Assignment Wind Energy

OFFSHORE RENEWABLES TECHNOLOGIES / MARINE RENEWABLE TECHNOLOGIES

OE44170/CIEM4305

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1 Part 1: The Wind Resource

We obtained wind data from the wind farm *Hecate Strait Offshore (400 MW)* in Canada. The latitude and longitude of the location are 53.1832 and -130.8310, respectively. The datasets we obtained give wind speeds at 10 m and 50 m heights.

1.1 Histogram and Empirical PDF

The mean and standard deviation of the wind speed at 10 meters are:

- $\mu = 6.49 \ \ m/s$
- $\sigma = 3.53 \ \ m/s$

Figure 1 illustrates the histogram of the wind speed at 10 m with bin sizes of 1 m/s.

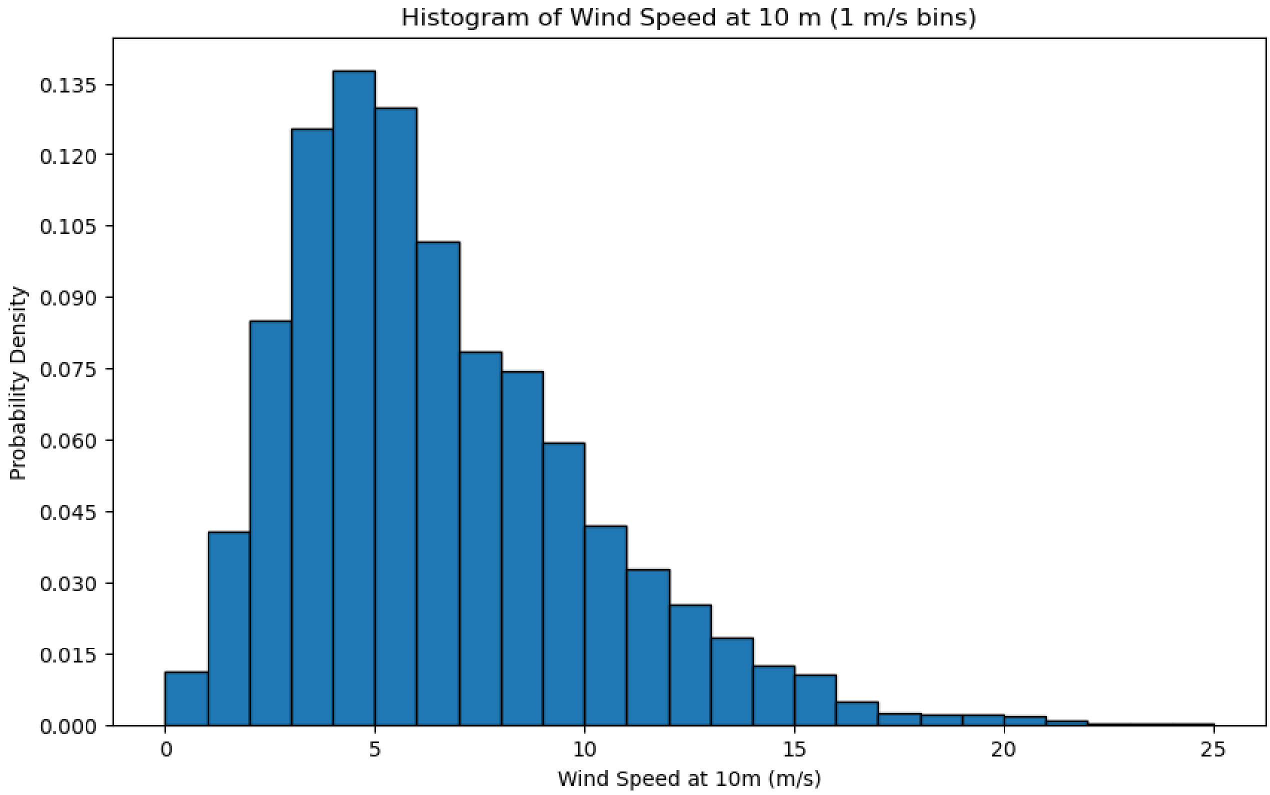

Figure 1: Histogram of wind speed at 10 meters height

1.2 Estimation of Weibull Parameters Using Method of Least Squares

First, the wind data set is used to obtain an empirical cumulative distribution function. This CDF is illustrated in Figure 2.

