

# Recursion & Backtracking

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DI

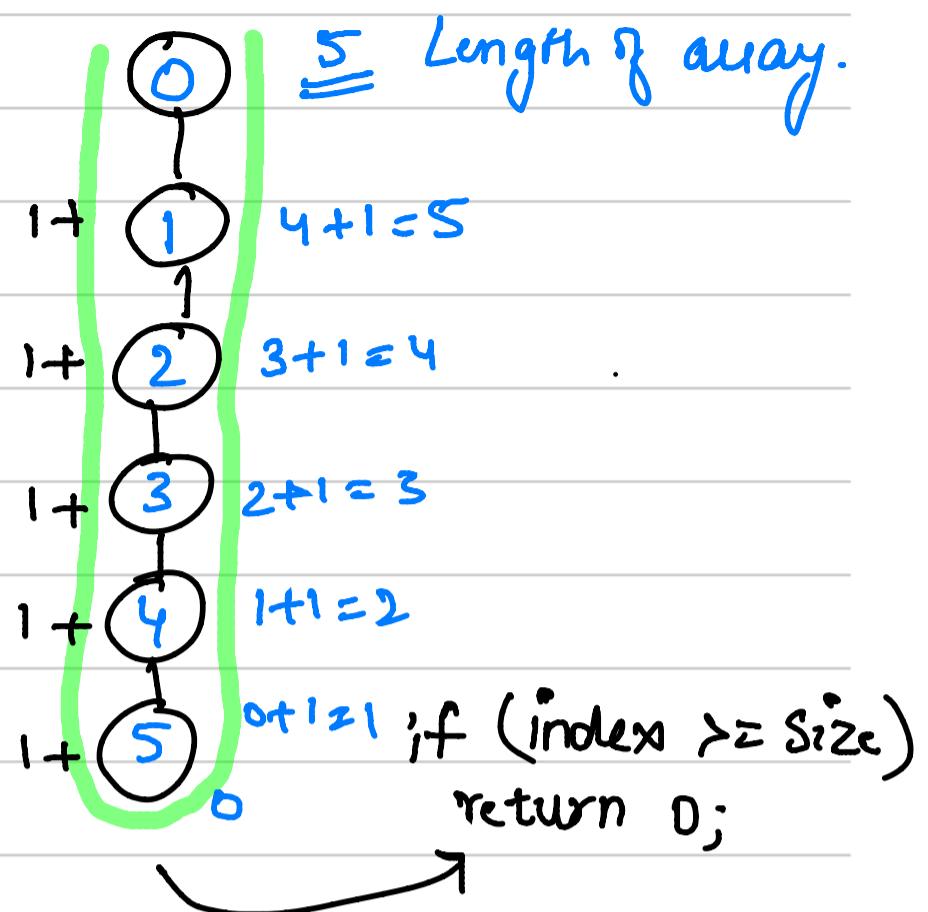
## Recursion

- ① Length of an array

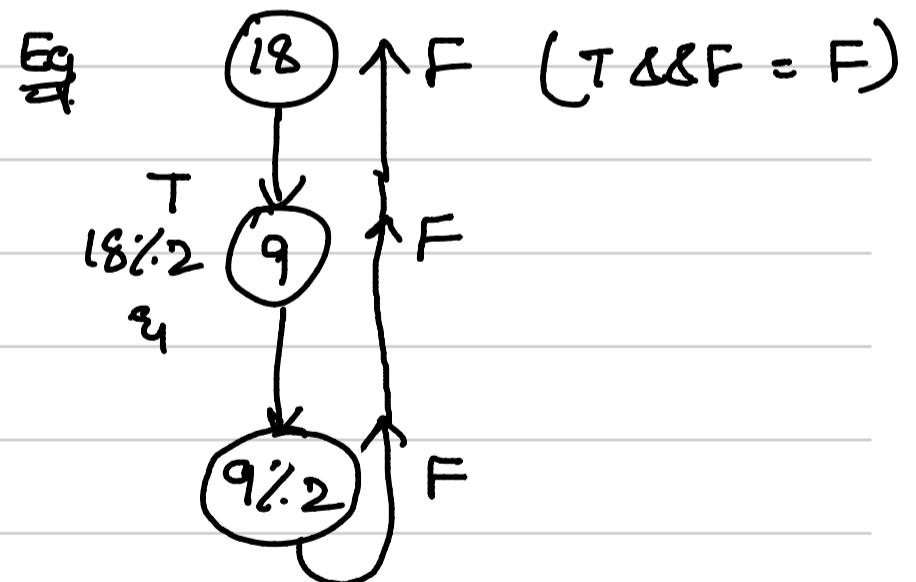
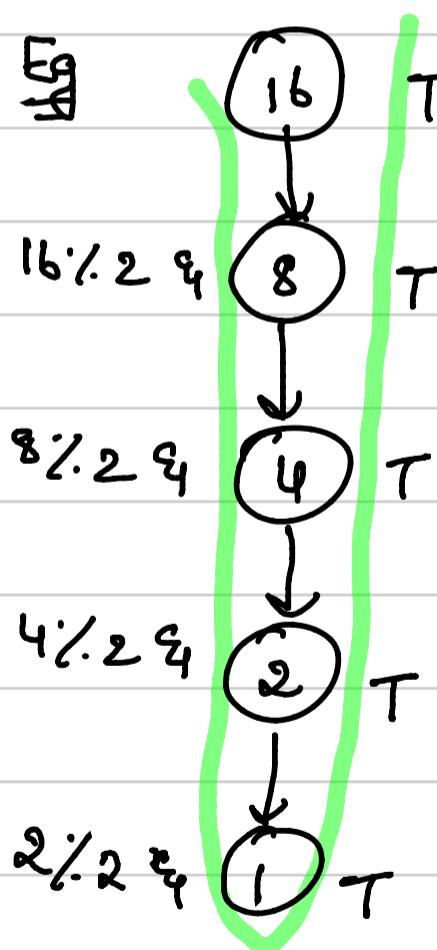
[20, 10, 40, 50, 30]  
0 1 2 3 4

$$TC = O(n)$$

$$SC = O(n).$$



- ① Power of 2  $\rightarrow 2^x = 2^0 \cdot 2^1 \cdot 2^2 \dots 2^n$ .



$$\underline{TC = O(\log_2 n)}$$

if 1 then  
return true.

