

CALCULUS
DEGREE IN SOFTWARE ENGINEERING
WORKSHEET 8. LOCAL EXTREMA

1. Find and classify the local extrema of the following two-variable function

$$f(x, y) = x^4 + y^4 - 2x^2 + 4xy - 2y^2.$$

2. Study if $f(x, y) = xy \sin x$ has a local extremum at any of the following points:
 $P_1 = (\pi/2, 0)$, $P_2 = (\pi/4, \pi/2)$ or $P_3 = (0, \pi/2)$.

3. Study if

$$f(x, y, z) = x^2 + 2y^2 + 3z^2 - xyz.$$

has a local extremum at the points $(0, 0, 0)$ and $(1, 2, 3)$. Classify them.

4. Find and classify the local extrema of $f(x, y) = x^4 + y^4 + 6x^2y^2 + 8x^3$.

5. Calculate the local maxima and minima of the following function

$$f(x, y) = x^2y + 2xy - y^2 - 3y.$$

6. What are the local extreme values of $f(x, y) = 4xy - x^4 - y^4$. Classify them.

7. Find and classify the local extrema of $f(x, y, z) = x^2 + y^2 + z^2 + xy - x + y + z$.

8. Has the function $f(x, y) = x^2y + y^2x$ any maximum or minimum value ?

9. Find the minimum distance from the the surface $z = \frac{1}{xy}$ to the origin of coordinates.

10. Find and classify the local extrema of $f(x, y) = xy - x^2 - y^2 - 2x - 2y + 4$