CALCULUS
DEGREE IN SOFTWARE ENGINEERING
CHAPTER 14. A BASIC TABLE OF INDEFINITE INTEGRALS.

In this chapter, we apply the methods introduced in Chapter 13 to the construction of a basic table of indefinite integrals. p is any real number

$$1) \quad \int x^p \, dx = \frac{x^{p+1}}{p+1} + C$$

if $p \neq -1$

$$2) \quad \int x^{-1} \, dx = \ln |x| + C$$

$$3) \quad \int e^{px} \, dx = \frac{e^{px}}{p} + C$$

$$4) \quad \int a^{px} \, dx = \frac{a^{px}}{p \ln a} + C$$

These first integrals are immediate and you can check that they are correct by differentiating the right-hand side. Note that the absolute value in 2) allows us to integrate the reciprocal function even if x is negative.

$$5) \quad \int \ln x \, dx = x \, \ln x - x + C$$

By parts, as calculated in Chapter 12.

$$6) \quad \int \cos px \, dx = \frac{\sin px}{p} + C$$

$$7) \quad \int \sin px \, dx = -\frac{\cos px}{p} + C$$

$$8) \quad \int \sec^2 px \, dx = \frac{\tan px}{p} + C$$

$$9) \quad \int \csc^2 px \, dx = -\frac{\cot px}{p} + C$$

$$10) \quad \int \sec px \, \tan px \, dx = \frac{\sec px}{p} + C$$

$$11) \quad \int \csc px \, \cot px \, dx = -\frac{\csc px}{p} + C$$

In all these integrals, we have used the bacic formulas for differentiating elementary trigonometric functions.

$$12)\quad \int \tan px\,dx = -\frac{-\ln(|\cos px|)}{p} + C$$

In this case, we write

$$\int \tan px\,dx = \int \frac{\sin px}{\cos px}\,dx$$

and make the substitution $t = \cos px$. Thus

$$\int \frac{\sin px}{\cos px}\,dx = -\int \frac{dt}{pt} = -\frac{\ln|t|}{p} + C = -\frac{\ln|\cos px|}{p} + C$$

$$13)\quad \int \cot px\,dx = \frac{\ln(|\sin px|)}{p} + C$$

Now, we write

$$\int \cot px\,dx = \int \frac{\cos px}{\sin px}\,dx$$

and make the substitution $t = \sin px$. Thus,

$$\int \frac{\cos px}{\sin px}\,dx = \int \frac{dt}{pt} = \frac{\ln|t|}{p} + C = \frac{\ln|\sin px|}{p} + C$$

$$14)\quad \int \sec px\,dx = \frac{\ln|\sec px + \tan px|}{p} + C$$

The proof of 14) and 15) will be carried out in the next chapter, together with other important trigonometric integrals.

$$15)\quad \int \csc px\,dx = -\frac{\ln|\csc px + \cot px|}{p} + C$$

$$16)\quad \int \frac{dx}{\sqrt{1-x^2}}$$

We make the substitution $x = \sin t$. Hence,

$$\int \frac{dx}{\sqrt{1-x^2}} = \int dt = t + C = \arcsin x + C$$

Maybe, you knew the derivative of arcsine by heart and could guess the result.

$$17)\quad \int \frac{dx}{1+x^2} = \arctan x + C$$

The same as before with the change $x = \tan t$

We will finish with three indefinite integrals for whose solution we will have to combine different methods

$$18) \int \arctan x \, dx$$

We integrate by parts, with $u = \arctan x$, $du = \dfrac{dx}{1+x^2}$, $dv = dx$ and $v = x$. Using the Integration by Parts Formula

$$\int \arctan x \, dx = x \, \arctan x - \int \frac{xdx}{1+x^2}$$

If we make the substitution $t = 1 + x^2$

$$\int \frac{xdx}{1+x^2} = \int \frac{dt}{2t} = \frac{\ln t}{2} + C = \frac{\ln(1+x^2)}{2} + C$$

and

$$\int \arctan x \, dx = x \, \arctan x - \frac{\ln(1+x^2)}{2} + C$$

We do something similar in our next integral

$$19) \int \arcsin x \, dx$$

now, $u = \arcsin x$, $du = \dfrac{dx}{\sqrt{1-x^2}}$, $dv = dx$ and $v = x$. Using the Integration by Parts Formula

$$\int \arcsin x \, dx = x \, \arcsin x - \int \frac{xdx}{\sqrt{1-x^2}}$$

With the substitution $t = 1 - x^2$

$$\int \frac{xdx}{\sqrt{1-x^2}} = \int -\frac{dt}{2t^{1/2}} = -\sqrt{t} + C = -\sqrt{1-x^2} + C$$

finally,

$$\int \arcsin x \, dx = x \, \arcsin x + \sqrt{1-x^2} + C$$

The last indefinite integral of this table is

$$20) \int \frac{dx}{x^2-1} = 1/2 \ln \left| \frac{x-1}{x+1} \right| + C$$

This is the integral of a rational function- the general metod for solving this kind of integrals will be studied in Chapter 15, for now, we will show that the integrand must be decomposed in the sum of partial fractions

$$\frac{1}{x^2-1} = \frac{A}{x-1} + \frac{B}{x+1}$$

Note that $x^2 - 1 = (x-1)(x+1)$. Placing all the fractions under a common denominator, we find
$1 = A(x+1) + B(x-1)$
From this equation, we can obtain the undetermined coefficients A and B. $A = 1/2$, $B = -1/2$.
The integral is

$$\int \frac{dx}{x^2-1} = \int \frac{dx}{2(x-1)} - \int \frac{dx}{2(x+1)} = 1/2 \ln|x-1| - 1/2 \ln|x+1| + C$$

and this is the same as 20).
In the next chapter, we will study different methods that enable us to calculate more complicated indefinite integrals.