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Seminario

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P1

Demostrar que $z = y \ln(x^2 - y^2)$ satisface:

$$\frac{1}{x} \cdot \frac{\partial z}{\partial x} + \frac{1}{y} \cdot \frac{\partial z}{\partial y} = \frac{2}{y^2}$$

$$\frac{\partial z}{\partial x} = y \cdot \frac{2x}{x^2 - y^2}$$

$$\frac{\partial z}{\partial y} = \ln(x^2 - y^2) - y \cdot \frac{2y}{x^2 - y^2}$$

$$(d_x A) + (d_y A) + (d_z A) = \nabla A$$

$$z = y \ln(x^2 - y^2) \Rightarrow \ln(x^2 - y^2) = \frac{z}{y}$$

Así obtenemos:

$$\frac{1}{x} \cdot \frac{\partial z}{\partial x} + \frac{1}{y} \cdot \frac{\partial z}{\partial y} = \frac{2y}{x^2 - y^2} + \frac{2}{y^2} - \frac{2y}{x^2 - y^2} = \frac{2}{y^2}$$

$$\Rightarrow \boxed{\frac{1}{x} \cdot \frac{\partial z}{\partial x} + \frac{1}{y} \cdot \frac{\partial z}{\partial y} = \frac{2}{y^2}}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = g_b$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = g_b \circ g_b = (g \circ g)_b$$

P2

Sea $h = f \circ g$, donde $g: \mathbb{R}^3 \rightarrow \mathbb{R}^2$ es $g \in C^{(2)}$ y

$g(x, y, z) = (u, v)$ con $u = x^2 + y^2 + z^2$, $v = x + y + z$.

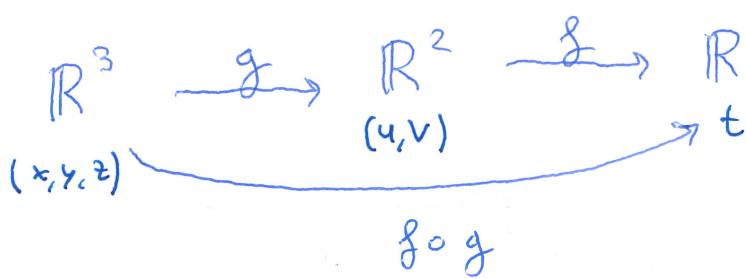
a) Probar que $\|\nabla h\|^2 = 4u(D_1 f)^2 + 4v D_2 f \cdot D_2 f + 3(D_3 f)^2$

$$g: \mathbb{R}^3 \rightarrow \mathbb{R}^2$$

$$f: \mathbb{R}^2 \rightarrow \mathbb{R} \text{ continua } \quad x = \begin{cases} \frac{\partial}{\partial u} & \text{si } u \\ \frac{\partial}{\partial v} & \text{si } v \end{cases}$$

$$(f \circ g)(x) = f(g(x)) : \mathbb{R}^3 \rightarrow \mathbb{R} \quad \text{sup continua} \quad L7$$

$$\frac{\partial}{\partial x} = \frac{\partial f}{\partial u} \cdot \frac{\partial u}{\partial x} + \frac{\partial f}{\partial v} \cdot \frac{\partial v}{\partial x}$$



$$\frac{\partial}{\partial x} = \frac{\partial f}{\partial u} \cdot \frac{\partial u}{\partial x} + \frac{\partial f}{\partial v} \cdot \frac{\partial v}{\partial x}$$

$$\frac{\partial f}{\partial u} \cdot x + (\frac{\partial f}{\partial v}) \cdot 1 = \frac{\partial f}{\partial u} \cdot x$$

Sabemos que:

$$\nabla h = (D_1 h, D_2 h, D_3 h) \Rightarrow \|\nabla h\|^2 = (D_1 h)^2 + (D_2 h)^2 + (D_3 h)^2$$

Calculamos los derivados parciales de h con la regla de la cadena.

$$Dg = \left(\frac{\partial f}{\partial u}, \frac{\partial f}{\partial v} \right) = \left(\frac{\partial f}{\partial u}, \frac{\partial f}{\partial v} \right) = \frac{\partial f}{\partial u} \cdot \frac{\partial u}{\partial x} + \frac{\partial f}{\partial v} \cdot \frac{\partial v}{\partial x} = \frac{\partial f}{\partial u} \cdot x + \frac{\partial f}{\partial v} \cdot 1$$

$$dg = \begin{pmatrix} D_1 g_1, D_2 g_1, D_3 g_1 \\ D_1 g_2, D_2 g_2, D_3 g_2 \\ D_1 g_3, D_2 g_3, D_3 g_3 \end{pmatrix}$$

$$\frac{\partial f}{\partial u} = \frac{\partial f}{\partial u} \cdot \frac{1}{x} + \frac{\partial f}{\partial v} \cdot \frac{1}{x}$$

$$d(f \circ g) = df \circ dg = (D_1 f, D_2 f) \begin{pmatrix} D_1 g_1, D_2 g_1, D_3 g_1 \\ D_1 g_2, D_2 g_2, D_3 g_2 \end{pmatrix} =$$

$$= (D_1 f \cdot D_1 g_1 + D_2 f \cdot D_1 g_2, \\ D_1 f \cdot D_2 g_1 + D_2 f \cdot D_2 g_2, \\ D_1 f \cdot D_3 g_1 + D_2 f \cdot D_3 g_2) =$$

$$= (D_1 h, D_2 h, D_3 h) \Rightarrow$$

$$\begin{cases} D_1 h = D_1 f \cdot D_1 g_1 + D_2 f \cdot D_1 g_2 \\ D_2 h = D_1 f \cdot D_2 g_1 + D_2 f \cdot D_2 g_2 \\ D_3 h = D_1 f \cdot D_3 g_1 + D_2 f \cdot D_3 g_2 \end{cases}$$

