CALCULUS
DEGREE IN SOFTWARE ENGINEERING
WORKSHEET 5. DEFINITE INTEGRALS. AREAS AND VOLUMES

1. Calculate the area of the regions bounded by the circle $x^2 + y^2 = 1$ and the hyperbola $4x^2 - 2y^2 = 1$.

2. Calculate the area of an ellipse with semiaxes a and b.

3. Calculate the area of the region bounded by the curve $y^2 = (x-1)(x-2)^2$.

4. Find the area of the region of the first quadrant enclosed by the circle $x^2 + y^2 = 3$ and the parabolas $x^2 = 2y,\ y^2 = 2x$ with $x^2 + y^2 \leq 3$.

5. Obtain the value of the area enclosed by the curve $x^3 + x^2 - y^2 = 0$.

6. Determine the volume of the circular cone of height h whose base has a radius R.

7. Find the volume of a sphere of radius R.

8. Find the volume of a paraboloid of revolution whose base has area S and whose height is equal to h.

9. Calculate the volume of the solid obtained by revolving the region bounded by the two branches of the curve $(y-x)^2 = x^3$ and the line $x = 1$ about the x-axis.

10. Ω is the region enclosed by the curves $\sqrt{x} + \sqrt{y} = 1$ and $x + y = 1$. Calculate the volume of the solid generated by rotating Ω around the x-axis.

11. Calculate the volume of the solid generated by rotating the plane figure bounded by the curve $y = \sqrt{x}$, the x-axis and the vertical line $x = 4$ about the x-axis.

12. The semicircle $x^2 + y^2 = a^2,\ y \geq 0$ is revolved around the x-axis. Find the volume of the generated solid.

13. Consider the plane figure bounded by the curve $y = x^3$, the x-axis and the line $x = 1$. Find the volume of the solid generated by revolving this region about the y-axis.

14. Let D be a region enclosed by the x-axis, the circle $y = \sqrt{2 - x^2}$ and the parabola $y = x^2$ in the first quadrant. We revolve this region about the y-axis. Calculate the volume of the generated solid.