Practice 1. Introduction to Matlab

1. What is Matlab?

The first version of Matlab dates from the 70's, and was designed as a support tool for courses of *Theory of Matrices*, *Linear Algebra* and *Numerical Analysis*. The name Matlab is an acronym: " Matrix Laboratory." Today, Matlab is a very powerful program, with a manageable environment, which includes scientific tools, technical calculations, graphical display and a high level programming language.

2. Elementary operations and variables

The way of representing numbers and of operating in Matlab is the same as in pocket calculators. For example:

`3` `-99` `.001` `9.63` `1.62e-20`

Note that the decimal separator (point) is used instead of a comma. The usual operations are performed with the same symbols and in the same sequence as in calculators. Note that e-20 means 10^{-20}.

addition	subtraction	multiplication	division	power
a+b	a-b	a*b	a/b	a^b

In order to execute a command, you must press the key **Intro** ↩ . For example, to calculate the value of $3 + 5 \times 2 + 1$, the following instruction is carried out

```
>> 3+5*2+1
```

and the following answer (ans) is obtained

```
ans =
14
```

This means that the result is saved in the variable `ans`. In contrast,

```
>> s=(3+5)*2+1
```

indicates to Matlab that the result of this operation is to be saved in the variable `s`. Notice the difference with the previous case, the parentheses alter the value.

2.1. Rules to name variables

- The variable name can have up to 63 characters (31 in previous versions), which can be letters, numbers and the hyphen to underscore (_).
- The first character must be a letter. `lado2` is a valid name, but `2lado` is not.

- Matlab is case sensitive: the variable `Base` is different from the variable `base`.
- A variable name cannot have blank spaces: `side1` is valid, but `side 1` is not.
- There are names that should be avoided because they have their own meaning in MATLAB: `ans`, `pi`, `Inf`, ...

2.2. Punctuation marks and cursor movements

- You can define multiple variables in one line if separated by commas. For example

```
>> base = 2, height = 3, area = base * height
```

- The variables can also be separated by semicolons. In that case the command is carried out but the result does not appear in the command window:

```
>> base = 5; height = 2; area = base * height
```

- The percent symbol `%` is used to write comments in the same line. The comments do not change the results and are usually written in scripts (programs) to remind us of what we are doing.
- The up and down arrow keys allow us to recover previous commands. The left and right arrow keys allow us to move to left and right in the same line without modifications. The Inicio and Fin keys can also be used for moving to the beginning and end of a line respectively.

3. The desktop

The window of MATLAB displays a desktop divided into several parts:

- **Command Window** is the window where you enter commands
- This window can be toggled with the window **Workspace** which provides information about the current variables.
- You can also see the window **Current Folder** that displays the contents of the current directory.
- All the commands are recorded in the **Command History**.

If we want to clear the command window (**Command Window**) we can do this by using `clc`. It should be borne in mind that this does not affect variables that are already in use.

If you double-click a line of **Command History** the line is carried out in the command window. If we click and drag the line with the mouse, we can correct it before it runs.

The command **whos** allows us to see the different variables and their corresponding class and number of bytes.

4. How to find help

The commands `doc` and `helpwin` or `help` are used to get information about a given subject. For instance,

```
>> doc ans
>> help ans
```

provide information about `ans`.

If you do not know the exact command on which you wish further information, you can simply type `helpwin` to open a help window **Help** in which a list of topics, an index of terms and a word search appear, among other things. You can also write `help sqrt` to obtain information about `sqrt`, the square root, (or any other function) directly on the command window. Try `help elfun` to obtain a list of elementary functions. Note that `exp(1)` is the number e=2.71828... and any exponential can be written in the same way, not as $e^\wedge$ (the argument). The natural logarithm is `log`. For instance

```
>> exp(1)
```

```
>> ans=2.7183
```

```
>> log(ans)
```

```
>> ans=1
```

```
>> log10(100)
```

```
>> ans=2
```

5. Formats

When MATLAB presents the results, it chooses a default format with a maximum of 3 digits for the integer part and 4 for the decimal part. If the number needs to be shown with more digits, Matlab uses the *exponential notation*. This is the option *short* of the command `format`. For example:

```
>> format short
>> pi
ans =
3.1416
```

Try

```
>> 10*pi
>> 100*pi
>> 1000*pi
```

Note that the letter **e** between two numbers means that the first number is multiplied by 10 raised to the second:

```
>> 2e3
>> 2e-3
```

The most used formats are:

`format short`	3 digits is the maximum for the integer part and 4 for the decimal part; if the number is greater it is converted to the exponential notation.
`format long`	2 digits for the integer part and 15 for the decimal; if the number is greater it is converted to the exponential notations.

