1. Orthogonality

Exercise 1 *Consider the following scalar product in* $\mathbb{R}^2$*:*

$$(x_1, x_2) \cdot (y_1, y_2) = x_1 y_1 - x_2 y_1 - x_1 y_2 + 4 x_2 y_2$$

Find out the orthogonal projection of the vector $(1,0)$ *onto* $(-1,1)$.

Exercise 2 *Compute the orthogonal projection of the vector* $\vec{v} = (1,2,3)$ *onto the plane*

$$\mathscr{H} = (x,y,z) \in \mathbb{R}^3 | x - y + z = 0$$

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Exercise 3 *Find out* $\mathscr{U}$*, the linear subspace of* $\mathbb{R}^3$*, orthogonal to the plane*

$$\mathscr{H} = (x,y,z) \in \mathbb{R}^3 | x + 2y - z = 0$$

Exercise 4 *Let* $\mathscr{U} = \{(x,y,z,t) \in \mathbb{R}^4 | x - z = 0, x + y + z = 0\}$. *Considering the standard scalar product in* $\mathbb{R}^4$*, compute the subspace* $\mathscr{H}$ *orthogonal to* $\mathscr{U}$.

Exercise 5 *Considering the following scalar product in* $\mathscr{C}^0[-1,1]$*:*

$$f \cdot g = \int_{-1}^{1} f(x) \cdot g(x) dx$$

a) In the linear subspace $\mathbb{R}_2[X]$*, compute the Gram matrix referred to the standard basis* $\mathscr{B} = \{1, x, x^2\}$.

b) Approximate the function $f(x) = e^x$ *by second degree polynomials, in the interval* $[-1,1]$.

c) Approximate the function $g(x) = \cos x$ *by second degree polynomials, in the interval* $[-1,1]$.

Exercise 6 ***Exercise 7*** *Using the Least Squares algorithm, find the first degree polynomial model* $y = ax + b$*, that best adjust the following data* $\{x_i, y_i\}_{i=1}^m$

x	3	4	5	6	7	8	9	10	11	12
y	4.5	5.5	5.7	6.6	7	7.7	8.5	8.7	9.5	9.7

Use the previous model to predict the quantity of y when utilizing $x = 30$ *(kilograms).*