Permutation and Combination - 1

Career Launcher

QA - 30

CEX-Q-0231/18

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Number of Questions : 30

Fundamental Principle of Counting

1. A thief is standing outside of a room, having 2 doors and 3 windows. In how many ways the thief can
   A. Enter the room from a window and exit from a door?
   B. Steal the only TV present inside the room and come outside again?

2. A new flag is to be designed with six vertical stripes using some or all of the colours yellow, green, blue and red. Then, the number of ways this can be done so that no two adjacent stripes have the same colour is

   [CAT – 2004]

   (1) 12 × 81 (2) 16 × 192
   (3) 20 × 125 (4) 24 × 216

3. How many five digit numbers are there such that the digits at hundreds' place, unit place and ten-thousand's place form a Geometric Progression, not necessarily in the same order?

   (1) 3300 (2) 2700
   (3) 1300 (4) 1700

4. If I write all the numbers from 100 to 1000, for how many times I will have to write the digit 2?

   (1) 100 (2) 280
   (3) 900 (4) 480

5. Let n be the number of ways in which 5 men and 6 women can stand in a queue such that all the women stand consecutively. Let m be the number of ways in which the same 11 persons can stand in a queue such that exactly 5 women stand consecutively. The value of $\frac{m}{n}$ is

   [PGDBA – 2018]

   (1) 5 (2) 6
   (3) $\frac{5}{6}$ (4) $\frac{6}{5}$

6. 5 boys and 5 girls are to be arranged in a row. Find the difference between the number of possible arrangements when no two girls are sitting together, and when boys and girls are alternate.

   (1) 0 (2) $(5!)^2$
   (3) $4 \times (5!)^2$ (4) 5!

7. A palindrome is a number that reads the same from left to right as it does from right to left. How many six digit palindromes are there which are even?

   (1) 900 (2) 500
   (3) 9×10^5 (4) 400

8. In a staircase, there are 4 steps. A person can take 1 step, 2 steps, 3 steps or 4 steps. In how many ways can he reach the top?

   (1) 8 (2) 6
   (3) 4 (4) 3

9. There are 5 Eclairs, 4 KitKat, 3 Melodies and 7 more chocolates in which all are distinct. Find the number of ways in which a child can select one or more chocolates?
(1) 7679 (2) 15360
(3) 7680 (4) 15359

10. How many four digit numbers can be formed with the digits 0, 1, 2 and 7 so that at least one of the digits is repeated?
(1) 192 (2) 96
(3) 144 (4) 174

11. In how many ways can 6 identical balls be placed in 3 identical baskets such that no box is empty?
(1) 2 (2) 3
(3) 6 (4) 10

12. There are 9 elements given as AABBBCCCC. In how many ways any 7 elements can be chosen?
(1) 20 (2) 11
(3) 10 (4) 6

13. There are 4 married couples in a club. The number of ways of choosing a committee of 3 members so that no couple appears in the committee is
(1) 4 (2) 8
(3) 16 (4) 32

14. How many four digit numbers which are divisible by 6, can be formed using the digits 0, 2, 3, 4, 6 such that no digit is used more than once and 0 does not occur in the left-most position? **[CAT - 2017]**

15. How many distinct 5 × 5 tables are there such that each entry is either 0 or 1 and each row sum and each column sum is 4? **[PGDBA – 2018]**
(1) 64 (2) 32
(3) 120 (4) 96

Permutations and Combinations

16. How many sets of two distinct numbers can be formed using the numbers between 0 and 180 (both inclusive) so that their average is 60?
(1) 180 (2) 60^3
(3) 60 (4) None of these

17. In how many ways can 8 distinct fruits (including Apple, Banana and Mango) be kept in a row such that apple always comes before banana, and banana always comes before mango?
(1) 8! – 7 (2) 5!
(3) $\frac{8!}{3!}$ (4) None of these

18. In how many different ways 6 players can be selected from 11 distinct players if
A. Two particular players are always selected?
B. Two particular players are never selected?