## Power of 2

```
C++ ▾  
class Solution {  
public:  
    bool isPowerOfTwo(int n) {  
        if(n==1) return true; //need to write it first else it might c  
        if(n<=0 || n%2!=0) return false;  
        return isPowerOfTwo(n/2);  
    }  
};
```

## ② Power of 3

```
C++ ▾  
class Solution {  
public:  
    bool isPowerOfThree(int n) {  
        if(n==1) return true; //need to write it first else it might  
        if(n<=0 || n%3!=0) return false;  
        return isPowerOfThree(n/3);  
    }  
};
```

## ③ Power of 4

```
C++ ▾  
class Solution {  
public:  
    bool isPowerOfFour(int n) {  
        if(n==1) return true; //need to write it first else it might c  
        if(n<=0 || n%4!=0) return false;  
        return isPowerOfFour(n/4);  
    }  
};
```

## D2 Subsets

④ Given an integer array nums, generate all the subsets. (subsequences)

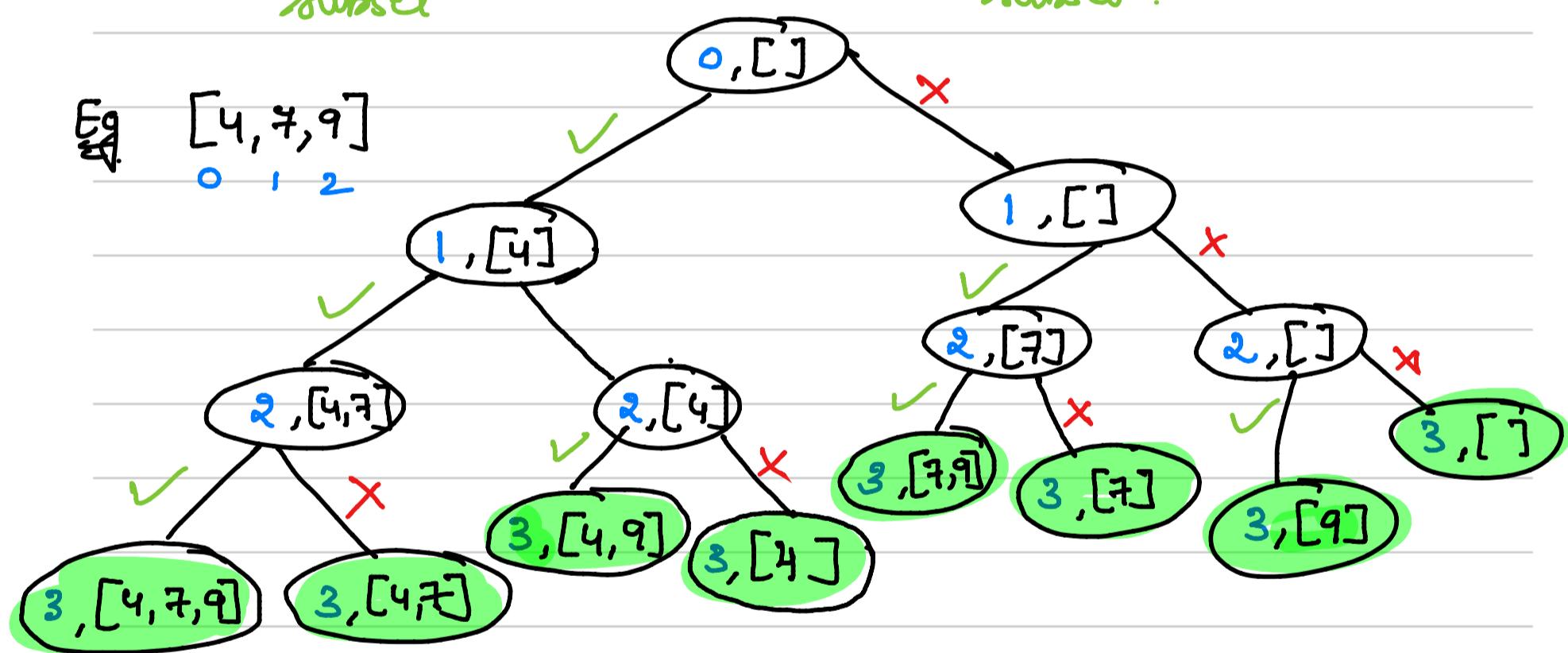
If size =  $n$  then no. of subsets =  $2^n$ .

Eg.  $\text{nums} = [1, 2, 3]$

$\text{Set} = [\underline{[ ]}, \underline{[1]}, \underline{[1, 2]}, \underline{[1, 3]}, \underline{[1, 2, 3]}, \underline{[2]}, \underline{[2, 3]}, \underline{[3]}]$

For every, element  
 can be a part of subset      cannot be a part of subset.

Eg.  $[4, 7, 9]$



$\Rightarrow [ [4, 7, 9], [4, 7], [4, 9], [4], [7, 9], [7] ]$   
 $[9], [ ] ]$ .

\* Once index is greater than or equal to size then store in result

$Tc = O(2^n) \rightarrow$  as there are 2 possibilities at every element.

