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S3-Euclidean Spaces
Design of scalar products II

Linear Algebra

Ingeniería del Software-Universidad de Oviedo

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Scalar product in $C^0_{[a,b]}$

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Scalar product in $C^0[a,b]$ and in $M_{m\times n}(\mathbb{R})$.

$$\vec{f},\vec{g} \in C^0[a,b] \Longrightarrow \vec{f}\cdot\vec{g} \overset{def}{=} \int_a^b f(x)\,g(x)\,dx$$

$$\|\vec{f}\|_2^2 \overset{def}{=} \vec{f}\cdot\vec{f} \Longrightarrow \|\vec{f}\|_2 = \sqrt{\underbrace{\int_a^b f^2(x)\,dx}_{\geq 0}}$$

$\int_a^b f^2(x)\,dx > 0$ and it's only 0 if $f(x) = 0(x)$

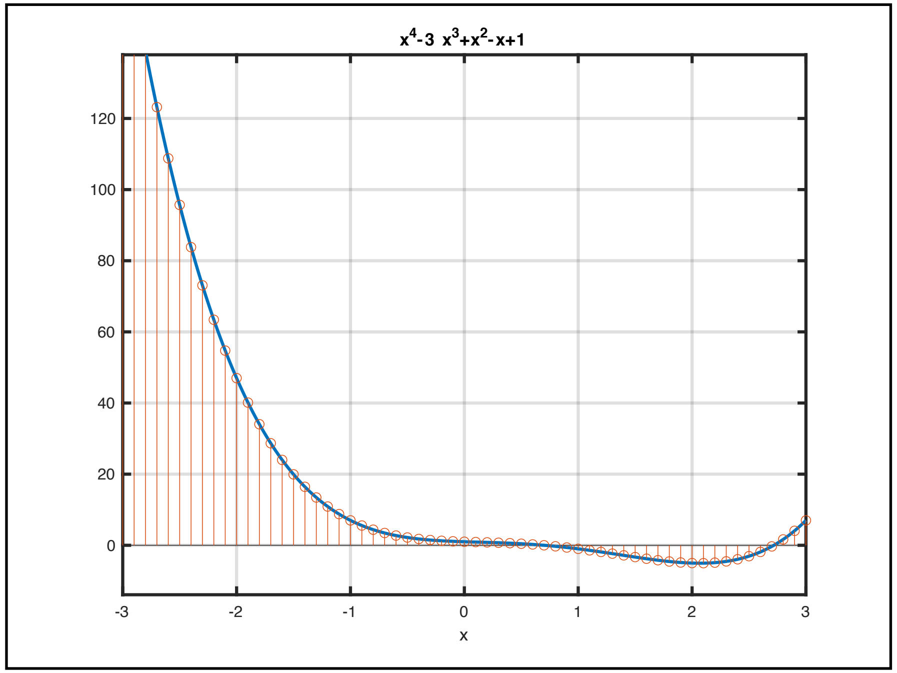
$$\cos(\vec{f},\vec{g}) = \frac{\vec{f}\cdot\vec{g}}{\|\vec{f}\|\cdot\|\vec{g}\|}$$

Physically if f is an analogic signal, then its norm $\|\vec{f}\|_2$ is its energy in $[a,b]$

$\downarrow$

content of information

Scalar product in $C^0_{[a,b]}$



Analogic signal $\xrightarrow{\text{Sampling}}$ Digital signal

$f(x) \xrightarrow{\text{Sampling}} \mathbf{f} \in \mathbb{R}^N$

$g(x) \longrightarrow \mathbf{g} \in \mathbb{R}^N$

$\longrightarrow \mathbf{f}.\mathbf{g}$

When the sampling interval is approaching to 0 $\longrightarrow$ $\langle f, g\rangle = \int_a^b f(x)g(x)dx$

$$cos\langle \boldsymbol{f},\ \boldsymbol{g}\rangle = \frac{\boldsymbol{f}\cdot \boldsymbol{g}}{\|\boldsymbol{f}\|\|\boldsymbol{g}\|} = \frac{\int_a^b f(x)g(x)dx}{\sqrt{\int_a^b f^2(x)dx}\sqrt{\int_a^b g^2(x)dx}}$$