Para poner $D_i h$ en términos de $D_k f$, necesitamos calcular $D_i g_j$.

$$g(x, y, z) = (x^2 + y^2 + z^2, x + y + z)$$

$$\begin{array}{lcl} D_1 g_1 = 2x & ; & D_2 g_1 = 2y & ; & D_3 g_1 = 2z \\ D_1 g_2 = 1 & ; & D_2 g_2 = 1 & ; & D_3 g_2 = 1 \end{array}$$

Por tanto:

$$(D_1 h)^2 = 4x^2 \cdot (D_1 f)^2 + (D_2 f)^2 + 4x \cdot D_1 f \cdot D_2 f$$

$$(D_2 h)^2 = 4y^2 \cdot (D_1 f)^2 + (D_2 f)^2 + 4y \cdot D_1 f \cdot D_2 f$$

$$(D_3 h)^2 = 4z^2 \cdot (D_1 f)^2 + (D_2 f)^2 + 4z \cdot D_1 f \cdot D_2 f$$

Sumando los tres:

$$(D_1 h)^2 + (D_2 h)^2 + (D_3 h)^2 =$$

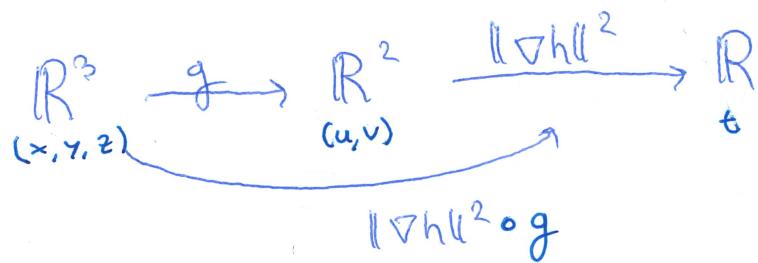
$$= 4 \underbrace{(x^2 + y^2 + z^2)}_u (D_1 f)^2 + 4 \underbrace{(x+y+z)}_v (D_2 f)^2 + 3 \cdot D_1 f \cdot D_2 f =$$

$$= 4u(D_1 f)^2 + 4v(D_2 f)^2 + 3 \cdot D_1 f \cdot D_2 f$$

$$\Rightarrow \boxed{\|\nabla h\|^2 = 4u(D_1 f)^2 + 4v(D_2 f)^2 + 3 \cdot D_1 f \cdot D_2 f} \quad \square$$

b) Calcular $\frac{\partial (\|\nabla h\|^2)}{\partial x}$

Podemos tomar la siguiente interpretación de $\|\nabla h\|^2$ como función:



$$\frac{\partial (\|\nabla h\|^2)}{\partial x} \text{ es en realidad } D_1(g \circ \|\nabla h\|^2) :$$

$$d \|\nabla h\|^2 = (D_1 \|\nabla h\|^2, D_2 \|\nabla h\|^2)$$

$$dg = \begin{pmatrix} D_1 g_1 & D_2 g_1 & D_3 g_1 \\ D_1 g_2 & D_2 g_2 & D_3 g_2 \end{pmatrix}$$

$$d(\|\nabla h\|^2 \circ g) = (D_1 \|\nabla h\|^2, D_2 \|\nabla h\|^2) \begin{pmatrix} D_1 g_1 & D_2 g_1 & D_3 g_1 \\ D_1 g_2 & D_2 g_2 & D_3 g_2 \end{pmatrix}$$

$$= (D_1(\|\nabla h\|^2 \circ g), D_2(g \circ \|\nabla h\|^2)) \Rightarrow$$

$$\Rightarrow D_1(\|\nabla h\|^2 \circ g) = D_1(\|\nabla h\|^2) \cdot D_1 g_1 + D_2(\|\nabla h\|^2) D_1 g_2 \frac{6}{x^6}$$

Reescribimos la información (que tenemos hasta ahora):

$$\|\nabla h\|^2 = 4u \cdot \left(\frac{\partial g}{\partial u} \right)^2 + 4v \left(\frac{\partial g}{\partial v} \right)^2 + 3 \cdot \left(\frac{\partial g}{\partial u} \right) \cdot \left(\frac{\partial g}{\partial v} \right)$$

$$\frac{\partial}{\partial x}(\|\nabla h\|^2 \circ g) = \frac{\partial}{\partial u}(\|\nabla h\|^2) \frac{\partial u}{\partial x} + \frac{\partial}{\partial v}(\|\nabla h\|^2) \frac{\partial v}{\partial x} +$$

$$(Pues g = (u, v) \rightarrow g_1 = u, g_2 = v \text{ con } u = \sqrt[6]{x^6+y^6})$$

funciones de \mathbb{R}^3 a \mathbb{R}^2). Calculemos:

$$\frac{\partial}{\partial u}(\|\nabla h\|^2) = \begin{cases} \text{Interpretamos como función de una variable,} \\ u, \text{ y aplicamos las reglas de derivación} \\ \text{conocidas.} \end{cases}$$

$$= 4 \cdot \left(\frac{\partial g}{\partial u} \right)^2 + 8u \cdot \left(\frac{\partial g}{\partial u} \right) \cdot \left(\frac{\partial^2 g}{\partial u^2} \right) + 8v \cdot \left(\frac{\partial g}{\partial v} \right) \cdot \left(\frac{\partial^2 g}{\partial v \cdot \partial u} \right) +$$

$$+ 3 \cdot \left(\frac{\partial g}{\partial u} \right) \cdot \left(\frac{\partial^2 g}{\partial v \cdot \partial u} \right) + 3 \cdot \left(\frac{\partial^2 g}{\partial u^2} \right) \cdot \left(\frac{\partial g}{\partial v} \right)$$

$\frac{\partial}{\partial v}(\|\nabla h\|^2) =$ (análogo al caso anterior debido a la simetría de la función).

$$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x}(x^2+y^2+z^2) = 2x$$

$$\frac{\partial v}{\partial x} = \frac{\partial}{\partial x}(x+y+z) = 1$$

Por tanto:

$$\Leftrightarrow (\mathcal{S} \mathcal{H} \mathcal{D} \mathcal{I} \circ g)_{,A} = (\mathcal{S} \mathcal{H} \mathcal{D} \mathcal{I} \circ g)_{,A} =$$

$$\frac{\partial}{\partial x} (\mathcal{S} \mathcal{H} \mathcal{D} \mathcal{I} (\|\nabla h\|^2 \circ g))_{,A} = \mathcal{S} \mathcal{H} \mathcal{D} \mathcal{I} \cdot (\mathcal{S} \mathcal{H} \mathcal{D} \mathcal{I})_{,A} = (\mathcal{S} \mathcal{H} \mathcal{D} \mathcal{I} \circ g)_{,A}$$
$$= 2x \cdot \left[4 \cdot \left(\frac{\partial g}{\partial u} \right)^2 + \left(\frac{\partial^2 g}{\partial u^2} \right) \left(8u \left(\frac{\partial g}{\partial u} \right) + 3 \left(\frac{\partial g}{\partial v} \right) \right) + \right. \\ \left. + \left(\frac{\partial^2 g}{\partial v^2} \right) \left(3 \left(\frac{\partial g}{\partial u} \right) + 8v \left(\frac{\partial g}{\partial v} \right) \right) \right] + \\ + \left[4 \cdot \left(\frac{\partial g}{\partial v} \right)^2 + \left(\frac{\partial^2 g}{\partial v^2} \right) \left(8v \left(\frac{\partial g}{\partial v} \right) + 3 \left(\frac{\partial g}{\partial u} \right) \right) + \right. \\ \left. + \left(\frac{\partial^2 g}{\partial u^2} \right) \left(3 \left(\frac{\partial g}{\partial v} \right) + 8u \left(\frac{\partial g}{\partial u} \right) \right) \right]$$

$$\left(\text{álgebra da álgebra das contrapositivas} \right) = (\mathcal{S} \mathcal{H} \mathcal{D} \mathcal{I}) \frac{6}{\sqrt{6}}$$

$$+ \left(\frac{\frac{26}{\sqrt{6}} - \frac{26}{\sqrt{6}}}{\omega_6 \cdot v_6} \right) VH + \left(\frac{\frac{26}{\sqrt{6}}}{\omega_6 v_6} \right) \cdot VH + \mathcal{S} \left(\frac{26}{\omega_6} \right) \cdot H = \\ \left(\frac{26}{\sqrt{6}} \right) \left(\frac{26}{\omega_6} \right) \mathcal{S} + \left(\frac{26}{\omega_6 \cdot v_6} \right) \left(\frac{26}{\omega_6} \right) \mathcal{S} +$$

$$\text{é o efeito visto em } \mathcal{S} \text{ (opção)} = (\mathcal{S} \mathcal{H} \mathcal{D} \mathcal{I}) \frac{6}{\sqrt{6}}$$

(álgebra da álgebra)

$$xS = (\mathcal{S} + \mathcal{Q} + \mathcal{X}) \frac{6}{\sqrt{6}} = \frac{36}{\sqrt{6}}$$

$$-Q = (\mathcal{S} + \mathcal{Q} + \mathcal{X}) \frac{6}{\sqrt{6}} = \frac{36}{\sqrt{6}}$$

P3

Hallerstads $u = f(x, y, z)$, t.g. $(f'_x, f'_y, f'_z) = F$.

donde $F(x, y, z) = (6xy \cos(z), 3x^2 \cos(z), -3x^2 y \sin(z))$.