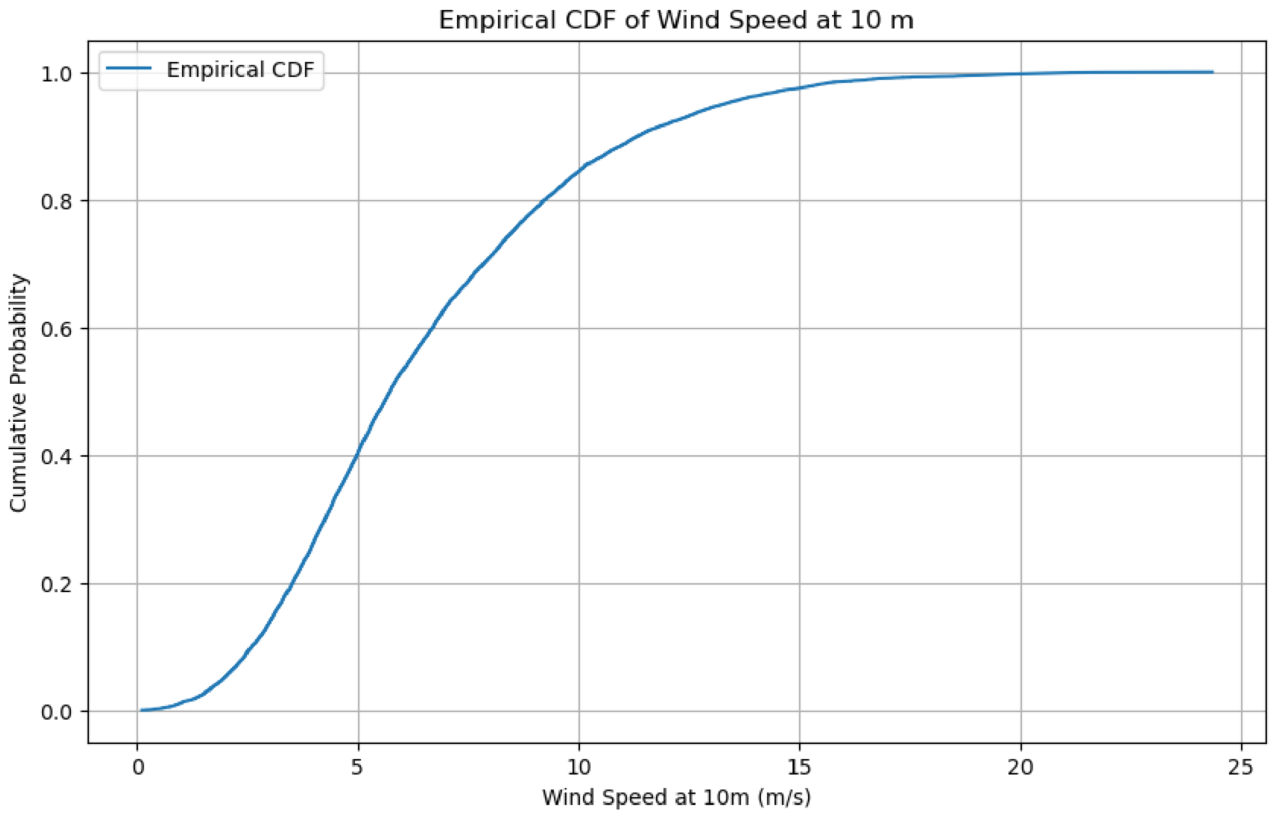

Figure 2: Empirical cumulative distribution function of the wind data set

The cumulative distribution function of the Weibull distribution is as follows:

$$F(v) = \int_0^v f(v)\, dv = 1 - e^{-\left(\frac{v}{c}\right)^k} \tag{1}$$

Rearranging the terms and taking the natural logarithm on both sides yields:

$$\ln\big(1 - F(v)\big) = -\left(\frac{v}{c}\right)^k \tag{2}$$

v are the sorted wind speeds. Multiplying both sides by -1 and then taking the natural logarithm again gives:

$$\ln\big(-\ln(1 - F(v))\big) = k \ln v - k \ln c \tag{3}$$

This can be expressed as a linear equation:

$$y = ax + b \tag{4}$$

where:

$$y = \ln\big(-\ln(1 - F(v))\big), \quad x = \ln v, \quad b = -k \ln c. \tag{5}$$

Figure 3 illustrates the data set on the new x and y axes. In addition, linear regression is performed, obtaining the slope and intercept.

