Problem 1

Given,

$P(B) = 0.001 \Rightarrow P(\neg B) = 1 - 0.001 = 0.999$

$P(E) = 0.002 \Rightarrow P(\neg E) = 1 - 0.002 = 0.998$

	E	$\neg E$
B	$P(A/B,E)$	$P(A/B,\neg E)$
$\neg B$	$P(A/\neg B,E)$	$P(A/\neg B,\neg E)$

Using this table,

$P(A/B,E) = 0.95 \Rightarrow P(\neg A/B,E) = 1 - 0.95 = 0.05.$

$P(A/B,\neg E) = 0.94 \Rightarrow P(\neg A/B,\neg E) = 1 - 0.94 = 0.06.$

$P(A/\neg B,E) = 0.29 \Rightarrow P(\neg A/\neg B,E) = 1 - 0.29 = 0.71$

$P(A/\neg B,\neg E) = 0.001 \Rightarrow P(\neg A/\neg B,\neg E) = 1 - 0.001 = 0.999$

Using table for John

$P(J/A) = 0.90 \Rightarrow P(\neg J/A) = 1 - 0.90 = 0.10$

$P(J/\neg A) = 0.05 \Rightarrow P(\neg J/\neg A) = 1 - 0.05 = 0.95$

Using table for Mary

$P(M/A) = 0.7 \Rightarrow P(\neg M/A) = 1 - 0.7 = 0.3$

$P(M/\neg A) = 0.01 \Rightarrow P(\neg M/\neg A) = 1 - 0.01 = 0.99.$

Thus the joint probability of

$P(\neg \text{John Calls}, \text{Mary Calls}, \neg \text{Alarm}, \text{Earthquake}, \neg \text{Burglary})$.

$= P(\neg J/A) \cdot P(M/A) \cdot P(\neg A/\neg B, E) \cdot P(E) \cdot P(\neg B)$

$= 0.95 \cdot 0.01 \cdot 0.71 \cdot 0.002 \cdot 0.999$.

$= \cancel{0.0000128}$ $\underline{\underline{0.0000135}}$

Problem 2:

a	P(a)
0	0.2
1	0.8

b	P(b)
0	0.86
1	0.14

c	P(c)
0	0.48
1	0.52

a	b	P(a, b)
0	0	0.14
0	1	0.06
1	0	0.72
1	1	0.08

a	c	P(a, c)
0	0	0.12
0	1	0.08
1	0	0.36
1	1	0.44

a	c	$P(c/a) = P(a,c)/P(a)$.
0	0	0.6
0	1	0.4
1	0	0.45
1	1	0.55.

a	b	c	$P(c/a,b) = P(a,b,c)/P(a,b)$.	
0	0	0	0.084/0.14 =	0.6.
0	0	1	0.056/0.14 =	0.4
0	1	0	0.036/0.06 =	0.6
0	1	1	0.024/0.06 =	0.4
1	0	0	0.324/0.72 =	0.45
1	0	1	0.396/0.72 =	0.55
1	1	0	0.036/0.08 =	0.45
1	1	1	0.044/0.08 =	0.55

(2)

a	c	$P(c/a,b)$	$P(c/a)$
0	0	0.6	0.6
0	1	0.4	0.4
1	0	0.45	0.45
1	1	0.55	0.55

$P(c/a,b) = P(c/a)$.

~~(3)~~ When $c=0$, $P(c/a,b)$ is $0.6/0.45$

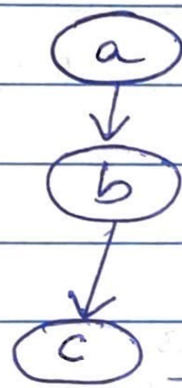
but $P(c=0) = 0.48$

When $c=1$, $P(c/a,b)$ is $0.4/0.55$

but $P(c=1) = 0.52$.

$$\underline{\underline{P(c/a,b) \neq P(c)}}.$$

(3)

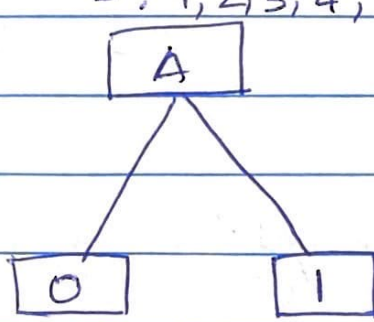


b	c	$P(c/b) = P(b,c)/P(b)$
0	0	$0.408/0.86 = 0.474$
0	1	$0.452/0.86 = 0.526$
1	0	$0.072/0.14 = 0.514$
1	1	$0.068/0.14 = 0.486$

Problem (3). For A:

+: 6, 7, 8, 10, 11, 13

−: 1, 2, 3, 4, 5, 9, 12, 14.