$$\mathbb{R}^3 \xrightarrow{F} \mathbb{R}^3 \quad y \quad \mathbb{R}^3 \xrightarrow{u} \mathbb{R}^2$$

$$F(x, y, z) = (6xy \cos(z), 3x^2 \cos(z), -3x^2 y \sin(z)) = \frac{16}{x^6} \times$$

$$= \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right) \stackrel{\text{stegene og konstanter}}{=} \nabla f$$

$$\frac{\partial f}{\partial x} = 6xy \cos(z) \Rightarrow f = \int 6xy \cos(z) dx \Rightarrow$$

$$\Rightarrow f = 3x^2 y \cos(z) + C \leftarrow (x^2, y) \leftarrow ((x, y))$$

$$\frac{\partial f}{\partial y} = 3x^2 \cos(z) \Rightarrow f = \int 3x^2 \cos(z) dy \leftarrow \text{med}$$

$$\Rightarrow f = 3x^2 y \cos(z) + C \text{for at etvidig retur}$$

$$\frac{\partial f}{\partial z} = -3x^2 y \sin(z) \Rightarrow f = \int -3x^2 y \sin(z) dz \Rightarrow \frac{16}{x^6}$$

$$\Rightarrow f = 3x^2 y \cos(z) + C \quad \begin{matrix} t = \frac{16}{x^6} \\ s = \frac{16}{x^6} \end{matrix} \quad \begin{matrix} \leftarrow xt = u \\ \leftarrow yt = v \end{matrix}$$

Una de estos $\frac{16}{x^6}$ podría ser $\frac{16}{x^6} \cdot x$ \leftarrow tomado

$$C = 0$$

$$\frac{16}{x^6} \cdot t = \frac{16}{x^6} \leftarrow$$

$$u = f(x, y, z) = 3x^2 y \cos(z)$$

: stemtaget A

$$\frac{16}{x^6} \cdot t \cdot x = \frac{16}{x^6} \cdot x \leftarrow \frac{16}{x^6} \cdot t = \frac{16}{x^6}$$

7

P4

a) $z = f(x, y)$ función diferenciable

que $f(tx, ty) = t^m \cdot f(x, y)$ para $x, y, t \in \mathbb{R}$, $m > 0$.

demostar: $\frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = m z$

$$x \cdot \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y} = m f \quad (f(x, y) = (x, y)^T)$$

Consideramos la siguiente composición de funciones:

$$\begin{array}{ccc} \mathbb{R}^3 & \xrightarrow{g} & \mathbb{R}^2 \\ (x, y, t) & \mapsto (u, v) & \mapsto f(u, v) = t^m \cdot f(x, y) \\ \text{con } g(x, y, t) = (tx, ty) & = (u, v) & \left(\begin{array}{l} u=tx \\ v=ty \end{array} \right) \end{array}$$

Por tanto, aplicando la regla de la cadena:

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial u} \cdot \frac{\partial u}{\partial x} + \frac{\partial f}{\partial v} \cdot \frac{\partial v}{\partial x} \quad \text{con } f(u, v) = t^m \cdot f(x, y)$$

$$\begin{cases} u = tx \rightarrow \frac{\partial u}{\partial x} = t \\ v = ty \rightarrow \frac{\partial v}{\partial x} = 0 \end{cases}$$

$$\Rightarrow \frac{\partial f}{\partial x} = t \cdot \frac{\partial f}{\partial u} \quad \text{obtenemos } \frac{\partial f}{\partial x} = x \cdot \frac{\partial f}{\partial u} \text{ al ser } u = tx$$

Análogamente:

$$\frac{\partial f}{\partial y} = t \cdot \frac{\partial f}{\partial v} \Rightarrow y \cdot \frac{\partial f}{\partial y} = y \cdot t \cdot \frac{\partial f}{\partial v}$$

$$(t^m \cdot f(x, y))_y = (x, y, t) f_y = 0$$

Calculamos además: $\frac{\partial f}{\partial t}$

$$\frac{\partial f}{\partial t} = \frac{26}{x^6} \cdot x + \frac{26}{x^6} \cdot x \quad \text{sup } t \rightarrow 0$$

Por una parte:

$$\frac{\partial}{\partial t} (f(tx, ty)) = \frac{\partial}{\partial t} (t^m \cdot \underbrace{f(x, y)}_{\text{de repetir}}) = m \cdot t^{m-1} \cdot f(x, y) =$$
$$= m \cdot \frac{t^m}{t} \cdot f(x, y) = \frac{m}{t} \cdot f(tx, ty) (= \frac{1}{t} \cdot m \cdot f(tx, ty))$$

Por otra parte:

$$\frac{\partial f}{\partial t} = \frac{\partial f}{\partial u} \cdot \frac{\partial u}{\partial t} + \frac{\partial f}{\partial v} \cdot \frac{\partial v}{\partial t} \quad \text{con}$$

$$\left\{ \begin{array}{l} u = tx \rightarrow \frac{\partial u}{\partial t} = x \\ v = ty \end{array} \right.$$

$$\left\{ \begin{array}{l} v = ty \rightarrow \frac{\partial v}{\partial t} = y \end{array} \right. \quad \text{obtenemos el resultado}$$

$$\Rightarrow \frac{\partial f}{\partial t} = \frac{\partial f}{\partial u} \cdot x + \frac{\partial f}{\partial v} \cdot y = x \cdot t \cdot \frac{\partial f}{\partial u} \cdot \frac{1}{t} + y \cdot t \cdot \frac{\partial f}{\partial v} \cdot \frac{1}{t} =$$
$$= x \cdot \frac{\partial f}{\partial x} \cdot \frac{1}{t} + y \cdot \frac{\partial f}{\partial y} \cdot \frac{1}{t} \quad (2)$$

