Seminario T6 (corrección)

$$x^2 + y^2 + z^2 - 2xy - 2xz + 2yz + \sqrt{6}\,x = 0$$

(Afín euclídeo)

Parte cuadrática: f. bilineal y h. autoadjunta →

$$M_{\beta c}(f) = M_{\beta c}(h) = \begin{pmatrix} 1 & -1 & -1 \\ -1 & 1 & 1 \\ -1 & 1 & 1 \end{pmatrix}$$

Diagonalizamos con vectores propios:

$$P_A(x) = \begin{vmatrix} x-1 & 1 & 1 \\ 1 & x-1 & -1 \\ 1 & -1 & x-1 \end{vmatrix} = x^2(x-3)$$

$$V(0) = \{ v \in \mathbb{R}^3 : A \cdot v = \bar{0} \} \Rightarrow$$

$$\begin{pmatrix} 1 & -1 & -1 \\ -1 & 1 & 1 \\ -1 & 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 0 \Rightarrow \begin{cases} x - y - z = 0 \\ -x + y + z = 0 \\ \cancel{-x + y + z = 0} \end{cases} \xrightarrow{\cdot -1} \begin{cases} x - y - z = 0 \\ \cancel{x - y - z = 0} \end{cases}$$

$$\Rightarrow V(0) = \{ (\lambda + \mu, \mu, \lambda) \mid \lambda, \mu \in \mathbb{R} \} = \langle (1,0,1), (1,1,0) \rangle$$

Gram - Schmidt:

$$v_1' = (1,1,0)$$

$$v_2'' = v_2' - \frac{v_2' \cdot v_1'}{v_1' \cdot v_1'} \cdot v_1' = \left(\tfrac{1}{2}, -\tfrac{1}{2}, 1\right)$$

Normalizamos:

$$w_1 = \frac{v_1'}{\|v_1'\|} = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right) \quad y \quad w_2 = \frac{v_2'}{\|v_2'\|} = \left(\sqrt{\tfrac{2}{3}} \cdot \tfrac{1}{2}, \sqrt{\tfrac{2}{3}} \cdot \tfrac{-1}{2}, \sqrt{\tfrac{2}{3}}\right)$$

$V(3) = \{v \in \mathbb{R}^3 : Av = 3v\} \rightarrow$

$$\begin{pmatrix} 1 & -1 & -1 \\ -1 & 1 & 1 \\ -1 & 1 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = 3\begin{pmatrix} x \\ y \\ z \end{pmatrix} \longrightarrow \dots \longrightarrow V_3 = (1, -1, -1)$$

$(V(3) = \langle (1, -1, -1) \rangle)$

Como $\forall i \neq j$, $V(\lambda_i) \perp V(\lambda_j)$, nos basta normalizar V_3.

$$W_3 = \left(\frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}\right)$$

Así: $\{w_1, w_2, w_3\}$ es ortonormal para el prod. escalar, entonces

$$D = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 3 \end{pmatrix} \longrightarrow \{w_3, w_1, w_2\} \text{ da } D = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$\left(D = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix}\right.$ con reordenaciones de la base cambiamos λ_1, λ_2 y λ_3).

Así: $D = P^{-1}AP = {}^tPAP$ $\quad \left(P = \begin{pmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & \sqrt{\frac{2}{3}}\cdot\frac{1}{2} \to \frac{1}{\sqrt{6}} \\ -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & \sqrt{\frac{2}{3}}\cdot\frac{-1}{2} \to -\frac{1}{\sqrt{6}} \\ -\frac{1}{\sqrt{3}} & 0 & \sqrt{2/3} \to \frac{2}{\sqrt{6}} \end{pmatrix} \text{ ortogonal}\right)$

(P ortogonal porque sus columnas forman una base ortonormal para el prod. escalar estándar).

Obteniendo:

$Q((x', y', z')) = 3(x')^2$ y

$$\begin{aligned} L((x', y', z')) &= \sqrt{6}\,x = \\ &= \sqrt{6}\left(\frac{1}{\sqrt{3}}x' + \frac{1}{\sqrt{2}}y' + \frac{1}{\sqrt{6}}z'\right) = \\ &= \sqrt{2}\,x' + \sqrt{3}\,y' + z' \end{aligned}$$

72 Completar el cuadrado.

Así:

$3(x')^2 + \sqrt{2}\,x' + \sqrt{3}\,y' + z' =$

$= 3\left(x' + \frac{\sqrt{2}}{6}\right)^2 + \sqrt{3}\,y' + z' - \frac{1}{6} = 0 \Rightarrow$

$\Rightarrow 3(x'')^2 + \sqrt{3}\,y'' + z'' - \frac{1}{6} = 0$

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} -\frac{\sqrt{2}}{6} \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} x'' \\ y'' \\ z'' \end{pmatrix} \longrightarrow \begin{cases} x' = -\frac{\sqrt{2}}{6} + x'' \\ y' = 0 + y'' \\ z' = 0 + z'' \end{cases}$$

Tomamos ahora: (con objetivo de obtener P ortogonal)

$$\begin{cases} x''' = x'' \\ y''' = \alpha(\sqrt{3}y'' + z'') \\ z''' = ax'' + by'' + cz'' \end{cases} \rightarrow P^t = P^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \sqrt{3}\alpha & \alpha \\ a & b & c \end{pmatrix} \Rightarrow$$

$$P = \begin{pmatrix} 1 & 0 & a \\ 0 & \sqrt{3}\alpha & b \\ 0 & \alpha & c \end{pmatrix}$$ y queremos que sea ortogonal $\Rightarrow$

(V_i es la i-ésima columna)

