}{1N \cdot 1m}$$

$$\text{Volt} = 1V = \frac{1J}{1C} = \frac{1N \cdot 1m}{1C}$$

$$\varepsilon_0 \left(\frac{F}{m}\right) = \left(\frac{C^2}{Nm^2}\right)$$

Pararellel-plate capacitor



$$\Delta V_{ba} = -\int E\,dl = -E\,d$$

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0} \rightarrow V_a - V_b = V_{ab} = \boxed{V = E \cdot d = \frac{Q\,d}{A\varepsilon_0}}$$

$\sigma = \frac{Q\,(C)}{A\,(m^2)}$

$$\boxed{C = \frac{Q}{V} = \frac{Q}{\frac{Qd}{A\varepsilon_0}} = \varepsilon_0 \frac{A}{d}}$$

Coaxial capacitor

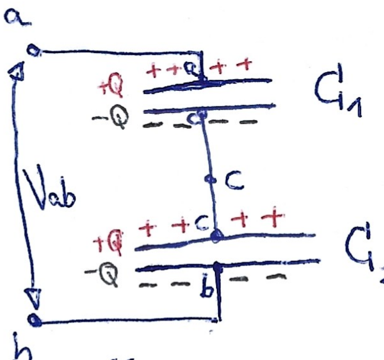
$$\boxed{V = \frac{Q}{2\pi\varepsilon_0 L} \ln\left(\frac{b}{a}\right)}$$

b → radio de −Q

a → radio de +Q

$$\boxed{C = \frac{Q}{V_{ab}} = \frac{2\pi\varepsilon_0 L}{\ln\left(\frac{b}{a}\right)}}$$

Capacitors in series:



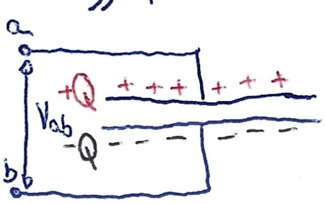
$$V_1 = V_{ac} = \frac{Q_1}{C_1} = \frac{Q}{C_1}$$

$$V_2 = V_{cb} = \frac{Q_2}{C_2} = \frac{Q}{C_2}$$

$$\boxed{V_{ab} = V_{ac} + V_{cb} = Q\left(\frac{1}{C_1} + \frac{1}{C_2}\right)}$$

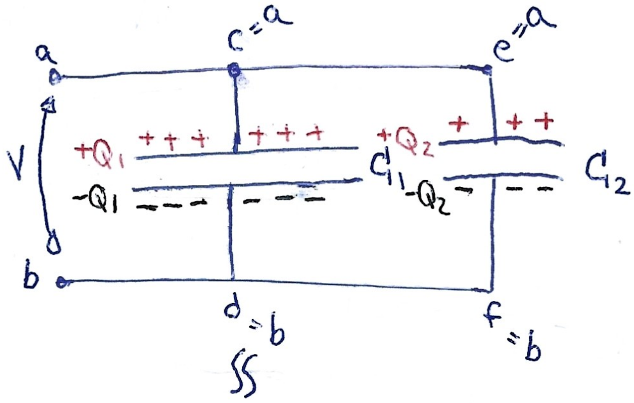
$$\frac{\cancel{Q}}{C_{eq}} = \cancel{Q}\left(\frac{1}{C_1} + \frac{1}{C_2}\right) \rightarrow \boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}}$$

⇕ equivalente



$$C_{eq} \rightarrow \boxed{V_{ab} = \frac{Q}{C_{eq}}}$$

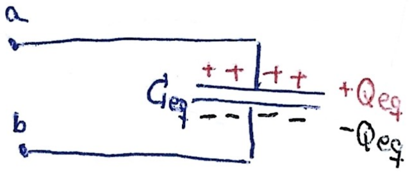
Capacitors in parallel:



$$V_{cd} = V_1 = \frac{Q_1}{C_1} = V \quad \boxed{V_1 = V_2}$$

$$V_{ef} = V_2 = \frac{Q_2}{C_2} = V$$

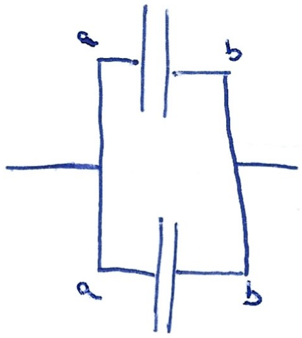
⇓



$$V = \frac{Q_{eq}}{C_{eq}}$$

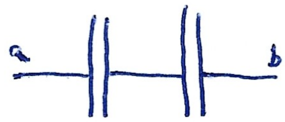
$$\begin{cases} Q_{eq} = Q_1 + Q_2 \\ Q_{eq} = V \cdot C_{eq} \\ Q_1 = V C_1 \\ Q_2 = V C_2 \end{cases} \to \boxed{C_{eq} = C_1 + C_2}$$