19. India and South Africa play one-day international cricket series until one team wins 4 matches. No match ends in a draw. Find in how many ways the series can be won?
(1) 24 (2) 35
(3) 70 (4) 120

20. There are 6 toys and 3 boxes, the toys are being placed in the boxes such that no box is empty. In how many ways this can be done if
A. toys are distinct and boxes are also distinct?
B. toys are distinct but boxes are identical?

21. In how many ways can three digit numbers be formed choosing the digits from 1, 1, 2, 3, 4?
(1) $\frac{{}^5P_3}{2!}$ (2) ${}^4P_3$
(3) 4^3 (4) ${}^4P_3 + {}^3C_1 \times \frac{3!}{2!}$

22. In how many ways can 8 letters be put in 8 differently addressed envelopes such that exactly 5 letters go into wrong envelopes?
(1) ${}^{8}C_{5} \times 44$ (2) 9!
(3) 44 (4) 5^5

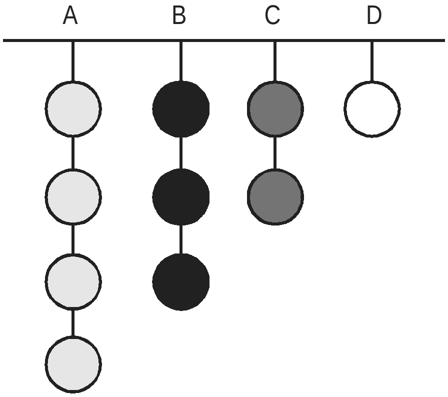
23. How many three digit integers are there in which the first digit (from left) is greater than the second digit and the second digit is greater than the third digit?

24. Five coins are tossed simultaneously. Find the total number of cases in which the number of heads will be greater than the number of tails.

25. A student is allowed to select at most n books from a collection of (2n+1) distinct books. If the total number of ways in which she can select at least one book is 255, find the value of n.
(1) 1 (2) 3
(3) 4 (4) 6

26. In how many ways can we distribute 92 identical chocolates among 12 boys such that each one gets at least two chocolates, but distinct number of chocolates?
(1) 2 × 12! (2) 12!
(3) ${}^{68}C_{11}$ (4) 12! × 3

27. The following figure shows the clay target in a shooting competition. All the discs in column A are Red, column B are Black, column C are green and column D are White. To win the competition, a shooter must break all 10 of these targets (using pistol and only 10 bullets) and in so doing must always break existing target at the bottom of a column. In how many different orders can the shooter break these 10 targets?



28. 12 persons are sitting in a row. Find the number of ways of selecting 5 persons such that no two of them are consecutive.
(1) 40 (2) 56
(3) 80 (4) None of these

29. In how many ways three numbers can be selected from a set of 43 consecutive natural numbers such that their sum will be a multiple of 3?
(1) 455 (2) 4123
(3) 1183 (4) 2940

30. What is(are) the possible number of way(s) to select three numbers from the first 15 natural numbers such that the selected numbers are in arithmetic progression?
(1) 1 (2) 5^2
(3) 7^2 (4) 15^2

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Answers and Explanations

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1	–	**2**	1	**3**	1	**4**	2	**5**	1	**6**	3	**7**	4	**8**	1	**9**	4	**10**	4
11	2	**12**	4	**13**	4	**14**	–	**15**	3	**16**	3	**17**	3	**18**	–	**19**	3	**20**	–
21	4	**22**	1	**23**	–	**24**	–	**25**	3	**26**	1	**27**	–	**28**	2	**29**	2	**30**	3

1. A. Since, the thief is standing outside the room, he can enter into the room from a window in 3 ways and come out from a door in 2 ways.
i.e. total (3 × 2) = 6 ways.
B. The thief can enter the room in 5 ways, and come out in 5 ways (from a widow or from a door)
Thus, 5 × 5 = 25 ways.

