Problems 1 (my attempts)

1

Convert 14432004_5 to radix 8, 4 and 16 (and 10)

- To base 10:

$$14432004_5 = 4\cdot5^0 + 2\cdot5^3 + 3\cdot5^4 + 4\cdot5^5 + 4\cdot5^6 + 1\cdot5^7 =$$

$$= 4 + 5^3\cdot(2+5\cdot(3+5\cdot(4+5\cdot(4+5\cdot(1))))) =$$

(1) $$=((((5\cdot1+4)\cdot5+4)\cdot5+3)\cdot5+2)\cdot5^3+4 =$$

$$= 155254_{10}$$

- To base 4 (method 1)

$(4\cdot2=8=5+3=13_5)$ $(4\cdot4=16=15+1=31_5)$
$(4\cdot3=12=10+2=22_5)$

$14432004_5 \div 4_5 = 2220223$
14
13
02
20
20
24
2

$2220223 \div 4_5 = 302303$
02
20
22
02
23
1

$302303 \div 4_5 = 34200$
32
13
00
03
3

$34200 \div 4_5 = 4411$
32
10
10
1

$4411 \div 4_5 = 1101$
04
01
11
2

$1101 \div 4_5 = 122$
20
21
3

$122 \div 4_5 = 14$
32
1

$14 \div 4_5 = 2$
1

$2 \div 4_5 = 0$
2

//

$$\Rightarrow \boxed{14432004_5 = 211321312_4}$$

To binary: $4 = 2^2 \rightarrow$ groups of two ($3 = 11$)

$$\underbrace{10}_{2}\underbrace{01}_{1}\underbrace{01}_{1}\underbrace{11}_{3}\underbrace{10}_{2}\underbrace{01}_{1}\underbrace{11}_{3}\underbrace{01}_{1}\underbrace{10}_{2}{}_2 = 211321312_4 = 14432004_5$$

To radix 8 $\rightarrow$ groups of 3: ($2^3 = 8$, and $100 = 4$)

$$\underbrace{100}_{4}\underbrace{101}_{5}\underbrace{111}_{7}\underbrace{001}_{1}\underbrace{110}_{6}\underbrace{110}_{6}{}_2 = \boxed{457166_8} = 14432004_5$$

To radix 16 $\rightarrow$ groups of 4: ($2^4 = 16$, $1000 = 8$)

$$\underbrace{0010}_{2}\underbrace{0101}_{5}\underbrace{1110}_{\substack{14 \\ \shortparallel \\ E}}\underbrace{0111}_{7}\underbrace{0110}_{6}{}_2 = 0x25E76 = \boxed{25E76_H}$$

($10 = A$, $B = 11$, $C = 12$, $D = 13$, $E = 14$, $F = 15$)

2

Convert 2448175_9 to hexadecimal (division), to base 7 (product) to binary, base 4 and octal, base 6, base 10

$$\boxed{2448175_9} = 5 \cdot 9^0 + 7 \cdot 9^1 + 1 \cdot 9^2 + 8 \cdot 9^3 + 4 \cdot 9^4 + 4 \cdot 9^5 + 2 \cdot 9^6 =$$

$$= 5 + 9 \cdot (7 + 9 \cdot (1 + 9 \cdot (8 + 9 \cdot (4 + 9 \cdot (4 + 9 \cdot (2)))))) =$$

$$= (((((9 \cdot 2 + 4) \cdot 9 + 4) \cdot 9 + 8) \cdot 9 + 1) \cdot 9 + 7) \cdot 9 + 5 =$$

$$= \boxed{1331303_{10}}$$

To base 6 we multiply:

$$(((((9\cdot 2+4)\cdot 9+4)\cdot 9+8)\cdot 9+1)\cdot 9+7)\cdot 9+5$$

$$9\cdot 2+4 = 34_6$$

$$34_6 \cdot 9+4 = 534_6$$

$$534_6 \cdot 9+8 = 12242_6$$

$$12242_6 \cdot 9+1 = 204031_6$$

$$204031_6 \cdot 9+7 = 3100454_6$$

$$3100454_6 \cdot 9+5 = 44311235_6$$

$9 = 13_6$ | $7 = 11_6$
$12 = 20_6$ | $8 = 12_6$
$11 = 15_6$
$15 = 23_6$

(1)

$$\begin{array}{r} 2 \\ \cdot\ 13 \\ \hline 10 \\ +\ 2 \\ \hline 30 \end{array}$$

(2)

$$\begin{array}{r} 34 \\ \cdot\ 13 \\ \hline 150 \\ +\ 34 \\ \hline 530 \end{array}$$

(3)

$$\begin{array}{r} 13 \\ \cdot\ 534 \\ \hline 100 \\ +\ 43 \\ 11\ 3 \\ \hline 12230 \end{array}$$

(4)

$$\begin{array}{r} 13 \\ \cdot\ 12242 \\ \hline 30 \\ +\ 100 \\ 30 \\ 30 \\ 13 \\ \hline 204030 \end{array}$$

