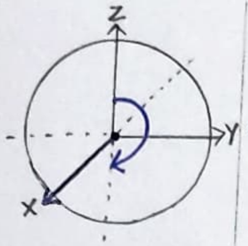
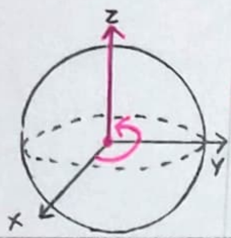
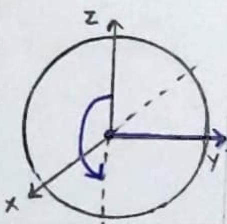
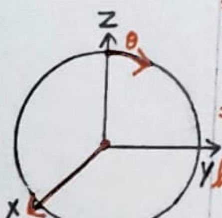
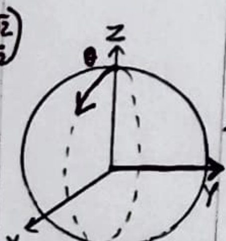
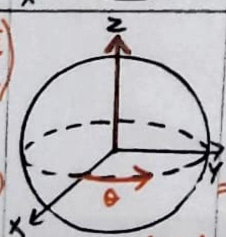
DSC TIET - Basics of Quantum Computing

Quantum Gates:- maps one quantum state to another

:- moves a point on bloch sphere to another point

clockwise on Bloch Sphere = +ve angle

anticlockwise on Bloch Sphere = −ve angle

| Name and Operation | Matrix | Application | Bloch Sphere | Unitary | Hermitian |
|---|---|---|---|---|---|
| **Pauli X Gate**<br>• 180° rotation on x axis<br>• Quantum Bit Flip | $\sigma_x = \begin{pmatrix}0&1\\1&0\end{pmatrix}$<br>$= \vert 0\rangle\langle 1\vert + \vert 1\rangle\langle 0\vert$ | $\sigma_x\vert 0\rangle = \begin{pmatrix}0&1\\1&0\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}0\\1\end{pmatrix} = \vert 1\rangle$<br>$\sigma_x\vert 1\rangle = \begin{pmatrix}0&1\\1&0\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}1\\0\end{pmatrix} = \vert 0\rangle$ |  | $\sigma_x\sigma_x^\dagger = \begin{pmatrix}0&1\\1&0\end{pmatrix}\begin{pmatrix}0&1\\1&0\end{pmatrix} = \begin{pmatrix}1&0\\0&1\end{pmatrix} = I$<br>hence, unitary | $\sigma_x^\dagger = \begin{pmatrix}0&1\\1&0\end{pmatrix}$<br>$\sigma_x = \sigma_x^\dagger$<br>hence, hermitian |
| **Pauli Z Gate**<br>• 180° rotation around z axis<br>• Quantum Phase Flip | $\sigma_z = \begin{pmatrix}1&0\\0&-1\end{pmatrix}$<br>$= \vert 0\rangle\langle 0\vert - \vert 1\rangle\langle 1\vert$ | $\sigma_z\vert +\rangle = \begin{pmatrix}1&0\\0&-1\end{pmatrix}\begin{pmatrix}1/\sqrt{2}\\1/\sqrt{2}\end{pmatrix} = \begin{pmatrix}1/\sqrt{2}\\-1/\sqrt{2}\end{pmatrix} = \vert -\rangle$<br>$\sigma_z\vert -\rangle = \begin{pmatrix}1&0\\0&-1\end{pmatrix}\begin{pmatrix}1/\sqrt{2}\\-1/\sqrt{2}\end{pmatrix} = \begin{pmatrix}1/\sqrt{2}\\1/\sqrt{2}\end{pmatrix} = \vert +\rangle$ |  | $\sigma_z\sigma_z^\dagger = \begin{pmatrix}1&0\\0&-1\end{pmatrix}\begin{pmatrix}1&0\\0&-1\end{pmatrix} = \begin{pmatrix}1&0\\0&1\end{pmatrix} = I$<br>hence, unitary | $\sigma_z^\dagger = \begin{pmatrix}1&0\\0&-1\end{pmatrix}$<br>$\sigma_z = \sigma_z^\dagger$<br>hence, hermitian |
