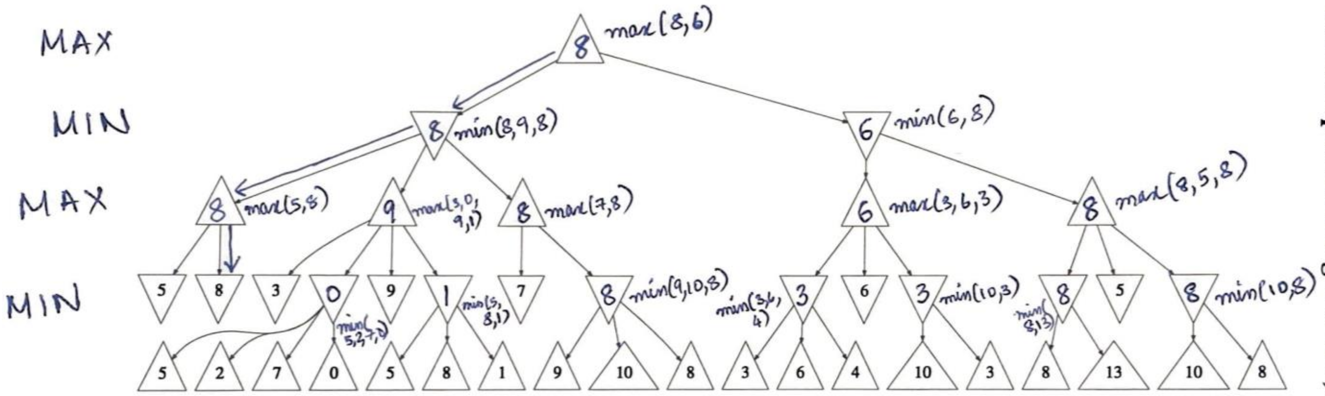
CSCE 625 Homework 2

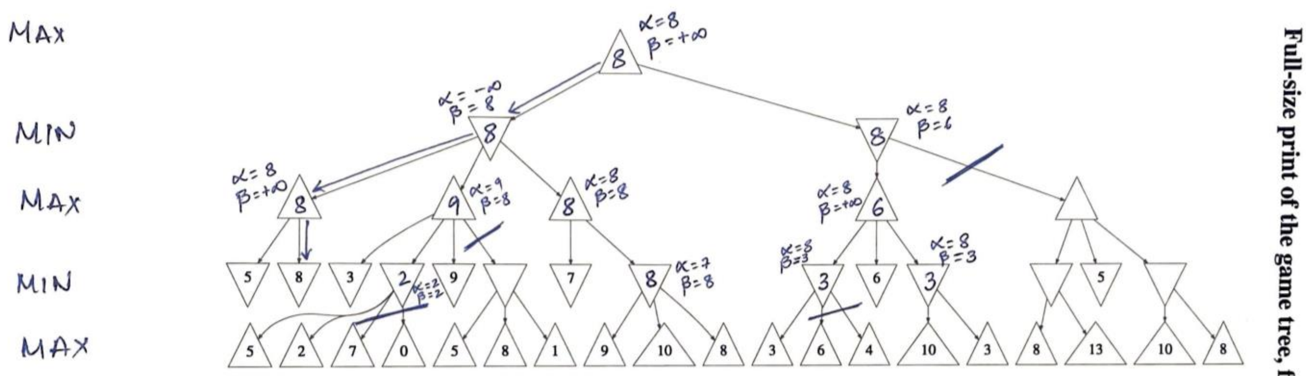
Search

Problem 1 –



The highlighted path is the optimal solution for the MAX node at the root.

Problem 2 –



The highlighted path is the optimal solution for the MAX node at the root.

Problem 3 –

A DFS (Depth First Search) becomes suboptimal when we have an infinitely deep tree and take the wrong direction, the algorithm gets stuck going deeper and deeper and never completes. But in MinMax search, we use game tree where we have the knowledge of the entire game state space. So, the algorithm provides an optimal solution by exhaustively exploring the game tree while alternating between maximizing and minimizing the player roles. This considers all the possible outcomes and makes choices that in turn provide the best possible outcome.

Problem 5 –

$((P \vee S) \wedge (\neg S \vee Q)) \rightarrow (P \vee Q)$ can be simplified as: $\neg ((P \vee S) \wedge (\neg S \vee Q)) \vee (P \vee Q)$

P	Q	S	(P ∨ S)	(¬S ∨ Q)	(P ∨ S) ∧ (¬S ∨ Q)	(P ∨ Q)	((P ∨ S) ∧ (¬S ∨ Q)) → (P ∨ Q)
T	T	T	T	T	T	T	T
T	T	F	T	T	T	T	T
T	F	T	T	F	F	T	T
T	F	F	T	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	F	T	F	T	T
F	F	T	T	F	F	F	T
F	F	F	F	T	F	F	T

Problem 6 –

1) Convert $(Q \wedge \neg P) \vee (Q \wedge R) \vee S$ into conjunctive normal form (CNF):

Distribution of ∨ over ∧:

$(Q \wedge \neg P) \vee (Q \wedge R) \vee S = (Q \vee S) \wedge (\neg P \vee S) \wedge (Q \vee R)$

Now the expression is in CNF.