2. 1 The first strip can be of any of the four colours, The 2nd can be of any colour except that of the first (i.e. 3). Similarly, each subsequent strip can be of any colour except that of the preceding strip i.e. 3.
Hence, the number of ways = $4 \times 3^5 = 12 \times 81$.

3. 1 Number is of the form $a_1\ a_2\ a_3\ a_4\ a_5$, where $a_1 \neq 0$
Possible geometric progressions are
(1, 1, 1), (2, 2, 2), (3, 3, 3) ... (9, 9, 9) and
(1, 2, 4), (1, 3, 9), (2, 4, 8), (4, 6, 9)
$\therefore$ Total number of ways = 9 × (1 × 10 × 1 × 10 × 1) + 4 × (3! × 10 × 10) = 3300.

4. 2 Here, we need to find the number of times we write 2 while writing all the three digit numbers.

1st	2nd	3rd

If we fix 2 in the first place, then 2nd and 3rd places can be filled in 10 ways each.
i.e. 10 × 10 = 100 times
If we fix 2 in the second place, then 1st and 3rd places can be filled in 9 and 10 ways respectively.
i.e. 9 × 10 = 90 times, and if we fix 2 in the third place, then 1st and 2nd places can be filled in 9 and 10 ways respectively.
i.e. 9 × 10 = 90 times.
Thus, total 100 + 90 + 90 = 280 times.

5. 1 We will make a graph of 6 women in which all of them are arranged in consecutive.
Hence, n = 6! × 6! (since the women can be arranged in 6! ways inside their group).
In second case, we have to keep exactly 5 women consecutively. So we will first select 5 women out of 6 women. In $^6C_5$ = 6 ways and women in group can be arranged in 5! ways.
Hence, a favourable case can be as
$\_\,m_1\,\_\,m_2$ 5 women $m_3\,\_\,m_4\,\_\,m_5\,\_$
And the sixth women can be seated in 5 ways.
So, m = 6 × 5! × 6! × 5 = 6! × 6! × 5
Hence, $\frac{m}{n} = 5.$

6. 3 **Case I:** When boys and girls are alternate
One of the possible arrangements is shown below
$B_1G_1B_2G_2B_3G_3B_4G_4B_5G_5$
The boys and the girls can be arranged in 5! ways each i.e. total 5! × 5! ways.
But the arrangement can also start with a girl at first position.
Thus, total 2 × 5! × 5! ways.
Case II: When no two girls are together
We will fix the boys' position first
$B_1 \quad B_2 \quad B_3 \quad B_4 \quad B_5$
The girls can be seated in $^6C_5$ × 5! ways.
Whereas, the boys can be arranged in 5! ways.
Thus, total = $^6C_5$ × 5! × 5! = 6 × 5! × 5! ways.
Hence, required difference = 6 × 5! × 5! – 2 × 5! × 5!
= 4 × 5! × 5!

7. 4 We need to select digits only for unit's, ten's and hundred's places. At remaining three places these digits will be repeated. Unit's place can be filled in 4 ways (because 0 cannot come at unit's place) and ten's and hundred's places can be filled in 10 ways each.
$\therefore$ Total number of ways = 10 × 10 × 4 = 400.

8. 1 If the person uses 1 step at a time, then 1 way.
If the person uses 2 steps at a time, then 1 way.
If the person uses 2, 1, 1 step, then $\frac{3!}{2!} = 3$ ways.
If the person uses 3, 1 step, then 2 ways.
If the person uses 4 steps, then 1 way.
i.e. total = 1 + 1 + 3 + 2 + 1 = 8 ways.

