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S4-Linear Applications matrices and linear systems

Linear Algebra

Ingeniería del Software-Universidad de Oviedo

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Review: From continuous to discrete least squares



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$$f(x) = \lg(x) \quad \text{in } [0, 1].$$

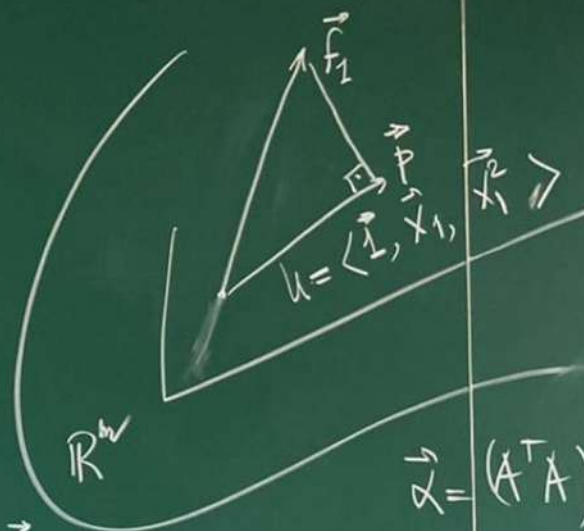
$$\vec{x}_1 = [0; 0.01; 1] \in \mathbb{R}^m \quad \text{Sampling in } x$$

$$\vec{f}_1 = f(x_1) \leadsto \text{Sampling of } f(x) \text{ in } x_1 \in \mathbb{R}^m$$

$$x_i \in \vec{x}_1 \quad f(x_i) \simeq a_0 + a_1 \cdot x_i + a_2 x_i^2$$

$$i = 1, \dots, m$$

$$\begin{pmatrix} 1 & x_1 & x_1^2 \\ 1 & x_2 & x_2^2 \\ \vdots & \vdots & \vdots \\ 1 & x_m & x_m^2 \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ a_2 \end{pmatrix} = \vec{f}_1$$



$$A \cdot \vec{x} = \vec{f}_1$$

↓ LS

$$A^T A \vec{x} = A^T \vec{f}_1$$

$$\vec{x} = (A^T A)^{-1} (A^T \vec{f}_1)$$

$$\vec{p} = A \cdot \vec{x}$$

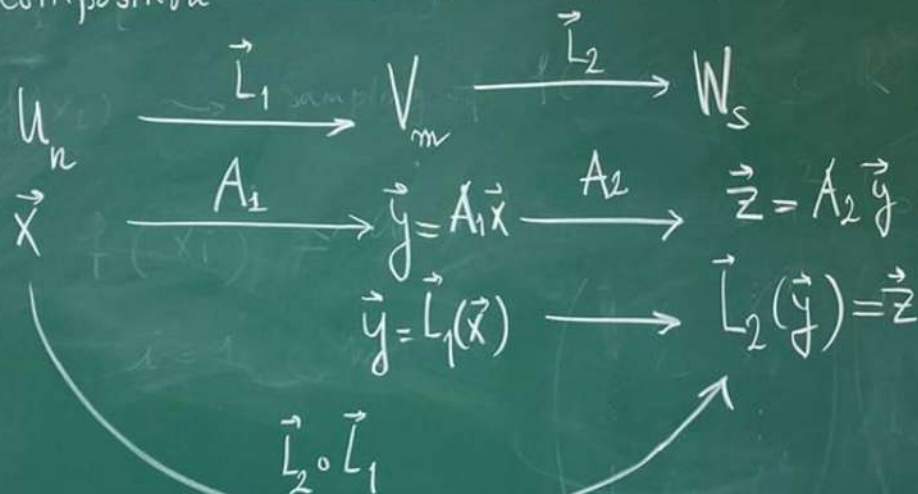
From continuous to discrete least squares



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Linear Applications - Operations

(2) Composition \longrightarrow Product of matrices



$$\vec{z} = \vec{L}_2(\vec{y}) = \vec{L}_2(\vec{L}_1(\vec{x})) =$$

$$\stackrel{\text{def}}{=} \vec{L}_2 \circ \vec{L}_1(\vec{x})$$

$$\vec{z} = A_2 \vec{y} = A_2 \cdot A_1 \vec{x}$$

$\vec{L}_2 \circ \vec{L}_1$ is represented by

$$\begin{array}{ccc}
 A_3 = A_2 \cdot A_1 & \rightsquigarrow & M(\mathbb{R}) \\
 \downarrow & & \downarrow \\
 (s \times m) \cdot (m, n) & & s \times n \\
 \downarrow & & \\
 s \times n & &
 \end{array}$$