probado
página
anterior

$$\left(\cancel{x} = \frac{u}{t} \Leftrightarrow \cancel{x} = u \right)$$

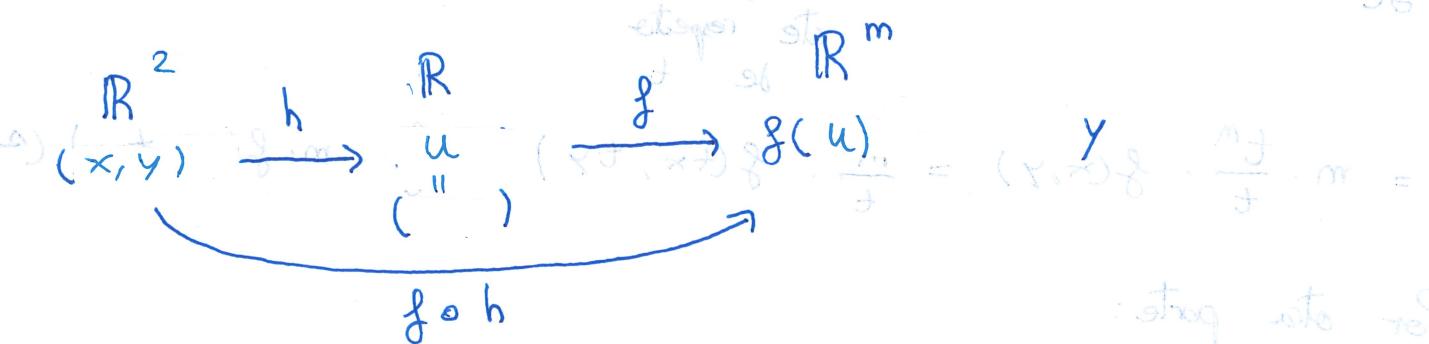
$$\text{Juntando (1) y (2): } x \cdot \frac{26}{x^6} - f \cdot x = x \cdot \frac{26}{x^6} \quad \text{P}$$

$$\cancel{\frac{1}{t}} \cdot m \cdot f = x \cdot \frac{\partial f}{\partial x} \cdot \cancel{\frac{1}{t}} + y \cdot \frac{\partial f}{\partial y} \cdot \cancel{\frac{1}{t}} \Rightarrow$$

$$\Rightarrow \boxed{x \cdot \frac{\partial f}{\partial x} + y \cdot \frac{\partial f}{\partial y} = m \cdot f = m \cdot z}$$

b) $z = (x+y) \cdot f(y/x)$ con f función arbitraria.
 Demostrar que $x \cdot \frac{\partial z}{\partial x} + y \cdot \frac{\partial z}{\partial y} = z$.

Consideremos: $(x,y) \cdot f \circ h = ((x,y) \cdot f) \frac{y}{x} = ((x,y) \cdot f) \frac{y}{x}$



$$(x,y) \xrightarrow{z} (x+y) \cdot f(y/x)$$

Calculamos los derivados parciales de z .

$$\begin{aligned} \frac{\partial z}{\partial x} &= \frac{\partial}{\partial x}((x+y) \cdot f) = f + \frac{\partial f}{\partial x} = \\ &= f + \frac{\partial f}{\partial u} \cdot \frac{\partial u}{\partial x} = f - \frac{\partial f}{\partial u} \cdot \frac{y}{x^2} \Rightarrow \end{aligned}$$

$$\left(u = \frac{y}{x} \Rightarrow \frac{\partial u}{\partial x} = -\frac{1}{x^2} \right)$$

$$\Rightarrow \frac{\partial z}{\partial x} \cdot x = x \cdot f - \frac{\partial f}{\partial u} \cdot \frac{y}{x} \quad : (S) \times (R) \text{ elaboremos}$$

$$= f \cdot x + \frac{\partial f}{\partial u} \cdot \frac{y}{x} \cdot x = f \cdot m$$

$$\boxed{f \cdot m = f \cdot m = \frac{y}{x} \cdot x + \frac{y}{x} \cdot x} \quad F$$

$$\frac{\partial z}{\partial y} = \frac{1}{x} \left[(x+y)^{0.8} f + \frac{\partial f}{\partial y} \right] = \frac{1}{x} + \frac{1}{x} \cdot \frac{1}{(x+y)^{0.2}} = \frac{1}{x} + \frac{1}{x(x+y)^{0.2}}$$

$$(u = \frac{y}{x} \Rightarrow \frac{\partial u}{\partial y} = \frac{1}{x})$$

$$\Rightarrow \frac{\partial z}{\partial y} \cdot y = y \cdot f + \frac{\partial f}{\partial u} \cdot \frac{y}{x}$$

Por tanto:

$$\frac{\partial z}{\partial x} \cdot x + \frac{\partial z}{\partial y} \cdot y = x \cdot f + \frac{\partial f}{\partial u} \cdot \frac{y}{x} + y \cdot f$$

$$= (x+y) f = 2 \Rightarrow$$

$$\Rightarrow \boxed{\frac{\partial z}{\partial x} \cdot x + \frac{\partial z}{\partial y} \cdot y = 2} = \frac{w}{x} ; \quad x = \frac{w}{x}$$

$$\frac{(dof)6}{x} \cdot \frac{1}{(dof)} \cdot x = \frac{6(dof)}{x} : x = \frac{56}{x}$$

