1. Basis

Exercise 1 *Finding the parametric and implicit equations, the basis, and the dimension:*

a) $U_1 = \{(x,y,z) \in \mathbb{R}^3 \mid x+y+z=0\}$

b) $U_2 = \{(x,y,z) \in \mathbb{R}^3 \mid x=y, x-y-z=0\}$

c) $U_3 = \{p(x) = ax^2+bx+c \in \mathbb{R}_2[x] \mid a-2b+c=0\}$

d) $U_4 = \{(x,y,z) \in \mathbb{R}^3 \mid x+y-z=0, \quad x^2+y^2=0\}$

e) $U_5 = \{(x,y,z) \in \mathbb{R}^3 \mid 2x+3y+z=0, 3y+z=0\}$

f) $U_6 = \{(x,y,z) \in \mathbb{R}^3 \mid x-2y+z=0, x-y-z=0\}$

g) $U_7 = \langle \vec{v}_1 = (1,1,1,0,0), \vec{v}_2 = (2,-1,0,1,3), \vec{v}_3 = (3,0,1,-1,2), \vec{v}_4 = (2,2,2,-2,-1) \rangle$

Exercise 2 *Considering the following linear subspaces in* $\mathbb{R}^4$*:*

a) $V_1 = \{(x,y,z,t) \mid x+y+z=0\}$

b) $V_2 = \langle (1,1,1,1), (1,2,3,4) \rangle$

c) $V_3 = \{(x,y,z,t) \mid x=\alpha+\beta, y=\alpha+\gamma,\ z=\gamma+\delta,\ t=\alpha+\delta\}$

Check if the vector vector $\vec{v} = (1,0,-1,-2)$ *belongs to any of them. Compute the coordinates of* $\vec{v}$ *in a basis set of the subspace where it belongs to.*

Exercise 3 *Finding the coordinates of the vectors:*

a) $\vec{w} = (1,0,-1)$ *in the basis set of* $\mathbb{R}^3$, $\mathscr{B} = \{\vec{u}_1 = (1,1,0), \vec{u}_2 = (1,1,1), \vec{u}_3 = (0,1,1)\}$

b) $p(x) = 2-x+3x^2$ *in the basis* $\mathscr{B} = \{1+x, 1-x, x^2\}$

c) $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ *in the basis set* $\mathscr{B} = \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \right\}$

Exercise 4 *Show that the vector system* $S = \{(1,1,1), (1,3,1), (-2,1,3)\}$ *is a basis set of* $\mathbb{R}^3$*. Find the coordinates of the vector* $\vec{v} = (1,1,2)$ *in the basis* S*.*
