1. Diagonalization of Endomorphisms

Ejercicio 1 *Diagonalize the following matrices, when possible:*

a) $A = \begin{pmatrix} -2 & 0 & 0 \\ 4 & 2 & -2 \\ 4 & 1 & -1 \end{pmatrix}$. *(Answer: eigenvalues:* $-2, 0, 1$*)*

b) $A = \begin{pmatrix} 0 & 0 & 0 \\ 4 & -2 & -6 \\ 0 & 0 & 1 \end{pmatrix}$. *(Answer: eigenvalues:* $-2, 0, 1$*)*

c) $A = \begin{pmatrix} 2 & -2 & -4 \\ 0 & 1 & 0 \\ 0 & -3 & -2 \end{pmatrix}$. *(Answer: eigenvalues:* $-2, 1, 2$*)*

d) $A = \begin{pmatrix} -6 & -4 & -4 \\ 2 & 0 & 1 \\ 6 & 6 & 5 \end{pmatrix}$. *(Answer: eigenvalues:* $-2, -1, 2$*)*

Ejercicio 2 *Obtain a matrix A whose eigenvalues are* $\lambda_1 = 1$ *with algebraic multiplicity* $m_1 = 2$*, and* $\lambda_2 = 2$ *with* $m_2 = 1$*. The corresponding eigenvectors are* $\vec{v}_1 = (1,1,1)$ *and* $\vec{v}_2 = (1,2,1)$ *associated to* λ_1*, and* $\vec{v}_3 = (0,1,2)$ *associated to* λ_2*.*

Ejercicio 3 *Considering an endomorphism* T *in* $\mathbb{R}^4$ *such as:*

1. *An eigenvector associated to the eigenvalue* 1 *is* $(1,1,0,-1)$.
2. *An eigenvector associated to the eigenvalue* 2 *is el* $(0,1,0,0)$.
3. *The images of* $(0,1,1,1)$ *and* $(0,-1,1,2)$ *are respectively* $(0,4,-1,2)$ *and* $(0,0,-2,3)$.

Is T *diagonalizable? Verify the conditions of the Theorem.*

Ejercicio 4 *Defining* T *in* $\mathbb{R}_3[x]$ *such as* $T(ax^3+bx^2+cx+d) = dx^3+cx^2+bx+a$*. Diagonalize* T *if possible.*

Ejercicio 5 *Considering the endomorphism* $T : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ *with:*

- $\lambda = 2$ *is an eigenvalues of* T *and the associated eigenspace* $S_T(\lambda)$ *is the plane* $x+y=0$.
- $T(\vec{v}) = \vec{w}$ *being* $\vec{v} = (3,2,1)$ *and* $\vec{w} = (6,4,7)$.

Diagonalize T *if possible.*

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2. Orthogonal Diagonalization

Ejercicio 6 *Verify if the following matrices are definite positive. Are they diagonalizable in an orthonormal basis?*

a) $A = \begin{pmatrix} 4 & -4 & 0 \\ -4 & 4 & 0 \\ 0 & 0 & 4 \end{pmatrix}$

b) $A = \begin{pmatrix} 8 & -8 & 0 \\ -8 & 8 & 0 \\ 0 & 0 & 2 \end{pmatrix}$

Ejercicio 7 *Considering the following matrices A, are A^tA and AA^t diagonalizable? Find both orthogonal basis formed by eigenvectors of A^tA and AA^t, respectively.*

a) $A = \begin{pmatrix} 0 & 2 & 1 \\ -1 & -1 & 0 \end{pmatrix}$

b) $A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \\ -1 & 0 \end{pmatrix}$