Linear Applications



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Linear Applications - Operations

$$\vec{y} = A \cdot \vec{x} = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} =$$

$$\Rightarrow y_j = a_{j1}x_1 + a_{j2}x_2 + \dots + a_{jn}x_n$$

j -th coordinate of \vec{y}

$$\vec{z} = A_2 \cdot \vec{y} = \begin{pmatrix} b_{11} & b_{12} & \dots & b_{1m} \\ b_{21} & b_{22} & \dots & b_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ b_{s1} & b_{s2} & \dots & b_{sm} \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{pmatrix} =$$

\downarrow $s \times m$ $\xrightarrow{k\text{th}}$

$$z_k = b_{k1}y_1 + b_{k2}y_2 + \dots + b_{km}y_m =$$

$$= b_{k1} \cdot (a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n) +$$

$$+ b_{km} (a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n).$$

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Linear Applications - Operations

$$\vec{y} = A \cdot \vec{x} = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} =$$

$k=1, s$

$$\begin{aligned} z_k = & (b_{k1} a_{11} + b_{k2} a_{21} + \dots + b_{km} a_{m1}) \cdot x_1 + \\ & + (b_{k1} a_{12} + b_{k2} a_{22} + \dots + b_{km} a_{m2}) \cdot x_2 + \\ & + (b_{k1} a_{1n} + b_{k2} a_{2n} + \dots + b_{km} a_{mn}) x_n \end{aligned}$$

$$\vec{z} = A_2 \cdot \vec{y} = \begin{pmatrix} b_{11} & b_{12} & \dots & b_{1m} \\ b_{21} & b_{22} & \dots & b_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ b_{s1} & b_{s2} & \dots & b_{sm} \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{pmatrix} =$$

$s \times m$
matrix

$$\Rightarrow \vec{z}_k = \begin{pmatrix} B_{k1} \\ B_{k2} \\ \vdots \\ B_{kn} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} \rightarrow \begin{pmatrix} b_{k1} a_{11} + \dots + b_{km} a_{m1} \\ \vdots \\ b_{k1} a_{1n} + \dots + b_{km} a_{mn} \end{pmatrix}$$

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Linear Applications - Operations

Conclusion $\rightarrow \vec{L}_2 \circ \vec{L}_1$ is represented by $C = A_2 \cdot A_1$ and

$$\text{the term } C(i,j) = \vec{L}_2(i,:) \cdot \vec{L}_1(:,j)$$

\Downarrow
 i -th row vector of A_2

\Downarrow
 j -th column vector of A_1

Matrix Product

③ Transpose

$$\textcircled{B_1} \quad U_n \xrightarrow{\vec{L}_1} V_m \textcircled{B_2}$$

$A \rightsquigarrow \vec{L}_1 \text{ in } B_1, B_2$

Adjoint or transpose

$$V_m \xrightarrow{\vec{L}_1^T} U_n$$

will be represented by A^T in B_2, B_1 .

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Linear Applications - Operations

$$\boxed{C} \quad \mathbb{R}_{B_1}^2 \xrightarrow{\vec{L}_1} \mathbb{R}_{B_2}^3 \xrightarrow{\vec{L}_2} \mathbb{R}_{B_1}^2$$

$$\vec{L}_1(x, y) = (x+y, x-y, y)$$

$$\vec{L}_2(x, y, z) = (x+y+z, x-z)$$

$$A_1 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \\ 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & -1 \end{bmatrix}$$

$$C = A_2 \cdot A_1 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & -1 \\ 0 & 1 \end{bmatrix} =$$

$$= \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}$$

$\sim \vec{L}_2 \circ \vec{L}_1$ in B_1, B_1

$$\vec{L}_2(\vec{L}_1(x, y)) = \vec{L}_2(x+y, x-y, y) =$$

$$= (x+y+x-y+y, x+y-x) = (2x+y, x)$$

Proof

$$\vec{L}_2 \circ \vec{L}_1(x, y) =$$

$$= (x+y+x-y+y,$$

$$x+y-x) = (2x+y, x)$$

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Linear Applications - Operations

$$\boxed{L} \quad \mathbb{R}_{B_1}^2 \xrightarrow{\vec{L}_1} \mathbb{R}_{B_2}^3 \xrightarrow{\vec{L}_2} \mathbb{R}_{B_1}^2$$

$$\vec{L}_1(x, y) = (x+y, x-y, y)$$

$$\vec{L}_1^T : \mathbb{R}^3 \xrightarrow{A_1^T} \mathbb{R}^2$$

$$(x, y, z) \longrightarrow (x+y, x-y+z)$$

$$\vec{L}_1^T = A_1^T \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 1 \end{bmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$= \begin{pmatrix} x+y, x-y+z \end{pmatrix}$$

Finding $\vec{L}_1^T \rightsquigarrow \vec{L}_2^T$

$$\vec{L}_1^T : \mathbb{R}_{B_2}^3 \longrightarrow \mathbb{R}_{B_1}^2$$

$$(x, y, z) \longrightarrow \vec{L}_1^T(x, y, z) =$$