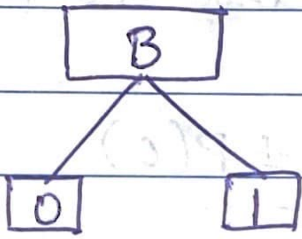


+: 6, 7, 8, 10 | +: 11, 13

−: 3, 4, 12 | −: 1, 2, 5, 9, 14

For B :

+ : 6, 7, 8, 10, 11, 13
− : 1, 2, 3, 4, 5, 9, 12, 14 .

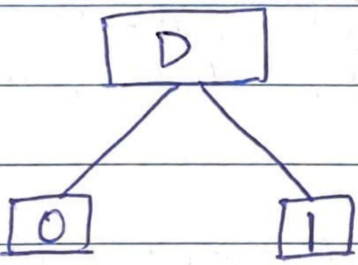


+ : 6, 10, 11, 13
− :

+ : 7, 8
− : 1, 2, 3, 4, 5, 9, 12, 14 .

For D :

+ : 6, 7, 8, 10, 11, 13
− : 1, 2, 3, 4, 5, 9, 12, 14 .



+ : 7, 8, 10, 13
− : 1, 5

+ : 6, 11
− : 2, 3, 4, 9, 12, 14 .

④ Entropy before split (E) : $-\frac{6}{14}\log_2\frac{6}{14} - \frac{8}{14}\log_2\frac{8}{14}$

$\boxed{\text{Entropy} = \sum_{i \in C} -P_i \log P_i}$

$= -\frac{6}{14}(-1.222) - \frac{8}{14}(-0.807)$

$= \underline{0.985}$

$$\text{Gain} = \text{Entropy}(E) - \sum_{v \in \text{Values}(A)} \frac{|E_v|}{E} \text{Entropy}(E_v)$$

For A:

$$\text{Entropy}_{A=0} : -\frac{4}{7}\log_2\frac{4}{7} - \frac{3}{7}\log_2\frac{3}{7}$$

$$= 0.985.$$

$$\text{Entropy}_{A=1} : -\frac{2}{7}\log_2\frac{2}{7} - \frac{5}{7}\log_2\frac{5}{7}$$

$$= 0.863$$

$$\text{Info Gain} = 0.985 - \left(\frac{7}{14} \times 0.985 + \frac{7}{14} \times 0.863\right)$$

$$= 0.985 - 0.9245$$

$$= \underline{\underline{0.0605}} \approx 0.061$$

For B:

$$\text{Entropy}_{B=0} : 0.$$

$$\text{Entropy}_{B=1} : -\frac{2}{10}\log_2\frac{2}{10} - \frac{8}{10}\log_2\frac{8}{10}$$

$$= 0.722$$

$$\text{Info Gain} = 0.985 - \left(\frac{10}{14} \times 0.722 + \frac{4}{14} \times 0\right).$$

$$= 0.985 - 0.516$$

$$= \underline{\underline{0.469}}$$

For D

$$\text{Entropy}_{D=0} : -\frac{4}{6}\log_2\frac{4}{6} - \frac{2}{6}\log_2\frac{2}{6} = 0.918$$

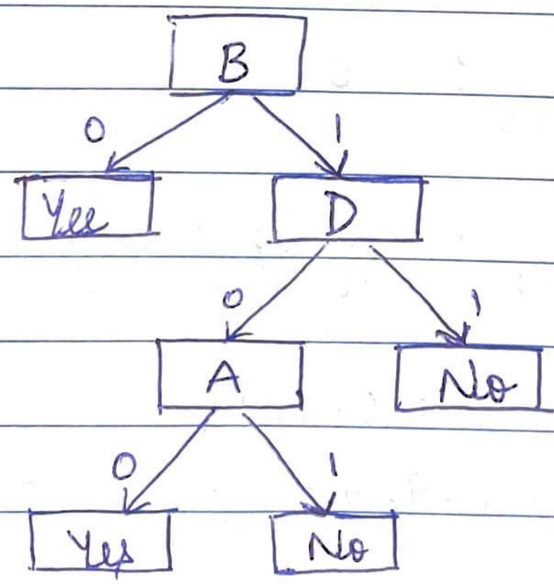
$$\text{Entropy}_{D=1} : -\frac{2}{8}\log_2\frac{2}{8} - \frac{6}{8}\log_2\frac{6}{8} = 0.811.$$

$$\text{Info Gain} : 0.985 - \left(\frac{6}{14} \times 0.918 + \frac{8}{4} \times 0.811\right)$$

$$= 0.128.$$

We will choose 'B' as the gain is maximized in that case.

