Exercise 1 *Using the polar form to calculate:*

a) $(1+i)^2$

b) $(1+\sqrt{3}i)^4$

c) $\left(\sqrt{3}-i\right)^8$

d) $(-2+2i)^{10}$

e) i^{2020}

f) $\sqrt[3]{i}$

g) $\sqrt[4]{-1}$

Exercise 2 *Convert to Cartesian (rectangular) form the following complex numbers:*

a) $e^{i\pi/6}$,

b) $e^{-1+i\pi/3}$,

c) $e^{-i\pi/4}$,

d) $\dfrac{1-e^{\pi i/2}}{1+e^{\pi i/2}}$,

e) $e^{\pi i}\left(1-e^{-\pi i/3}\right)$,

f) $\dfrac{1-i^3}{(1+i)^3}$.

Exercise 3 *Expressing complex numbers exponentially:*

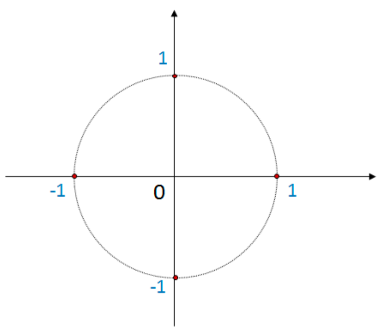
$$-\sqrt{3}+i, \quad -2-2\sqrt{3}i, \quad \sqrt{3}-i, \quad 3\sqrt{2}+3\sqrt{2}i$$

Exercise 4 *Find $x \in \mathbb{R}$ such as $z = \frac{x^2+2i}{8-i}$ is:*

a) *pure immaginary*

b) *real*

Exercise 5 *a)* *Plot on a unit radius circumference the following angles:* $\frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{6}, \frac{5\pi}{4}, \frac{2\pi}{3}, -\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$.



b) *Plot the following complex numbers:* $z_1 = -1+i$, $z_2 = 1+i$, $z_3 = \frac{1+i}{\sqrt{2}}$, $z_4 = -\sqrt{3}+i$, $z_5 = \frac{1}{2}(-\sqrt{3}-i)$, $z_6 = \frac{1}{2}(1-\sqrt{3}i)$.

Exercise 6 *Graphically represent and express in polar, binomial, Cartesian and exponential form the following complex numbers:*

$$\frac{2_{\pi/6}3_{\pi/6}}{2_{2\pi/3}1_{-\pi/3}}; \quad \frac{i^{32}i^{17}}{i^2 i^3}$$

Exercise 7 *Plot the following sets of complex numbers:*

a) $A = \{ z \in \mathbb{C} \quad / \quad |z| = 5, \quad Im(z) = 3 \}$

b) $B = \{ z \in \mathbb{C} \quad / \quad |z - 1 + i| = 2 \}$

c) $C = \{ z \in \mathbb{C} \quad / \quad |z - 2| < 1 \}$.

d) $D = \left\{ z \in \mathbb{C} \quad \Big/ \quad \text{Real}\left(\frac{z+1}{z-1}\right) > 1 \right\}$

Mark the pure complex numbers.

Exercise 8 *What does it mean (geometrically) the multiplication of a complex number z by i? And the multiplication by $2i$? Now rotate $z = 3 + i$ by $\frac{\pi}{4}$ radians counterclockwise. Give the rectangular form of the resulting complex number.*

Exercise 9 *Factorizing the following polynomials*

a) $p(z) = z^3 - 4z^2 + 6z - 4$,

b) $p(z) = z^2 - 2iz + 1$.

Exercise 10 *Solve the following equations:*

a) $z^8 - 1 = 0$

b) $z^3 + i = 0$

c) $z^2 - 6z + 10 = 0$

Exercise 11 *Finding a polynomial that has at least the roots $z = 1, \quad z = 2 + i$.*

Exercise 12 *Finding the intersection points between the circumference $x^2 + y^2 = 1$ and the line $y = x - 3$.*

Exercise 13 *If z_1, z_2 are the roots of the equation with real coefficients $z^2 + az + b = 0$, prove that ${z_1}^n + {z_2}^n$ is a real number for any natural value of n. In the particular case of the equation $z^2 - 2z + 2 = 0$, express that sum as a function of n.*