2) Convert $\neg(\neg P \vee Q) \vee R$ into conjunctive normal form (CNF):

De Morgan's Law:

$\neg(\neg P \vee Q) \vee R = (P \wedge \neg Q) \vee R$

Distribution of ∨ over ∧:

$(P \wedge \neg Q) \vee R = (P \vee R) \wedge (\neg Q \vee R)$

Now the expression is in CNF.

3) Convert $\neg((\neg P \rightarrow Q) \wedge (R \vee S))$ into disjunctive normal form (DNF):

Implication Law:

$\neg((\neg P \rightarrow Q) \wedge (R \vee S)) = \neg((P \vee Q) \wedge (R \vee S))$

De Morgan's Law:

$\neg((P \vee Q) \wedge (R \vee S)) = (\neg(P \vee Q) \vee \neg(R \vee S))$

De Morgan's Law:
$(\neg(P \vee Q) \vee \neg(R \vee S)) = ((\neg P \wedge \neg Q) \vee (\neg R \wedge \neg S))$

Now the expression is in DNF.

Problem 7 –

Premise 1:
$R \rightarrow W = \neg R \vee W$

Premise 2:
$R \vee (\neg P \wedge S) = (R \vee \neg P) \wedge (R \vee S)$

Premise 3:
$S \rightarrow (P \vee Q) = \neg S \vee P \vee Q$

Premise 4:
$S \vee R$

Negated Conclusion:

Negating $(Q \vee W)$ gives us $\neg Q \wedge \neg W$.

Clauses:
C1: $\neg R \vee W$
C2: $R \vee \neg P$
C3: $R \vee S$
C4: $\neg S \vee P \vee Q$
C5: $S \vee R$
C6: $\neg Q$
C7: $\neg W$

Problem 8 –

C1: $\neg R \vee W$
C2: $R \vee \neg P$
C3: $R \vee S$
C4: $\neg S \vee P \vee Q$
C5: $S \vee R$
C6: $\neg Q$
C7: $\neg W$

Using C3 and C4 we get
1. $R \vee P \vee Q$

Using 1 and C2 we get
2. $R \vee Q$

Using 2 and C1 we get
3. $Q \vee W$

Using C6 and 3 we get

4. W

Using 4 and C7 we get

False

Thus, by resolution we can derive False with the above-mentioned steps.

Problem 9 –

1. $\neg\forall x((\exists y Q(x,y)) \rightarrow P(x))$

To convert to prenex normal form, we first need to move the negation inwards

$\neg\forall x((\exists y\ Q(x, y)) \rightarrow P(x)) = \exists x\ \neg((\exists y\ Q(x, y)) \rightarrow P(x))$

Using implication law

$\exists x\ \neg((\exists y\ Q(x, y)) \rightarrow P(x)) = \exists x\ \neg(\neg(\exists y\ Q(x, y)) \vee P(x))$

Using De Morgan's law and then moving the Quantifiers ahead

$\exists x\ \neg(\neg(\exists y\ Q(x, y)) \vee P(x)) =$ $\boldsymbol{\exists x\ \exists y\ (Q(x, y) \wedge \neg P(x))}$

2. $\forall x \neg(\exists y\ Q(x, y) \wedge \neg R(x))$

Moving the negation inwards and use De Morgan's law

$\forall x\ \neg(\exists y\ Q(x, y) \wedge \neg R(x)) = \forall x(\neg(\exists y\ Q(x, y)) \vee R(x))$

Moving negation inwards and then moving the Quantifiers ahead

$\forall x(\neg(\exists y\ Q(x, y)) \vee R(x)) =$ $\boldsymbol{\forall x\ \forall y\ (\neg Q(x, y) \vee R(x))}$

3. $\neg\exists x(\neg(\forall y\ Q(x, y)) \vee \neg P(x))$

Moving the negation inward. Starting with the outer most one.