| **Pauli Y Gate**<br>• 180° rotation around y axis<br>• Bit and Flip | $\sigma_y = \begin{pmatrix}0&-i\\i&0\end{pmatrix}$<br>$= i(\vert 1\rangle\langle 0\vert - \vert 0\rangle\langle 1\vert)$<br>$= i\sigma_x\sigma_z$ | $\sigma_y\vert 0\rangle = \begin{pmatrix}0&-i\\i&0\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}0\\i\end{pmatrix} = i\begin{pmatrix}0\\1\end{pmatrix} = i\vert 1\rangle$<br>$\sigma_y\vert 1\rangle = \begin{pmatrix}0&-i\\i&0\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}-i\\0\end{pmatrix} = -i\begin{pmatrix}1\\0\end{pmatrix} = -i\vert 0\rangle$<br>$= -i\vert 0\rangle$ |  | $\sigma_y\sigma_y^\dagger = \begin{pmatrix}0&-i\\i&0\end{pmatrix}\begin{pmatrix}0&-i\\i&0\end{pmatrix} = \begin{pmatrix}-i^2&0\\0&-i^2\end{pmatrix} = \begin{pmatrix}1&0\\0&1\end{pmatrix} = I$<br>hence, unitary | $\sigma_y^\dagger = \begin{pmatrix}0&-i\\i&0\end{pmatrix}$<br>$\sigma_y^\dagger = \sigma_y$<br>hence, hermitian |
| $R_x(\theta)$ - rotate about x axis by θ degree<br>$R_x(\theta) = e^{-i\frac{\theta}{2}\sigma_x}$<br>$= \cos\frac{\theta}{2} I - i\sin\frac{\theta}{2}\sigma_x$<br>$= \cos\frac{\theta}{2}\begin{pmatrix}1&0\\0&1\end{pmatrix} - i\sin\frac{\theta}{2}\begin{pmatrix}0&1\\1&0\end{pmatrix}$<br>$= \begin{pmatrix}\cos\theta/2 & -i\sin\theta/2\\ -i\sin\theta/2 & \cos\theta/2\end{pmatrix}$ | $R_x(\theta)$<br>$\begin{pmatrix}\cos\theta/2 & -i\sin\theta/2\\ -i\sin\theta/2 & \cos\theta/2\end{pmatrix}$ | $R_x(\pi)\vert 0\rangle = \begin{pmatrix}0&-i\\-i&0\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}0\\-i\end{pmatrix} = -i\vert 1\rangle$<br>$-i$ is global phase and is ignorable.<br>$R_x(\pi)$ is same as $\sigma_x$ |  | $R_x(\theta)R_x^\dagger(\theta)$<br>$= \begin{pmatrix}\cos\frac{\theta}{2} & -i\sin\frac{\theta}{2}\\ -i\sin\frac{\theta}{2} & \cos\frac{\theta}{2}\end{pmatrix}\begin{pmatrix}\cos\frac{\theta}{2} & i\sin\frac{\theta}{2}\\ i\sin\frac{\theta}{2} & \cos\frac{\theta}{2}\end{pmatrix}$<br>$= \begin{pmatrix}1&0\\0&1\end{pmatrix} = I$<br>hence, unitary | $R_x^\dagger(\theta) = \begin{pmatrix}\cos\theta/2 & i\sin\theta/2\\ i\sin\theta/2 & \cos\theta/2\end{pmatrix}$<br>• depends on angle θ |
| $R_y(\theta)$ - rotate about y axis by θ degree<br>$R_y(\theta) = e^{-i\frac{\theta}{2}\sigma_y}$<br>$= \cos\frac{\theta}{2} I - i\sin\frac{\theta}{2}\sigma_y$<br>$= \begin{pmatrix}\cos\theta/2 & -\sin\theta/2\\ \sin\theta/2 & \cos\theta/2\end{pmatrix}$ | $R_y(\theta)$<br>$\begin{pmatrix}\cos\theta/2 & -\sin\theta/2\\ \sin\theta/2 & \cos\theta/2\end{pmatrix}$ | $R_y(\frac{\pi}{2})\vert 0\rangle = \begin{pmatrix}1/\sqrt{2} & -1/\sqrt{2}\\ 1/\sqrt{2} & 