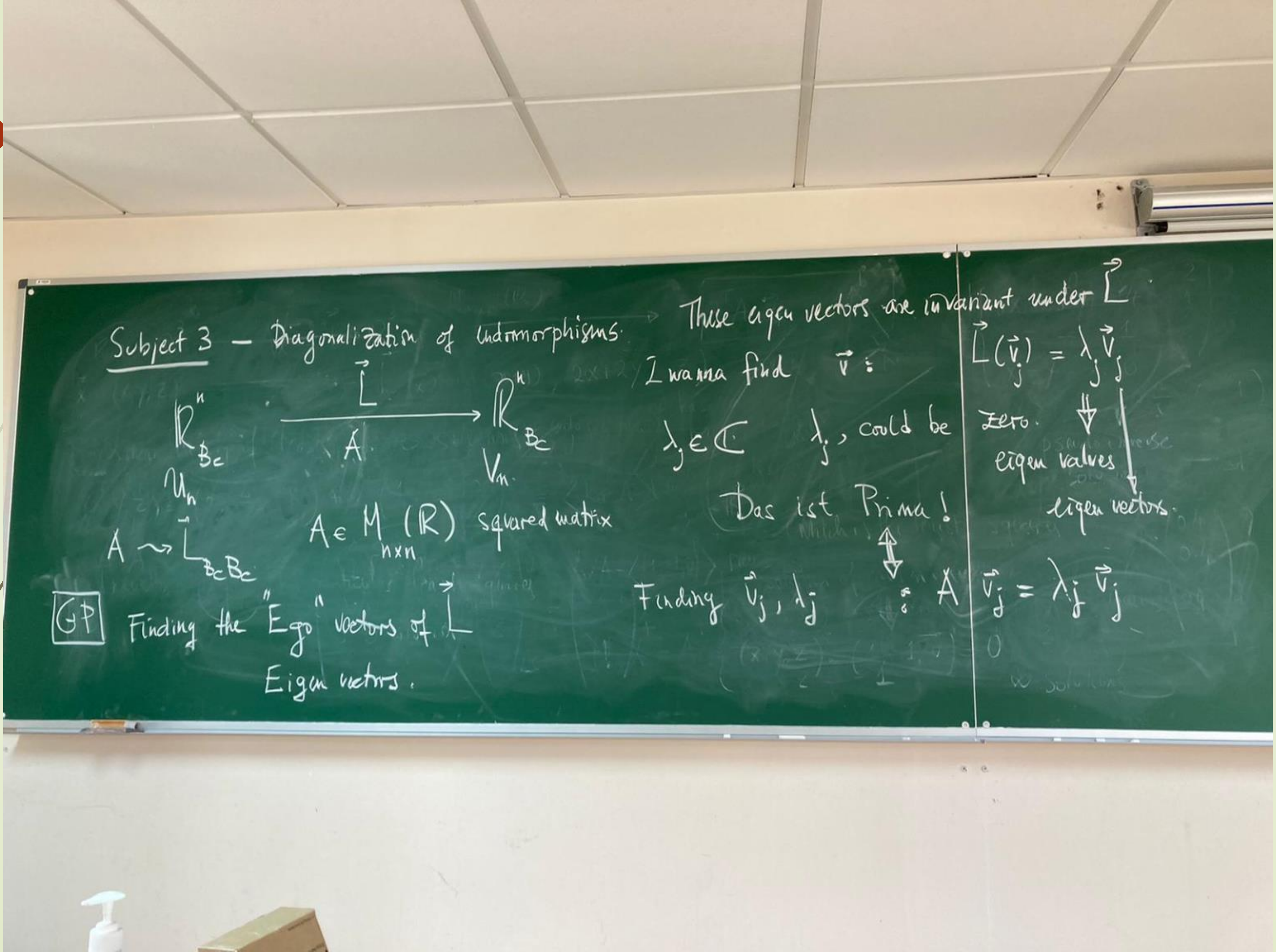
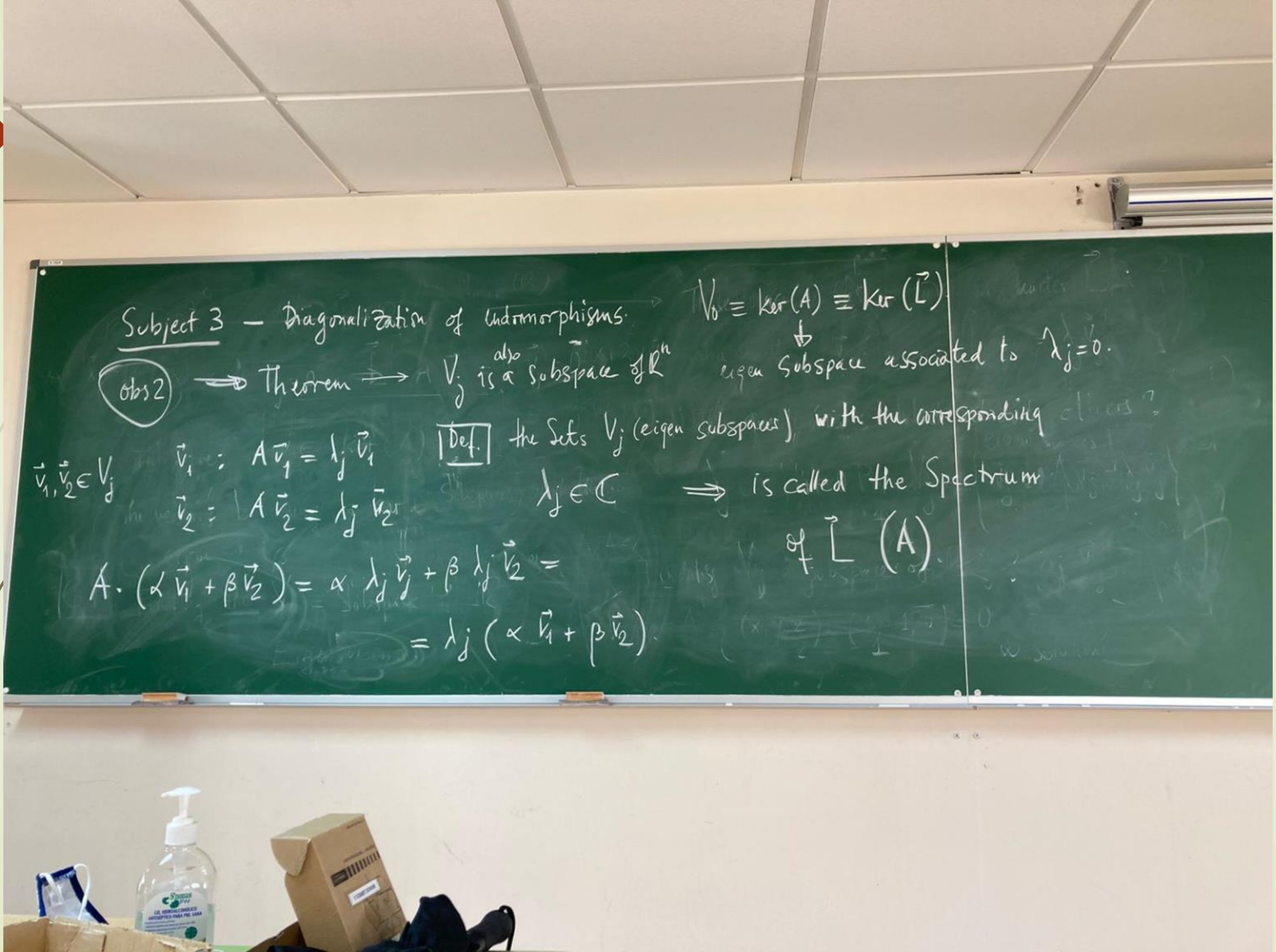
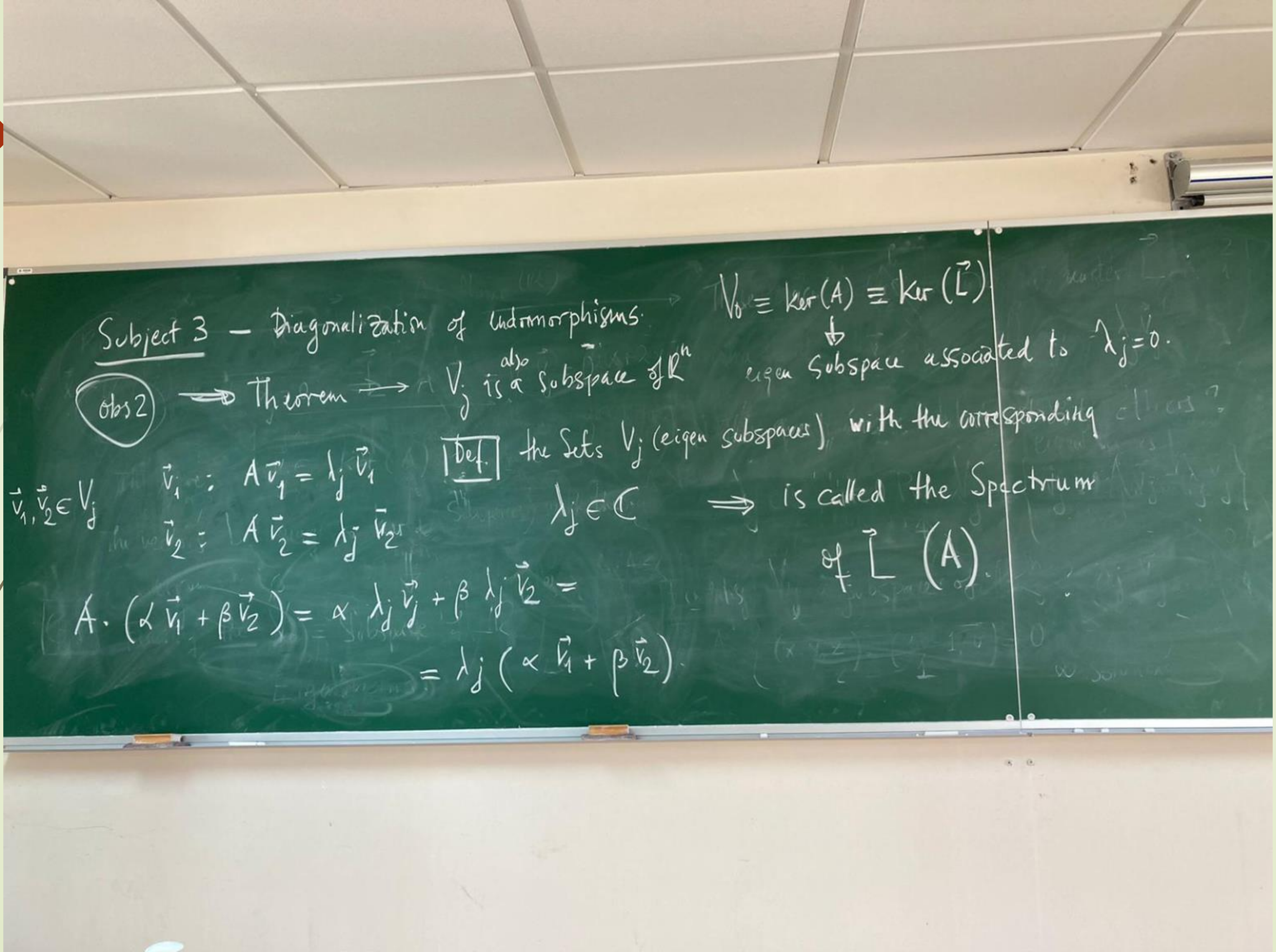
Diagonalization of Endomorphisms

Session 1


Subject 3 — Diagonalization of endomorphisms.
$\mathbb{R}^n_{B_c} \xrightarrow[A]{\vec{L}} \mathbb{R}^n_{B_c}$
U_n
V_n
$A \rightsquigarrow \vec{L}_{B_c B_c}$
$A \in M_{n\times n}(\mathbb{R})$ squared matrix
GP
Finding the "Ego" vectors of $\vec{L}$