$\neg\exists x\ (\neg(\forall y\ Q(x, y)) \vee \neg P(x)) = \forall x \neg(\neg(\forall y\ Q(x, y)) \vee \neg P(x))$

Using De Morgan's law

$\forall x\ \neg(\neg(\forall y\ Q(x, y)) \vee \neg P(x)) = \forall x((\forall y\ Q(x, y) \wedge P(x))$

Moving the Quantifiers ahead

$\forall x\ ((\forall y\ Q(x, y) \wedge P(x)) =$ $\boldsymbol{\forall x\ \forall y(Q(x, y) \wedge P(x))}$

Problem 10 –

a) $\forall x\ \exists y\ P(x, y)$

i) $\exists y$ can be f(x)

Hence $\forall x\ \exists y\ P(x, y)$ can be written as $\boldsymbol{\forall x\ P(x, f(x))}$

b) $\exists x\ \exists y\ \forall z\ P(x, y) \wedge Q(y, z)$

i) $\exists x$ can be written as f(x)

ii) $\exists y$ can be written as g(x)

Hence $\exists x\ \exists y\ \forall z\ P(x, y) \land Q(y, z)$ can be written as $\mathbf{\forall z\ P(f(x), g(x)) \land Q(g(x), z)}$

c) $\forall x\ \exists y\ \exists z\ P(x, y, z) \land Q(y, z)$
   i) $\exists y$ can be written as $f(x)$
   ii) $\exists z$ can be written as $g(x)$

   Hence $\forall x\ \exists y\ \exists z\ P(x, y, z) \land Q(y, z)$ can be written as $\mathbf{\forall x\ P(x, f(x), g(x)) \land Q(f(x), g(x))}$

d) $\forall x\ \forall y\ \exists z\ P(x, y) \land Q(x, y, z)$
   i) $\exists z$ can be written as $f(x, y)$

   Hence $\forall x\ \forall y\ \exists z\ P(x, y) \land Q(x, y, z)$ can be written as $\mathbf{\forall x\ \forall y\ P(x, y) \land Q(x, y, f(x, y))}$

Problem 11 –

Remove implications:
$\forall x\ [\neg(\neg\exists z(P(z) \land Q(x, z))) \lor \exists y\ R(x, y)]$

Move negation inwards:
$\forall x\ [\ (\exists z(P(z) \land Q(x, z))) \lor \exists y\ R(x, y)]$

Move quantifiers to front:
$\forall x\ \exists z\ \exists y\ [\ (P(z) \land Q(x, z)) \lor R(x, y)]$

Convert to CNF:
$\forall x\ \exists z\ \exists y\ [\ (P(z) \lor R(x, y)) \land (Q(x, z) \lor R(x, y))\ ]$

Using skolemization:
- $\exists z$ can be written as $f(x)$
- $\exists y$ can be written as $g(x)$

$\forall x\ \exists z\ \exists y\ [\ (P(f(x)) \lor R(x, g(x))) \land (Q(x, f(x)) \lor R(x, g(x)))\ ]$

Final Standard Form:
   $\mathbf{\forall x\ \exists z\ \exists y\ [\ (P(f(x)) \lor R(x, g(x))) \land (Q(x, f(x)) \lor R(x, g(x)))\ ]}$

Problem 12 –

1) $P(x, y) \lor Q(x)$
Replacing $\{x/f(A)\}$, we get $P(f(A), y) \lor Q(f(A))$

2) $P(x, y) \lor Q(x)$
Replacing $\{x/A, y/f(z)\}$, we get $P(A, f(z)) \lor Q(A)$

3) $P(x, y) \lor Q(x)$
Replacing $\{y/x\}$, we get $P(x, x) \lor Q(x)$

Problem 13 –

1) P(x, f(x), y) and P(A, f(g(w)), z)

   The above pair is not unifiable.

2) P(x, A) and P(y, y)

   - The 1st disagreement set is {x/y}
     Thus the sets would be P(y,A) & P(y,y)

   - The 2nd disagreement set is {y/A}
     Thus the sets would be P(A,A) & P(A,A)

   Hence the unifier is {} ◦ {x/y} ◦ {y/A} = {x/A , y/A}
   The unified expression is P(A,A)

3) P(x, f(g(x)), g(A), w) and P(A, f(y), y, y)
   - The 1st disagreement set is {x/A}
     Thus the sets would be P(A, f(g(A)), g(A), w) and P(A, f(y), y, y)

   - The 2nd disagreement set is {y/g(A)}
     Thus the sets would be P(A, f(g(A), g(A), w) and P(A, f(g(A)), g(A), g(A))

   - The 3rd disagreement set is {w/g(A)}
     Thus the sets would be P(A, f(g(A)), g(A), g(A)) and P(A, f(g(A), g(A), g(A))

   Hence the unifier is {} ◦ {x/A} ◦ {y/g(A)} ◦ {w/g(A)} = {x/A , y/g(A), w/g(A)}
   The unified expression is P(A, f(g(A)), g(A), g(A))

4) P(x, f(x)) and P(A, f(B))

   The above pair is not unifiable