1/\sqrt{2}\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}1/\sqrt{2}\\1/\sqrt{2}\end{pmatrix} = \vert +\rangle$<br>$R_y(\frac{\pi}{2})\vert +\rangle = \begin{pmatrix}1/\sqrt{2} & -1/\sqrt{2}\\ 1/\sqrt{2} & 1/\sqrt{2}\end{pmatrix}\begin{pmatrix}1/\sqrt{2}\\1/\sqrt{2}\end{pmatrix} = \begin{pmatrix}0\\1\end{pmatrix} = \vert 1\rangle$<br>$R_y(\pi/2)$ applied 2 times is same as $\sigma_y$, if we ignore the global phase. |  | $R_y(\theta)R_y^\dagger(\theta) =$<br>$\begin{pmatrix}\cos\theta/2 & -\sin\theta/2\\ \sin\theta/2 & \cos\theta/2\end{pmatrix}\begin{pmatrix}\cos\theta/2 & \sin\theta/2\\ -\sin\theta/2 & \cos\theta/2\end{pmatrix}$<br>$= \begin{pmatrix}1&0\\0&1\end{pmatrix} = I$<br>hence, unitary | $R_y^\dagger(\theta) = \begin{pmatrix}\cos\theta/2 & \sin\theta/2\\ -\sin\theta/2 & \cos\theta/2\end{pmatrix}$<br>depends on angle θ |
| $R_z(\theta)$ - rotate about z axis by θ deg<br>$R_z(\theta) = e^{-i\frac{\theta}{2}\sigma_z}$<br>$= \cos\frac{\theta}{2} I - i\sin\frac{\theta}{2}\sigma_z$<br>$= \begin{pmatrix}\cos\theta/2 - i\sin\theta/2 & 0\\ 0 & \cos\theta/2 + i\sin\theta/2\end{pmatrix}$ | $R_z(\theta)$<br>$\begin{pmatrix}\cos\frac{\theta}{2} - i\sin\frac{\theta}{2} & 0\\ 0 & \cos\frac{\theta}{2} + i\sin\frac{\theta}{2}\end{pmatrix}$ | $R_z(\frac{\pi}{2})\vert +\rangle = \begin{pmatrix}\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}} & 0\\ 0 & \frac{1+i}{\sqrt{2}}\end{pmatrix}\begin{pmatrix}1/\sqrt{2}\\1/\sqrt{2}\end{pmatrix}$<br>$= \begin{pmatrix}(1-i)/2\\(1+i)/2\end{pmatrix}$ apply $R_z(\pi/2)$ again<br>$= \begin{pmatrix}\frac{1-i}{\sqrt{2}} & 0\\ 0 & \frac{1+i}{\sqrt{2}}\end{pmatrix}\begin{pmatrix}1-i\\1+i\end{pmatrix}\frac{1}{2}$<br>$= \begin{pmatrix}(1-i)^2\\(1+i)^2\end{pmatrix}\frac{1}{2\sqrt{2}} = \begin{pmatrix}-i\\i\end{pmatrix}\frac{1}{\sqrt{2}} = -i\vert -\rangle$<br>ignoring the global phase, applying $R_z(\pi/2)$ 2 times is same as $\sigma_z$ |  | $R_z(\theta)R_z^\dagger(\theta)$<br>$= \begin{bmatrix}(\cos\frac{\theta}{2} - i\sin\frac{\theta}{2})^2 & 0\\ 0 & (\cos\frac{\theta}{2} + i\sin\frac{\theta}{2})^2\end{bmatrix}$ -ve<br>$= \begin{pmatrix}\cos^2\frac{\theta}{2} + \sin^2\frac{\theta}{2} & 0\\ 0 & \cos^2\frac{\theta}{2} + \sin^2\frac{\theta}{2}\end{pmatrix} = I$<br>hence UNITARY | $R_z^\dagger(\theta) = \begin{pmatrix}\cos\frac{\theta}{2} + i\sin\frac{\theta}{2} & 0\\ 0 & \cos\frac{\theta}{2} - i\sin\frac{\theta}{2}\end{pmatrix}$ -ve<br>~~$R_z(\theta) = R_z^\dagger(\theta)$~~<br>~~hence, hermitian~~<br>depends on angle θ |

