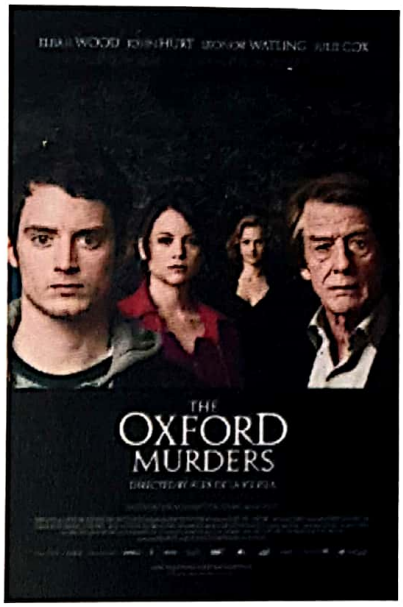
Nos toca presentar

Group 2.2 and 1.2

DISCRETE MATHEMATICS 2019/20



THE OXFORD MURDERS

"The Oxford Murders" is a 2008 film directed by Álex de la Iglesia. This thriller film is adapted from the novel "Crímenes imperceptibles" by Argentine mathematician and writer Guillermo Martínez, winner of the Planeta Argentina Prize 2003. He gained a PhD in mathematical logic at the University of Buenos Aires. After his degree in Argentina, he worked for two years in a postdoctoral position at the Mathematical Institute, Oxford.

In 1993, Martin (Elijah Wood), a US student at the University of Oxford, wants Arthur Seldom (John Hurt) as his thesis supervisor. He idolises Seldom and has learned all about him. He takes accommodation in Oxford at the house of Mrs. Eagleton (Anna Massey), an old friend of Seldom. Also in the house is her daughter, Beth (Julie Cox), who is her full-time caregiver, which she resents bitterly. After a lecture the two men enter the house together and find Martin's landlady murdered. Seldom tells the police that he had received a note with his friend's address marked as "the first of a series". As Seldom is an authority on logical series, he argues that a serial killer is using murder as a way to challenge his intelligence. It appears that the serial killer can be stopped only if somebody can decode the next symbol in the sequence. Martin is joined by Seldom on the quest to solve the cryptic clues.

In chapter 24 of the book a clue is given through a phone call: "The fourth of the sequence is the Tetraktys. Ten points in the blind triangle". The Tetraktys and the triangular numbers, in general, were very important to the Pythagoreans. Among the many relationships of numbers that have fascinated man are those that suggest (or were derived from) the arrangement of points representing numbers into series of geometrical figures. Such numbers, known as figurate or polygonal numbers, appeared in 15th-century arithmetic books and were probably known to the ancient Chinese; but they were of especial interest to the ancient Greek mathematicians. To the Pythagoreans (*c.* 500 bc), numbers were of paramount significance; everything could be explained by numbers, and numbers were invested with specific characteristics and personalities. Among other properties of numbers, the Pythagoreans recognized that numbers had "shapes." Thus, the triangular numbers were visualized as points or dots arranged in the shape of a triangle.

TASKS:

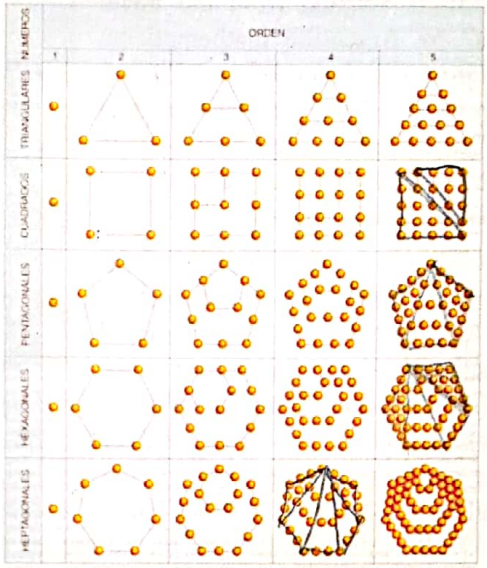
1. Give the definition of the triangular numbers and the first numbers of the sequence.
2. Give (and explain) the formulas for the triangular numbers:
   a) The explicit formula.
   b) The recurrence formula.
   c) Some properties.
3. Prove that triangular numbers cannot finish by 2, 4, 7 or 9.

Pictorial

¿? (4) Analyze the following statement: "If you take eight times a triangular number and afterwards you add one, it turns out a square".

(5) Polygonal numbers: A polygonal number is a number represented as dots or pebbles arranged in the shape of a regular polygon.

$\Omega = \{1, 2, \dots$



$\binom{n+1}{2} = T(n)$

a) Obtain the explicit formulas for the square, pentagonal and hexagonal numbers as well as the formula for any polygonal number.
b) Obtain the recurrence relationship for the polygonal numbers.
¿? c) Find some properties about these sequences of numbers.

$\Omega = \{1, 2, \dots, n+1\}$

One side → $\binom{n+1}{2}$

Subsets of two elements → $\binom{n+1}{2}$

$A(1) = \{B \subset \Omega : |B| = 2, 1 \in B\}$
$A(2) = \{B \subset \Omega : |B| = 2, 2 \in B, B \notin A(1)\}$
$\vdots$
$A(k) = \{B \subset \Omega : |B| = 2, k \in B, B \subset A(k-1)\}$

$A(n) = \dots$
$A(n+1) = \dots$

Other side

$A(k) = \{B \subset \Omega : |B| = 2, k \in B, B \notin A_n(k-1)\}$

$|A(k)| = n+1-k$

$A(1) = n$
$A(2) = n-1$
$\dots A(n) = 2$
$A = \dots$
$A(n+1) = \dots$