



## 1. Linear Applications 2

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**Exercise 1** Let  $f : \mathbb{R}^4 \longrightarrow \mathbb{R}_2[X]$  be the linear application defined as:

$$\begin{aligned} f(1, 0, 0, 0) &= 1 + x, \\ f(1, 1, 0, 0) &= 2x, \\ f(1, 1, 1, 0) &= -1 - x, \\ f(0, 0, 0, -1) &= 2 + 3x - x^2. \end{aligned}$$

- a) Obtain the matrix of the linear transformation  $f$  referred to the standard basis of  $\mathbb{R}^4$  and  $\mathbb{R}_2[X]$ , respectively.
  - b) Compute the image of the vector  $\vec{v} = (1, -1, 1, -2)$ .
  - c) Compute the Kernel of  $f$ . Is  $f$  an injective transformation?
  - d) Compute the Range (image space) of  $f$ . Is  $f$  a surjective transformation?
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**Exercise 2** Let  $f : \mathbb{R}^3 \longrightarrow \mathbb{R}^2$  be a linear transformation such as:

$$\text{Ker}(f) = \langle (1, 0, 0), (0, 1, 1) \rangle$$

and  $f(0, 0, 2) = (2, 2)$ .

Obtain the matrix of the linear transformation  $f$  referred to the standard basis of  $\mathbb{R}^3$  and  $\mathbb{R}^2$ , respectively. Compute the image of the vector  $\vec{v} = (-1, 3, 5)$ .

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