Parallel:



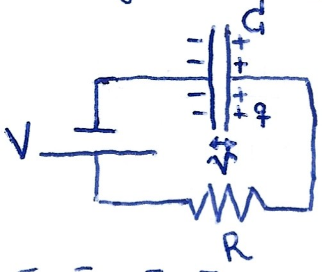
$\to \boxed{V \text{ same}}$

Series:



$\to \boxed{Q \text{ same}}$

Energy stored in capacitors



$$V = \frac{Q}{C} \qquad \begin{matrix} v \le V \\ q \le Q \end{matrix}$$

$$v = \frac{q}{C}$$

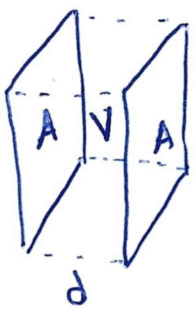
$$W = Q\,\Delta V = \int dW = \int dq \cdot v = \int dq\, \frac{q}{C} =$$

$$= \int_0^Q \frac{q\,dq}{C} = \frac{1}{C} \cdot \frac{1}{2} q^2 \Big]_0^Q = \boxed{\frac{1}{2}\frac{Q^2}{C}} = \underset{\text{electrical potencial energy}}{U}$$

$$U = \frac{1}{2}\frac{\overset{C \cdot V}{Q^2}}{C} = \frac{1}{2} C V^2 \underset{C = \varepsilon_0 \frac{A}{d}}{=} \frac{1}{2}\left(\frac{\varepsilon_0 A}{d}\right) V^2 \underset{V = E \cdot d}{=} \frac{1}{2}\left(\frac{\varepsilon_0 A}{\cancel{d}}\right)\left(E^2 d^{\cancel{2}}\right) = \boxed{\frac{1}{2}\varepsilon_0 E^2 (A \cdot d)}$$

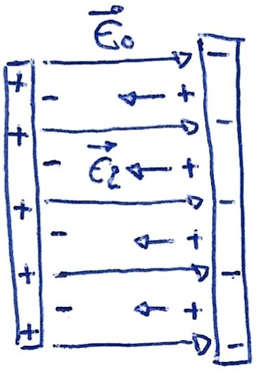
$(A \cdot d)$ Volume between plates

Pararel-plates capacitor



Energy density: $u = \dfrac{U}{\text{Volume}} = \dfrac{U}{A \cdot d} = \boxed{\dfrac{1}{2}\varepsilon_0 E^2}$ $\quad (J/m^3) \to (N/m^2)$

Dielectrics: reduction of the voltage



$$E = E_0 - E_i$$

$$E < E_0$$

$$V = E \cdot d < E_0 \cdot d = V_0 \rightarrow \boxed{V < V_0}$$

$$\frac{Q}{C} = V < V_0 = \frac{Q}{C_0}$$

$$\boxed{C > C_0}$$

V decreases
Q increases
C increases

$$C = k C_0$$

$$V = \frac{V_0}{k}$$

$$E = \frac{E_0}{k}$$

$$k > 1$$

$$E_i = \left(1 - \frac{1}{\varepsilon_r}\right) E_0$$

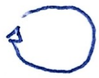
$$\varepsilon_r = k \varepsilon_0$$

$$U = \frac{U}{k} \qquad U = \frac{U}{k}$$

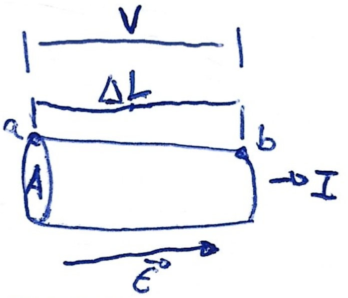
Current Intensity (Ampere)

$$I = \frac{dQ}{dt} \left(\frac{C}{s}\right)$$

The current is defined by the movement of positive charges

Direct Current (DC): Batery 

Alternating Current (AC): Redeléctrica 

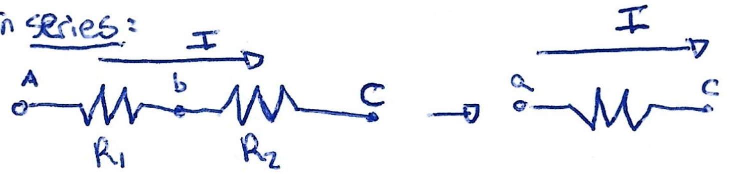


$$\boxed{V = I \cdot R}$$

$$R(\Omega) = \frac{\rho L}{A} \qquad \rho(\Omega \cdot m)$$

Resistors

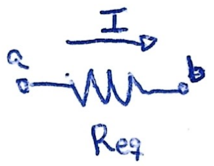
In series:

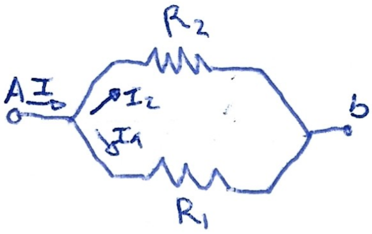


$$\left.\begin{aligned} V_{AB} &= I \cdot R_1 \\ V_{BC} &= I R_2 \end{aligned}\right\} V_{ac} = V_{ab} + V_{bc} = I \cdot R_1 + I R_2 = I(R_1 + R_2)$$

$$V_{ac} = I R_{eq} \rightarrow \boxed{R_{eq} = R_1 + R_2}$$

In parallel:





$$\boxed{I = I_1 + I_2}$$

$$\begin{cases} V_{ab} = I_1 \cdot R_1 \rightarrow I_1 = \dfrac{V_{ab}}{R_1} \\ V_{ab} = I_2 R_2 \rightarrow I_2 = \dfrac{V_{ab}}{R_2} \end{cases}$$

$$\frac{\cancel{V_{ab}}}{R_{eq}} = \frac{\cancel{V_{ab}}}{R_1} + \frac{\cancel{V_{ab}}}{R_2} \rightarrow \boxed{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}}$$

Power: $P_R = VI = I^2 R = \dfrac{V^2}{R}$ $(W = V \cdot A)$
in resistors.

Batery

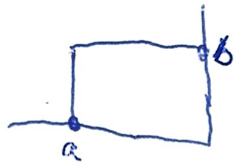
$P = \varepsilon I$

Voltemeter and Ammeter:

ideal (V) $\rightarrow R = \infty$ $\rightarrow$ In parallel

ideal (A) $\rightarrow R = 0$ $\rightarrow$ In series

Kirchhoff's rules:



2 Junctions

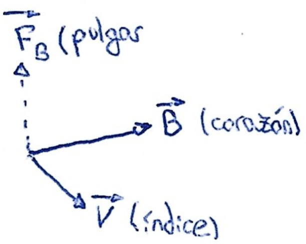
- Junction: Point where 3 or more cables join

$$\boxed{\sum I_{entering} = \sum I_{leaving}}$$

- Loop: Close circuit

$$\boxed{\sum V = 0}$$

Magnetic Force



$\vec{F}_B = q\, \vec{V} \times \vec{B}$ (T)

$F_B = q \cdot V \cdot B \cdot \sin\theta$

$(T) = \left(\dfrac{kg}{A \cdot s^2}\right)$

Lorentz's Force

$\vec{F}_\varepsilon = q \cdot \vec{\varepsilon}$

$\vec{F}_B = q\,(\vec{V} \times \vec{B})$

$$\boxed{\vec{F} = \vec{F}_\varepsilon + \vec{F}_B = q\,(\vec{\varepsilon} + \vec{V} \times \vec{B})}$$

$\underset{}{\overset{\text{índice}}{\vec{A}}} \times \overset{\text{corazón}}{\vec{B}} = \overset{\text{pulgar}}{\vec{C}}$

$$\vec{B} = \frac{\mu_0}{4\pi} \cdot q \frac{\vec{v} \times \vec{u}_r}{r^2}$$

$$\boxed{\frac{F_E}{F_B} = \frac{c^2}{v^2}}$$

speed of light

$$\boxed{c^2 = \frac{1}{\mu_0 \varepsilon_0}}$$

$$\boxed{T \cdot m^2 = Wb}$$

Magnetic Flux

$$\phi_B = \int d\phi_B = \int \vec{B} \cdot d\vec{A} \quad (T \cdot m^2)$$

Closed surface

$$\oint_A \vec{B} \cdot d\vec{A} = 0$$

Porque las líneas de campo son cerradas

Faraday's law

$$\mathcal{E} = -N \frac{d\phi_B}{dt} \qquad \vec{B} = \vec{B}(t) \qquad \phi_B = \int_A B(t)\, dA \cdot \cos\theta$$

$$\phi_B = B \cdot S \cdot \cos\theta$$

$$\mathcal{E} = -\frac{d\phi_B}{dt} \neq 0 \rightarrow \begin{cases} B(t) \\ S(t) \\ \theta(t) \end{cases}$$

$$i = \frac{\mathcal{E}}{R} = -\frac{1}{R} \cdot \frac{\Delta\phi_B}{\Delta t}$$