9. 4 The number of ways in which the child can choose or not choose an Eclairs = 5 + 1 = 6.
Similarly, for KitKat and Melodies, there are 5 and 4 ways available.
Now, for 7 distinct chocolates, there would be 2 ways for each.
Thus, total no. of ways in which the person can choose one or more chocolates = 6 × 5 × 4 × 2^7 – 1 = 15359

10. 4 Total four-digit numbers that can be formed with 0, 1, 2, 7 = 3 × 4 × 4 × 4 = 192
Out of these, in 3 × 3 × 2 × 1 = 18 numbers, no digit is repeated.
⇒ In remaining 192 – 18 = 174 numbers, we will have at least one digit repeated.

11. 2 Since there should not be any empty basket, this is the case of partitions.
i.e. 6 can be divided in three parts as
(1 + 2 + 3), (2 + 2 + 2) and (1 + 1 + 4)
Thus, there are 3 ways to fill the baskets with balls.

12. 4 Selecting 7 elements is as good as rejecting 2 elements. So, we can reject 2 elements as AA or BB or CC or AB or BC or AC, i.e. 6 ways.

13. 4 Since, there can be 3 members in a committee, there can be at most one couple.
The number of ways in which a couple is definitely selected = ${}^4C_1 \times {}^6C_1$ = 24 ways.
Total number of selecting 3 people from 8 people = ${}^8C_3$
∴ Total cases where no couple is there
= ${}^8C_3$ – 24 = 32.

14. 50 For a number to be divisible by 6, it must be even and it's digit sum must be a multiple of 3.

Case I:

			0

6 4 2 → 3! ways
or 2 3 4 → 3! ways
i.e. 12 ways

Case II:

			2

3 6 4 → 3! ways
or 4 6 0 → 2 × 2 × 1 = 4 ways[Since 0 cannot appear at first position]
or 3 4 0 → 2 × 2 × 1 = 4 ways[Since 0 cannot appear at first position]

i.e. 14 ways.

Case III:

			4

2 3 6 → 3! ways
or 2 6 0 → 2 × 2 × 1 = 4 ways
or 2 3 0 → 2 × 2 × 1 = 4 ways

i.e. 14 ways.

Case IV:

			6

2 3 4 → 3! ways = 6 ways
2 4 0 → 4 ways

i.e. 10 ways
Total = 12 + 14 + 14 + 10 = 50 ways

15. 3

0	1	1	1	1
1	0	1	1	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	0

This is a possible 5 × 5 table with each entry either 0 or 1 and sum of each row or column is 4.
We can arrange the rows or column in 5! = 120 ways to get every possible combination.

16. 3 Let a and b be two numbers.
Then, a + b = 120
Since a and b both cannot be equal to greater than 60.
Let $0 \le a \le 59$ and $61 \le b \le 120$
The total number of ways in which 'a' can be chosen = ${}^{60}C_1$ = 60
The value of b depends on the value of a and there is one value of b corresponding to one value of a.
So, total number of sets having two numbers = 60

17. 3 The arrangement of 8 distinct fruits, in a row, is given by 8! ways.
But the condition is Apple(A) must be before Banana(B), and B must be before Mango(M).
The possible cases of the arrangement of these three fruits can be given as,
ABM, AMB, BMA, BAM, MAB, MBA (Not necessarily conjecutive)
Here, out of 3! Arrangements, only 1 arrangement is valid.

So, out of 8! Arrangements, there would be $\frac{8!}{3!}$ arrangements valid.

18. A. When two particular players are always selected, then we will select those 2 players first, i.e. 1 way, and then we need to select 4 more players out of 9 available players.
i.e. $^9C_4$ ways.

B. When two particular players are never selected, then we will reject those 2 players first, i.e. 1 way, and then we need to select all the 6 players out of 9 available players.
i.e. $^9C_6$ ways.

19. 3 Taking I for India and S for South Africa. We can arrange I and S to show the wins for India and South Africa respectively.
Let, South Africa won the series, then last match is always won by South Africa, and it will be fixed.
Thus, following are the possible ways in which this can be done

	Wins for India	Wins for S.A.	No. of ways
1	0	4	1
2	1	4	4!/3! = 4
3	2	4	5!/2!3! = 10
4	3	4	6!/3!3! = 20

So, total number of ways = 35
In the same number of ways India can win the series.
Thus, total number of ways = 35 × 2 = 70.