You can consult other forms of presentation of results with `doc format` . Regardless of the format in which a calculation is shown on screen, the computer performs all calculations with 16 significant figures (double precision).

vpa(x) uses variable-precision floating-point arithmetic (VPA) to evaluate each element of the symbolic input x to at least d significant digits, where d is the value of the digits function. The default value of digits is 32. For instance

```
>> vpa(pi,20)
ans=3.1415926535897932385
```

one integer (3) plus 19 decimal digits.

6. Some mathematical functions

MATLAB has a wide range of functions — a complete list can be obtained with the command `doc elfun` — corresponding to the most used mathematical functions. Examples of these functions are:

Scientific Notation	Name in MATLAB	Meaning
$\|x\|$	`abs(x)`	Absolute value of x
$\sin x$	`sin(x)`	sine of x
$\cos x$	`cos(x)`	cosine of x
$\tan x$	`tan(x)`	tangent of x
$\sin^{-1} x$	`asin(x)`	arcsine of x
$\cos^{-1} x$	`acos(x)`	arccosine of x
$\tan^{-1} x$	`atan(x)`	arctangent of x
e^x	`exp(x)`	exponential of x
$\ln x$	`log(x)`	logarithm with base e of x
$\sqrt{x}$	`sqrt(x)`	square root of x

In the trigonometric functions of this list, the angle is always expressed in radians. It is possible to express the angle in degrees (See for example the command `sind`).

For instance, to calculate $sin(\pi/2)$, $sin(\pi)$, e^4 and $\sqrt{2}$ we write:

```
>> sin(pi/2)
>> sin(pi)
>> exp(4)
>> sqrt(2)
```

Why is sin(pi) different from zero ? Try sin(sym(pi)).

7. Symbolic expressions

The capabilities of MATLAB can be expanded by installing various modules (*toolboxes*). One, called SYMBOLIC MATH TOOLBOX, allows symbolic computation, i.e. enables us to manipulate variables without using their numerical approximations.

To use the symbolic calculation module SYMBOLIC MATH TOOLBOX is necessary to establish *symbolic objects* representing the symbolic variables. For *misuse of language* , the *symbolic objects* of MATLAB are also called *symbolic variables*.

Among others, the module SYMBOLIC MATH TOOLBOX allows you to perform the following tasks:

`syms x y z`	Creates the symbolic variables `x`, `y`, `z`.
`solve(Expr)`	Calculates the *zeros* of `Expr`.
`solve(Expr,z)`	Calculates the values of `z` that make `Expr` equal to zero.
`subs(S,x,a)`	Substitutes in `S` the variable `a` for `x`.
`pretty(S)`	Presents the expression S in elegant form.
`double(S)`	Calculates the numerical value of a symbolic expression.
`expand(S)`	Expands the expression `S` as the product of its factors.
`factor(S)`	Factorizes, if possible, the expression `S`.
`simplify(S)`	Simplifies a symbolic expression.

7.1. Example

1. Solve the equation $x^3 + 3x^2 - 4 = 0$.

   We have to calculate the *zeros* of $p(x) = x^3 + 3x^2 - 4$.

```
>> syms x
>> p = x^3+3*x^2-4;
>> solve(p,x)
```

2. Use `factor` to factorize the polynomial p of the previous exercise.

```
>> factor(p)
```

   Did you expect this result?

3. Calculate the value of the polynomial p at $x = \sqrt{2}$.

```
>> s1 = subs(p,x,sqrt(2))
```

   However, if we want a numerical expression for the answer, we have to use the command `double`, that writes the result with double precision

```
>> double(s1)
ans =

   4.828427124746190
```

4. We can use a `function handle` to represent a function. It has some advantages and enables us to manipulate the functions in more general ways. Compare with the previous example

```
>> clear
>> syms x
>> p = @(x) x^3+3*x^2-4;
>> solve(p,x)
```

```
>> factor(p,x)
```

```
>> s1=p(sqrt(2))
```

Now we could try to locate the zero of this function in a given interval. For example in $[0\,,2]$

```
>> fzero(p,[0 2])

ans=1
```

However, what happens in $[-3,-1]$?

