Problem 1:

① All Aggies are college students.

$$\forall x \, (Aggie(x) \longrightarrow College(x)).$$

② All ~~Aggies~~ Aggies are diligent.

$$\forall x \, (Aggie(x) \longrightarrow Diligent(x)).$$

③ For all college students there is some Starbucks that they have tea at.

$$\forall x \, (College(x) \longrightarrow \exists y \, (Starbucks(y) \wedge Tea(x,y))).$$

④ For all Starbucks where there is someone who is diligent and has ~~the~~ tea there, that Starbucks is in a library.

$$\forall x (\exists y \, (Diligent(y) \wedge Tea(y,x)) \longrightarrow (Starbucks(x) \wedge Library(x))).$$

⑤ Everyone who has tea in a library, they are brilliant.

$$\forall x (\forall y (Tea(y,x) \wedge Library(x)) \rightarrow Brilliant(y)).$$

⑥ Conclusion: All Aggies are brilliant.

$$\forall x \, (Aggie(x) \longrightarrow Brilliant(x)).$$

From ①,

(i) $\neg Aggie(x) \vee College(x)$.

From ②,

(ii) $\neg Aggie(x) \vee Diligent(x)$

From ③

$\neg Colleg(x) \vee (Starbucks(b) \wedge Tea(x, b))$

(iii) $\neg College(x) \vee Starbucks(b)$

(iv) $\neg College(x) \vee Tea(x, b)$.

From ④

$\forall x (\neg \exists y (Diligent(y) \wedge Tea(y, x)) \vee (Starbucks(x) \wedge Library(x)$

$\forall x \forall y (\neg Diligent(y) \vee \neg Tea(y, x)) \vee (Starbucks(x) \wedge Library(x)))$

(v) $\neg Diligent(y) \vee \neg Tea(y, x) \vee Starbucks(x)$.

(vi) $\neg Diligent(y) \vee \neg Tea(y, x) \vee Library(x)$

From ⑤

$\forall x \exists y \neg Tea(y, x) \vee \neg Library(x) \vee Brilliant(y)$.

(vii) $\neg Tea(a, x) \vee \neg Library(x) \vee Brilliant(a)$.

From ⑥ $\exists x\, Aggie(x) \wedge \neg Brilliant(x)$

(viii) $Aggie(c)$

(ix) $\neg Brilliant(c)$.

Using shorthands, we have :

1. $\neg A(x) \vee C(x)$
2. $\neg A(x) \vee D(x)$
3. $\neg C(x) \vee S(b)$
4. $\neg C(x) \vee T(x, b)$.
5. $\neg D(y) \vee \neg T(y, x) \vee S(x)$.
6. $\neg D(y) \vee \neg T(y, x) \vee L(x)$.
7. $\neg T(a, x) \vee \neg L(x) \vee B(a)$
8. $A(c)$.
9. $\neg B(c)$.

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10. From 6 & 7

    $\neg D(a) \vee \neg T(a, x) \vee B(a)$

11. From 4 & 10

    $\neg C(a) \vee \neg D(a) \vee B(a)$.

12. From 1 & 11

    $\neg A(a) \vee \neg D(a) \vee B(a)$.

13. From 2 & 12

    $\neg A(a) \vee B(a)$

14. From 8 & 13

    $B(a)$

15. From 9 & 14

    False

Using shorthands, we have the following:

① $\neg A(x) \vee \cancel{\#} C(x)$.

Problem 2

The CNF forms of the equations

1. $M \rightarrow I$

   $\neg M \vee I$

2. $\neg M \rightarrow (\neg I \wedge L)$

   $\neg(\neg M) \vee (\neg I \wedge L)$

   $M \vee (\neg I \wedge L)$.

   $(M \vee \neg I) \wedge (M \vee L)$.

3. $(I \vee L) \rightarrow H$

   $\neg(I \vee L) \vee H$

   $(\neg I \wedge \neg L) \vee H$

   $(\neg I \vee H) \wedge (\neg L \vee H)$

4. $H \rightarrow G$

   $\neg H \vee G$

The conclusions to be drawn are

(a) M (b) G (c) H

So, we will negate each one of them and use it for resolution.

The clauses are

1. $\neg M \vee I$
2a. $M \vee \neg I$
2b. $M \vee L$
3a. $\neg I \vee H$
3b. $\neg L \vee H$
4. $\neg H \vee G$