(5)

$$\begin{array}{r} 13 \\ \cdot\ 204031 \\ \hline 13 \\ +\ 43 \\ 00 \\ 100 \\ 00 \\ 30 \\ \hline 3100443 \end{array}$$

$$\Rightarrow \boxed{2448175_9 = 44311235_6}$$

(6)

$$\begin{array}{r} 13 \\ \cdot\ 3100454 \\ \hline 100 \\ 113 \\ 100 \\ +\ 00 \\ 00 \\ 13 \\ 43 \\ \hline 44311230 \end{array}$$

To base 7 (multiplying).

$$(((((9\cdot 2+4)\cdot 9+4)\cdot 9+8)\cdot 9+1)\cdot 9+7)\cdot 9+5$$

31_7

406_7

5216_7

65626_7

1255165_7

14213231_7

$9 = 12_7$
$12 = 15_7$
$10 = 13_7$
$7 = 10_7$

$$\Rightarrow \boxed{2448175_9 = 14213231_7}$$

①

$$\begin{array}{r} 12 \\ \cdot\ 2 \\ \hline 04 \\ +\ 2 \\ \hline 24 \end{array} \qquad \begin{array}{r} 24 \\ +\ 4 \\ \hline 31 \end{array}$$

②

$$\begin{array}{r} 12 \\ 31 \\ \hline 12 \\ +\ 36 \\ \hline 402 \end{array} \qquad \begin{array}{r} 402 \\ +\ 4 \\ \hline 406 \end{array}$$

③

$$\begin{array}{r} \cdot\ 12 \\ 406 \\ \hline 105 \\ +\ 00 \\ 51 \\ \hline 5205 \\ +\ 11 \\ \hline 5216 \end{array}$$

④

$$\begin{array}{r} 12 \\ 5216 \\ \hline 105 \\ +\ 12 \\ 24 \\ 63 \\ \hline 65625 \\ +\ 1 \\ \hline 65626 \end{array}$$

⑤

$$\begin{array}{r} \cdot\ 12 \\ 65626 \\ \hline 105 \\ 24 \\ 105 \\ 63 \\ 105 \\ \hline 1154145_7 \\ +\ 10_7 \\ \hline 1154155 \end{array}$$

⑥

$$\begin{array}{r} \cdot\ 12 \\ 1154155 \\ \hline 63 \\ 63 \\ 12 \\ 51 \\ 63 \\ 12 \\ 12 \\ \hline 14213223 \\ +\ 5_7 \\ = 14213231_7 \end{array}$$

To base 16 (dividing)

$(16 = 9 + 7 \to 16 = 17_9)$

$(5 \cdot 7 = 35 = 36 - 1 = 40 - 1 = 38_9)$

$(3 \cdot 7 = 21 = 18 + 3 = 20 + 3 = 23_9)$

$(4 \cdot 7 = 28 = 31_9)$

$(6 \cdot 7 = 42 = 50_9 - 3 = 46_9) = (36 + 6 = 40 + 6_9)$

$(7 \cdot 7 = 49 = 50_9 + 4 = 54_9)$

$(7 \cdot 8 = 56 = 54_9 + 2 = 62_9)$

$(2 \cdot 7 = 14 = 9 + 5 = 15_9)$

(Pienso: $2 \cdot 7 = 15_{16} \to 5$ al 0_9 hay 4, + los que falten, ...)

$2448175_9 \; | \; 17_9$

064 → 136121

118

021

-17

037

25

7

$136121_9 \; | \; 17_9$

021 → 7117

032

141

06

$7117 \; | \; 17_9$

01 → 401

17

0

$401 \; | \; 17_9$

041 → 22

05

$22 \; | \; 17_9$

04 → 1

$1 \; | \; 17$

1 → 0

$\Rightarrow \boxed{2448175_9 = 145067_{16}} = 145067_H = 0 \times 145067$

To binary from 16 $\to 16 = 2^4 \to$ groups of 4 $(1111 = 15)$

$$\underbrace{1}_{0001}\;\underbrace{4}_{0100}\;\underbrace{5}_{0101}\;\underbrace{0}_{0000}\;\underbrace{6}_{0110}\;\underbrace{7}_{0111}{}_{16} = \boxed{101000101000001100111_2}$$

To base four, $4 = 2^2 \to$ groups of 2 $(11 = 3)$

$$\underbrace{1}_{1}\underbrace{01}_{1}\underbrace{00}_{0}\underbrace{01}_{1}\underbrace{01}_{1}\underbrace{00}_{0}\underbrace{00}_{0}\underbrace{01}_{1}\underbrace{10}_{2}\underbrace{01}_{1}\underbrace{11}_{3} = \boxed{11011001213_4}$$

To base 8 from binary ($8=2^3 \to$ groups of 3).

$$\underbrace{101}_{5}\underbrace{000}_{0}\underbrace{101}_{5}\underbrace{000}_{0}\underbrace{001}_{1}\underbrace{100}_{4}\underbrace{111}_{7} = \boxed{5050147_8}$$

3

0.14433021_5 to bases: 10, (normal method + dividing), 16 (multiplying), bases 2, 4 and 8.