$Sc \approx O(2^n)$

## Code

```
class Solution {
public:
    void generateAllSubsets(vector<int>&nums, int currentIndex, vector<int>&res, vector<vector<int>> &powerSet){
        // base condition
        if(currentIndex >= nums.size()){
            powerSet.push_back(res);
            return;
        }
        int currentVal = nums[currentIndex];
        res.push_back(currentVal);
        generateAllSubsets(nums, currentIndex+1, res,powerSet);

        // remove the currentVal (not considering)
        res.pop_back();
        generateAllSubsets(nums, currentIndex+1, res,powerSet);
    }

    vector<vector<int>> subsets(vector<int>& nums) {
        vector<vector<int>> powerSet;
        vector<int> res;
        generateAllSubsets(nums, 0, res, powerSet);
        return powerSet;
    }
};
```

⑤ Combination sum :-  $\text{nums} = [2, 3, 5]$  target = 8  
 $\underset{0, 1, 2}{}$

Sol.  $[[2, 2, 2, 2], [2, 3, 3], [3, 5]]$

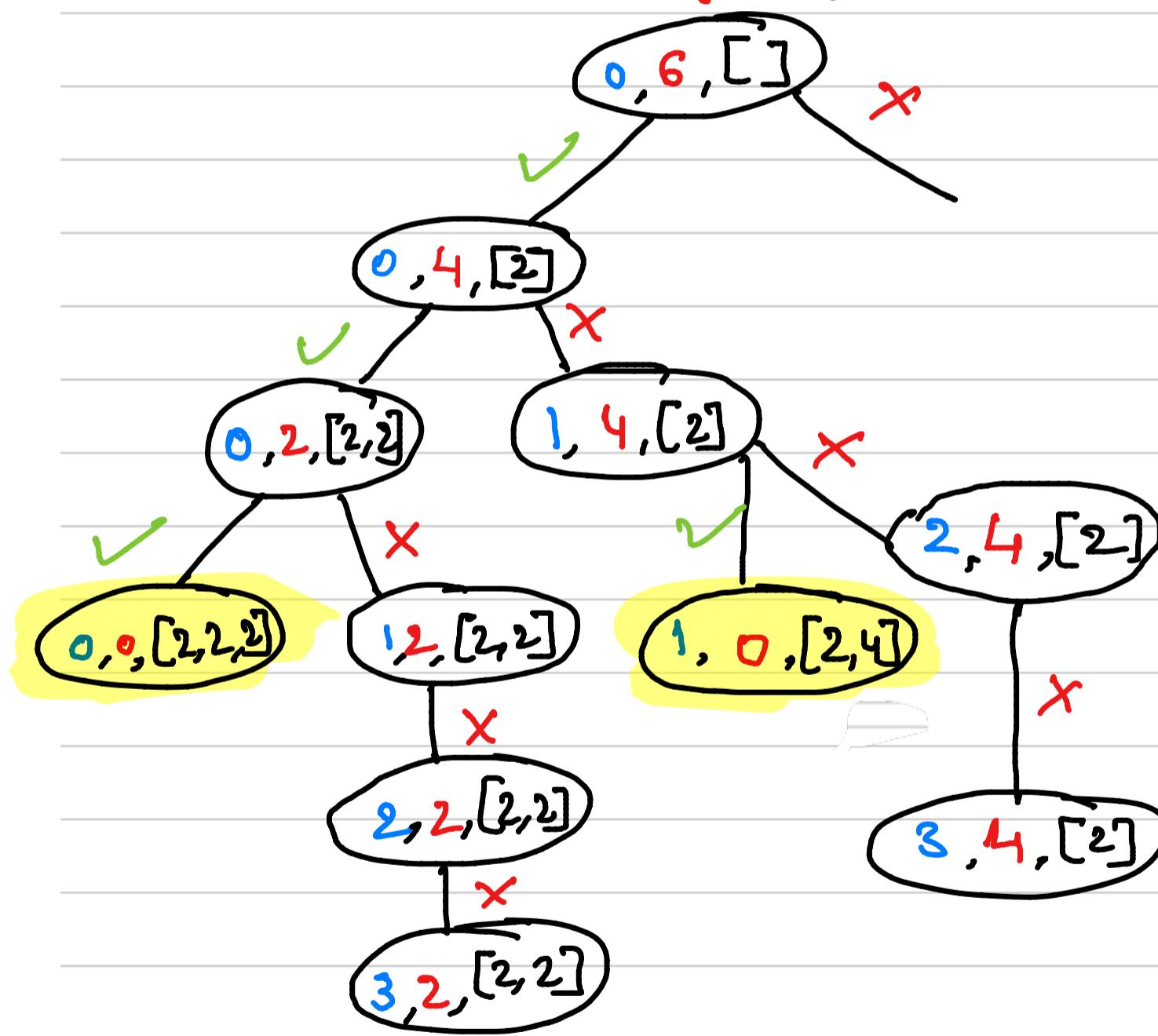
Eg  $[2, 4, 5]$   
 $\underset{0, 1, 2}{}$   
target = 6

For every index  $\Rightarrow$  CI, t

CI, t - nums[CI]

CI+1, t

Index Target Subset



$\Rightarrow [[2, 2, 2], [2, 4]]$

\* Store the result when target sum = 0

## Code →

```
class Solution {
public:
    void totalWays(vector<int>& candidates, int target, int curr, vector<vector<int>>& res, vector<int>& aux ) {
        if(curr==candidates.size()){
            if(target==0){
                res.push_back(aux);
            }
            return;
        }
        // feasible only if curr value is less than the target
        if(candidates[curr]<=target){
            aux.push_back(candidates[curr]);
            totalWays(candidates, target-candidates[curr], curr+1, res, aux);
            aux.pop_back();
        }
        // back-tracking
        totalWays(candidates, target, curr+1, res, aux);
    }

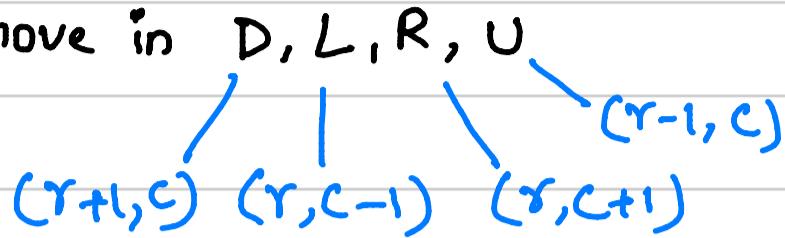
    vector<vector<int>> combinationSum(vector<int>& candidates, int target) {
        vector<vector<int>> res;
        vector<int> aux;
        totalWays(candidates, target, 0, res, aux);
        return res;
    }
};
```

