Exercises from Unit 1

Exercise 1

Using 8 bits to represent signed numbers (2's Complement)

a) Represent in binary $-17_{(10)}$ Sol: 11101111

b) Represent in binary $-22_{(10)}$ Sol: 11101010

c) Represent in binary (-17) + (-22)   Sol: 11011001

d) Make 2's Complement to the point C   Sol: 39

Exercise 2

Using 1 byte to represent signed numbers:

a) Represent in binary $-74_{(10)}$ Sol: 10110110

b) Represent in binary +128   Sol: Out of Range [-128, +127]

Exercise 3

a) Using 1 byte to represent unsigned numbers, represent in binary $156_{(10)}$

      Sol: 10011100

b) Using 1 byte to represent signed numbers, represent in binary $156_{(10)}$

      Sol: Out of Range

c) Using 8 bits to represent unsigned numbers, represent in binary $-100_{(10)}$

      Sol: 10011100

Exercise 4

a) Using A, B, and C as variables, write an expression that represents "At least one of the three variables is greater than or equals to 0, but it does not happen that A is greater than B, and B is greater than or equals to C simultaneously."

       Sol: (A>=0 or B>=0 or C>=0) and not (A>B and B>=C)

b) Evaluate the expression given the following interpretations:

       Interpretation 1) A=5 ; B=7; C=2   Sol: True

       Interpretation 2) A=6 ; B=-2 ; C=-3   Sol: False

Exercise 5

a) Using A, B, and C as variables, write an expression that represents "A is greater than or equal to 1 and less than 14, or B belongs to the interval [0..2]".

       Sol: (a>=1 and a<14) or (b>=0 and b<2)

b) Negate the previous expression and simplify it up to the point that all the negations are eliminated.

       Sol: not ((a>=1 and a<14) or (b>=0 and b<2)) =

                    (not (a>=1 and a<14)) and (not (b>=0 and b<2)) =

                    (not (a>=1) or not (a<14)) and (not (b>=0) or not (b<2)) =

                    (a<1 or a>=14) and (b<0 or b>=2)

Exercise 6

a) Using A, and B as variables, write an expression that represents "Neither A nor B are less than zero and only one of the two is less than 10".

Sol: (a>=0 and b>=0) and ((a<10 and b>=10) or (a>=10 and b<10))

b) Negate the previous expression and simplify it up to the point that all the negations are eliminated.

Sol: not ((a>=0 and b>=0) and ((a<10 and b>=10) or (a>=10 and b<10)))=

(not (a>=0 and b>=0)) or (not ((a<10 and b>=10) or (a>=10 and b<10))) =

(not (a>=0)) or (not (b>=0)) or (not ((a<10 and b>=10) and (not (a>=10 and b<10))) =

(a<0 or b<0) or ((not ((a<10) or not (b>=10)) and ((not (a>=10) or not (b<10))) =

(a<0 or b<0) or ((a>=10 or b<10) and (a<10 or b>=10))

Exercise 7

a) Using A, and B as variables, write an expression that represents "One of the two variables is greater than the other, and the smaller of them is greater than zero".

Sol: (a>b and b>0) or (b>a and a>0)

b) Negate the previous expression and simplify it up to the point that all the negations are eliminated.

Sol: not ((a>b and b>0) or (b>a and a>0)) =

not (a>b and b>0) and not (b>a and a>0) =

(not(a>b) or not (b>0)) and (not (b>a) or not(a>0)) =

(a<=b or b<=0) and (b<=a or a<=0)

Exercise 8

a) Using A, and B as variables, write an expression that represents "One of the two variables is negative, but not both".

Sol 1: (a<0 or b<0) and not (a<0 and b<0)

Sol 2: (a<0 and b>=0) or (a>=0 and b<0)

(You can check that the two solutions above are equivalent, because they produce the same truth table)

b) Negate the previous expression and simplify it up to the point that all the negations are eliminated.