$\boxed{0.14433021_5} =$

$= 1\cdot5^{-1}+4\cdot5^{-2}+4\cdot5^{-3}+3\cdot5^{-4}+3\cdot5^{-5}+\cancel{0\cdot5^{-6}}+2\cdot5^{-7}+1\cdot5^{-8} = (1)$

$= \boxed{0'39778816_{10}}$

(1)
$= 5^{-1}\cdot(1+5^{-1}(4+5^{-1}\cdot(4+5^{-1}(3+5^{-1}(3+5^{-1}(0+5^{-1}(2+5^{-1}\cdot(1))))))))$

$=((((((1\cdot5^{-1}+2)5^{-2}+3)5^{-1}+3)5^{-1}+4)5^{-1}+4)5^{-1}+1)5^{-1}$

$2'2$

$3'088$

$3'6176$

$4'72352$

① 1 | 5; 10 → 0'2; 0

② 2'2 | 25; 220 → 0'088; 200; 00

③ 3'088 | 5; 30 → 0'6176; 08; 38; 30; 0

④ 3'6176 | 5; 36 → 0'72352; 11; 17; 26; 10; 0

⑤

$4'72352 \;\lfloor 5$

$0'944704$

47
22
23
35
02
20
0

$0'944704 + 4 = 4'944704$

⑥

$4'944704 \;\lfloor 5$

$0'9889408$

49
44
44
47
20
04
40
0

$0'9889408 + 1 = 1'9889408$

⑦

$1'9889408 \;\lfloor 5$

$0'39778816$

19
48
38
39
44
40
08
30
0

$$\Rightarrow \boxed{0.14433021_5 = 0'39778816_{10}}$$

To base 16 (multiplying): $(16_{10} = 31_5)$

$$u_{-1} = \lfloor 0.14433021 \cdot 31_5 \rfloor = 11_5 = 6_{10} = 6_{16}$$

①

$$\begin{array}{r} 0.14433021 \\ \cdot \quad 31 \\ \hline + \; 14433021 \\ 104404113 \\ \hline 11.14024201 \end{array}$$

②

$$\begin{array}{r} 0.14024201 \\ \cdot \quad 31_5 \\ \hline + \; 14024201 \\ 102133103 \\ \hline 10.40410231 \end{array}$$

8

$$u_{-2} = \lfloor 0.14024201 \cdot 31_5 \rfloor = 10_5 = 5_{16}$$

$u_{-3} = \lfloor 0.40410231 \cdot 31_5 \rfloor = 23_5 = 10+3_{10} = D$

$$\begin{array}{r} 0.40410231 \\ \cdot \quad 31 \\ \hline + 40410231 \\ 222231243 \\ \hline 23.13223211 \end{array} \qquad \begin{array}{r} 0.13223211 \\ \cdot \quad 31 \\ \hline + 13223211 \\ 100230133 \\ \hline 10.21030041 \end{array}$$

$10 = A_{16}$
$11 = B$
$12 = C$
$13 = D$
$14 = E$
$15 = F$

$u_{-4} = \lfloor 0.13223211 \cdot 31_5 \rfloor = 10_5 = 5_{16}$

$u_{-5} = \lfloor 0.21030041 \cdot 31_5 \rfloor = 12_5 = 7_{16}$

$$\begin{array}{r} 0.21030041 \\ \cdot \quad 31 \\ \hline + 21030041 \\ 113140223 \\ \hline 12.02432321 \end{array}$$

$u_{-6} = \lfloor 0.02432321 \cdot 31_5 \rfloor = \dots$

$$\boxed{0.14433021_5 \simeq 0.65D57_{16}}$$

(Seguir o hasta que pidan, o hasta que haya repetición o hasta que se acabe el número).

Comprobación:

$6 \cdot 16^{-1} + 5 \cdot 16^{-2} + 13 \cdot 16^{-3} + 5 \cdot 16^{-4} + 7 \cdot 16^{-5} = 0'3977880478$

$0.14433021_5 = 0'39778816_{10}$ (iba bien)

(Habíamos calculado bien 6 cifras).

Now we convert to base 2 (from base 16).

Note: Important!

$0.65D57_{16}$ ($16 = 2^4 \to$ groups of four.)

If it was an integer, we would make the groups from the point towards the left. (and add needed zeros to the left.

$\overbrace{00d_1d_2}\ \overbrace{\dots}\ d_k\,.$

But since it is a floating number, we do it from the point towards the right.