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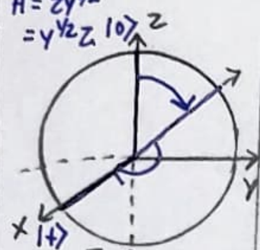
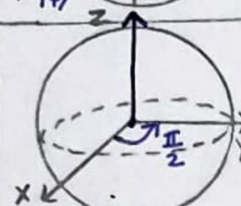
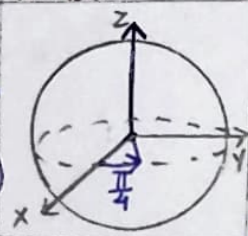
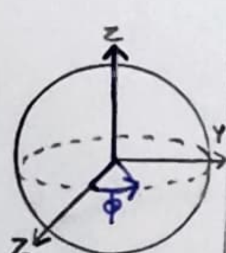
| Name and Operation | Matrix | Application | Bloch Sphere | Unitary | Hermitian |
|---|---|---|---|---|---|
| Hadamard<br>$H = XY^{1/2}$ 90° about y<br>$= Y^{-1/2}X$ then 180° about x<br>or<br>$H = ZY^{-1/2} = Y^{1/2}Z$<br>creates equal superposition of 2 basis states | $H = \frac{1}{\sqrt{2}}\begin{pmatrix}1&1\\1&-1\end{pmatrix}$ | $H\lvert 0\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1&1\\1&-1\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix}$<br>$= \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix} = \lvert +\rangle$<br>$H\lvert 1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1&1\\1&-1\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix}$<br>$= \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix} = \lvert -\rangle$ | let qubit be at $\lvert 0\rangle$ initially<br>Bloch sphere shows<br>$H = ZY^{-1/2}$<br>$= Y^{1/2}Z\,\lvert 0\rangle$<br> | $HH^\dagger = \frac{1}{2}\begin{pmatrix}1&1\\1&-1\end{pmatrix}\begin{pmatrix}1&1\\1&-1\end{pmatrix}$<br>$= \frac{1}{2}\begin{pmatrix}2&0\\0&2\end{pmatrix} = I$<br>hence, unitary | $H^\dagger = \frac{1}{\sqrt{2}}\begin{pmatrix}1&1\\1&-1\end{pmatrix} = H$<br>hence, hermitian |
| S Gate<br>$R_Z(\pi/2) = \begin{pmatrix}1&0\\0&i\end{pmatrix}$<br>$S^2 = Z$<br>90° rotation about Z | $S = \begin{pmatrix}1&0\\0&i\end{pmatrix}$ | $S\lvert 0\rangle = \begin{pmatrix}1&0\\0&i\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix}$<br>$= \begin{pmatrix}1\\0\end{pmatrix} = \lvert 0\rangle$<br>$S\lvert +\rangle = \begin{pmatrix}1&0\\0&i\end{pmatrix}\begin{pmatrix}1/\sqrt{2}\\1/\sqrt{2}\end{pmatrix}$<br>$= \begin{pmatrix}1/\sqrt{2}\\1/\sqrt{2}\,i\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\i\end{pmatrix} \xrightarrow[S]{\text{apply}} \begin{pmatrix}1&0\\0&i\end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix}1\\i\end{pmatrix} = \lvert -\rangle$ so applying S two times is same as Z gate |  | $SS^\dagger = \begin{pmatrix}1&0\\0&i\end{pmatrix}\begin{pmatrix}1&0\\0&-i\end{pmatrix}$<br>$= \begin{pmatrix}1&0\\0&1\end{pmatrix} = I$<br>hence, unitary | $S^\dagger = \begin{pmatrix}1&0\\0&-i\end{pmatrix}$<br>S gate is not it's inverse.<br>inverse $= S^\dagger$<br>Not hermitian |