Scalar product in $C^0_{[a,b]}$

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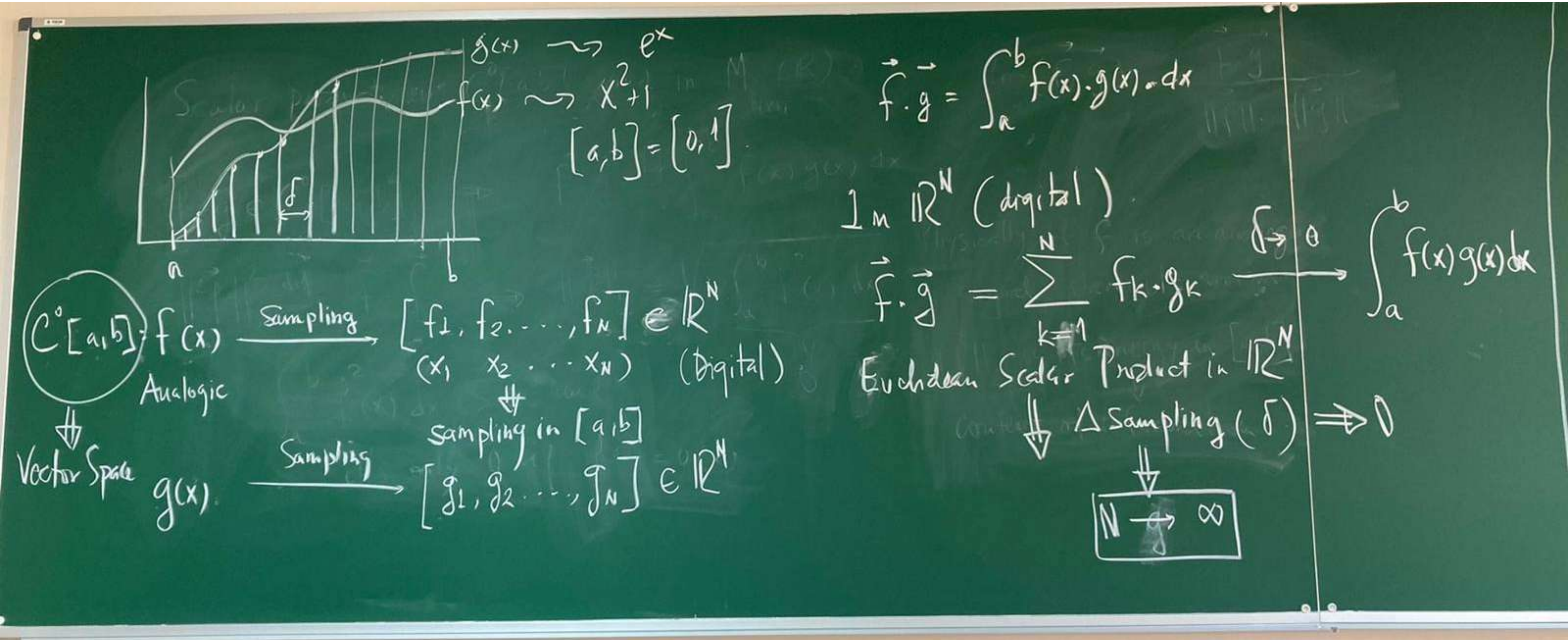