20. A-540, B-90

A. Again, let us start with the distributions.
Scenario I: (1, 2, 3): This can be done in $^6C_3 \times {^3C_2} \times 3!$ ways. First select 3 out of 6, and then 2 out of the remaining 3. After we have done this, the toys can go into the three distinct boxes in 3! ways. **360 ways**
Scenario II: 1, 1, 4: This can be done in $^6C_4 \times 3!$ ways. Once we select 4 out of 6, the other two go get broken up as 1 and 1. Now, we have something akin to ABCD, E and F to be allotted into 3 distinct boxed. This can be done in 3! ways. **90 ways**
Scenario III: 2, 2, 2: This should be $^6C_2 \times {^4C_2}$ ways. The idea we are using here is simple – select 2 out of 6 for the first box and then select 2 out of 4 for the second box. **90 ways.**
Total number of ways = 360 + 90 + 90 = **540 ways.**
Now, this question can be rephrased wonderfully like this:
How many onto functions can be defined from {a, b, c, d, e, f} to {1, 2, 3}?
You can solve the above question by thinking of all functions from the first set to the second and subtracting the non–onto functions from that. Needless to say, we would get the same answer.

B. First let us think of the distributions. The boxes can have
1, 2, 3: This can be done in $^6C_3 \times {^3C_2}$ ways. First select 3 out of 6, and then 2 out of the remaining 3. This is nothing but distributing 6 as 3, 2, 1 which can be done in $\frac{6!}{2! \times 3! \times 1!}$ ways
1, 1, 4: This can be done in $^6C_4$ ways. Once we select 4 out of 6, the other two go into one box each. Since the boxes are identical, we do not have to worry about selecting anything beyond the first set of 4 toys.
2, 2, 2: This looks like it could be $^6C_2 \times {^4C_2}$ ways. But this will carry some multiple counts. The idea we are using here is simple – select 2 out of 6 and then select 2 out of 4.
When we do this, a selection of AB, and then CD will get counted. This will get accounted as AB, CD, EF. However, we will also be counting a selection of CD, AB, EF, and EF, AB, CD. Since the boxes are identical, all these selections are effectively the same. So, number of ways would be $\frac{^6C_2 \times {^4C_2}}{3!}$
So, total number of ways of doing this would be 60 + 15 + 15 = **90** ways.

21. 4 For a 3-digit number:
Case (i): All digits different = $^4P_3$

Case (ii): Two 1's and one other number $= {^3C_1} \times \frac{3!}{2!}$

Hence, total number of ways = $^4P_3 + {^3C_1} \times \frac{3!}{2!}$.

22. 1 This is the problem of derangement.
In this case, we will first select the 3 correct letters which are going into right envelopes.
The number of ways in which this can be done = $^8C_3$
The rest 5 letters are going into wrong envelopes, which is given by derangement of 5, i.e. D(5)

Since, $D(n) = n!\left\{\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} + \ldots. \frac{1}{n!}\right\}$

$\Rightarrow$ D(5) = 44
Thus, total number of ways in which this can happen = $^8C_3 \times 44$ ways.

23. There are 10 digits available, i.e. from 0 to 9
If we select any three digits, the condition will automatically be satisfied.
Thus, $^{10}C_3 = 120$ ways.

