

MAGNETIC FIELD CREATED BY ELECTRIC CURRENTS: BIOT-SAVART'S LAW

Goals of the experiment:

- Measure the magnetic field \vec{B} along the axis of a circular coil with N turns.
- Calculate the number of turns N from the measurement of \vec{B} at the centre of the coil.
- Measure the magnetic field along the axis of a solenoid.

1. Theoretical background.

The magnetic field $d\vec{B}$ produced by a current element $I d\vec{l}$ of section A and length dl , is given by Biot-Savart's law, expressed through the equation:

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I d\vec{l} \times \vec{u}_r}{r^2}, \quad (1)$$

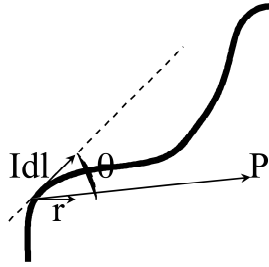


Fig. 1. Biot-Savart's law.

where $\mu_0 = 4\pi \times 10^{-7} \text{ Tm A}^{-1}$ is the permeability of free space, \vec{r} is the position vector of the point where the field is evaluated with regards to the current element and \vec{u}_r is its unit vector.

If the conductor through which the current flows is a circular loop with N turns of radius R and Biot-Savart's law is applied to find the magnetic field created in points on the symmetry axis (x) passing through the centre of the loop, as represented in Figure 2, the components of $d\vec{B}$ perpendicular to x add up to zero and the following equation is obtained

$$dB_x = dB \sin \theta = \frac{\mu_0}{4\pi} \frac{Idl}{x^2 + R^2} \frac{R}{\sqrt{x^2 + R^2}}, \quad (2)$$

where we have taken into account that $r^2 = x^2 + R^2$ and that $d\vec{l}$ and \vec{r} are perpendicular (so that $|d\vec{l} \times \vec{r}| = dl$).

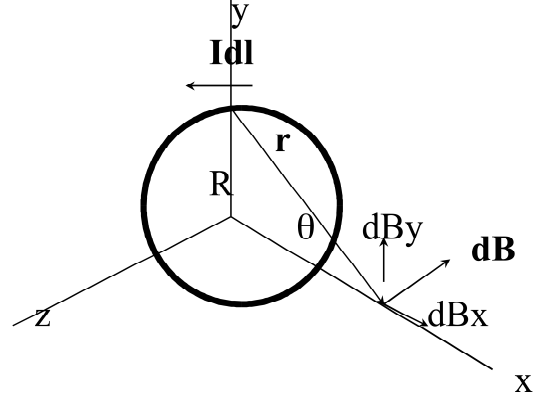


Fig. 2. Application of Biot-Savart's law to a circular coil.

For a winding with N turns, and using the previous equation to calculate the field created by a coil in points of its x axis, dB_x shall be integrated to find:

$$B_x = \frac{\mu_0}{2} \frac{R^2 N I}{(x^2 + R^2)^{\frac{3}{2}}}. \quad (3)$$

The value of \vec{B} at the centre of an N -turn circular coil of radius R is obtained by setting $x = 0$ in Eq. 3:

$$B = \frac{\mu_0 N I}{2R}. \quad (4)$$

2.1 Required material:

- A Helmholtz coil.
- DC power supply unit.
- Digital multimeter.
- Digital teslameter.
- Longitudinal Hall probe.
- Connecting cords.
- Meter scale, $l = 1000$ mm.
- Support rod with a barrel base and a right angle clamp.

2.2 Set-up:

The experimental set-up is comprised of a coil connected in series to a power supply through an ammeter that reads the current flowing through it and a teslameter equipped with a longitudinal probe that allows for the measurement of the magnetic field.

Before measurements begin we shall follow a few previous steps:

(1) Switch on the teslameter with the longitudinal probe connected. Using the zero adjustment knob set the reading in the teslameter screen to zero. Allow it to stabilise for a few minutes.

(2) Find the reference point for the measurement of lengths with the meter scale. For that purpose, locate and note down the place where the Hall probe finds a maximum value at the center of the coil. This point corresponds to a $x = 0$ distance, that will be taken as the reference.

24.5cm

(3) Measure the external and internal diameters of the coil and note them down with the corresponding accuracy:

$D = 41.3 \pm 0.1$ cm
 $d = 37.7 \pm 0.1$ cm

(4) Connect to the power supply and in series to an ammeter.

2.3 Magnetic field along the axis of the coil:

(1) The current will be set to 3 A and kept constant throughout this particular experiment.

(2) The axial Hall probe is used to measure the magnetic field along the axis of the coil from 20 cm to -20 cm in 5 cm steps. Note down the results in Table 1.

TABLE 1: MEASUREMENT OF THE MAGNETIC FIELD ALONG THE AXIS OF THE COIL.

x (cm)	B_{exp} (mT)	B_t (mT)
20	0.50	0.50
15	0.75	0.72
10	0.98	1.02
5	1.30	1.30
0	1.44	1.43
-5	1.30	1.30
-10	1.00	1.02
-15	0.70	0.72
-20	0.49	0.50

(3) Plot the magnetic field along the axis of the coil as a function of the position x on the axis and interpret the result.

(4) To find the theoretical value of the field we need I , the average radius R of the coil and the number of turns N , to substitute their values in Eq. 4.

(5) I is read from the ammeter. Let us recall that the error in this case is only of experimental nature, since only one measurement is performed. Hence, it depends on the scale used.

$I = 3.00 \pm 0.01$ A

(6) The average radius is found from the external and internal diameters D and d , respectively.

$R = 19.75 \pm 0.05$ cm

However, we still don't know the number of turns of the coil N .

2.4 Find out the number of turns of the coil:

Now we will vary the current and measure the magnetic field, so that we will be able to obtain the number of turns of the coil.

(1) Let's start with the current $I = 3A$ and we will decrease it, every $0.5A$, down to $I = 0.5A$. Measure the magnetic field at the centre of the coil by means of the axial Hall probe. Note down those va-

lues in Table 2.

TABLE 2: CURRENT DEPENDENT MAGNETIC FIELD MEASUREMENT, AT THE CENTRE OF THE COIL, IN ORDER TO OBTAIN THE NUMBER OF TURNS.

I (A)	B (mT)
0.50	0.25
1.00	0.48
1.50	0.73
2.00	0.97
2.50	1.20
3.00	1.44

(2) (7) Perform a linear least-squares-fit of the form $y = ax + b$, where $y \equiv B$, $x \equiv I$, and obtain the values of a and b , as well as their uncertainties (e.g., by

using the LINEST() function on Microsoft Excel¹).

(3) Obtain the number of turns N and its error ΔN from Eq. 4.

$$N = 149.98 \pm 8.41 \text{ vueltas}$$

(5) Does it make sense?

Yes, we counted all the lines and it was 154 so it makes sense

¹Or ESTIMACION.LINEAL() if using it in Spanish