CALCULUS
DEGREE IN SOFTWARE ENGINEERING
WORKSHEET 1. FUNCTIONS, LIMITS, CONTINUITY

1. Find the domain of the following functions

   (a) $y = \sqrt{x^2 + 2x - 15}$
   (b) $y = \sqrt{(x-1)(x-2)(x-3)}$
   (c) $y = \ln(-x^2 + 2x - 4)$
   (d) $y = \dfrac{1}{\sqrt{|x| - x}}$
   (e) $y = \tan(x)$
   (f) $y = \csc(x)$
   (g) $y = \dfrac{x+5}{x+3}$
   (h) $y = \dfrac{1}{x^2 - 3}$
   (i) $y = \sin^{-1} \dfrac{x}{1+x}$
   (j) $y = \cos^{-1} \dfrac{2x}{5+x}$
   (k) $y = \tan^{-1} \dfrac{x}{6+x}$
   (l) $y = \sin^{-1} \dfrac{x}{2-|x|}$
   (m) $y = \dfrac{1}{\sqrt{x^2-4}} + \dfrac{1}{\sqrt{25-x^2}}$

2. Compose the following functions, obtaining $f \circ g$ and $g \circ f$

I)	$f(x) = x + 5$	$g(x) = x^2 - 3$
II)	$f(x) = ln(x)$	$g(x) = sinx$
III)	$f(x) = x^2 + 9$	$g(x) = \sqrt{x}$
IV)	$f(x) = x^2 + 7$	$g(x) = \dfrac{1}{x+5}$
V)	$f(x) = x^3 + x$	$g(x) = x^{1/3}$

3. Calculate the limits of these functions :

I) $\lim\limits_{x\to\infty} x^4 - x^3 + 1$ II) $\lim\limits_{x\to+\infty} \dfrac{e^x + cosx}{e^x - cosx}$

III) $\lim\limits_{x\to 0} \dfrac{\sin x + x}{x}$ IV) $\lim\limits_{x\to -1} \dfrac{x}{x+1}$

V) $\lim\limits_{x\to 1} \dfrac{x^3 - 1}{x^2 - x}$ VI) $\lim\limits_{x\to\infty} \sqrt{x+2} - \sqrt{x}$

VII) $\lim\limits_{x\to-\infty} \dfrac{5x+10}{2x}$ VIII) $\lim\limits_{x\to 2} \left(\dfrac{2x+1}{2x+3}\right)^{\frac{x^2-1}{(x-2)^2}}$

4. Prove that $f(x) = \sin x + 2x - 1$ has at least a real zero.

5. Prove that the graphs of the functions $h(x) = \ln x$ ans $g(x) = e^{-x}$ intersect at least at one point.

6. Given the equation $x^3 + \lambda x^2 - 2x - 1 = 0$. Prove that

   (a) if $\lambda > 2$ the equation has at least a solution less than 1.

   (b) If $\lambda < 2$ there is a solution of the equation that is greater than 1.

7. Has the equation $ax^5 + bx^3 + cx + d = 0$ a real solution ? Use Bolzano´s theorem.

8. Let $f(x)$ be

$$f(x) = \begin{cases} x^2|x+2| & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ x^2 \sin \frac{1}{x} & \text{if } x > 0 \end{cases}$$

   Study the continuity of $f(x)$

9. Calculate a and b such that $f(x)$ is a continuous function at $x = 0$ and $x = 1$

$$f(x) = \begin{cases} e^x + a & \text{if } x < 0 \\ ax^2 + 2 & \text{if } 0 \leq x \leq 1 \\ \dfrac{b}{2x} & \text{if } x > 1 \end{cases}$$

10. The function $f(x) = \dfrac{x}{1 + e^{\frac{1}{x+1}}}$ has a discontinuity at $x = -1$. Is the discontinuity removable? Analyze also the behaviour of the function at ∞ and $-\infty$