(6)

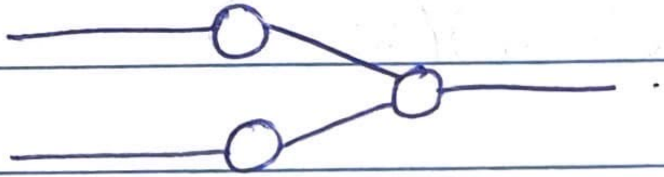


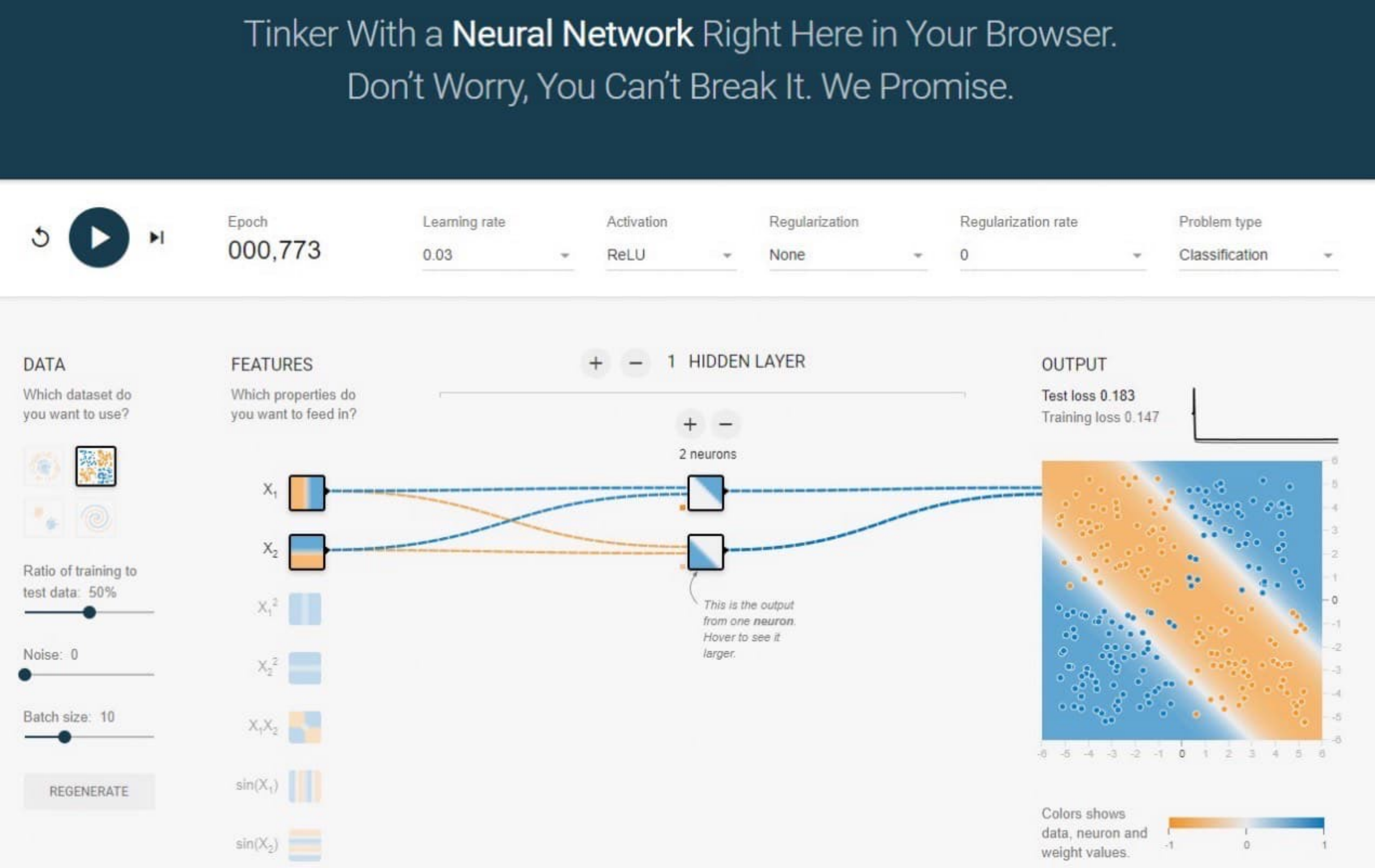
Tried 2 novel inputs :→ [1, 1, 1, 1, 1] : Output - 'No'

→ [0, 0, 0, 0, 0] : Output - 'Yes'.