Eigen vectors.
These eigen vectors are invariant under $\vec{L}$
I wanna find $\vec{v}$:
$\lambda_j \in \mathbb{C}$
λ_j, could be zero.
Das ist Prima!
Finding $\vec{v}_j$, λ_j : $A\,\vec{v}_j = \lambda_j\,\vec{v}_j$
$\vec{L}(\vec{v}_j) = \lambda_j \vec{v}_j$
eigen values
eigen vectors.


Subject 3 – Diagonalization of endomorphisms.
obs 2 ⟹ Theorem ⟶ V_j is also a subspace of $\mathbb{R}^n$
Def. the Sets V_j (eigen subspaces) with the corresponding
$\lambda_j \in \mathbb{C}$ ⟹ is called the Spectrum
of $\vec{L}$ (A).
$\vec{v}_1, \vec{v}_2 \in V_j$
$\vec{v}_1 : A\vec{v}_1 = \lambda_j \vec{v}_1$
$\vec{v}_2 : A\vec{v}_2 = \lambda_j \vec{v}_2$
$A \cdot (\alpha \vec{v}_1 + \beta \vec{v}_2) = \alpha \lambda_j \vec{v}_j + \beta \lambda_j \vec{v}_2 =$
$= \lambda_j (\alpha \vec{v}_1 + \beta \vec{v}_2)$.