D3

## ⑥ Rat in a maze

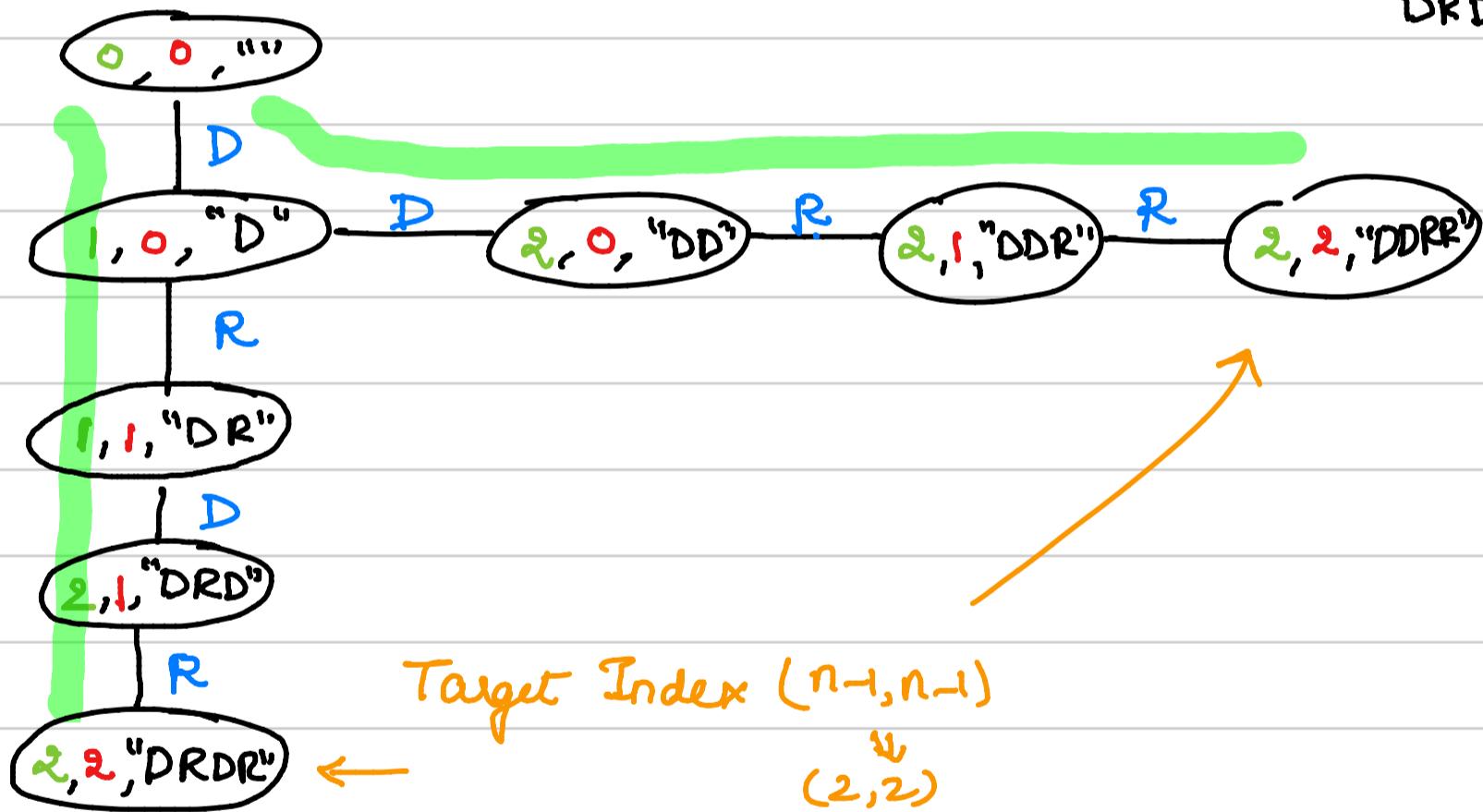
Generate all the ways to go from  $(0, 0)$  to  $(n-1, n-1)$

- \* At any cell we can move in  $D, L, R, U$



Eg  $n=3$   $\begin{bmatrix} 0 & 1 & 2 \\ 1, 0, 0 \\ 1, 1, 0 \\ 2, 1, 1 \end{bmatrix} \Rightarrow DRDR, DDRR$

$N=4$   $\begin{bmatrix} 0 & 1, 0, 0, 0 \\ 1, 1, 0, 1 \\ 1, 1, 0, 0 \\ 0, 1, 1, 1 \end{bmatrix}$   
[DDRD, DRDR]



- \* Before making any call from cell change its state
- \* while returning, **UNDO** the changes made (Backtracking.)