24. 16 The number of heads are greater than the number of tails. The following cases can be made

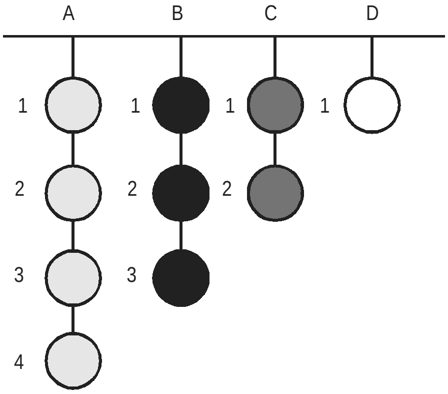
No. of tails	No. of heads	Possible arrangement	No. of ways
0	5	HHHHH	1
1	4	THHHH	5!/4! = 5
2	3	TTHHH	5!/2!3! = 10

Hence, total number of ways for the possibility = 16

25. 3 The student can select 1 book or 2 books or 3 books and so on up to n books.
The number of ways can be given by
$^{2n+1}C_0 + {}^{2n+1}C_1 + {}^{2n+1}C_2 + {}^{2n+1}C_3 + \ldots + {}^{2n+1}C_n$
$= 255 + {}^{2n+1}C_0$
$\Rightarrow\ 2^{2n} = 2^8$.
$\Rightarrow\ n = 4$

26. 1 Since, 2 + 3 + 4 +…+ 13 = 90
Thus, first boy will be getting 2 chocolates, second boy will be getting 3 chocolates and so on.
This can be done in 12! ways.
Also, we have to keep the number of chocolates distinct and we are still left with 2 more chocolates. Either these 2 chocolates can be given to the boy who is getting 13 chocolates or we can give 1 chocolate each to the boys with 12 and 13 chocolates(to keep number of chocolates distinct)
Thus, 2 × 12! ways.

27. $\dfrac{10!}{4!3!2!1!}$
We number the targets as 1, 2, 3, 4 as shown below.



The total number of arrangements would be 10!
But A4 must be before A3, A3 must be before A2 and A2 must be before A1. i.e. out of 4! Arrangements, only 1 arrangement would be valid. Similarly, for columns B, C and D, out of 3!, 2! and 1! arrangements respectively, only 1 arrangement would be valid in each case.

Thus, total number of ways $= \dfrac{10!}{4!3!2!1!}$

28. 2 $\underline{A}\ \underline{B}\ \underline{C}\ \underset{x}{D}\ \underline{E}\ \underset{x}{F}\ \underline{G}\ \underset{x}{H}\ \underline{I}\ \underset{x}{J}\ \underline{K}\ \underset{x}{L}$
Let the 12 persons be A, B, C, D, E, F, G, H, I, J, K and L. We will select 5 persons which are not consecutive first. Suppose we have selected D, F, H, J and L.
Now, we are left with A, B, C, E, I, G and K, and we have to place D, F, H, J and L in such a way that they are not consecutive.
Now, when 7 people A, B, C, E, G, I and K are seated then there would be 8 places in which D, F, H, J and L can be seated.
This can be done in $^8C_5 = 56$ ways.

29. 2 In the set of 43 consecutive numbers, there would be 14 numbers in the form of 3n + 1, 14 numbers in the form of 3n + 2 and 15 numbers in the form of 3n (**Note:** the number of numbers of the given forms depends upon the starting point of these 43 numbers, but there won’t be any effect in the number of ways in which we can make a + b + c divisible by 3).
Here, we can select three numbers which are in the form of 3n, or three numbers which are in the form of 3n + 1, or three numbers which are in the form of 3n + 2 and one number each from each category because there sum would be a multiple of 3.
So, total number of ways in which this can be done is $^{14}C_3 + {}^{14}C_3 + {}^{15}C_3 + {}^{14}C_1{}^{14}C_1{}^{15}C_1 = 4123$ ways.

30. 3 Let the selected numbers be a, b and c. Since, these are in AP, a + c = 2b.
Here, RHS is always even, so LHS must be even.
This is possible if both a and c are even or both are odd.
Since, from 1 to 15, there are 8 odd numbers and 7 even numbers, so number of ways of selection
$= {}^8C_2 + {}^7C_2 = 28 + 21 = 7^2$.