$.\overbrace{d_1d_2d_3d_4}\,\overbrace{d_5 \dots}\ \dots\ \overbrace{d_kd_{k+1}00}$

Thus:

$$\boxed{0.\underbrace{0110}_{6}\ \underbrace{0101}_{5}\ \underbrace{1101}_{13=D_{16}}\ \underbrace{0101}_{5}\ \underbrace{0111}_{7}\ {}_2}$$

(Checked with calc: $= 0.3977880478_{10}$)

We make groups of 3 to convert to base 8 ($= 2^3$).

$$0.\underbrace{011}_{3}\,\underbrace{001}_{1}\,\underbrace{011}_{3}\,\underbrace{101}_{5}\,\underbrace{010}_{2}\,\underbrace{101}_{5}\,\underbrace{11\ }_{6}\,{}_2 =$$

$$= \boxed{0.3135256_8}$$

(Checked with calc: $= 8^{-1}\cdot 3 + \dots + 6\cdot 8^{-7} = 0.3977880478_{10}$)

And to base 4: ($2^2 \to$ groups of 2)

$$\vec{0}.\underbrace{01}_{1}\underbrace{10}_{2}\underbrace{01}_{1}\underbrace{01}_{1}\underbrace{11}_{3}\underbrace{01}_{1}\underbrace{01}_{1}\underbrace{01}_{1}\underbrace{01}_{1}\underbrace{11}_{3}\,{}_2 =$$

$$= \boxed{0.1211311113_4}$$

(Checked with calculator:
$= 1\cdot 4^{-1} + \dots + 3\cdot 4^{-10} =$
$= 0.3977880478$)

4 Convert the following number to base 6 (general method).

$$12221.12\overline{10}_3 = x_6$$

$$10000_3 \cdot x_3 = 122211210.\overline{10}_3$$

$$\ominus\quad 100_3 \cdot x_3 = 1222112.\overline{10}_3$$

$$x_3(10000_3 - 100_3) = (122211210 - 1222112)_3 \qquad (*)$$

(1)

$$\begin{array}{r} 10000_3 \\ -\ 100_3 \\ \hline 2200_3 \end{array}$$

$$\begin{array}{r|l} 2200_3 & 20_3 = 6 \\ -20 & \\ \hline 20 & \\ -20 & \\ \hline 00 & \\ \underset{\sim}{0} & \end{array} \qquad \begin{array}{r|l} 110 & 20_3 = 6 \\ \underset{\sim}{0} & \end{array} \qquad \begin{array}{r|l} 2 & 20_3 = 6 \\ \underset{\sim}{2} & \end{array} \qquad \begin{array}{r|l} 0 & 20_3 \end{array}$$

$20_3 \cdot 2 = 110_3$

$20_3 \cdot 3 = 200_3$

$\Rightarrow 2200_3 = 200_6$

(2)

$$\begin{array}{r} 122211210 \\ -\ 1222112 \\ \hline 120212021_3 \end{array}$$

$$\begin{array}{r|l} 120212021 & 20_3 \\ -110 & 2122101 \\ \hline 102 & \\ -20 & \\ \hline 121 & \\ -110 & \\ \hline 112 & \\ -110 & \\ \hline 20 & \\ -20 & \\ \hline 02 & \\ -00 & \\ \hline 21 - 20 = \underset{\sim}{1} & \end{array}$$

$$\begin{array}{l|l} 2122101 & 20_3 \\ \hline -20 & 102220 \\ \quad 12 & \\ \quad -0 & \\ \quad 122 & \\ \quad -110 & \\ \quad\; 121 & \\ \quad\; -110 & \\ \qquad 110 & \\ \qquad -110 & \\ \qquad\quad 01 & \\ \qquad\quad -0 & \\ \qquad\quad \underbrace{1} & \end{array}
\qquad
\begin{array}{l|l} 102220 & 20_3 \\ \hline -20 & 1222 \\ \; 122 & \\ \; -110 & \\ \quad 122 & \\ \quad -110 & \\ \qquad 120 & \\ \qquad -110 & \\ \qquad \underbrace{10} & \\ \qquad = 3_6 & \end{array}
\qquad
\begin{array}{l|l} 1222 & 20_3 \\ \hline -110 & 22 \\ \; 122 & \\ \; -110 & \\ \quad \underbrace{12} & \\ \quad = 3+2 = 5_6 & \end{array}
\qquad
\begin{array}{l|l} 22 & 20_3 \\ \hline -20 & 1 \\ \underbrace{2} & \end{array}
\qquad
\begin{array}{l|l} 1 & 20_3 \\ \hline \underbrace{1} & 0 \end{array}
\qquad
\begin{array}{l|l} 0 & 20_3 \end{array}$$

$$\Rightarrow 120212021_3 = 125311_6 \quad (= 11563_{10})$$

So our equation is:

$$(\circledast) \to \quad X_6 \cdot 200_6 = 125311_6 \Rightarrow X_6 = \frac{125311_6}{200_6}$$