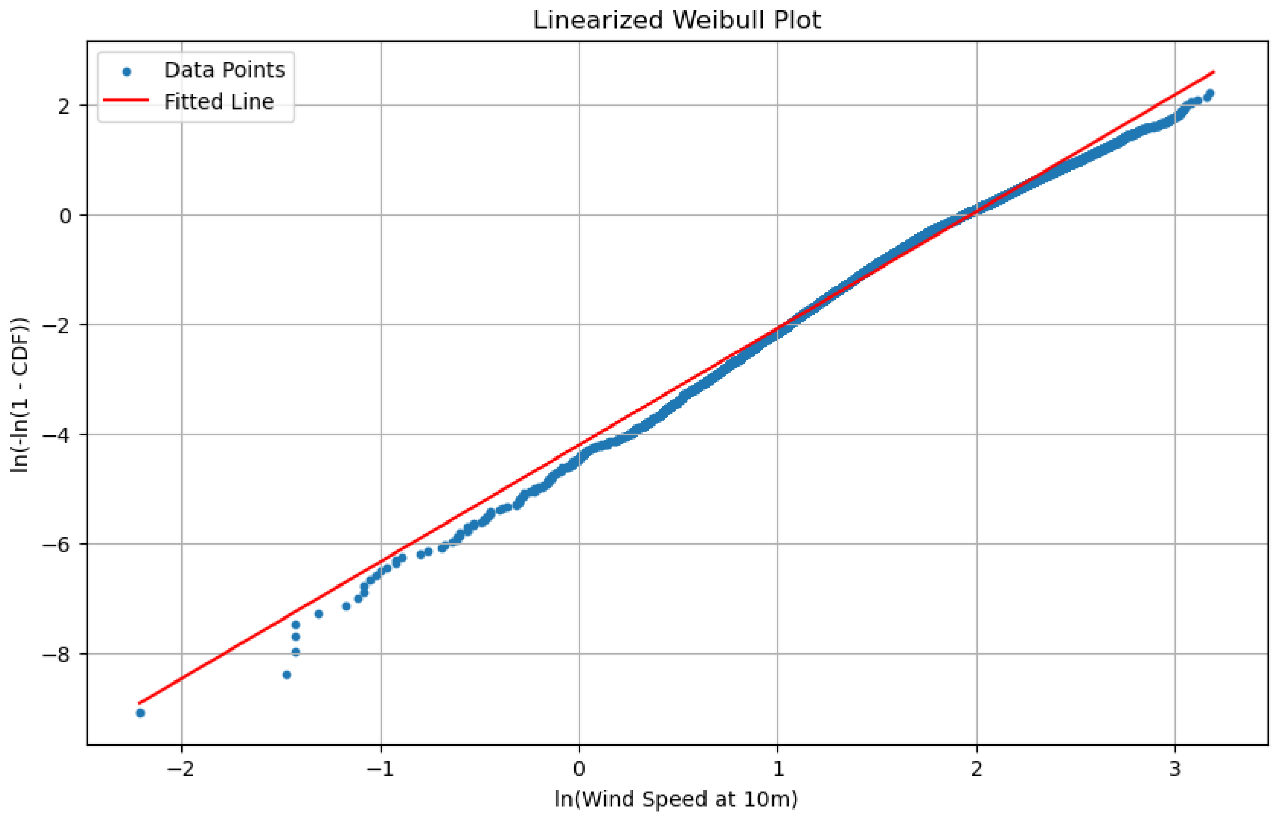

Figure 3: Linearised Weibull distribution with a fitted linear expression, $a = 2.1295$, $b = -4.2209$

Using a and b, we can obtain the shape factor and the scale factor used in the Weibull distribution.

$$k = a = 2.1295 \tag{6}$$

$$c = exp\left(\frac{b}{a}\right) = exp\left(\frac{-4.2209}{2.1295}\right) = 7.2577 \tag{7}$$

The shape factor and scale factor are now substituted into the Weibul function. Figure 4 illustrates the histogram of the wind speed, the Weibull distribution based on the least-squares method and the Weibull distribution based on a built-in fitting function in Python.

