

1. Scalar Product

Exercise 1 Consider the following scalar product in \mathbb{R}^2 :

$$(x_1,x_2)\cdot(y_1,y_2)=x_1y_1-x_2y_1-x_1y_2+4x_2y_2$$

Compute the Gram matrix and the associated quadratic form. Compute $(1,0)^2$ and $(-1,1)^2$.

Exercise 2 Compute the Gram matrix and the scalar product associated to the following quadratic form:

$$q(x_1, x_2) = 3x_1^2 - x_1x_2 + 2x_2^2$$

Compute $(1,2) \cdot (-1,1)$.

Exercise 3 a) Compute the Gram matrix respect of the standard basis of \mathbb{R}^3 , associated to the scalar product:

$$\vec{u} \cdot \vec{v} = x_1 y_1 + x_1 y_3 + 2x_2 y_2 - x_2 y_3 + x_3 y_1 - x_3 y_2 + 2x_3 y_3$$

b) Using the matrix form $(\vec{u} \cdot \vec{v} = X^t GY)$, compute $(1,2,3) \cdot (1,0,-1)$.

Exercise 4 a) Compute the Gram matrix respect of the standard base of \mathbb{R}^3 associated to the scalar product:

$$(x_1,x_2)\cdot(y_1,y_2) = x_1y_1 + x_1y_2 + x_2y_1 + 2x_2y_2 + 3x_3y_3 - x_1y_3 - x_3y_1$$

(taking into account that $g_{ij} = is$ the coefficient of $x_i y_j$ in the scalar product)

b) Compute the Gram matrix respect of the standard base of \mathbb{R}^3 associated to the quadratic form:

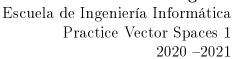
$$q(x_1, x_2, x_3) = 2x_1^2 + x_2^2 + x_3^2 - 2x_1x_2 + x_1x_3$$

(taking into account that the elements in the diagonal g_{ii} = are the coefficients of x_i^2 and the rest $g_{ij} = \frac{1}{2} coeff x_i x_j$, with $i \neq j$)

Exercise 5 Obtain the Gram matrix in each case, and decide which of them is actually a scalar product:

a)
$$\vec{u} \cdot \vec{v} = x_1 y_1 - x_2 y_2 - x_1 y_2 + 4x_2 y_2$$
, in \mathbb{R}^2

b)
$$\vec{v}^2 = 2x^2 + 2xy + 2y^2 - 2yz + 3z^2$$
 in \mathbb{R}^3 .





Exercise 6 We consider the following scalar product in $\mathbb{R}_2[X]$:

$$p \cdot q = \int_0^1 p(x) \ g(x) \ dx.$$

- a) Compute the Gram matrix referred to the standard basis $\mathbb{R}_2[X]$.
- b) Compute the angle between the polynomials $p_1 = -x + 2x^2$ y $p_2 = 1 + x + x^2$.

Exercise 7 Considering the scalar product in $\mathbb{R}^3 \times \mathbb{R}^3 \longrightarrow \mathbb{R}$

$$\vec{u} \cdot \vec{v} = x_1 y_1 + (a-1)x_1 y_2 + x_1 y_3 + 2ax_2 y_2 - x_2 y_3 + x_3 y_1 - x_3 y_2 + bx_3 y_3,$$

dónde a y b son parámetros reales y los vectores están expresados en la base canónica de \mathbb{R}^3 .

- a) Obtain the Gram matrix and compute the real values of a and b which give us a scalar product.
- b) Choosing a pair of values, compute the Gram matrix in $B = \{(1,1,1), (0,1,1), (0,0,1)\}.$
- c) Compute ||(1,0,1)||.