$$\left(= \frac{11563}{72} = 160.597\overline{2} \right)$$

$200 \cdot 1 = 200_6$	$200 \cdot 4 = 1200_6$
$200 \cdot 2 = 200 + 200 = 400_6$	$200 \cdot 5 = 1400_6$
$200 \cdot 3 = 1000_6$	

$$\begin{array}{l|l} 125311 & 200 \\ \hline -1200 & 424.333 \\ \quad 531 & \\ \quad -400 & \\ \quad\; 1311 & \\ \quad\; -1200 & \\ \qquad 1110 & \\ \qquad -1000 & \\ \qquad\; 1100 & \\ \qquad\; -1000 & \\ \qquad\quad 1000 & \\ \qquad\quad -1000 & \\ \qquad\qquad \underbrace{0} & \end{array}$$

$$\Rightarrow \boxed{\begin{array}{l} 12221.12\overline{10}_3 = \\ = 424.333_6 \end{array}}$$

(Checked with calculator that both are equal to: $160.597\overline{2}_6$ //)

Let's conver it to base 10 using it's particular method:

$$\boxed{424.\overline{3}_6} = 4 + 2\cdot 6^1 + 4\cdot 6^2 + 3\cdot 6^{-1} + 3\cdot 6^{-2} + 3\cdot 6^{-3} = \frac{11563}{72} =$$

$$= \boxed{160.597\overline{2}_{10}} //$$

And also: (method of infinite sum).

$12221.121\overline{0}_3 \longrightarrow$ First the part that is not periodic:

$$12221.12_3 = 1\cdot 3^0 + 2\cdot 3^1 + 2\cdot 3^2 + 2\cdot 3^3 + 3^4 + 3^{-1} + 2\cdot 3^{-2} =$$

$$= 1445/9 = 160'\overline{5}$$

$$0.00\overline{10}_3 = 3^{-3} + 3^{-5} + 3^{-7} + 3^{-9} + 3^{-11} + \dots =$$

$$= 3^{-3}\cdot\left(1 + 3^{-2} + 3^{-4} + 3^{-6} + 3^{-8} + \dots\right) =$$

$$= 3^{-3}\cdot \sum_{n=0}^{\infty} 3^{-2n} = 3^{-3}\cdot \sum_{n=0}^{\infty} (3^{-2})^n = 3^{-3}\cdot \frac{1}{1-3^{-2}} =$$

$$= \frac{1}{24} = 0.041\overline{6}$$

$$\Rightarrow \frac{1445}{9} + \frac{1}{24} = \frac{11563}{72} = 160.597\overline{2}_{10}$$

$$\Rightarrow \boxed{12221.121\overline{0}_3 = 160.597\overline{2}_{10}}$$

5 Suppose a 20 bit machine that saves 6 bits for the exponent. Convert the first number to base 10 and the second to hexadecimal coded IEEE 754. ($c' = 31 = \frac{2^6}{2} - 1$)

10010110011011011111 (i)

(ii) -21344_{10}

(i)

$$\underset{9}{1|001}\ \underset{6}{011|0}\ \underset{6}{011|0}\ \underset{D}{1101}\ \underset{F}{1111} \rightarrow \boxed{0\times 966DF = 966DF_{16}}$$

$$\begin{cases} s = 1 \\ c = 001011 = 1+2+8 = 11 \\ f = 0011011011111 = 2^{-3} + 2^{-4} + 2^{-6} + 2^{-7} + 2^{-9} + 2^{-10} + 2^{-11} + 2^{-12} + \\ \quad + 2^{-13} = 0.2147216797 \end{cases}$$

$$\rightarrow \boxed{n = (-1)^1 \cdot 2^{11-31} \cdot (1 + 0.2147216797) = \\ = -1.1584488868 \cdot 10^{-6}}$$

(ii)

-21344_{10}

((*) Note: $21344 = (1+f) \cdot 2^{14}$)

$(-) \Rightarrow s = 1$

$\frac{21344}{2^{14}} = 1 + f \Rightarrow c - c' = 14 \Rightarrow c = 14 + 31 = 45 =$

$= \underline{1}\ \underline{0}\ \underline{1}\ \underline{1}\ \underline{0}\ \underline{1}$ //

(*)

$f = 1 + 0.32734375$ (max of 13 bits).

