CALCULUS
DEGREE IN SOFTWARE ENGINEERING
WORKSHEET 6. FUNCTIONS OF SEVERAL VARIABLES. LIMITS AND CONTINUITY

1 Functions. Limits and continuity

1. Study the domains of the following functions:

   (a) $(1-x^2-y^2)^{\frac{1}{2}}$

   (b) $(x.y)^{\frac{1}{2}}$

   (c) $\dfrac{(x^2+y^2)}{(x^2-y^2)}$

   (d) $\dfrac{(e^x-e^y)}{e^x+e^y}$

2. Draw the level curves of the functions

   (a) x^2+y^2

   (b) x^2-y^2

3. Analyze the existence of the limit at the origin for the following functions and calculate it when it exists

   a) $f(x,\ y)=xy\sin(\dfrac{y}{x})$ b) $f(x,\ y)=\dfrac{(x^2-y^2)}{(x^2+y^2)}$ c) $f(x,\ y)=\dfrac{xy^2}{(x^2+y^4)}$

   d) $f(x,\ y)=\dfrac{xy}{(x^2+y^2)^{\frac{1}{2}}}$ e) $f(x,\ y)=e^y\dfrac{\sin x}{x}$ f) $f(x,\ y)=\dfrac{2xy^2}{(x^2+y^2)}$

4. Calculate the limits:

   (a) $\displaystyle\lim_{(x,\,y)\to(1,\,1)}\frac{x^2-y^2}{x-y}$

   (b) $\displaystyle\lim_{(x,\,y)\to(1,\,1)}\frac{(3-x-y)(x-1)(y-1)}{xy-x-y+1}$

5. Study the continuity of the functions:

(a) $$f(x,y) = \begin{cases} \dfrac{x^2y^2}{x^2y^2+(x-y)^2} & \text{if} \quad (x,y) \neq (0,0) \\ 1 & \text{if} \quad (x,y) = (0,0) \end{cases}$$

(b) $$f(x,y) = \begin{cases} (x^2+y^2)\sin\left(\dfrac{1}{x^2+y^2}\right)^{\frac{1}{2}} & \text{if} \quad (x,y) \neq (0,0) \\ 0 & \text{if} \quad (x,y) = (0,0) \end{cases}$$

(c) $$f(x,y) = \begin{cases} \dfrac{2x^5+4x^2y^3-2y^5}{(x^2+y^2)^2} & \text{if} \quad (x,y) \neq (0,0) \\ 0 & \text{if} \quad (x,y) = (0,0) \end{cases}$$

(d) $$f(x,y) = \begin{cases} \dfrac{x^5-x^3y^2+y^6}{(x^2+y^2)^2} & \text{if} \quad (x,y) \neq (0,0) \\ 0 & \text{if} \quad (x,y) = (0,0) \end{cases}$$

(e) $$f(x,y) = \begin{cases} \dfrac{x^4-y^4}{(x^2+y^2)^2} & \text{if} \quad (x,y) \neq (0,0) \\ 0 & \text{if} \quad (x,y) = (0,0) \end{cases}$$

6. We define $$f(x,y) = \begin{cases} \dfrac{xy^3}{(x^2+y^6)} & \text{if} \quad (x,y) \neq (0,0) \\ k & \text{if} \quad (x,y) = (0,0) \end{cases}$$

(a) Calculate the limit of $f(x,y)$ as (x,y) approaches $(0,0)$ along the curve $x = y^3$.

(b) Prove that, regardless of the value of k, f is not continuous at $(0,0)$.