Seminario 1 (Josu Pérez Zorroenandia)

2 (L.E.: 1)

Let $b \geq 2$, $b \in \mathbb{Z}$. Taking a base b numeral system, $n \in \mathbb{N}$, n is uniquely expressed by digits in $\{0, \dots, b-1\}$.

We can express n on a basis b such as:

$n_b = d_{k-1} d_{k-2} \dots d_0$ so that $n = d_0 \cdot b^0 + \dots + d_{k-1} b^k$

Note that:

$$b^k \leq n_b < b^{k+1} \Rightarrow \log_b(b^k) \leq \log_b(n_b) < \log_b(b^{k+1}) \Rightarrow$$

$$\Rightarrow k \leq \log_b(n_b) < k+1$$

If $n_b = b^k$, we need $k+1$ digits and thus:

$$\boxed{d_b(n) = \lfloor \log_b(n_b) \rfloor + 1}$$

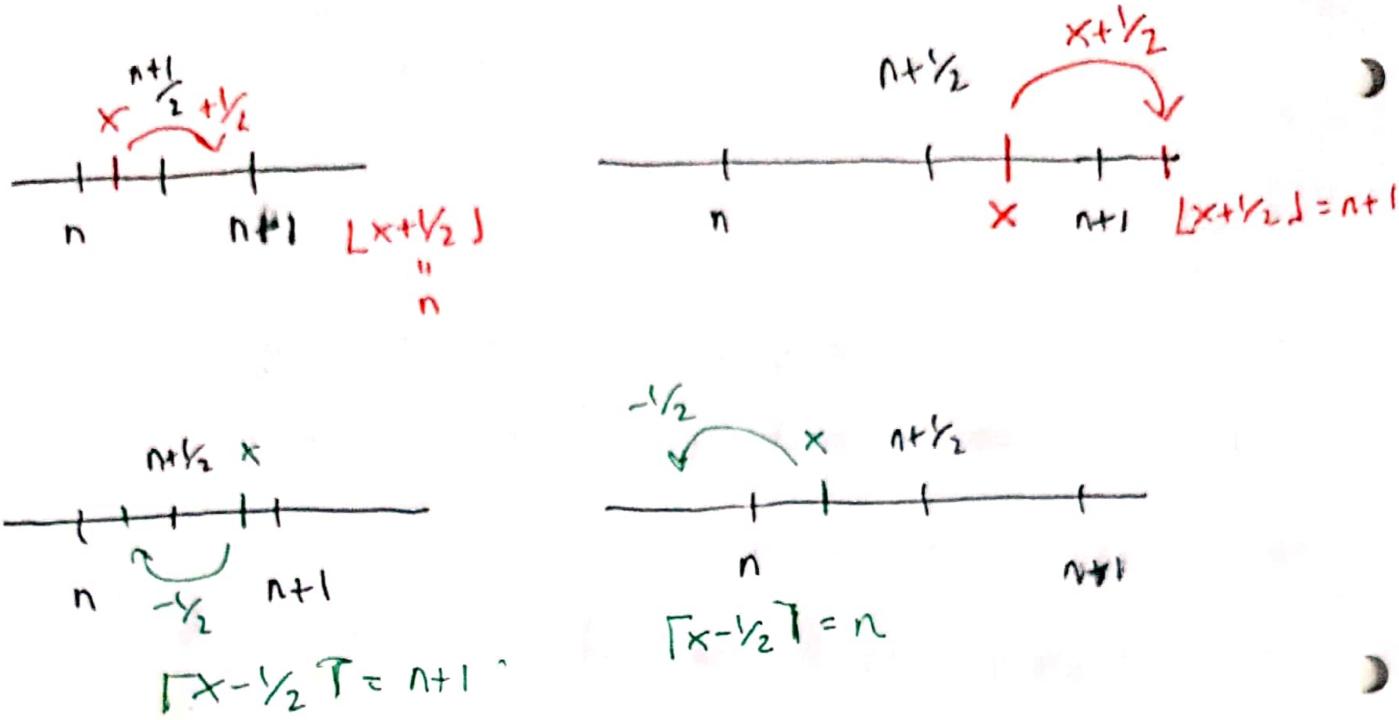
11 (L.E.: 1)

We consider two cases:

If $\{x\} < 1/2 \Rightarrow \varepsilon(x) = \lfloor x \rfloor \Rightarrow \varepsilon(x) = \lfloor x + 1/2 \rfloor$

If $\{x\} > 1/2 \Rightarrow \varepsilon(x) = \lceil x \rceil \Rightarrow \varepsilon(x) = \lfloor x + 1/2 \rfloor$

$$\Longrightarrow \varepsilon(x) = \lfloor x + 1/2 \rfloor$$



$\boxed{10}$ (L.E.: 3)

We define:

$\Omega = \{a_1, \dots, a_m, b_1, \dots, b_n\}$

$A = \{a_1, \dots, a_m\}$

$B = \{b_1, \dots, b_n\}$

The answer is to take every possible pair of subsets A', B' s.t. $A', B' \neq \emptyset$ and $A' \subset A$ and $B' \subset B$ and join them. How many possible nonempty subsets do we have?

$|\mathcal{P}(A) \setminus \emptyset| = 2^m - 1$

$|\mathcal{P}(B) \setminus \emptyset| = 2^n - 1$

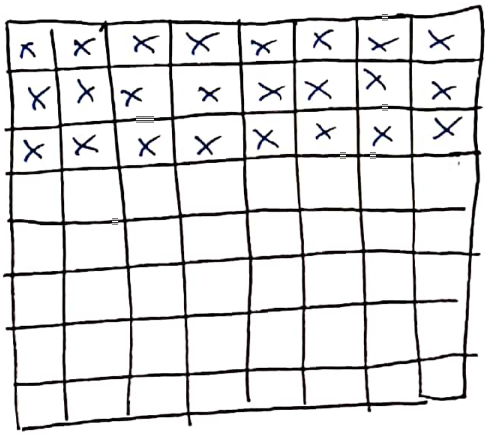
By the product rule: $\boxed{\text{Total} = (2^m - 1)(2^n - 1)}$

$\boxed{6}$ (L.E.: 3)

Since A, B, C have to be together and in that particular order, this problem is equivalent to arrange $n-3+1=n-2$ people. Hence:

$$\boxed{P(n-2) = (n-2)!}$$

$\boxed{9}$ (L.E. 4)

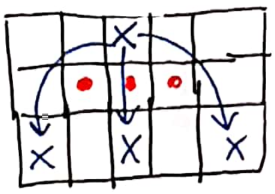


Rules:

A piece can jump over another to an empty square:

* HORIZONTALLY
* VERTICALLY
* DIAGONALLY

Note that all pieces will preserve their original row's parity. That means, whichever it is the move a piece performs, the destination-row's parity is always the same as the original row's parity.



Since the first three numbers of our board are odd, even, odd (1, 2, 3) the last three should be too, but they are even, odd, even (6, 7, 8). Therefore there is no possible solution.