CALCULUS
DEGREE IN SOFTWARE ENGINEERING
WORKSHEET 3. FUNCTION GRAPHING AND OPTIMIZATION

1. Plot the function $f(x) = ln(x^2 - 3x + 2)$, studying previously its domain, asymptotes, the intervals on which f is increasing and on which f is decreasing and its concavity.

2. Given the function $f(x) = \dfrac{x}{e^x - 1}$

   (a) Find its domain and symmetries. Classify its discontinuities.

   (b) Calculate its asymptotes.

   (c) Prove that $x = 0$ is the only real solution of the equation $-xe^x + e^x - 1 = 0$.

   (d) On what intervals is f increasing or decreasing ?

   (e) Sketch the curve.

3. Sketch the graph of $f(x) = \dfrac{x^2}{1 + x^2}$, studying the intervals on which it is increasing or decreasing , calculating its maxima and minima, analyzing its concavity, asymptotes, etc.

4. Plot the function $f(x) = x^4 - 2x^2$, studying the intervals on which it is increasing or decreasing , calculating its maxima and minima, analyzing its concavity, asymptotes, etc.

5. Sketch the graph of $f(x) = \dfrac{1}{1 + x^2}$ and $g(x) = \dfrac{x^2 - 4}{x^3}$, following the same steps as in the previous exercises.

6. Let $f(x) = xe^{\frac{1}{x}}$

   (a) Calculate $\lim\limits_{x \to 0} f(x)$. Is f continuous at $x = 0$?

   (b) Find the local maxima and minima of the function. Identify on which intervals the function is increasing and on which it is decreasing.

   (c) Find the inflection points. Study the concavity of the graph.

   (d) Identify and classify the asymptotes. Sketch the curve.

7. We want to make a closed box, whose base is a square, so that the area of its surface measures A cm^2. Find the dimensions of the box that make the maximum volume. Solve the same problem for an open box.

8. Given a circle of radius R, find the sides of the rectangle with the largest area that can be inscribed in the circle. Solve the same problem for an ellipse of semiaxes a and b.

9. Calculate the length of the sides of an isosceles triangle with the largest area whose perimeter measures L meters.

10. Find the volume of the largest right circular cone that can be inscribed in a sphere of radius 3.