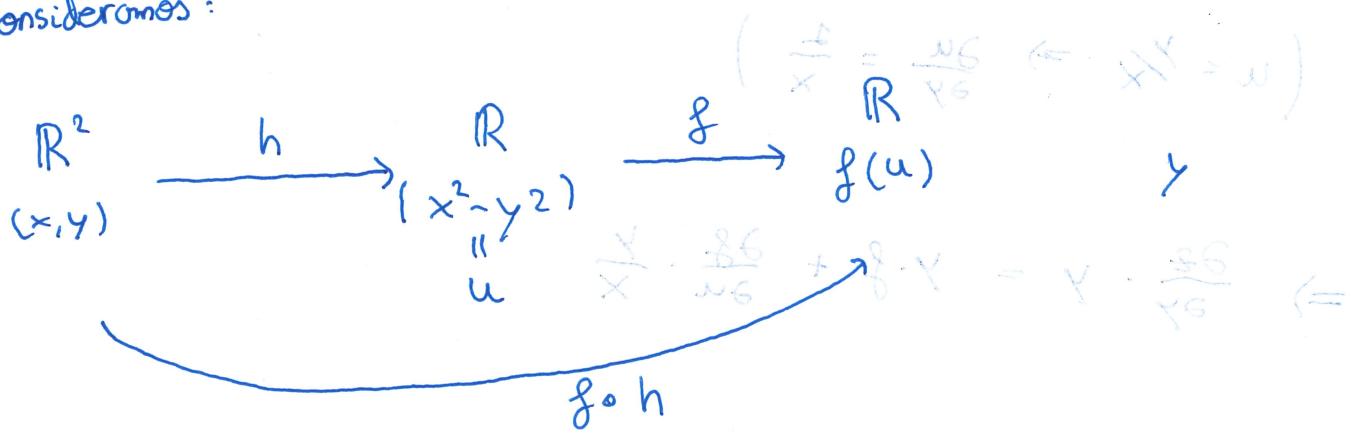
$$\frac{(dof)6}{w} \cdot \frac{w}{(dof)} = \frac{w}{x} \cdot \frac{(dof)6}{w} \cdot \frac{1}{(dof)} \cdot w =$$

$$\frac{(dof)6}{w} \cdot \frac{w}{(dof)} = \frac{w}{x} \cdot \frac{1}{x} (=$$

PS Si $z = \frac{y}{f(x^2-y^2)}$ [demonstrar que es] $\frac{\partial z}{\partial x} = \frac{y}{x^2}$

$$\frac{1}{x} \cdot \frac{\partial z}{\partial x} + \frac{1}{y} \cdot \frac{\partial z}{\partial y} = \frac{2}{y^2} \cdot \frac{y}{x^2} + 0 = \frac{2}{x^2} + 0 = \frac{2}{x^2}$$

Consideremos:



$$R^2 \xrightarrow{h} (x^2 - y^2, u) \xrightarrow{f} f(u) = y \cdot (x^2 - y^2)$$

Calcularemos los derivados parciales de z . Pero f antes los de u .

$$\frac{\partial u}{\partial x} = 2x \quad ; \quad \frac{\partial u}{\partial y} = -2y \quad ; \quad \frac{\partial f}{\partial u} = \frac{1}{(f \circ h)^2} \cdot \frac{\partial f}{\partial u}$$

$$\frac{\partial z}{\partial x} = y \cdot \frac{\partial ((f \circ h)^{-1})}{\partial x} = -y \cdot \frac{1}{(f \circ h)^2} \cdot \frac{\partial (f \circ h)}{\partial x} =$$

$$= -y \cdot \frac{1}{(f \circ h)^2} \cdot \frac{\partial (f \circ h)}{\partial u} \cdot \frac{\partial u}{\partial x} = -\frac{2xy}{(f \circ h)^2} \cdot \frac{\partial (f \circ h)}{\partial u}$$

$$\Rightarrow \frac{1}{x} \cdot \frac{\partial z}{\partial x} = -\frac{2y}{(f \circ h)^2} \cdot \frac{\partial (f \circ h)}{\partial u}$$

$$\frac{\partial z}{\partial y} = (f \circ h)^{-1} + y \cdot \frac{\partial(f \circ h)}{\partial y} = (\times, \times) \text{ 2.6}$$

$$= (f \circ h)^{-1} + y \cdot \frac{1}{(f \circ h)^2} \cdot \frac{\partial(f \circ h)}{\partial y} = \frac{(\times, \times) \text{ 2.6}}{x^2}$$

$$= (f \circ h)^{-1} - \frac{y}{(f \circ h)^2} \cdot \frac{\partial(f \circ h)}{\partial u} \cdot \frac{\partial u}{\partial y} = \text{reduz zu } \frac{(\times, \times) \text{ 2.6}}{x^2}$$

$$= (f \circ h)^{-1} + \frac{2y^2}{(f \circ h)^2} \cdot \frac{\partial(f \circ h)}{\partial u} \Rightarrow \text{consideren nur } \frac{2y^2}{(f \circ h)^2}$$

$$\Rightarrow \frac{1}{y} \cdot \frac{\partial z}{\partial y} = \frac{1}{y} \cdot (f \circ h)^{-1} + \frac{2y}{(f \circ h)^2} \cdot \frac{\partial(f \circ h)}{\partial u} = \frac{(\times, \times) \text{ 2.6}}{x^2} = (\pm, \times) \text{ 8}$$

