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THEORY OF AUTOMATA AND DISCRETE MATHEMATICS

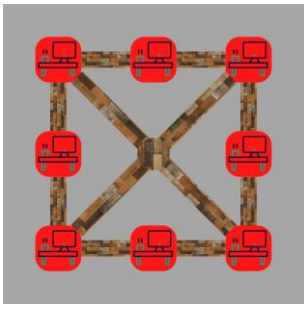
GRAPHS EXERCISE

2023-2024

Exercise 1:

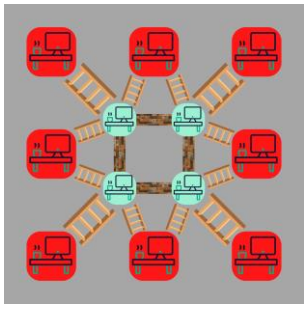
Imagine a **multi-level building** designed with interconnected rooms that form a pyramid, each facilitating communication and movement among various departments or teams within a company. The building has three levels:

The ground floor comprises a large square area with eight rooms (red colour) arranged in a square pattern. Each room is connected to the rooms near them, allowing easy access between adjacent rooms. Also to save time between very distant rooms, it has been decided to join the rooms that are on the diagonals (as shown in the image the diagonal paths collide in the middle).



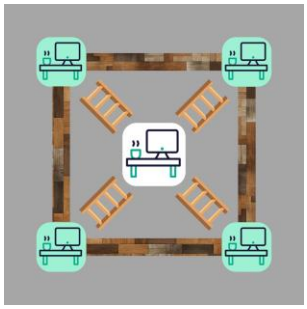
First Floor

The second level consists of four rooms (blue colour) arranged in a smaller square pyramid shape directly above the corner rooms of the ground floor. These rooms are designated for specialized or executive departments requiring direct communication with specific areas on the ground floor. (Shown in the image)



Second Floor + First Floor

At the very top of the building sits a single room, representing the executive office or central command center. This room is connected to the four rooms directly below it on the second.



Last Floor + Second Floor

<u>You have just been hired!</u>

Your only job is to scrub all the hallways and stairs in the building. But remember, you can't step on the mops because you would have wasted your time and would have to start all over again.

Analyze the building and show if it is possible a route that starts and ends in the room on the top floor (the boss room).

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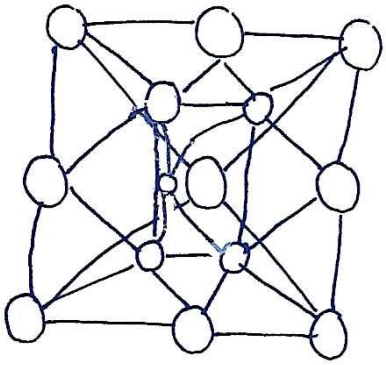
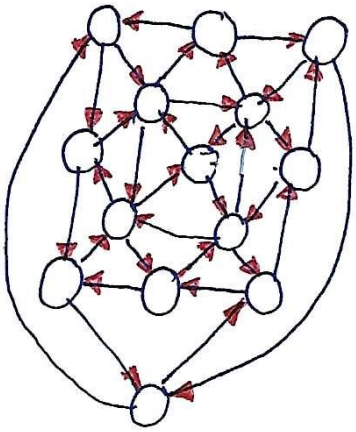
Exercise 1:

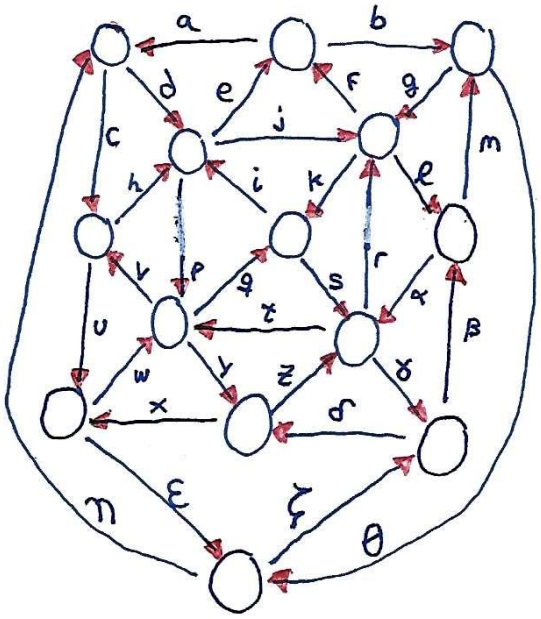


⇒ Solution



(In red, the chosen direction)

In order for don't passing twice through the same path, you must search for an Eulerian cycle. For this, we can start in the office in the last floor.