$u_{-1} = \lfloor 2 \cdot 0.302734375 \rfloor = \lfloor 0.60546875 \rfloor = 0$

$u_{-2} = \lfloor 2 \cdot 0.60546875 \rfloor = \lfloor 1.2109375 \rfloor = 1$

$u_{-3} = \lfloor 2 \cdot 0.2109375 \rfloor = \lfloor 0.421875 \rfloor = 0$

$u_{-4} = \lfloor 2 \cdot 0.421875 \rfloor = \lfloor 0.84375 \rfloor = 0$

$u_{-5} = \lfloor 2 \cdot 0.84375 \rfloor = \lfloor 1.6875 \rfloor = 1$

$u_{-6} = \lfloor 2 \cdot 0.6875 \rfloor = \lfloor 1.375 \rfloor = 1$

$u_{-7} = \lfloor 2 \cdot 0.375 \rfloor = \lfloor 0.75 \rfloor = 0$

$u_{-8} = \lfloor 2 \cdot 0.75 \rfloor = \lfloor 1.5 \rfloor = 1$

$u_{-9} = \lfloor 2 \cdot 0.5 \rfloor = \lfloor 1.0 \rfloor = 1$

$u_{-10} = \lfloor 2 \cdot 0.0 \rfloor = \lfloor 0.0 \rfloor = 0$

$u_{-11} = \lfloor 2 \cdot 0.0 \rfloor = \lfloor 0.0 \rfloor = 0$

$u_{-12} = \lfloor 2 \cdot 0.0 \rfloor = \lfloor 0.0 \rfloor = 0$

$u_{-13} = \lfloor 2 \cdot 0.0 \rfloor = \lfloor 0.0 \rfloor = 0$

Note: if it had decimals:

$\frac{-21344.275}{2^{14}} = 1 + f$

$\rightarrow$ (...)

$\Rightarrow 0.302734375 = 0.0100110110000_2$

(Checked with calculator) so:

$-21344_{10} = (-1)^{-1} \cdot 2^{45-31} \cdot (1 + 0.302734375)$ (w)

$\Rightarrow \boxed{1 \mid 101101 \mid 0100110110000_2}$ (in this system)

$\overbrace{1101}\overbrace{1010}\overbrace{1001}\overbrace{1011}\overbrace{0000} \leftarrow$

D A 9 B 0

$\rightarrow$ $-21344_{10} \rightsquigarrow$ 0xDA9B in IEEE754 (with this specifications).

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Homework: cost of evaluating a polynomial (nested or usual).

For the $\boxed{\text{usual way:}}$

$$a_0 x^n + a_1 x^{n-1} + \dots + a_{n-1} x^1 + a_n$$

First we compute the powers of x, which takes a total $(n-2+1 = n-1)$ $n-1$ multiplications. Then we perform $\forall i$, $a_{n-i} \cdot x^i$ $(i = 1, n)$, which takes another n multiplications. Finally we perform n sums:

$$\boxed{\begin{array}{ll} 2n-1 & \text{multiplications} \\ n & \text{sums} \end{array}} \Big\} \text{Total of } \boxed{3n-1 \text{ operations.}}$$

For the $\boxed{\text{nested manner:}}$

$$(\dots(((x \cdot a_0 + a_1)x + a_2)x + a_3) \cdot x \dots) \cdot x + a_n$$

For the i^{th} parenthesis we perform two operations: $\underset{\sim}{c} \cdot x$ and $\underset{\sim}{d} + a_i$. If we suppose a parenthesis after a_n, we would have a total of n parenthesis (note that each x has a parenthesis to its right). So:

$$\sum_{i=1}^{n} (2) = \boxed{2n \text{ operations}} \quad \left\{ \boxed{\begin{array}{ll} n & \text{sums} \\ n & \text{multiplications} \end{array}} \right.$$

Homework: number of additions in the Gauss elimination and backwards substitution method.

We perform as many operations as columns has A $(n), -1$. In Each step, we are going to change $n-i$ rows. For each row, we perform $(n+2-i)$ sums $(i=1, n+1 ; i=2, n ; \dots)$

So: (Actually $n+1-i$, because we know the first one is zero)

$$\sum_{i=1}^{n-1} \Big((n-i)(n+1-i) \Big) =$$

We take: $n-i = j$, so:
the sum goes from $1 \rightarrow n-1$
$n-i \in \{n-1, n-2, \dots, 1\} \Leftrightarrow j \in \{1, \dots, n-1\}$

$$= \sum_{j=1}^{n-1} j(j+1) = \sum_{j=1}^{n-1} j^2 + \sum_{j=1}^{n-1} j = \frac{(n-1)n(2n-1)}{6} + \frac{n(n-1)}{2} =$$

$$= \frac{(n-1)(n)}{6} (2n - \cancel{1} \overset{2}{+3}) = \frac{(n-1)(n)2(n+1)}{\cancel{6}\,3} = \boxed{\frac{(n^2-1)n}{3} \text{ sums}}$$

Then, for the backwards substitution, in the ith step we perform $i-1$ sums. And we go from $i=1$, $i=n$:

$$\sum_{i=1}^{n} (i-1) = \sum_{i=1}^{n} (i) - \sum_{i=1}^{n} (1) = \frac{n(n+1)}{2} - n = \frac{n}{2}(n+1-2) =$$

$$= \boxed{\frac{n(n-1)}{2} \text{ sums}}$$

So for this method we have a total of $\boxed{\frac{n(n-1)(2n+5)}{6} \text{ sums.}}$