Por tanto:

$$\frac{1}{x} \cdot \frac{\partial z}{\partial x} + \frac{1}{y} \cdot \frac{\partial z}{\partial y} = \text{f6}(\pm, \times) \text{ 8} \quad \left. \begin{array}{l} (\times) \text{ sd } \\ (\times, \pm) \end{array} \right\} = (\times, \times)$$

$$= -\frac{2y}{(f \circ h)^2} \cdot \frac{\partial(f \circ h)}{\partial u} + \frac{2y}{(f \circ h)^2} \cdot \frac{\partial(f \circ h)}{\partial u} + \frac{1}{y} \cdot (f \circ h)^{-1} = \text{f6}(\pm, \times) \text{ 8} \quad \left. \begin{array}{l} (\times) \text{ sd } \\ (\times, \pm) \end{array} \right\} = \frac{(\times, \times) \text{ 2.6}}{x^2} \quad \left. \begin{array}{l} (\times) \text{ sd } \\ (\times, \pm) \end{array} \right\} = (\times, \times)$$

$$= y \cdot (f \circ h)^{-1} \cdot \frac{1}{y^2} = \frac{1}{y^2} = \frac{2}{y^2}$$

Es decir:

$$\boxed{\frac{1}{x} \cdot \frac{\partial z}{\partial x} + \frac{1}{y} \cdot \frac{\partial z}{\partial y} = \frac{2}{y^2}}$$

$$\frac{(\pm, \times) \text{ 2.6}}{x^2} = ((\times) \text{ sd }, \times) \text{ 8}$$

$$\Delta = (\times) \text{ sd }$$

P6

$$G(x, y) = \int_{xy}^{x+y} \frac{\sin((x+y)t)}{t} dt$$

$$\text{Halla: } \frac{\partial G(x, y)}{\partial x} = \frac{(d\partial g)}{y6} + \frac{1}{y6} \cdot y + \frac{1}{t} (d\partial g) =$$

Dado que para calcular $\frac{\partial G(x, y)}{\partial x}$ suponemos $y = \text{cte}$, con esta suposición y teniendo en mente lo que vamos a hacer reescribimos:

$$\frac{(d\partial g)}{y6} + \frac{1}{y6} \cdot y + \frac{1}{t} (d\partial g) =$$

$$\left\{ \begin{array}{l} h_1(x) = xy \\ h_2(x) = x + \frac{y}{t} \\ g(x, t) = \frac{\sin((x+y)t)}{t} \end{array} \right. \Rightarrow \frac{(d\partial g)}{y6} + \frac{sy}{y6} + \frac{1}{t} (d\partial g) = \frac{sy}{y6} + \frac{1}{x} (d\partial g) =$$

$$G(x) = \int_{h_1(x)}^{h_2(x)} g(x, t) dt \Rightarrow = \frac{sy}{y6} \cdot \frac{1}{x} + \frac{sy}{x6} \cdot \frac{1}{x}$$

$$\Rightarrow \frac{\partial G(x, y)}{\partial x} = G'(x) = \underbrace{g(x, h_2(x)) \cdot h_2'(x)}_{= \frac{sy}{x}} + \underbrace{\frac{(d\partial g)}{y6} \cdot \frac{y}{x}}_{= -g(x, h_1(x)) \cdot h_1'(x)} + \underbrace{\frac{sy}{x6} \cdot \frac{1}{x}}_{= \frac{sy}{x^2}} + \underbrace{\int_{h_1(x)}^{h_2(x)} \left(\frac{\partial g}{\partial x} \right) dy}_{= \frac{1}{x}} =$$

Calculamos los distintos términos:

- $g(x, h_2(x)) = \frac{\sin((x+y)^2)}{(x+y)}$

$$\boxed{\frac{1}{x^2} = \frac{sy}{x6} \cdot \frac{1}{x} + \frac{sy}{x6} \cdot \frac{1}{x}}$$

- $h_2'(x) = 1$

- $\bullet \quad f(x, h_1(x)) = \frac{\sin((x+y) \cdot y)}{xy} \quad \begin{cases} x \neq 0 \\ y \neq 0 \end{cases}$
 $\quad \quad \quad h_1'(x) = y \quad \begin{cases} x \neq 0 \\ y \neq 0 \end{cases}$
 $\quad \quad \quad \int \frac{\partial f}{\partial x} dy = \frac{\sin((x+y) \cdot t)}{t} = f(x)$
 $\bullet \quad \frac{\partial f}{\partial x} = \cos((x+y) \cdot t) \Rightarrow \int \frac{\partial f}{\partial x} dy = \frac{\sin((x+y) \cdot t)}{t} = f(x)$
 $\bullet \quad \int_{h_1(x)}^{h_2(x)} \frac{\partial f}{\partial x} dy = f(x, h_2(x)) - f(x, h_1(x)) \quad (\text{ya calculados})$
 $\quad \quad \quad (\exists x \in \mathbb{R}) \frac{\partial f}{\partial x} \text{ es constante}$
 $\quad \quad \quad (\text{considerando constante en } x \neq 0 \text{ y } y \neq 0)$