7.2. Example

Solve the quadratic equation

$$ax^2+bx+c=0, \quad \text{with } a,b,c\in\mathbb{R}.$$

```
>> syms x a b c
>> E = a*x^2+b*x+c
>> s=solve(E,x)      % produces the two solutions
>> pretty(s)         % shows the expression in a clearer way
```

Note the importance of making clear what is the variable that should be isolated:

```
>> s=solve(E,b)      % Take care. b is isolated by this command
```

7.3. Example

Expand the polynomial $p(x)=2(x-1)-2(x-1)^2+(x-1)^3$ in powers of x. Factorize the polynomial and find its roots.

```
>> syms x
>> p = 2*(x-1) - 2*(x-1)^2 + (x-1)^3
>> pretty(p)
>> p = expand(p)
>> factor(p)
>> solve(p)
   ans =
      1
      2+i
      2-i
```

This means that p has a real root 1 and the complex roots $2+i$ y $2-i$ (they are conjugate).

7.4. Example

Given $f(x) = x^2 + 1$ and $g(x) = x^3 + 2x - 3$. Calculate $f + g$, $f - g$, $f \cdot g$, f/g, f^{-1}, $g \circ f$ y $f \circ g$.

```
>> syms x
>> f=x^2+1;
>> g=x^3+2*x-3;
>> f+g
>> f-g
>> f*g
>> f/g
>> finverse(f) % calculates the inverse function of f
>> compose(g,f) % calculates g composed with f
>> compose(f,g) % calculates f composed with g
```

Note that when we calculate the inverse of f, we only obtain the root with positive sign (f is not one-to-one). You cannot carry out these operations for function handles.

7.5. Example

Given $f = xe^{x^2-1}$. Calculate $f(2)$, $f(-5)$ y $f(2) \times f(-5)$.

We can do the example using symbolic variables (left) or numerical variables (right)

```
>> syms x
>> f=x*exp(x^2-1);
>> a=subs(f,x,2)
>> b=subs(f,x,-5)
>> a*b
```

```
>> x=2;
>> a=x*exp(x^2-1)
>> x=-5;
>> b=x*exp(x^2-1)
>> a*b
```

8. Exercises

1. Do the following operations:

   a) $\dfrac{10000}{400 + 6 \cdot 500}$ *Sol:* 2.9412

   b) $270^{\frac{1}{3}}(690 + 876)$ *Sol:* 10121.53

   c) $\dfrac{500\,(645 + 7843)}{45 + 9}$ *Sol:* 78592.59

   d) $\dfrac{21 + 78}{43^{\frac{1}{2}} + 80^3}$ *Sol:* 0.00019336

2. Calculate the value of these functions at the given points:

   a) $f(x) = \dfrac{x^2}{6x + x^3}$ at $x = 1$ and $x = -0.5$ *Sol:* $0.1429,\ -0.0800$

   b) $f(x) = \dfrac{\sin x}{1 + \cos x}$ at $x = 2$ and $x = 2^\circ$ (2 degrees). *Sol:* $1.5574,\ 0.0175$

   c) $f(x) = \ln|x + 2|$ at $x = 4$, $x = -2$ and $x = -10$. *Sol:* $1.7918, NaN,\ 2.0794$

   d) $f(x) = \dfrac{e^x}{e^{2x+1}}$ at $x = 5$ and $x = -2$. *Sol:* $0.0025,\ 2.7183$

3. Solve the equations:

   a) $x^3 - 3x + 2 = 0$ *Sol:* $1, 1, -2$

b) $x^4 - 2x + 1 = 0$ *Sol:* $1, 0.5437, -0.7718 + 1.1151i, -0.7718 - 1.1151i$

c) $\ln(x^2 - 1) = 1$ *Sol:* $-1.9283, 1.9283$

d) $\sin x = -1$ *Sol:* $\{-\pi/2 + 2k\pi; k \in \mathbb{Z}\}$

4. Let $f(x) = x \sin x$, $g(x) = x^2 - 1$ y $h(x) = e^{x+3}$. Calculate:

a) $h \circ g \circ f$ *Sol:* $e^{x^2 \sin^2 x + 2}$

b) $f \circ g \circ h$ *Sol:* $(e^{2(x+3)} - 1) \sin(e^{2(x+3)} - 1)$

c) $h^{-1} \circ h$ *Sol:* x

d) $(f + g) \circ h$ *Sol:* $e^{(x+3)} \sin e^{(x+3)} + e^{2(x+3)} - 1$

e) $f \circ (g + h)$ *Sol:* $(x^2 - 1 + e^{x+3}) \sin(x^2 - 1 + e^{x+3})$

f) $f(2) \times g(3)$ *Sol:* 14.5488

g) $(f(1) + g(4)) \times h(4)$ *Sol:* 17372.28