Loop1: ieadjfbgk

Loop 2: srlαγβmθζδzτq

Loop3: ηchpvuwyxε

1 Solution: ieadjfbgk srlαγβmθ ηchpvuwyxε ζδzτq

Loop1 | Loop 2 | Loop 3 | Loop2

There is a path where you doesn't waste time.

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Exercise 2:

This time we have been hired to buy the t-shirts of the **12 players** of a tournament in the game Axie Infinity Origins.

The rules you have to follow are as follows:

- Each team consists of six members: five form a row and the sixth is the captain who steps forward one step.
- Players facing each other in the row, as well as adjacent players in the row, cannot wear the same t-shirt shape.
- The captain must wear a different t-shirt shape than any other member of his team.
- In addition, captains of different teams may not wear the same t-shirt shape."

What is the minimal quantity of t-shirt shapes do you have to order for this unique tournament?

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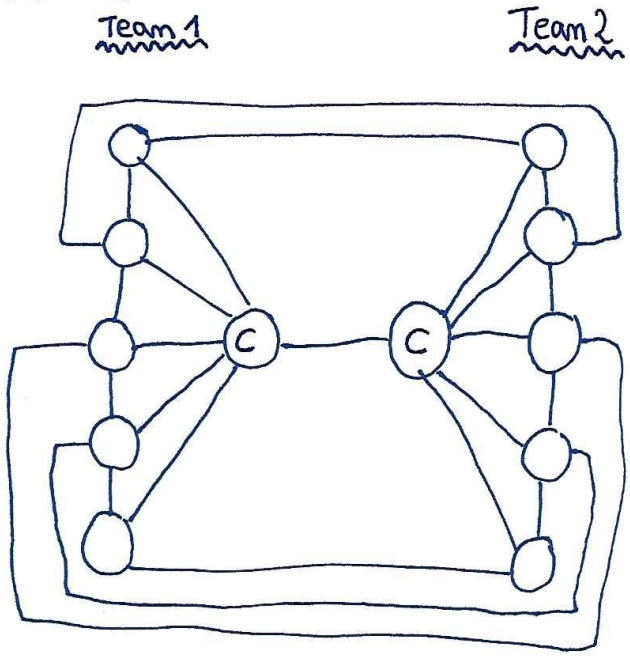
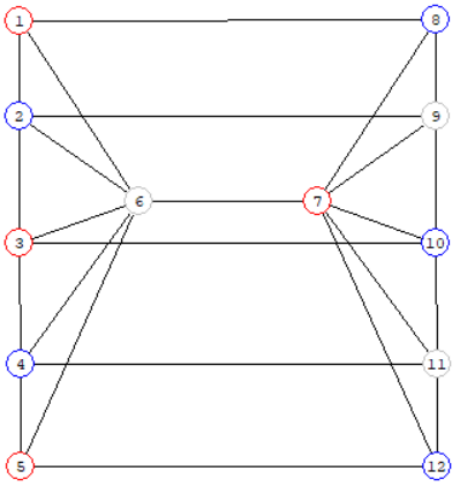
Exercise 2:



Each player is a vertex and each constraint is an edge.

A good idea to solve this exercise is to look for the minimal coloring of the graph, decreasing as much as possible the number of t-shirt shapes.

Already in advance, being a planar graph, we can say that the maximum number of shapes is 4. It only remains to see if it can be less.

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Chromatic Number = 3
Minimal Point Coloring is :
Point     1   2   3   4   5   6   7   8   9  10
Color     1   2   1   2   1   3   1   2   3   2

Point    11  12
Color     3   2
```

After using Grin, we can find that it is possible to buy only 3 shapes of t-shirts.