$V_1 \cdot V_1 = 1$ ✓

$V_1 \cdot V_2 = 0$ ✓

$V_1 \cdot V_3 = 0 \Rightarrow a = 0$

$V_2 \cdot V_2 = 0 \Rightarrow \alpha^2(4) = 1 \Rightarrow \alpha = \pm\frac{1}{2} \rightarrow \alpha = \frac{1}{2}$

$V_2 \cdot V_3 = 0 \Rightarrow \alpha(\sqrt{3}b + c) = 0 \underset{\alpha \neq 0}{\Rightarrow} c = -\sqrt{3}\,b$

$V_3 \cdot V_3 = 1 \Rightarrow b^2 + c^2 = 1 \Rightarrow 4b^2 = 1 \Rightarrow b = \pm\frac{1}{2} \rightarrow b =$

$\Rightarrow c =$

y así obtenemos: $$\begin{pmatrix} x'' \\ y'' \\ z'' \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & \frac{1}{2} \\ 0 & \frac{1}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix}}_{\text{ortogonal}} \begin{pmatrix} x''' \\ y''' \\ z''' \end{pmatrix}$$

La ecuación tras este cambio:

$$3(x''')^2 + \cancel{x}^{\left(\frac{1}{2}\right)^{-1}} \cdot y''' - \frac{1}{6} = 0 \Rightarrow$$

$$\Rightarrow 3(x''')^2 + 2(y''') - \frac{1}{6} = 0 \Rightarrow \frac{3(x''')^2}{2} + (y''') - \frac{1}{12} = 0$$

Un último cambio para simplificar totalmete:

$$\begin{cases} x^{iv} = x''' \\ y^{iv} = -y''' + 1/12 \\ z^{iv} = z''' \end{cases} \Rightarrow \begin{pmatrix} x''' \\ y''' \\ z''' \end{pmatrix} = \begin{pmatrix} 0 \\ 1/12 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x^{iv} \\ y^{iv} \\ z^{iv} \end{pmatrix} \Rightarrow$$

$$\Rightarrow \frac{3}{2}(x^{iv})^2 - (y^{iv}) = 0 \Rightarrow \boxed{3(x^{iv})^2 = 2(y^{iv})}$$

$\boxed{\text{Cilindro parabólico.}}$

El nuevo sistema de referencia es:

$$\boxed{R = \left(\left(\frac{-1}{6\sqrt{6}}, \frac{5}{12\sqrt{6}}, \frac{5}{12\sqrt{6}}\right), \left\{\left(\frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}\right), \left(\frac{-2}{\sqrt{6}}, \frac{-1}{\sqrt{6}}, \frac{-1}{\sqrt{6}}\right), \left(0, \frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)\right\}\right)}$$

Producto de matices, composición de isometrías ó sustituir en los ecuaciones.

(Afín)

Forma bilineal simétrica:

$$M_{\beta c}(f) \to \left(\begin{array}{ccc|ccc} 1 & -1 & -1 & 1 & 0 & 0 \\ -1 & 1 & 1 & 0 & 1 & 0 \\ -1 & 1 & 1 & 0 & 0 & 1 \end{array}\right) \xrightarrow{F_2' = F_2 - F_3} \left(\begin{array}{ccc|ccc} 1 & -1 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ -1 & 1 & 1 & 0 & 0 & 1 \end{array}\right) \longrightarrow$$

$$\xrightarrow{C_2' = C_2 - C_3} \left(\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ -1 & 1 & 0 & 0 & -1 & 1 \end{array}\right) \xrightarrow{F_3' = F_3 + F_1} \left(\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 \end{array}\right) \longrightarrow$$

$$\xrightarrow{C_3' = C_3 + C_1} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 \end{array}\right) \Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{pmatrix} \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix}$$

D P

Así:

$$f'((x',y',z')) = (x')^2 + \sqrt{6}x = (x')^2 + \sqrt{6}x' + \sqrt{6}z' = 0 \Rightarrow$$

$$\Rightarrow \left(x' + \frac{\sqrt{6}}{2}\right)^2 - \frac{3}{2} + \sqrt{6}\, z' = 0$$

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} -\sqrt{6}/2 \\ 0 \\ \frac{3\sqrt{6}}{12} \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1/\sqrt{6} \end{pmatrix} \begin{pmatrix} x'' \\ y'' \\ z'' \end{pmatrix} \Rightarrow$$

$$\Rightarrow f((x'',y'',z'')) = (x')^2 - (z'') = 0 \Rightarrow \boxed{(x')^2 = (z'')}$$

$\boxed{\text{Cilindro parabólico.}}$ Obtenemos el sistema de referencia nuevo:

$$\boxed{\begin{pmatrix} x \\ y \\ z \end{pmatrix}} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{pmatrix} \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{pmatrix} \left[\begin{pmatrix} -\sqrt{6}/2 \\ 0 \\ \frac{3\sqrt{6}}{12} \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1/\sqrt{6} \end{pmatrix} \begin{pmatrix} x'' \\ y'' \\ z'' \end{pmatrix} \right] =$$

$$= \boxed{\begin{pmatrix} -\sqrt{6}/4 \\ 0 \\ \frac{3\sqrt{6}}{\cancel{12}\,4} \end{pmatrix} + \begin{pmatrix} 1 & 0 & -1/\sqrt{6} \\ 0 & 1 & 0 \\ 0 & -1 & -1/\sqrt{6} \end{pmatrix} \begin{pmatrix} x'' \\ y'' \\ z'' \end{pmatrix}}$$

Aplicando la fórmula para el cambio de sistemas de referencia:

$$\boxed{\tilde{R} = \left(\left(\frac{-\sqrt{6}}{4}, 0, \frac{\sqrt{6}}{4}\right), \{(1,0,0), (0,1,-1), (-1/\sqrt{6}, 0, -1/\sqrt{6})\}\right)}$$