| T Gate<br>$R_Z(\pi/4) = \begin{pmatrix}1&0\\0&e^{i\pi/4}\end{pmatrix}$<br>$T^2 = S$<br>$T^4 = Z$ | $T = \begin{pmatrix}1&0\\0&e^{+i\pi/4}\end{pmatrix}$ | $T\lvert +\rangle = \begin{pmatrix}1&0\\0&\frac{1+i}{\sqrt{2}}\end{pmatrix}\begin{pmatrix}1/\sqrt{2}\\1/\sqrt{2}\end{pmatrix}$<br>$= \begin{pmatrix}1/\sqrt{2}\\(1+i)/2\end{pmatrix}$ applying again gives<br>$\begin{pmatrix}1/\sqrt{2}\\(1+i)^2/2\sqrt{2}\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\+\frac{2i}{2}\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\i\end{pmatrix}$<br>applying T twice is same as S |  | $TT^\dagger = \begin{pmatrix}1&0\\0&\frac{1+i}{\sqrt{2}}\end{pmatrix}\begin{pmatrix}1&0\\0&\frac{1-i}{\sqrt{2}}\end{pmatrix}$<br>$= \begin{pmatrix}1&0\\0&\frac{(1+i)(1-i)}{2}\end{pmatrix}$<br>$= \begin{pmatrix}1&0\\0&1\end{pmatrix} = I$<br>hence, unitary | $T^\dagger = \begin{pmatrix}1&0\\0&e^{-i\pi/4}\end{pmatrix}$<br>$= \begin{pmatrix}1&0\\0&\frac{1-i}{\sqrt{2}}\end{pmatrix}$<br>inverse $= T^\dagger$<br>not, hermitian |
| P Gate - Phase gate<br>$P(\phi) = \begin{pmatrix}1&0\\0&e^{i\phi}\end{pmatrix}$<br>• $\phi$ is a real number<br>• P performs $\phi$ rotation around z axis<br>$\phi = \pi \rightarrow$ Z gate<br>$\phi = 0 \rightarrow$ I gate (does nothing)<br>$\phi = \frac{\pi}{2} \rightarrow$ S gate<br>$\phi = \frac{\pi}{4} \rightarrow$ T gate | $P(\phi) = \begin{pmatrix}1&0\\0&e^{i\phi}\end{pmatrix}$ | This is a generalised Phase gate<br>And some examples of its application are I, T, S, Z gate.<br>$\phi = -\pi$ gives same output as Z gate<br>$\phi = -\frac{\pi}{2}$ is $S^\dagger$<br>$\phi = -\frac{\pi}{4}$ is $T^\dagger$ |  | $P(\phi)P^\dagger(\phi)$<br>$= \begin{pmatrix}1&0\\0&e^{i\phi}\end{pmatrix}\begin{pmatrix}1&0\\0&e^{-i\phi}\end{pmatrix}$<br>$= \begin{pmatrix}1&0\\0&1\end{pmatrix} = I$<br>hence, unitary. | $P^\dagger(\phi) = \begin{pmatrix}1&0\\0&e^{-i\phi}\end{pmatrix}$<br>depends on the angle $\theta$<br>I, Z $\rightarrow$ hermitian<br>S, T $\rightarrow$ not hermitian |
| U Gate<br>general single qubit gate<br>$U(\theta,\phi,\lambda) = \begin{bmatrix}\cos\frac{\theta}{2} & -e^{i\lambda}\sin\frac{\theta}{2}\\ e^{i\phi}\sin\frac{\theta}{2} & e^{i(\phi+\lambda)}\cos\frac{\theta}{2}\end{bmatrix}$<br>Every gate can be specified using this<br>Interesting Relation<br>$X = HZH$<br>$Z = HXH$ | | | | | |