CALCULUS
DEGREE IN SOFTWARE ENGINEERING
WORKSHEET 7. DIFFERENTIATION

1 Partial Derivatives . Directional derivatives.

1. Find the gradient at each point for the following scalar fields:

   a) $x^2 + y^2 \sin(xy)$ b) $e^x \cos y$ c) $x^2 y^3 z^4$

   d) $\dfrac{xy}{x^2 + y^2 + 5}$ e) $x^3 e^{x^2+y^2}$ f) $\ln(x^2 + 2y^2 - 3z^2)$

   g) $x^{(y^z)}$ h) $\dfrac{x^2 y^3}{x^2 + y^4}$ i) $e^{x+y^2} \cos(x + y)$

   j) $x^{(y^{z^2})}$ k) $\dfrac{x^2 y^5}{x + y}$ l) $sin(x^3 y^2 z^4)$

2. Calculate the directional derivatives of the following scalar fields at the given points and in the indicated directions:

   (a) $f(x, y, z) = x^2 + 2y^2 + 3z^2$ at $a = (1, 0, 0)$ in the direction of $\vec{v} = \vec{i} - \vec{j} + 2\vec{k}$
   (b) $g(x, y, z) = \left(\frac{x}{y}\right)^z$ at $a = (1, 1, 1)$ in the direction of $\vec{v} = 2\vec{i} + \vec{j} - \vec{k}$
   (c) $h(x, y) = \sin^{-1}(\frac{y}{x})$ at $a = (2, 1)$ in the direction of $\vec{v} = \vec{i} + 3\vec{j}$

3. Captain Alexandra has problems near the sunlit side of Mercury. The temperature of the spacecraft, when she is at the position (x, y, z) is $T(x, y, z) = e^{-x^2-2y^2-3z^2}$.
   Currently, she is situated at (1, 1, 1). In which direction must she travel to produce the fastest decrease of temperature?

4. Let $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ y $r = |\vec{r}|$. Prove that:

$$\nabla\left(\frac{1}{r}\right) = \frac{-\vec{r}}{r^3}$$

5. Calculate the second-order partial derivatives and check that the mixed derivatives are equal:

a) $f(x,\ y) = \dfrac{2xy}{(x^2+y^2)}$ b) $f(x,\ y,\ z) = e^z + xe^{-y} + \frac{1}{x}$

c) $f(x,\ y) = \cos(xy^2)$ d) $f(x,\ y) = e^{-xy^2} + x^4y^3$

e) $f(x,\ y) = (\cos^2 x + e^{-y})^{-1}$ f) $f(x,\ y,\ z) = ze^{xy} + yz^3x^2$