$V_0 \equiv \ker(A) \equiv \ker(\vec{L})$
eigen subspace associated to $\lambda_j = 0$.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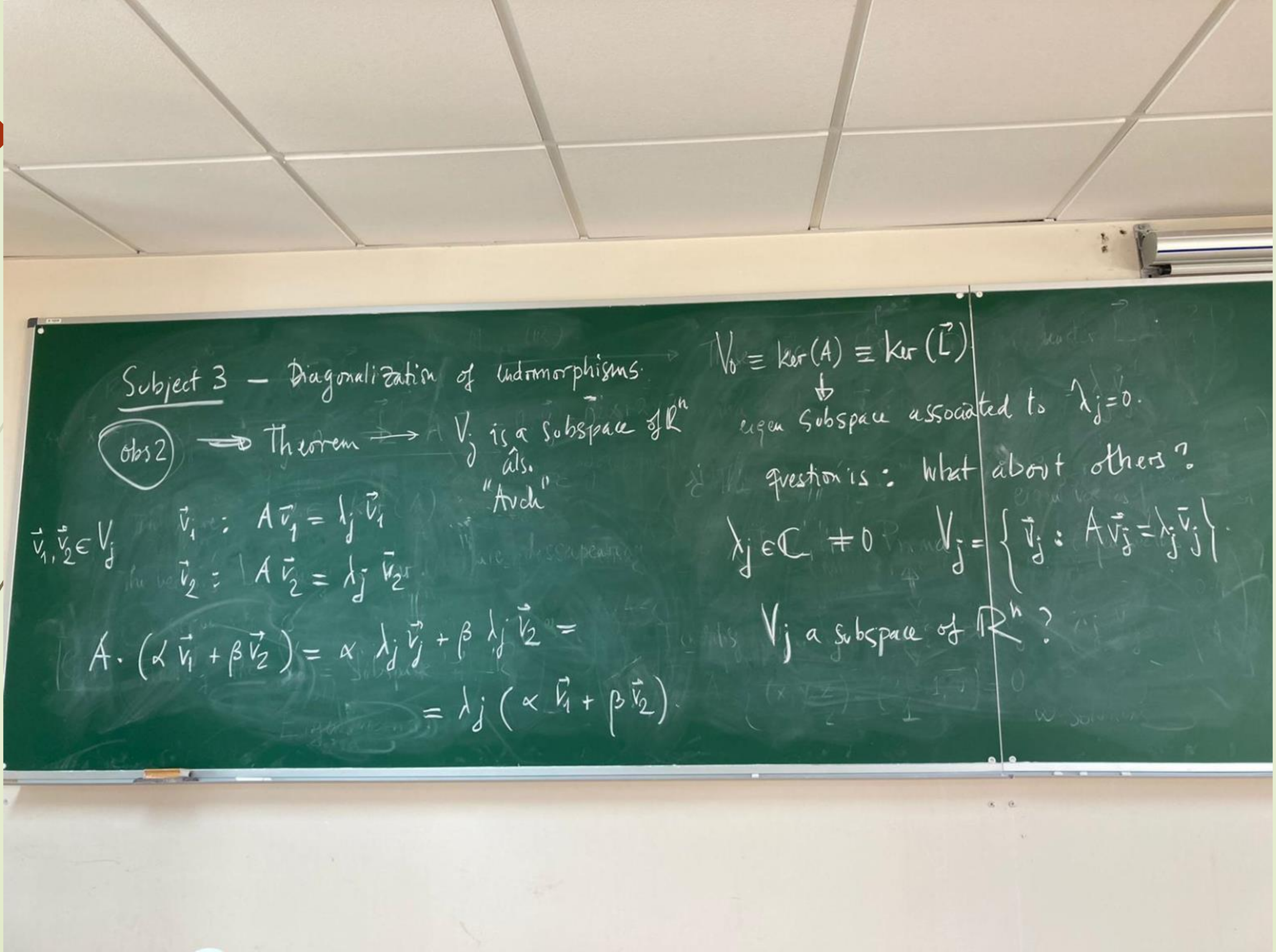
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$V_0 \equiv \ker(A) \equiv \ker(\vec{L})$
eigen subspace associated to $\lambda_j = 0$.
question is: what about others?
$\lambda_j \in \mathbb{C} \neq 0$ $V_j = \{\vec{v}_j : A\vec{v}_j = \lambda_j\vec{v}_j\}$
V_j a subspace of $\mathbb{R}^n$?