Es decir:

$$g'(x) = \frac{\partial f}{\partial x}(x, h_2(x)) = (h_2'(x) + 1) + \sin((x+h_2(x)) \cdot h_2'(x) + 1)$$

$$(\text{y } y \neq 0)$$

Obteniendo:

$$\frac{\partial G(x, y)}{\partial x} = \frac{2 \sin((x+y)^2)}{(x+y)} - \frac{\sin((x+y) \cdot y)}{xy} (y+2)$$

$$\frac{\partial G(x, y)}{\partial x} = \frac{2 \sin((x+y)^2)}{(x+y)} - \frac{\sin((x+y) \cdot y)}{xy} (y+2)$$

$$(\text{y } y \neq 0)$$

$$(\exists x \in \mathbb{R}) \left(\frac{26}{56} \right) \text{ constante}$$

considerando constante en $y \neq 0$

$$\frac{26}{56} + 2 = \left(\frac{x}{x+y} + y \right) \frac{6}{y} = (\exists x \in \mathbb{R}) \left(\frac{26}{56} \right) \frac{6}{y}$$

$\therefore g(x)$

P7

$$f(x, y, z) = \begin{cases} yz + \frac{x^2 + xz}{x^2 + y^2} & \text{si } x \neq 0 \text{ y } y \neq 0 \text{ y } z \neq 0 \\ z^2 & \text{si } x = 0 \text{ e } y = 0 \end{cases}$$

$$f = \frac{(z \cdot (y+z)) \text{ no se}}{z} = yz + \frac{z^2}{x^2 + y^2} \Leftrightarrow (z \cdot (y+z)) \text{ no se} = \frac{z^2}{x^2 + y^2}$$

Hallar:

$$\frac{\partial}{\partial x} \left(\frac{\partial f}{\partial y} \left(\frac{\partial f}{\partial z} \right) \right)(0, 0, 0)$$

(sabemos y) $((x), y, z) - ((0), y, z) = yz + \frac{z^2}{x^2 + y^2}$

Primero hallamos $\frac{\partial f}{\partial z}(x, y, z)$.Si $x \neq 0$ y $y \neq 0$, no tenemos problemas:

$$\frac{\partial f}{\partial z}(x, y, z) \cdot ((x), y, z) = \frac{\partial}{\partial z} \left(yz + \frac{x^2 + xz}{x^2 + y^2} \right) = (y) + \frac{x}{x^2 + y^2} = (x)^{\frac{1}{2}}$$

$(x \neq 0 \text{ y } y \neq 0)$

Si $x = 0$, $y = 0$:

$$\frac{\partial f}{\partial z}(x, y, z) = \lim_{h \rightarrow 0} \frac{f(0, 0, z+h) - f(0, 0, z)}{h} = \frac{\partial f}{\partial z}(z^2) = 2z$$

$$\Rightarrow \frac{\partial f}{\partial z}(x, y, z) = \begin{cases} y + \frac{x}{x^2 + y^2} & \text{si } x \neq 0 \text{ y } y \neq 0 \\ 2z & \text{si } x = 0 \text{ e } y = 0 \end{cases}$$

Hallaremos ahora $\frac{\partial}{\partial y} \left(\frac{\partial f}{\partial z} \right)(x, y, z)$.Si $x \neq 0$ y $y \neq 0$, no tenemos problemas.

$$\frac{\partial}{\partial y} \left(\frac{\partial f}{\partial z} \right)(x, y, z) = \frac{\partial}{\partial y} \left(y + \frac{x}{x^2 + y^2} \right) = 1 - \frac{2xy}{(x^2 + y^2)^2}$$

Si $x=0, y=0$:

$$\frac{\partial}{\partial y} \left(\frac{\partial f}{\partial z} \right) (x, y, z) = \lim_{h \rightarrow 0} \frac{\frac{\partial f}{\partial z}(0, h, z) - \frac{\partial f}{\partial z}(0, 0, z)}{h} =$$
$$= \lim_{h \rightarrow 0} \frac{h + 0 - 2z}{h}$$

$\not\exists$ si $z \neq 0$
 $\rightarrow 1$ si $z = 0$

Es decir:

$$\frac{\partial}{\partial y} \left(\frac{\partial f}{\partial z} \right) (x, y, z) = \begin{cases} 1 - \frac{2xy}{(x^2+y^2)^2} & \text{si } x \neq 0 \text{ o } y \neq 0 \\ 1 & \text{si } x = 0, y = 0, z = 0 \\ \not\exists & \text{si } x = 0, y = 0, z \neq 0 \end{cases}$$

Veamos si podemos calcular:

$$\frac{\partial}{\partial x} \left(\frac{\partial}{\partial y} \left(\frac{\partial f}{\partial z} \right) \right) (0, 0, 0) = \lim_{h \rightarrow 0} \frac{\frac{\partial}{\partial y} \left(\frac{\partial f}{\partial z} \right) (h, 0, 0) - \frac{\partial}{\partial y} \left(\frac{\partial f}{\partial z} \right) (0, 0, 0)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{1 - 0 - 1}{h} = \lim_{h \rightarrow 0} \frac{0}{h} = 0$$

Por tanto:

$$\boxed{\frac{\partial}{\partial x} \left(\frac{\partial}{\partial y} \left(\frac{\partial f}{\partial z} \right) \right) (0, 0, 0) = 0}$$