Entrywise Scalar Product in $M_{m\times n}(\mathbb{R})$

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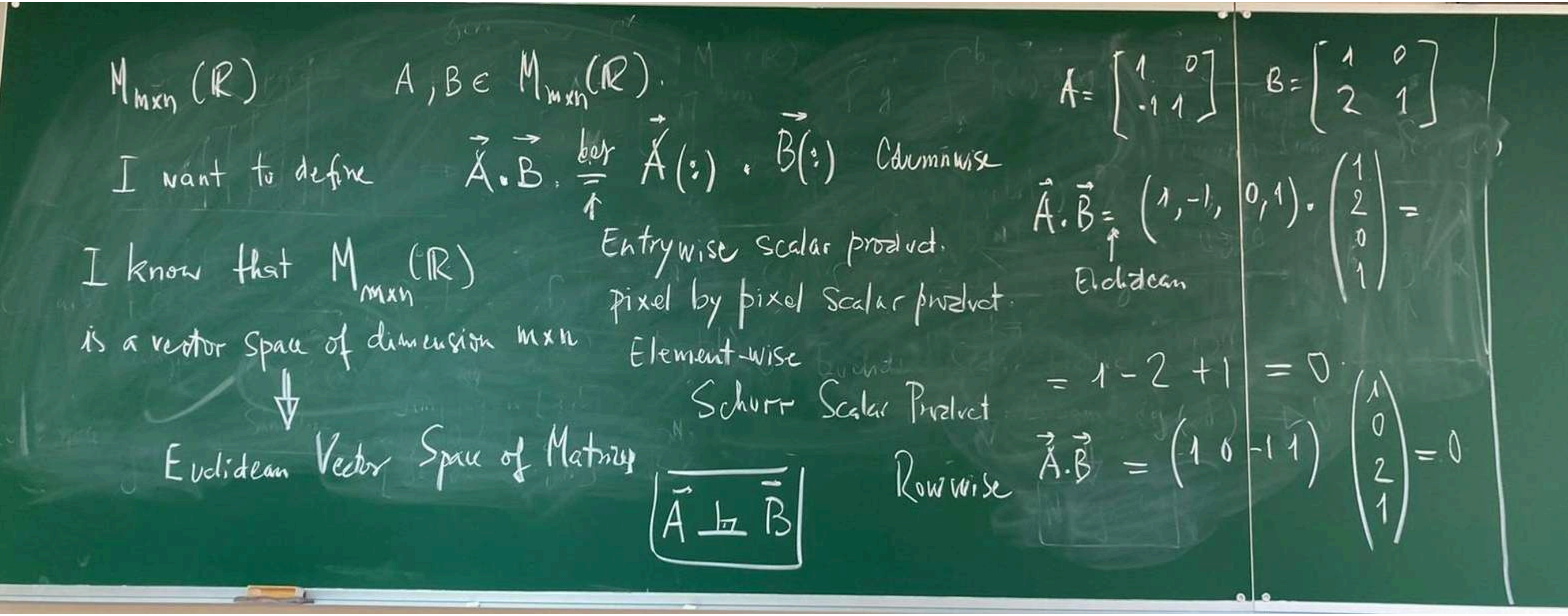

