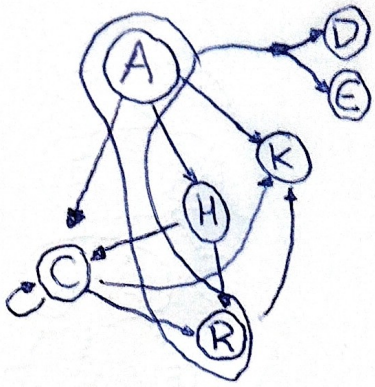
1. $F = \begin{cases} A \to CHK \\ C \to CKR \\ H \to CR \\ R \to K \\ AR \to DE \end{cases}$




$(A)^+ = \{A, C, H, K, R, D, E\} = R$

$\boxed{A \in F^+}$

$R = (A, C, D, E, H, K, R)$

- Remove trivial dependencies.

$F^+ = \begin{cases} A \to CHK \\ C \to KR \\ H \to CR \\ R \to K \\ AR \to DE \end{cases}$

- Remove attributes by transitivity:

$F^+ = \begin{cases} A \to H \\ C \to R \\ H \to C \\ R \to K \\ AR \to DE \end{cases}$

- Remove by pseudotransitivity: $A \to R$

$F^+ = \begin{cases} A \to HDE \\ H \to C \\ C \to R \\ R \to K \end{cases} = F_c$

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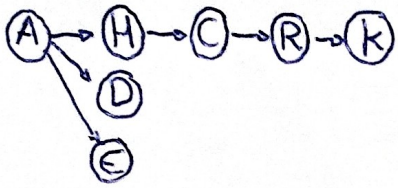
2. $R = (A, C, D, E, H, K, R, X)$

$F = \begin{cases} A \to HDE \\ C \to R \\ H \to C \\ R \to K \end{cases}$ (X)




As $A^+ \neq R$

we introduce X

$\{AX\}^+ = \{A, X, C, H, K, R, D, E\} = R$

Check minimality of $\{A, X\}$

- $A^+ \neq R$
- $X^+ \neq R$

$\vdash$ $\{A, X\}$ is a candidate key

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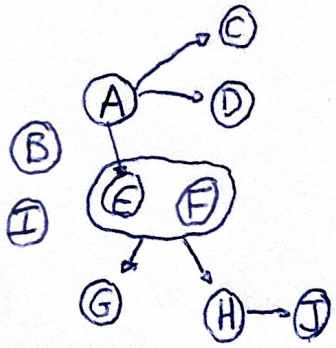
3. $R = (A, B, C, D, E, F, G, H, I, J)$

$F = \begin{cases} A \to CDE \Rightarrow R_1 \\ EF \to GH \Rightarrow R_2 \\ H \to J \Rightarrow R_3 \end{cases}$




Candidate keys:

- AFBI as $(A, F, B, I)^+ = R$

BCNF $R_1(\underline{A}, C, D, E)$ $F_1 = \{A \to CDE\}$

BCNF $R_2(\underline{E, F}, G, H)$ $F_2 = \{EF \to GH\}$

BCNF $R_3(\underline{H}, J)$ $F_3 = \{H \to J\}$

BCNF $R_4(\underline{AFBI})$ $F_4 = \{\}$

Lossless-Join:

$R = R_1 \bowtie R_2 \bowtie R_3 \bowtie R_4$

A, F, B, I

$\uparrow$

$R_1(C, D, E)$

(G, H)

|

(J)

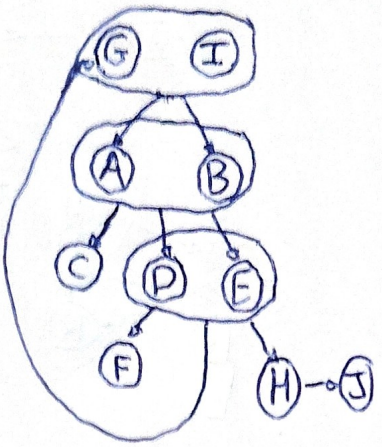
Dependency-preservation:

$F \leftarrow F_1 \cup F_2 \cup F_3 \cup F_4$

$F = \begin{cases} A \to CDE \\ EF \to GH \\ H \to J \end{cases}$

4. $R = (A, B, C, D, E, F, G, H, I, J)$

$$F = \begin{cases} GI \to AB \\ AB \to CDE \\ DE \to FGH \\ H \to J \end{cases}$$




Candidate keys:

- $GI$
- $IAB$
- $IDE$

BCNF $R_1 (\underline{GI}, A, B)$ $F_1 = \{GI \to AB\}$

BCNF $R_2 (\underline{AB}, C, D, E)$ $F_2 = \{AB \to CDE\}$

BCNF $R_3 (\underline{DE}, F, G, H)$ $F_3 = \{DE \to FGH\}$

BCNF $R_4 (\underline{H}, J)$ $F_4 = \{H \to J\}$

Lossless-Join:

$R = R_1 \bowtie R_2 \bowtie R_3 \bowtie R_4$

$(\underline{G, I}, A, B)$

$(\underline{A, B}, C, D, E)$

$(\underline{D, E}, F, G, H)$

$(\underline{H}, J)$

Dependency check:

$F = F_1 \cup F_2 \cup F_3 \cup F_4$

$$F = \begin{cases} GI \to AB \\ AB \to CDE \\ DE \to FGH \\ H \to J \end{cases}$$