NOTATION FOR REPRESENTING - 2 level qubits

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

where $|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $|1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

DIRAC NOTATION / BRA-KET NOTATION

$$\langle bra | ket \rangle$$

- standard notation for describing quantum states
- way of representing vectors in HILBERT space

HILBERT SPACE - Complex Multidimensional vector space

KET

$$|x\rangle = \begin{bmatrix} a \\ b \\ c \end{bmatrix} \begin{matrix} \hat{i} \text{ co-ordinate} \\ \hat{j} \text{ co-ordinate} \\ \hat{k} \text{ co-ordinate} \end{matrix}$$

★ a, b, c are complex numbers

★ ket is a quantum state

eg $|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $|1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $|a\rangle = \begin{bmatrix} 1+2i \\ 3-4i \\ 5i \end{bmatrix}$

BRA

complex conjugate transpose of ket $\langle x| = \begin{bmatrix} a & b & c \end{bmatrix}$

- complex conjugate

$$z = a+ib \rightarrow z^* = a-ib$$

- transpose

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \rightarrow A^T = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$$

eg - $|a\rangle = \begin{bmatrix} 1+2i \\ 3-4i \\ 5i \end{bmatrix} \rightarrow \langle a| = \begin{bmatrix} 1-2i & 3+4i & -5i \end{bmatrix}$

Conjugate transpose

$$\vec{v}^\dagger = (\vec{v}^T)^* = (\vec{v}^*)^T$$

INNER PRODUCT / DOT PRODUCT

$\langle \vec{v}| = \begin{bmatrix} v_1 & v_2 & \cdots & v_n \end{bmatrix}$ $|\vec{w}\rangle = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix}$

$$\langle \vec{v}|\vec{w}\rangle = \begin{bmatrix} v_1 & v_2 & \cdots & v_n \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix} = v_1w_1 + v_2w_2 + \cdots v_nw_n = \sum_{i=1}^{n} v_i w_i$$

$\langle \vec{a} | \vec{b} \rangle$

projection of $\vec{a}$ on $\vec{b}$

$$\langle \vec{a} | \vec{b} \rangle = |\vec{a}||\vec{b}| \cos(\theta)$$

θ: angle between $\vec{a}$ and $\vec{b}$

$$\theta = \cos^{-1}\left(\frac{\langle \vec{a} | \vec{b} \rangle}{|\vec{a}||\vec{b}|}\right)$$

Magnitude

$$\langle \vec{v} | \vec{v} \rangle = \begin{bmatrix} v_1 & v_2 & v_3 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = v_1^2 + v_2^2 + v_3^2 = |\vec{v}|^2 = \text{length}^2$$

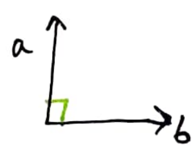
projection of $\vec{a}$ superimposes on $\vec{b}$

Normalization

$$\vec{v} \rightarrow \frac{\vec{v}}{\sqrt{\langle \vec{v} | \vec{v} \rangle}} = \frac{\vec{v}}{|\vec{v}|}$$

Normalised vector has unit length.
i.e. length = 1 $\quad \langle \vec{a} | \vec{a} \rangle = \text{length}^2 = (1)^2 = 1$

Orthogonal - vectors at right angle
- inner product is zero ($\cos 90° = 0$)



projection of $\vec{a}$ on $\vec{b}$ is zero

let $|a\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $\quad |b\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$$\langle a | b \rangle = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = 1 \times 0 + 0 \times 1 = 0$$

Basis

$$|r\rangle = \begin{bmatrix} a \\ b \end{bmatrix} = a \begin{bmatrix} 1 \\ 0 \end{bmatrix} + b \begin{bmatrix} 0 \\ 1 \end{bmatrix} = a|0\rangle + b|1\rangle$$

$|0\rangle$ and $|1\rangle$ are ortho-normal basis states

basis - vectors that we measure things against
like: x, y, z axis
- simple unit vector
- independent (orthogonal) linearly
- span every part of the space, i.e. every vector is a unique linear combination of basis vector.

PROOF:- $|0\rangle$ and $|1\rangle$ are orthonormal basis state

orthogonal $\quad \langle 0 | 1 \rangle = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = 1 \times 0 + 0 \times 1 = 0$

normalised $\quad \langle 0 | 0 \rangle = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = 1 \times 1 + 0 \times 0 = 1$

$\quad \langle 1 | 1 \rangle = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = 0 \times 0 + 1 \times 1 = 1$

So normalised $\langle \vec{a} | \vec{a} \rangle = 1$, orthogonal $\langle \vec{a} | \vec{b} \rangle = 0$

Also OUTER PRODUCT

$$|\vec{v}\rangle\langle \vec{w}| = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} \begin{bmatrix} w_1 & w_2 & w_3 \end{bmatrix} = \begin{bmatrix} v_1 w_1 & v_1 w_2 & v_1 w_3 \\ v_2 w_1 & v_2 w_2 & v_2 w_3 \\ v_3 w_1 & v_3 w_2 & v_3 w_3 \end{bmatrix}$$