(8) The screenshot for the XOR function linking up 3 perceptron units is attached below.

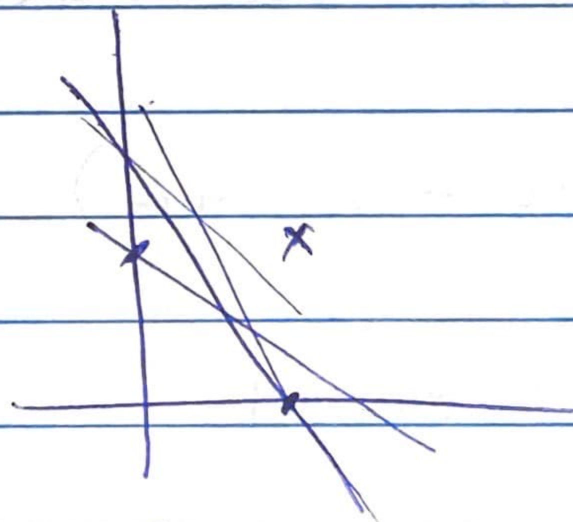
The neural n/w topology would look similar to



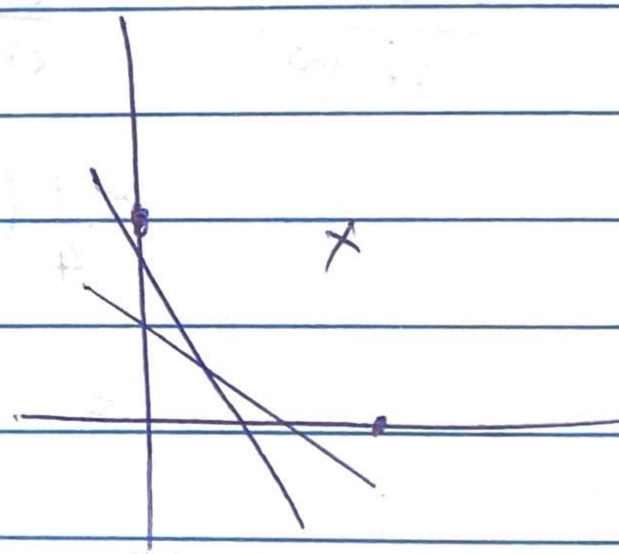


Tinker With a Neural Network Right Here in Your Browser.
Don't Worry, You Can't Break It. We Promise.
Epoch
000,773
Learning rate
0.03
Activation
ReLU
Regularization
None
Regularization rate
0
Problem type
Classification
DATA
Which dataset do you want to use?
Ratio of training to test data: 50%
Noise: 0
Batch size: 10
REGENERATE
FEATURES
Which properties do you want to feed in?
1 HIDDEN LAYER
2 neurons
This is the output from one neuron. Hover to see it larger.
OUTPUT
Test loss 0.183
Training loss 0.147
Colors shows data, neuron and weight values.

7. For AND and OR problem, there exists an inherent linear-separable property.



FOR AND.



For OR.

Since there is random initialisation of weights, it is possible that we get the proper decision boundaries mentioned above. Thus, we will have zero error without training in such a situation.

(9.) $E(w) = \frac{1}{4} w(w-2)(w-1)(w+2)$

$= \frac{1}{4} w(w^2-4)(w-1)$

$= \frac{1}{4} (w^3-4w)(w-1)$

$= \frac{1}{4} (w^4-4w^2+4w-w^3).$

$\frac{dE(w)}{dw} = \frac{d}{dw}\frac{1}{4}(w^4-w^3-4w^2+4w)$

$= \frac{1}{4}(4w^3-3w^2-8w+4)$

$= w^3 - \frac{3}{4}w^2 - 2w + 1$

$\Delta w = -\alpha\left(\frac{dE}{dw}\right).$

$= -\alpha\left(w^3 - \frac{3}{4}w^2 - 2w + 1\right).$