CALCULUS
DEGREE IN SOFTWARE ENGINEERING
WORKSHEET 2. DIFFERENTIATION

1. Calculate the derivative of the following fuctions
   (a) $y = x^5 + 6x^4 - 3x^3 + 2x^2 + 10x + 1$
   (b) $y = \dfrac{x^2 + 3x}{x + 5}$
   (c) $y = \sqrt{x^3 + 3x^2 + 5x}$
   (d) $y = x^x$
   (e) $y = cscx$
   (f) $y = \sin\left(\cos\left(x^2 + 2\right)\right)$
   (g) $y = \sqrt{\sin^{-1}(x - 1)}$
   (h) $y = \cos^{-1} \dfrac{2x}{5 + x}$
   (i) $y = x \tan x$
   (j) $y = \dfrac{x + \sin x}{x - \sin x}$

2. Given the function $f(x) = \dfrac{|x^2 - 4x| + 16}{x^2}$
   (a) Study the differentiability of $f(x)$. (Use the definition at $x_0 = 4$).
   (b) Determine the derivative function where it exists.
   (c) Analyse the intervals of increase-decrease of the function. Has it got an absolute maximum or minimum in its domain? Find its absolute maximum and minimum in the interval $[1, 6]$.

3. Find the points at which $f(x) = |x^2 + 6x + 8|$ has no derivative. Give reasons for the answer.

4. We know that $f : [0,\ 5] \to \mathbf{R}$

$$f(x) = \begin{cases} bx^2 + ax & 0 \le x \le 2 \\ c + \sqrt{x - 1} & 2 < x \le 5 \end{cases}$$

   is differentiable in $(0, 5)$ and $f(0) = f(5)$. What are the values of a, b and c ?

5. Calculate the following limits, applying a suitable method:

$$\text{I) } \lim_{x\to 0} \left(\frac{1}{ln(1 + x)} - \frac{1}{x} \right) \qquad \text{II) } \lim_{x\to +\infty} \frac{e^x + cosx}{e^x}$$

$$\text{III) } \lim_{x\to 0} \frac{e^{3x} - e^x}{\sin 2x} \qquad \text{IV) } \lim_{x\to 0^+} \frac{ln(\sin\ 2x)}{ln(\sin\ x)}$$

$$\text{V) } \lim_{x\to 0} \frac{\sin\ 5x}{\sin\ 2x} \qquad \text{VI) } \lim_{x\to 1} \frac{\tan(x^2 - 1)}{x - 1}$$

$$\text{VII) } \lim_{x\to 1} \frac{2^{x-1} - 1}{x - 1} \qquad \text{VIII) } \lim_{x\to \frac{1}{2}} \frac{ln(4x - 1)}{2x - 1}$$

$$\text{IX) } \lim_{x\to 0} \frac{ln(2x^2 + 1)}{2x} \qquad \text{X) } \lim_{x\to \pi/2} e^{\tan x}$$

6. Find the extreme values (absolute and local) of the functions and where they occur

   (a) $y = x^2 - 6x + 7$
   (b) $x(4-x)^3$
   (c) $x^2 + \frac{2}{x}$
   (d) $\sqrt{2x - x^2}$

7. Show that these functions have exactly one zero in the given interval

   (a) $f(x) = x^4 + 3x + 1 \quad [-2, -1]$
   (b) $f(x) = x^3 + \frac{3}{x^2} + 7 \quad (-\infty, 0)$
   (c) $f(x) = \tan x - \cot x - x \quad (0, \frac{\pi}{2})$
   (d) $f(x) = \frac{1}{1-x} + \sqrt{1+x} - 3 \quad (-1, 1)$

8. Find the value or values of c that satisfy $f(b) - f(a) = f'(c)(b-a)$ for the following functions and intervals

   (a) $x^{\frac{2}{3}} \quad [0, 1]$
   (b) $x + \frac{1}{x} \quad [0.5, 2]$
   (c) $x^{\frac{2}{3}} \quad [-1, 8]$
   (d) $x^{\frac{4}{5}} \quad [0, 1]$

   In one case you cannot apply the Mean Value Theorem. In which case and why not?

9. Write the Taylor expansion of the following functions at $x = 0$ with a remainder of order three (MacLaurin)

   (a) $\sin x$
   (b) e^x
   (c) $\cos x$
   (d) $ln(1+x)$
   (e) $\tan^{-1} x$