Code →

```
class Solution{
public:
void allPaths(int row, int col, int n, vector<vector<int>>&m, string ans, vector<string>&res){

    if(row<0 || row>=n || col<0 || col>=n || m[row][col]==0){
        return;
    }

    if(row==n-1 && col==n-1){
        res.push_back(ans);
        return;
    }

    m[row][col]= 0;
    allPaths(row+1, col,n,m,ans+"D",res);
    allPaths(row, col-1,n,m,ans+"L",res);
    allPaths(row, col+1,n,m,ans+"R",res);
    allPaths(row-1, col,n,m,ans+"U",res);
    m[row][col] = 1;

    return;
}

vector<string> findPath(vector<vector<int>> &m, int n) {
    string ans = "";
    vector<string> res;
    allPaths(0,0,n,m,ans,res);
    sort(res.begin(), res.end());
    return res;
}
};
```

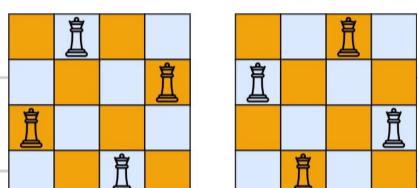
# N-queens

D4

return all configurations

- ⑦ If given  $n$ , then we should place  $n$ -queens in  $N \times N$  matrix, such that  $\text{No 2 queens} \rightarrow \text{share same row, column, diagonal}$

Eg  $n = 4$



Initially	0 1 2 3	$X_R \rightarrow \text{Bad Row}$
0	[., ., ., .]	$X_C \rightarrow \text{Bad column}$
1	[., ., ., .]	$X_D \rightarrow \text{Bad diagonal}$
2	[., ., ., .]	$X_N \rightarrow \text{Not possible}$
3	[., ., ., .]	

Step → Start from (0,0)

①

Column pos	0 1 2 3	0 1 2 3
0	[Q, ., ., .]	✓
1	[., ., ., .]	$X_R$ $X_D$ ✓ $X_D$
2	[., ., ., .]	
3	[., ., ., .]	

②

Column pos	0 1 2 3	0 1 2 3
0	[Q, ., ., .]	✓
1	[., ., Q, .]	$X_R$ $X_D$ ✓ $X_D$
2	[., ., ., .]	$X_R$ $X_D$ $X_C$ $X_D$
3	[., ., ., .]	↳ this says (1,2) is a bad config & so is (0,0) ∴ Backtrack.

③

Column pos	0 1 2 3	0 1 2 3
0	[., ., Q, .]	$X_N$ ✓
1	[., ., ., .]	$X_D$ $X_C$ $X_D$ ✓
2	[., ., ., .]	
3	[., ., ., .]	

④

Column pos	0 1 2 3	0 1 2 3
0	[., ., Q, .]	$X_N$ ✓
1	[., ., ., Q]	$X_D$ $X_C$ $X_D$ ✓
2	[., ., ., .]	✓ $X_C$ $X_D$ $X_C$
3	[., ., ., .]	

⑤

Column pos	0 1 2 3	0 1 2 3
0	[., ., Q, .]	$X_N$ ✓
1	[., ., ., Q]	$X_D$ $X_C$ $X_D$ ✓
2	[Q, ., ., .]	✓ $X_C$ $X_D$ $X_C$
3	[., ., ., .]	$X_C$ $X_C$ ✓ $X_C$

⑥

Column pos	0 1 2 3	0 1 2 3
0	[., ., Q, .]	$X_N$ ✓
1	[., ., ., Q]	$X_D$ $X_C$ $X_D$ ✓
2	[Q, ., ., .]	✓ $X_C$ $X_D$ $X_C$
3	[., ., ., Q]	$X_C$ $X_C$ ✓ $X_C$

↑ final result ∴ store & backtrack for other config.

## code →

```
● ○ ●
1 class Solution {
2 public:
3
4     bool valid_row(int curr_row, vector<vector<char>>&grid, int n){
5         for(int i = 0; i < n; i++){
6             if(grid[curr_row][i]=='Q')
7                 return false;
8         }
9         return true;
10    }
11
12    bool valid_col(int curr_col, vector<vector<char>>&grid, int n){
13        for(int i = 0; i < n; i++){
14            if(grid[i][curr_col]=='Q')
15                return false;
16        }
17        return true;
18    }
19
20    bool valid_diagonal(vector<vector<char>>&grid, int curr_row, int curr_col, int n){
21        int i = curr_row;
22        int j = curr_col;
23        while(i>=0 && j>=0){           // Top-left diagonal
24            if(grid[i][j]=='Q')
25                return false;
26            i--; j--;
27        }
28
29        i = curr_row;
30        j = curr_col;
31        while(i>=0 && j<n){          // Top-right diagonal
32            if(grid[i][j]=='Q')
33                return false;
34            i--; j++;
35        }
36
37        i = curr_row;
38        j = curr_col;
39        while(i<n && j>=0){          // Bottom-left diagonal
40            if(grid[i][j]=='Q')
41                return false;
42            i++; j--;
43        }
44
45        i = curr_row;
46        j = curr_col;
47        while(i<n && j<n){          // Bottom-right diagonal
48            if(grid[i][j]=='Q')
49                return false;
50            i++; j++;
51        }
52
53        return true;
54    }
55}
```



```
1  bool isValid(vector<vector<char>>&grid, int curr_row, int curr_col, int n){
2      return valid_row(curr_row, grid, n) && valid_col(curr_col, grid, n) && valid_diagonal(grid, curr_row, curr_col, n);
3  }
4
5  // Function to convert grid char to strings
6  vector<string> populate(vector<vector<char>>&grid, int n){
7      vector<string> result;
8      for(int i = 0; i<n; i++){
9          string temp = "";
10         for(int j=0; j<n; j++){
11             temp += grid[i][j];
12         }
13         result.push_back(temp);
14     }
15     return result;
16 }
17
18 void solve(vector<vector<char>>&grid, int curr_row, int n, vector<vector<string>>&ans){
19     if(curr_row==n){
20         vector<string> temp = populate(grid,n);
21         ans.push_back(temp);
22         return;
23     }
24     for(int curr_col=0; curr_col < n; curr_col++){
25         if(isValid(grid, curr_row, curr_col,n)){
26             grid[curr_row][curr_col] = 'Q';
27             solve(grid, curr_row+1, n, ans);
28             grid[curr_row][curr_col] = '.';
29         }
30     }
31 }
32
33 vector<vector<string>> solveNQueens(int n) {
34     vector<vector<string>> ans;
35     vector<vector<char>>grid(n, vector<char>(n, '.'));
36     solve(grid, 0, n, ans);
37     return ans;
38 }
39 };
```

### 13 N-Queens II

↳ need to find the total number of possibilities

\* everything is same as in N-Queens but return the no. of elements in the result.

