FIBONACCI NUMBERS

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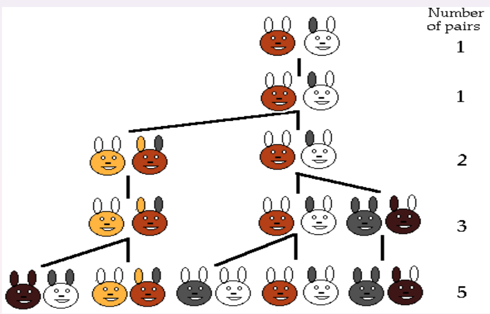
Fibonacci numbers

Leonardo de Pisa, Fibonacci (1175-1250), posed:

> Every pair of rabbits (at least two months old) gives birth to one pair of rabbits every month.
>
> How many pairs of rabbits are there each month if there was 1 at the beginning?



Answer:

Month:	1	2	3	4	5	6
Pairs:	1 1	1 1	$1+1$ $1+1=2$	$\underbrace{1+1}+1$ $2+1=3$	$\underbrace{1+1+1}+1+1$ $3+2=5$	$\underbrace{1+1+1+1+1}+1+1+1$ $5+3=8$

...

Fibonacci numbers: 1 1 2 3 5 8 13 21 34 55 89 144...

Fibonacci numbers and the golden ratio

Golden ratio or divine proportion :

$$\frac{x+y}{x} = \frac{x}{y} \Rightarrow (y=1) \quad x^2 - x - 1 = 0 \Rightarrow x = \frac{1 \pm \sqrt{1+4}}{2} \Rightarrow x = \frac{1+\sqrt{5}}{2}$$

Fibonacci numbers : $1 \quad 1 \quad \underbrace{1+1}_{2} \quad \underbrace{1+2}_{3} \quad \underbrace{2+3}_{5} \quad \underbrace{3+5}_{8} \quad \underbrace{5+8}_{13} \quad \underbrace{8+13}_{21} \quad \ldots$

F_n: nth Fibonacci number or number of pairs of rabbits in the nth month.

$$\boxed{\begin{array}{c} F_1 = 1 \\ F_2 = 1 \\ F_n = F_{n-2} + F_{n-1}, \quad n \geq 3 \end{array}}$$

Theorem : $F_n \sim \frac{1}{\sqrt{5}} \left(\frac{1+\sqrt{5}}{2}\right)^n$ Corollary : $\frac{F_{n+1}}{F_n} \sim \frac{1+\sqrt{5}}{2}$

$$\Downarrow$$

$\boxed{\text{The ratio between two consecutive Fibonacci numbers is around the golden ratio}}$

Fibonacci numbers in Nature

Fibonacci numbers: 1 1 2 3 5 8 13 21 34 55 89 144...