Sol 1:   not ((a<0 or b<0) and not (a<0 and b<0)) =

not (a<0 or b<0) or not (not (a<0 and b<0)) =

(not (a<0 or b<0)) or (a<0 and b<0) =

(a>=0 and b>=0) or (a<0 and b<0)

Sol 2:   not ((a<0 and b>=0) or (a>=0 and b<0)) =

not (a<0 and b>=0) and not (a>=0 and b<0)) =

(not a<0 or not b>=0) and (not a>=0 or not b<0) =

(a>=0 or b<0) and (a<0 or b>=0)

(You can check that the two solutions above are equivalent, because they produce the same truth table)

Exercise 9

a) Using A, B, and C as variables, write an expression that represents "Two and only two variables are equal to 0".

Sol: (a==0 and b==0 and c!=0) or (a==0 and b!=0 and c==0) or (a!=0 and b==0 and c==0)

b) Negate the previous expression and simplify it up to the point that all the negations are eliminated.

Sol:

not ((a==0 and b==0 and c!=0) or (a==0 and b!=0 and c==0) or (a!=0 and b==0 and c==0)) =

not (a==0 and b==0 and c!=0) and not (a==0 and b!=0 and c==0) and not (a!=0 and b==0 and c==0)) =

(not a==0 or not b==0 or not c!=0) and (not a==0 or not b!=0 or not c==0) and (not a!=0 or not b==0 or not c==0) =

(a!=0 or b!=0 or c==0) and (a!=0 or b==0 or c!=0) and (a==0 or b!=0 or c!=0)

Exercise 10

Write the truth table of the following expression: ((not (A and B) or not B) and C).

A	B	C	(partial expressions)	((not (A and B) or not B) and C)
0	0	0	…	0
0	0	1	…	1
0	1	0	…	0
0	1	1	…	1
1	0	0	…	0
1	0	1	…	1
1	1	0	…	0
1	1	1	…	0

Exercise 11

Simplify, using the laws of logic, as much as possible to reach DNF: not (p or not q) or ((r and s) or (r and not s))

Sol:	(not p and q) or ((r and s) or (r and not s)) =	De Morgan
	(not p and q) or (r and (s or not s)) =	Distributive
	(not p and q) or (r and True) =	Complements
	(not p and q) or r	T and F (DNF reached)

Exercise 12

Write the truth table of the following expression: A and ((C or B) and not B)

A	B	C	(partial expressions)	A and ((C or B) and not B)
0	0	0	…	0
0	0	1	…	0
0	1	0	…	0
0	1	1	…	0
1	0	0	…	0
1	0	1	…	1
1	1	0	…	0
1	1	1	…	0

Exercise 13

Negate the following expression and simplify it up to the point that all the negations are eliminated:
x!=y and (x!=z or z<0 or z>=5)

        Sol: not (x!=y and (x!=z or z<0 or z>=5)) =

                not x!=y or not ((x!=z or z<0 or z>=5)) =

                x==y or (not x!=z and not z<0 and not z>=5) =

                x==y or (x==z and z>=0 and z<5)

After simplifying the expression, if y=10, what are the possible values of x for the resulting expression to be true?

        Sol: x=[10, 0, 1, 2, 3, 4].

Exercise 14

Write the truth table of the following expression n: not Z or (Y or (X and Z))

A	B	C	(partial expressions)	A and ((C or B) and not B)
0	0	0	…	1
0	0	1	…	0
0	1	0	…	1
0	1	1	…	1
1	0	0	…	1
1	0	1	…	1
1	1	0	…	1
1	1	1	…	1

Exercise 15

Negate the following expression and simplify it up to the point that all the negations are eliminated:
(Z<A or X!=Z or Z>=B) and X!=Y

        Sol: not ((Z<A or X!=Z or Z >=B) and X!=Y) =

                    not ((Z<A or X!=Z or Z>=B)) or not X!=Y =

                    (Z>=A and X==Z and Z<B) or X==Y

After transforming the expression, if (Y=32, A=28, B=30), what are the possible values of X (put them in hexadecimal) for the transformed expression to be true?

        Sol: X=[28 ,29, 32] $_{(10)}$

                  X=[1C, 1D, 20] $_{(16)}$