D5

## Sudoku Solver

- ⑧ A sudoku solution must satisfy all of the following rules:

- 1 Each of the digits 1-9 must occur exactly once in each row.
- 2 Each of the digits 1-9 must occur exactly once in each column.
- 3 Each of the digits 1-9 must occur exactly once in each of the 9 3x3 sub-boxes of the grid.

Eg

5	3		7					
6		1	9	5				
9	8				6			
8			6					3
4		8	3					1
7		2						6
6				2	8			
		4	1	9				5
		8		7	9			



5	3	4	6	7	8	9	1	2
6	7	2	1	9	5	3	4	8
1	9	8	3	4	2	5	6	7
8	5	9	7	6	1	4	2	3
4	2	6	8	5	3	7	9	1
7	1	3	9	2	4	8	5	6
9	6	1	5	3	7	2	8	4
2	8	7	4	1	9	6	3	5
3	4	5	2	8	6	1	7	9

### Algorithm

① Let  $(i, j)$  be an empty cell

② for  $i$  from 1 to 9 :

if  $i$  is not in row, column,  $3 \times 3$  sub-grid :

    ③  $\text{grid}(r, c) = i$

    ④ recursively fill remaining empty cells.

    ⑤ if recursion is successful :

        return true

    ⑥  $\text{grid}(r, c) = \cdot$  (backtracking)

⑦ return false

## Code



```
1 class Solution {
2 public:
3     bool valid_row(vector<vector<char>>&board, int currRow, int currVal){
4         for(int i=0; i<9; i++){
5             if(board[currRow][i]==currVal+'0'){
6                 return false;
7             }
8         }
9         return true;
10    }
11
12    bool valid_col(vector<vector<char>>&board, int currCol, int currVal){
13        for(int i=0; i<9; i++){
14            if(board[i][currCol]==currVal+'0'){
15                return false;
16            }
17        }
18        return true;
19    }
20
21    bool valid_grid(vector<vector<char>>&board, int currRow, int currCol, int currVal){
22        int x = 3*(currRow/3);
23        int y = 3*(currCol/3);
24        for(int i=0; i<3; i++){
25            for(int j=0; j<3; j++){
26                if(board[x+i][y+j]== currVal+'0'){
27                    return false;
28                }
29            }
30        }
31        return true;
32    }
33
34    bool isValidCell(vector<vector<char>>&board, int currRow, int currCol, int currVal){
35        return valid_row(board, currRow, currVal) && valid_col(board, currCol, currVal) &&
36        valid_grid(board, currRow, currCol, currVal);
37    }
38
39
```



```
1
2     bool sudokuSolver(vector<vector<char>>&board, int currRow, int currCol){
3         if(currRow==9)
4             return true;
5
6         int nextRow = 0;
7         int nextCol = 0;
8
9         // find next possible row n column
10        if(currCol==8){
11            nextRow = currRow+1;
12            nextCol = 0;
13        } else {
14            nextRow = currRow;
15            nextCol = currCol+1;
16        }
17
18        // if not filled then call
19        if(board[currRow][currCol]!='.'){
20            return sudokuSolver(board, nextRow, nextCol);
21        }
22
23        // try all possibilities from 1 to 9 numbers
24        for(int currVal=1; currVal<10; currVal++){
25
26            // if valid then make the change
27            if(isValidCell(board, currRow, currCol, currVal)){
28                board[currRow][currCol] = '0'+currVal;
29
30                // if already solved then return true directly
31                if(sudokuSolver(board, nextRow, nextCol)==true)
32                    return true;
33
34                // backtracking
35                board[currRow][currCol] = '.';
36            }
37        }
38
39        return false;
40    }
41    void solveSudoku(vector<vector<char>>& board) {
42        sudokuSolver(board, 0, 0);
43    }
44};
```

D6

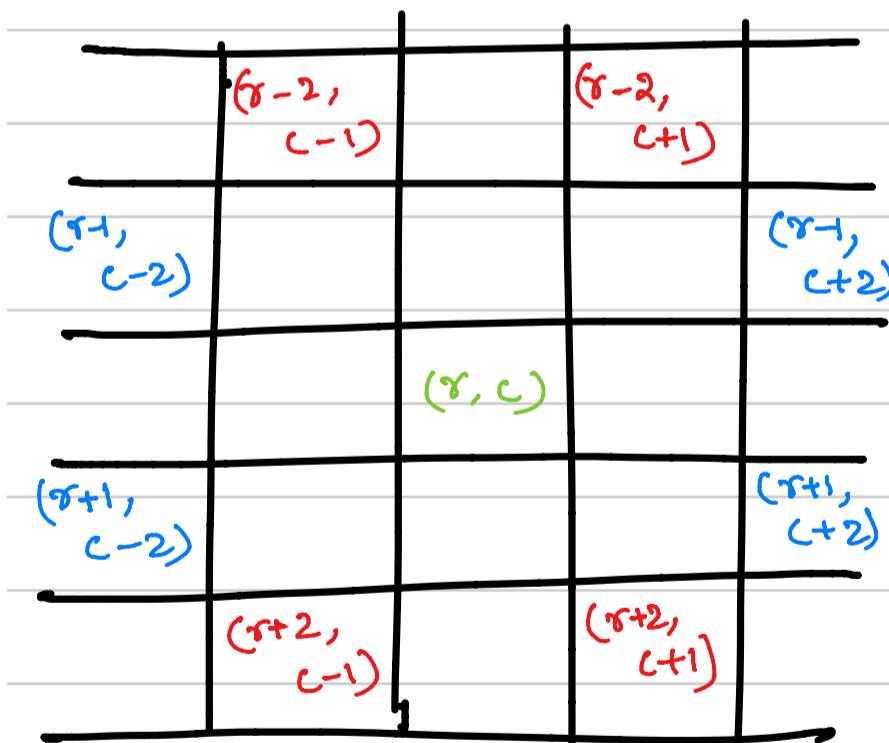
## Knight's tour problem.

