1. Linear Applications 3

Exercise 1 *Let $f : \mathbb{R}_2[x] \longrightarrow \mathbb{R}^3$ be the linear transformation defined as $f(a+bx+cx^2) = (b+c, b, 0, a)$.*

a) *Compute the matrix of f referred to the standard basis in both spaces.*

b) *Find out the kernel of f. Is f injective?*

c) *Find out the range of f. Is f surjective?*

d) *Is f bijective? Does f^{-1} exist? Compute $f^{-1}(1,2,3)$.*

Exercise 2 *Let $f : \mathbb{R}_2[x] \longrightarrow \mathbb{R}^4$ be the linear transformation defined by $f(a+bx+cx^2) = (b+c, b, 0, a)$. Compute the matrix referred to the basis $B_1 = \{1, 1-x, 1+x+x^2\}$ in $\mathbb{R}_2[x]$ and $B_2 = \{(1,0,0,0), (1,1,0,0), (0,1,1,0), (0,0,0,1)\}$ in $\mathbb{R}^4$.*

Exercise 3 *Considering the linear application $f : \mathbb{R}_3[x] \longrightarrow \mathscr{M}_2(\mathbb{R})$ defned by*

$$f(a_3x^3 + a_2x^2 + a_1x + a_0) = \begin{pmatrix} 3a_3 - a_2 & a_1 \\ 2a_2 + a_0 & a_2 \end{pmatrix}.$$

Obtain the matrix of f in the basis:

$$B = \{1, 1+x, 1+x+x^2, 1+x+x^2+x^3\}$$

and

$$C = \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ -1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix} \right\}$$

Exercise 4 *Considering the linear transformation $f : \mathbb{R}^4 \longrightarrow \mathbb{R}_2[x]$ with $f(1,0,0,0) = 1+x$, $f(1,1,0,0) = 2x$, $f(1,1,1,0) = -1-x$, $f(0,0,0,-1) = 2+3x-x^2$.*

Compute:

a) *The matrix of f referred to the standard basis in both spaces.*

b) *$f(1,-1,1,-2)$*

c) *The Kernel and the Image space of f, and classify f.*

d) *Let $p = 1+x^2$. Does p belong to $Im(f)$? Considering the linear system $AX = p$, classify the system and give a solution.*