$\left(\frac{(n-1)n(n+1)}{3} + \frac{n(n-1)}{2} = \dots \right)$

6 Matrix factorization

Factorize in $L \cdot U = A$ the following matrix:

$$A = \begin{pmatrix} 1 & 0 & 4 & -3 & 2 \\ 1 & 0 & 3 & 2 & 1 \\ -1 & -2 & 0 & 0 & 0 \\ 1 & 2 & 1 & 3 & -1 \\ 0 & 0 & 1 & -3 & 0 \end{pmatrix}$$

(Probably it doesn't follow the theorem)

$$L = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ l_{21} & 1 & 0 & 0 & 0 \\ l_{31} & l_{32} & 1 & 0 & 0 \\ l_{41} & l_{42} & l_{43} & 1 & 0 \\ l_{51} & l_{52} & l_{53} & l_{54} & 1 \end{pmatrix} \quad \text{and} \quad U = \begin{pmatrix} U_{11} & U_{12} & U_{13} & U_{14} & U_{15} \\ 0 & U_{22} & U_{23} & U_{24} & U_{25} \\ 0 & 0 & U_{33} & U_{34} & U_{35} \\ 0 & 0 & 0 & U_{44} & U_{45} \\ 0 & 0 & 0 & 0 & U_{55} \end{pmatrix}$$

$L_1 \cdot U^j$:

$$\begin{cases} U_{11} = 1 \\ U_{12} = 0 \\ U_{13} = 4 \\ U_{14} = -3 \\ U_{15} = 2 \end{cases}$$

$L_i \cdot U^1$:

$l_{21} \cdot 1 = 1$

$l_{31} \cdot 1 = -1$

$l_{41} \cdot 1 = +1$

$l_{51} \cdot 1 = 0$

$L_2 \cdot U^j$:

$1 \cdot 0 + U_{22} = 0 \Rightarrow U_{22} = 0$

$4 + U_{23} = 3 \Rightarrow U_{23} = -1$

$-3 + U_{24} = 2 \Rightarrow U_{24} = 5$

$2 + U_{25} = 1 \Rightarrow U_{25} = -1$

$L_i \cdot U^2$:

$0 + 0 \longrightarrow$ Since $U_{12} = U_{22} = 0 \rightarrow$ this doesn't add any information, how to follow?

(Let's do the example in class).

$$B = \begin{pmatrix} 1 & 1 & 0 & 3 \\ 2 & 1 & -1 & 1 \\ 3 & -1 & -1 & 2 \\ -1 & 2 & 3 & -1 \end{pmatrix}$$

$$\left[\begin{pmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 3 & 4 & 1 & 0 \\ -1 & -3 & & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 0 & 3 \\ 0 & -1 & -1 & -5 \\ 0 & 0 & 3 & 13 \\ 0 & 0 & 0 & \end{pmatrix} \right]$$

A lápiz llevar la cuenta de las matrices para hacer más rápido el producto.

$$L = \begin{pmatrix} 1 & 0 & 0 & 0 \\ \ell_{21} & 1 & 0 & 0 \\ \ell_{31} & \ell_{32} & 1 & 0 \\ \ell_{41} & \ell_{42} & \ell_{43} & 1 \end{pmatrix} \qquad U = \begin{pmatrix} U_{11} & U_{12} & U_{13} & U_{14} \\ 0 & U_{22} & U_{23} & U_{24} \\ 0 & 0 & U_{33} & U_{34} \\ 0 & 0 & 0 & U_{44} \end{pmatrix}$$

$L_1 \cdot U^j$:

$$\begin{cases} U_{11} = 1 \\ U_{12} = 1 \\ U_{13} = 0 \\ U_{14} = 3 \end{cases}$$

$L_i \cdot U^1$:

$$\begin{cases} \ell_{12} = 2 \\ \ell_{13} = 3 \\ \ell_{14} = -1 \end{cases}$$

$L_2 \cdot U^j$:

$$\begin{cases} 2 + U_{22} = 1 \Rightarrow U_{22} = -1 \\ 0 + U_{23} = -1 \Rightarrow U_{23} = -1 \\ 6 + U_{24} = 1 \Rightarrow U_{24} = -5 \end{cases}$$

$L_i \cdot U^2$:

$$\begin{cases} 3 - \ell_{32} = -1 \Rightarrow \ell_{32} = 4 \\ -1 - \ell_{42} = 2 \Rightarrow \ell_{42} = -3 \end{cases}$$

$L_3 \cdot U^j$:

$$\begin{cases} -4 + U_{33} = -1 \Rightarrow U_{33} = 3 \\ 9 - 20 + U_{34} = 2 \Rightarrow U_{34} = 13 \end{cases}$$