Entrywise Scalar Product in $\boldsymbol{M}_{m\times n}(\mathbb{R})$

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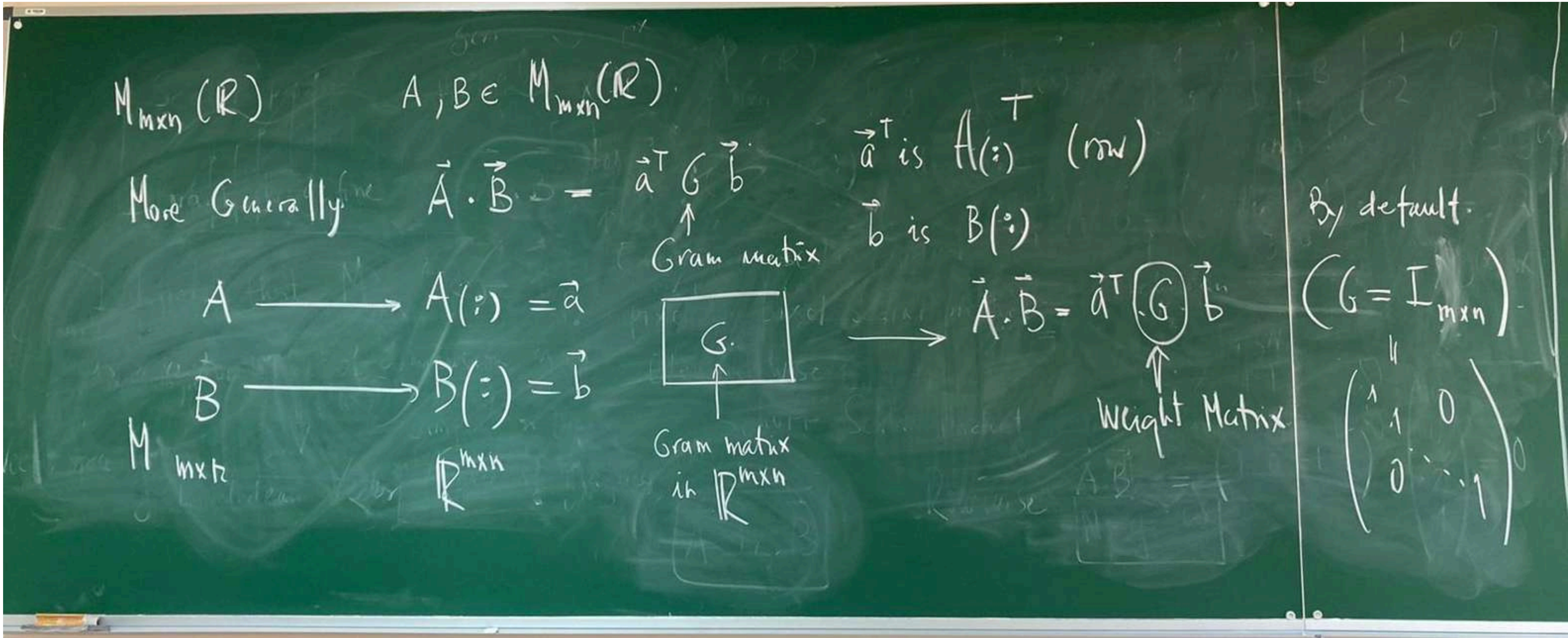

Cauchy-Schwartz

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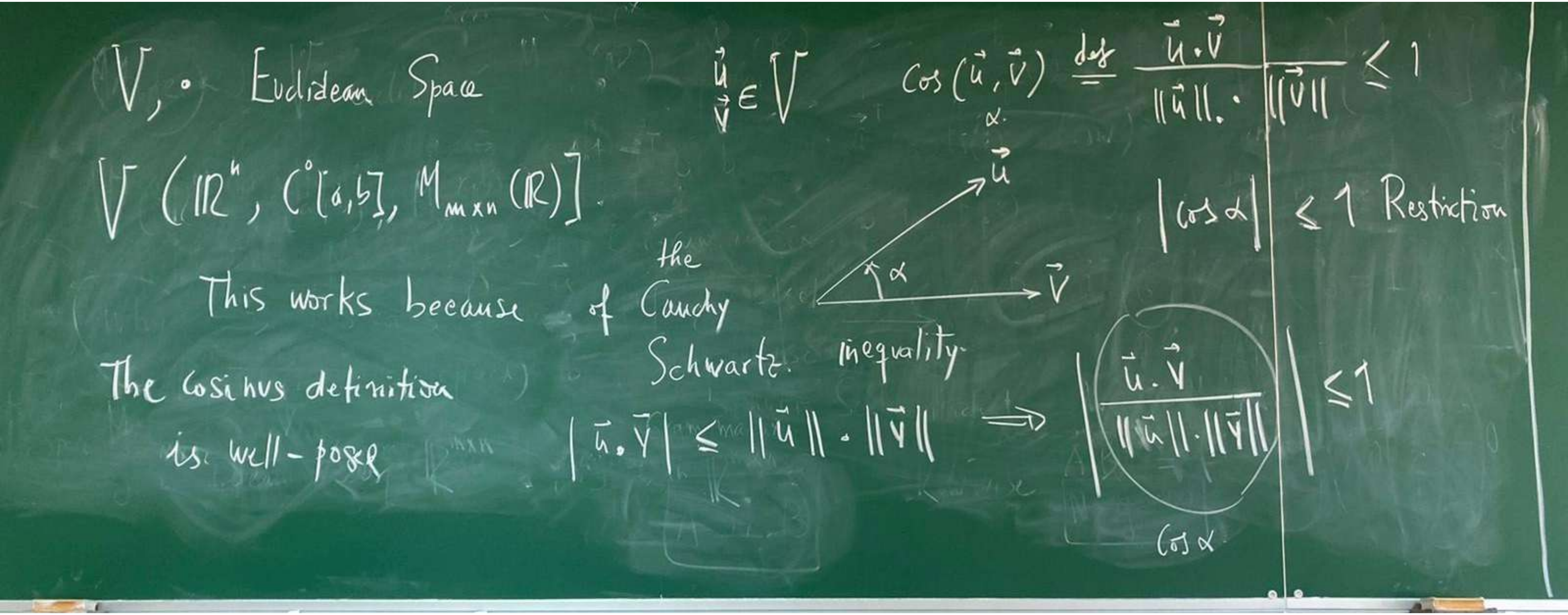