- ⑨ Given an  $n \times n$  board, print the order of each cell in which they are visited. ( $n >= 8$ )

For  $n = 8$ , the result is

0	59	38	33	30	17	8	63
37	34	31	60	9	62	29	16
58	1	36	39	32	27	18	7
35	48	41	26	61	10	15	28
42	57	2	49	40	23	6	19
47	50	45	54	25	20	11	14
56	43	52	3	22	13	24	5
51	46	55	44	53	4	21	12

Sol) For every cell  $(r, c)$  we have 8 possibilities,



- $(r-2, c-1)$
- $(r-2, c+1)$
- $(r+2, c-1)$
- $(r+2, c+1)$
- $(r-1, c-2)$
- $(r-1, c+2)$
- $(r+1, c-2)$
- $(r+1, c+2)$

- the rest is similar to rat-in-a-maze problem except that the value will be incremented by 1.

## Code →

```
● ● ●

1 #include <bits/stdc++.h>
2 using namespace std;
3
4 void display(vector<vector<int>>&grid){
5     for(auto i: grid){
6         for(auto j:i){
7             cout<<j<<" ";
8         }
9         cout<<"\n";
10    }
11 }
12
13 void KnightTour(vector<vector<int>> &grid, int currRow, int currCol,
14                           int upcomingVal, int n){
15     if(upcomingVal==n*n){
16         display(grid);
17         cout<<"\n";
18         return;
19     }
20
21     if(currRow<0 || currRow>=n || currCol<0 || currCol>=n
22         || grid[currRow][currCol]!=0){
23         return;
24     }
25
26     grid[currRow][currCol] = upcomingVal;
27
28     KnightTour(grid, currRow-2, currCol-1, upcomingVal+1, n);
29     KnightTour(grid, currRow-2, currCol+1, upcomingVal+1, n);
30     KnightTour(grid, currRow+2, currCol-1, upcomingVal+1, n);
31     KnightTour(grid, currRow+2, currCol+1, upcomingVal+1, n);
32     KnightTour(grid, currRow-1, currCol-2, upcomingVal+1, n);
33     KnightTour(grid, currRow-1, currCol+2, upcomingVal+1, n);
34     KnightTour(grid, currRow+1, currCol-2, upcomingVal+1, n);
35     KnightTour(grid, currRow+1, currCol+2, upcomingVal+1, n);
36
37     grid[currRow][currCol] = 0;
38     return;
39 }
40
41 int main() {
42     int n;
43     cin>>n;
44     vector<vector<int>>grid(n, vector<int>(n, 0));
45     KnightTour(grid, 0, 0, 1, n);
46     return 0;
47 }
48
```