$L_4 \cdot U^3$:

$$\{ 3 + 3\ell_{43} = 3 \Rightarrow \ell_{43} = 0$$

$L_4 \cdot U^4$:

$$\{ -3 + 15 + 0 + U_{44} = -1 \Rightarrow U_{44} = -13$$

$$\Rightarrow \underbrace{\begin{pmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 3 & 4 & 1 & 0 \\ -1 & -3 & 0 & 1 \end{pmatrix}}_{L} \underbrace{\begin{pmatrix} 1 & 1 & 0 & 3 \\ 0 & -1 & -1 & -5 \\ 0 & 0 & 3 & 13 \\ 0 & 0 & 0 & -13 \end{pmatrix}}_{U} = \underbrace{\begin{pmatrix} 1 & 1 & 0 & 3 \\ 2 & 1 & -1 & 1 \\ 3 & -1 & -1 & 2 \\ -1 & 2 & 3 & -1 \end{pmatrix}}_{B}$$

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$\boxed{10}$

b) 3 digit rounding, partial pivoting.

$$3.330x_1 + 15920x_2 + 10.333x_3 = 7953$$
$$2.2220x_1 + 16.710x_2 + 9.6120x_3 = 0.965$$
$$-1.5611x_1 + 5.1792x_2 - 1.6855x_3 = 2.714$$

Solutions: $[1, 0.5, -1]$

$$\left(\begin{array}{ccc|c} 3.330 & 15920 & 10.333 & 7953 \\ 2.2220 & 16.710 & 9.6120 & 0.965 \\ -1.5611 & 5.1792 & -1.6855 & 2.714 \end{array}\right)$$

pivot = $\max\{3.33, 2.222, |-1.5611|\}$

pivot = 3.330

$F_2' = F_2 - \frac{2.2220}{3.330} \cdot F_1$

$F_3' = F_3 + \frac{1.5611}{3.3330} F_1$

$$\xrightarrow[m_3 = 4.69 \cdot 10^{-1}]{m_2 = 6.667 \cdot 10^{-1}} \left(\begin{array}{ccc|c} 3.330 & 15920 & 10.333 & 7953 \\ 0 & a_1 & a_2 & a_3 \\ 0 & b_1 & b_2 & b_3 \end{array}\right)$$

$$a_1 = fl(16.710 - fl(m_2 \cdot 15920)) = -10600$$
$$a_2 = fl(9.6120 - fl(m_2 \cdot 10.333)) = 2.72$$
$$a_3 = fl(0.965 - fl(m_2 \cdot 7953)) = -5300$$

Análogamente $\rightarrow$ $b_1 = 7480$; $b_2 = 3.16$; $b_3 = 3730$

pivot $= \max\{|a_1|, |b_1|\} = |a_1| = 10600 \Rightarrow F_2 \leftrightarrow F_2 \rightarrow$

$$\rightarrow \left(\begin{array}{ccc|c} 3.330 & 15920 & 10.333 & 7953 \\ 0 & -10600 & 2.72 & -5300 \\ 0 & 7480 & 3.16 & 3730 \end{array}\right) \xrightarrow{F_3' = F_3 + 7.06 \cdot 10^{-1} \cdot F_2}$$

$$\rightarrow \left(\begin{array}{ccc|c} 3.330 & 15920 & 10.333 & 7953 \\ 0 & -10600 & 2.72 & -5300 \\ 0 & 0 & 5.08 & -10 \end{array}\right) \Rightarrow \left\{ x_3 = \frac{-10}{5.08} = -1.97 \right.$$

$$\left\{ x_2 = \underbrace{(-5300 + \underbrace{2.72 \cdot \overset{x_2}{\widetilde{1.97}}}_{5.36})}_{-5290} \cdot \frac{1}{-10600} \right. = 0.499 = 4.99 \cdot 10^{-1}$$

$$\left\{ x_1 = \underbrace{(7953 + \underbrace{\underbrace{10.333 \cdot 1.97}_{20.4} - \underbrace{15920 \cdot 0.499}_{-7940}}_{-7920})}_{33} \cdot \frac{1}{3.330} \right. = 9.91$$

$(9.91, 0.499, -1.98)$ $(x_2 ✓, x_3 \sim, x_1 ✗)$

The error:

$\left|\frac{9.91-1}{1}\right| = 8.91 \leq 5 \cdot 10^1 \rightarrow -1$ correct digits (all incorrect). (For x_1)

$\left|\frac{0.499-0.5}{0.5}\right| = 2 \cdot 10^{-3} \leq 5 \cdot 10^{-3} \rightarrow 3$ correct digits (all correct with this system) (For x_2)

$\left|\frac{-1+1.98}{-1}\right| = 0.98 \leq 5 \cdot 10^0 \rightarrow 0$ correct digits (with this system) (For x_3)

Homework: compute number of operations for forward and backwards substitution.

(We observe by symmetry they are going to be the same).

We suppose forward substitution: In the i-th step:

$$x_i = \frac{a_1 x_1 + \ldots + a_{i-1} x_{i-1}}{a_i} \Rightarrow \begin{cases} i-1 \text{ products} \\ i-1 \text{ sums} \\ 1 \text{ division} \end{cases} \rightarrow$$