Orthogonality

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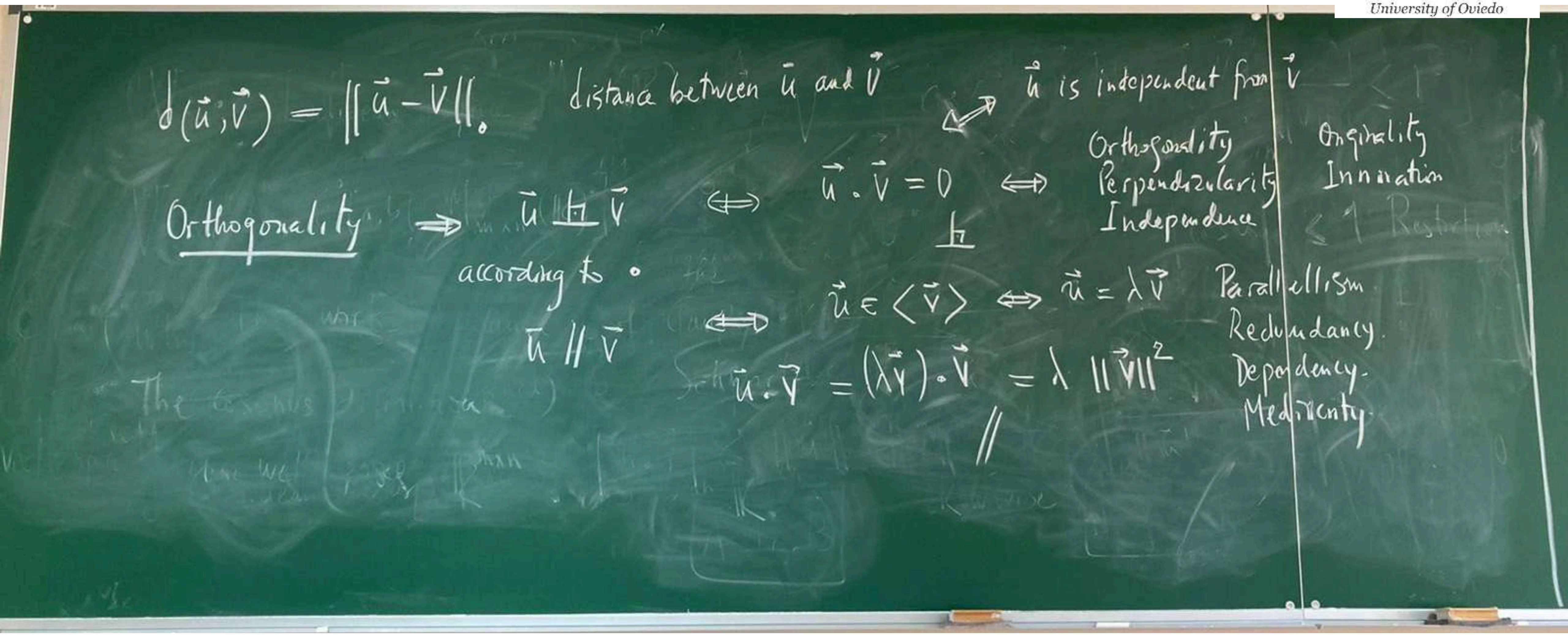

Orthogonality