## 10 Letter combination of a phone number

Eg digits = "23" → abc  
 0 1 2 → def  
 0 1 2

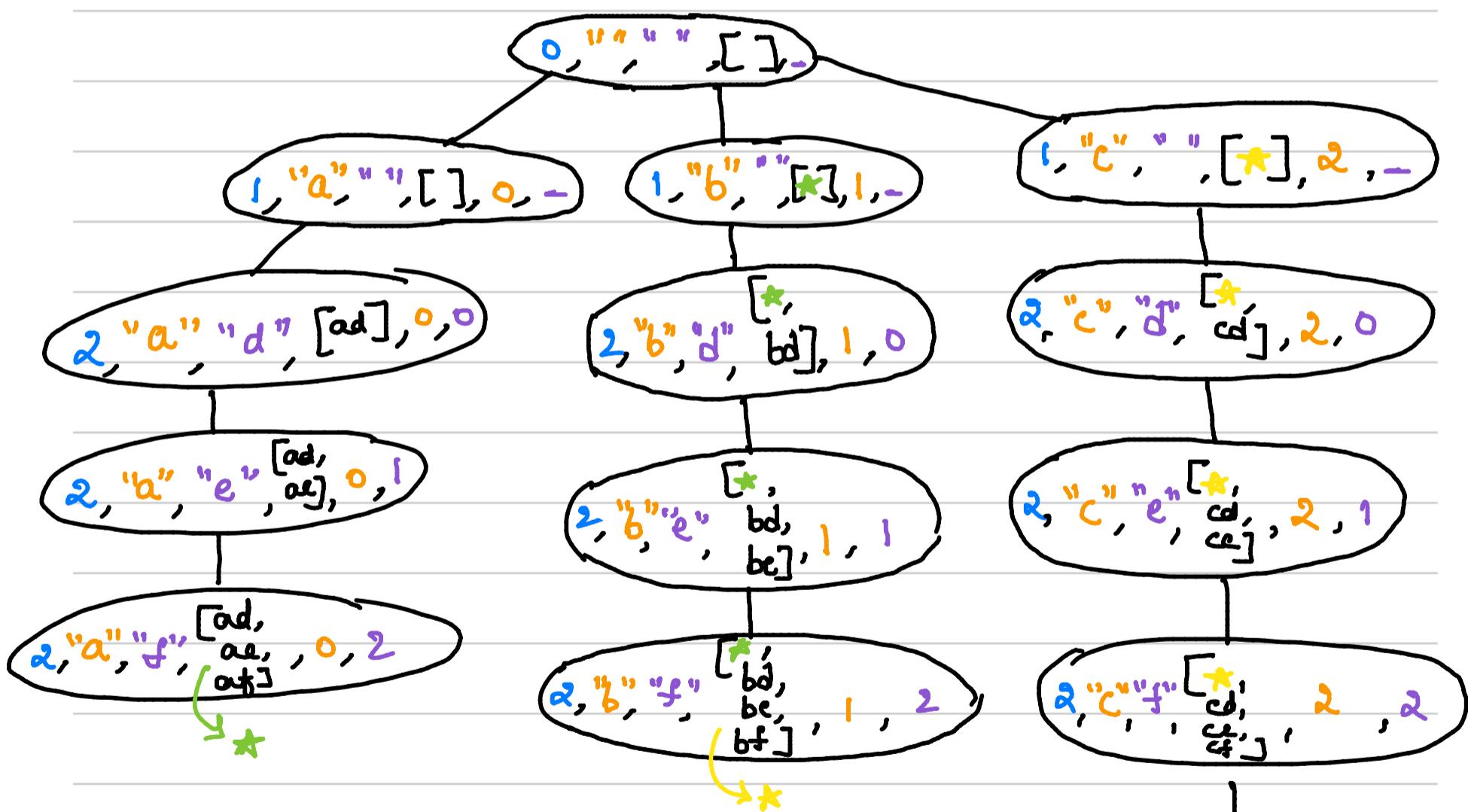
"2" → abc  
 0 1 2

"3" → def  
 0 1 2



- \* Initially create a map for numbers & their alphabets
- \* Then for each index in a string find all possibilities

//



final result.

[ad, ae, af, bd, be, bf, cd, ce, cf] ←

## Code →

```
1 class Solution {
2 public:
3     void findAll( map<char,string> &mapper, string digits,
4                 vector<string> &ans, string &s, int currentIndex){
5
6         if(currentIndex>=digits.length()){
7             ans.push_back(s);
8             return;
9         }
10
11         char currNum = digits[currentIndex];
12         string alpha = mapper[currNum];
13
14         for(int i=0; i<alpha.size(); i++){
15             s.push_back(alpha[i]);
16             findAll(mapper, digits, ans, s, currentIndex+1);
17             s.pop_back();
18         }
19         return;
20     }
21
22     vector<string> letterCombinations(string digits) {
23
24         map<char,string> mapper{
25             {'1', ""}, // 1 has no letters
26             {'2', "abc"}, // 2 has 3 letters
27             {'3', "def"}, // 3 has 3 letters
28             {'4', "ghi"}, // 4 has 3 letters
29             {'5', "jkl"}, // 5 has 3 letters
30             {'6', "mno"}, // 6 has 3 letters
31             {'7', "pqrs"}, // 7 has 4 letters
32             {'8', "tuv"}, // 8 has 3 letters
33             {'9', "wxyz"} // 9 has 4 letters
34         };
35         string s = "";
36         vector<string> ans;
37
38         // edge case
39         if(digits.size()==0){
40             return ans;
41         }
42         // else generate all possibilities
43         findAll(mapper, digits, ans, s, 0);
44         return ans;
45
46     }
47 }
```

11) Subsets → Same as subsets but no duplicates.

① using set<int>

Code →



```
1 class Solution {
2 public:
3     void allsubs(vector<int>& nums,int curr,
4                 vector<int>&ds, set<vector<int>>&ans)
5     {
6         if(curr>=nums.size()){
7             ans.insert(ds);
8             return;
9         }
10        int currval = nums[curr];
11        ds.push_back(currval);
12        allsubs(nums,curr+1,ds,ans);
13
14        // removing currentVal (not considering)
15        ds.pop_back();
16        allsubs(nums,curr+1,ds,ans);
17    }
18
19    vector<vector<int>> subsetsWithDup(vector<int>& nums) {
20        set<vector<int>>ans;
21        vector<int>vec;
22        sort(nums.begin(),nums.end());
23        allsubs(nums,0,vec,ans);
24        vector<vector<int>> res{ans.begin(), ans.end()};
25        return res;
26    }
27};
```

② without using lets

code →

```
● ● ●

1 class Solution {
2 public:
3     void allsubs(vector<int> &nums, int curr, vector<int> &ds,
4                  vector<vector<int>>& res){
5         res.push_back(ds); // storing initial answers
6         for(int i=curr; i<nums.size(); i++){
7             if(i>curr && nums[i]==nums[i-1]) continue; // avoiding duplicates
8             ds.push_back(nums[i]);
9             allsubs(nums, i+1, ds, res);
10            ds.pop_back();
11        }
12        return;
13    }
14
15    vector<vector<int>> subsetsWithDup(vector<int>& nums) {
16        vector<vector<int>> res;
17        vector<int> ds;
18        sort(nums.begin(), nums.end());
19        allsubs(nums, 0, ds, res);
20        return res;
21    }
22 };
23
24
```

12

## Combinational sum - II

→ same as Combinational Sum but no duplicates

Code →

```
● ● ●  
1 class Solution {  
2 public:  
3     void findAll(vector<int>& candidates, int target, int idx,  
4                     vector<vector<int>> &ans, vector<int> &ds){  
5  
6         if(target==0){  
7             ans.push_back(ds);  
8             return;  
9         }  
10  
11         for(int i = idx; i<candidates.size(); i++){  
12  
13             // avoid duplicates  
14             if(i>idx && candidates[i]==candidates[i-1]) continue;  
15  
16             if(candidates[idx]<=target){  
17                 ds.push_back(candidates[i]);  
18                 findAll(candidates, target-candidates[i], i+1, ans, ds);  
19                 ds.pop_back();  
20             }  
21         }  
22     }  
23  
24     vector<vector<int>> combinationSum2(vector<int>& candidates,  
25                                         int target){  
26         vector<vector<int>> ans;  
27         sort(candidates.begin(), candidates.end());  
28         vector<int> ds;  
29         findAll(candidates, target, 0, ans, ds);  
30         return ans;  
31     }  
32 };
```

### ⑬ N-Queens II

↳ need to find the total number of possibilities

- \* everything is same as in N-Queens but return the no. of elements in the result.