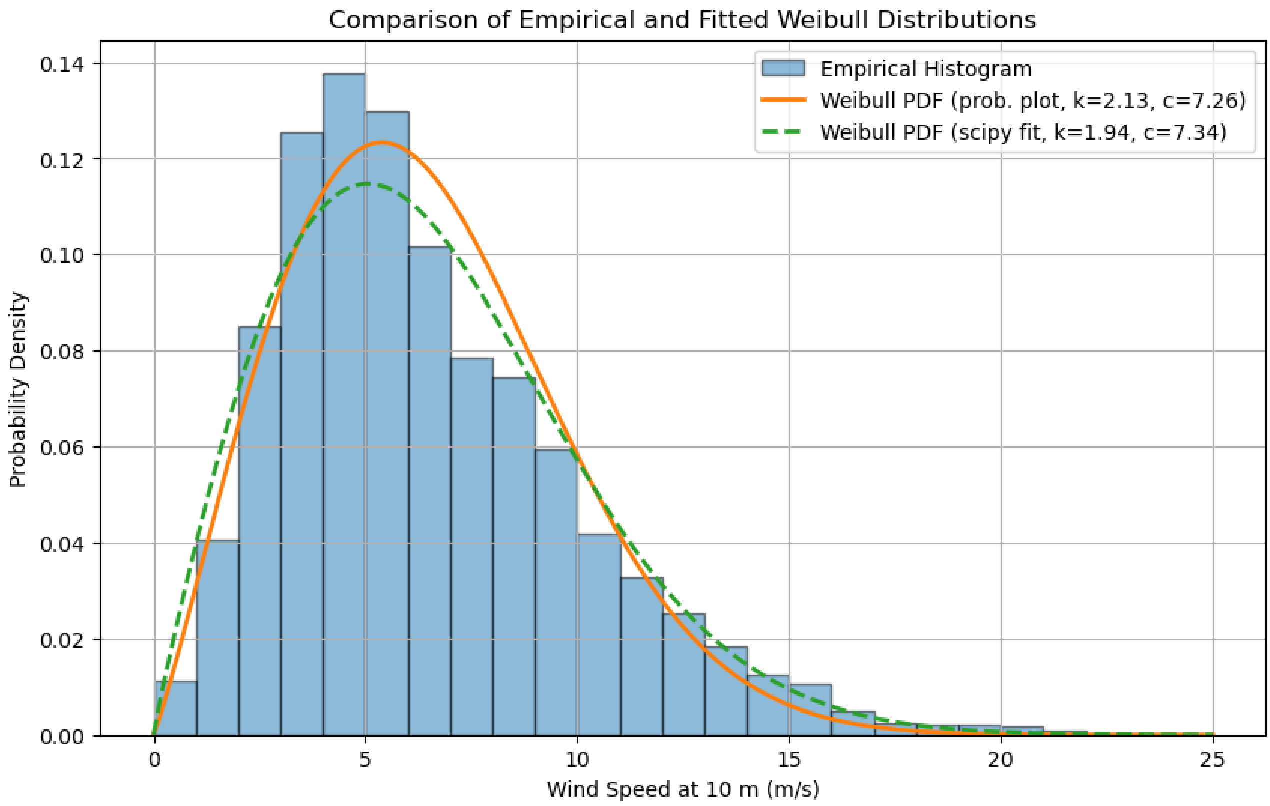

Figure 4: Comparison between Weibull functions following the least-squares method and a built-in fitting function in Python

Comparison between the two Weibull Distributions

The shape factor k using the Python fit is $\frac{2.13}{1.94} = 1.1$ times larger than the shape factor using the graphical method. Subsequently, the scale factor using the Python fit is $\frac{7.26}{7.34} = 0.99$. The Python fit is more accurate because it fits the data immediately. In the graphical method, we use a linear regression model to fit the data, which is less accurate. We see that the linear regression in Figure 3 model deviates from the data set close to the tails of the distribution. In Figure 4, the green curve seems to fit the data better than the orange curve. However, it is difficult to say which is better based on the graph.

Coefficient of Determination (R^2)

The coefficient of determination R^2 indicates how much of the data is explained by the Weibull distribution. If R^2 is close to 1, the Weibull distribution is in line with the data. If it is close to 0, the Weibull distribution with the chose parameters does not fit the data. The coefficient of determination is computed as follows:

$$R^2 = 1 - \frac{\sum_{i=1}^{n} \left(y_i - \hat{y}_i\right)^2}{\sum_{i=1}^{n} \left(y_i - \bar{y}\right)^2} \tag{8}$$

where:

$$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i, \quad y_i = \ln(-\ln(1 - F(v_i))), \quad \hat{y}_i = a x_i + b. \tag{9}$$

The parameters used in the equation for the coefficient of determination are as follows:

- y_i is the observed transformed value from the dataset
- $\hat{y}_i$ are the predicted values from the fitted regression line
- $\bar{y}$ is the mean of the observed y_i values

Filling in all data, we obtain $R^2 = 0.9905$. This indicates a good fit, as the coefficient of determination is close to 1. This means that around 99 % of the data is explained by the Weibull distribution and about 1 % is random noise.

RMSE and Reflection

THe Root mean squared error explains how the total average error is between the data points and the fitted curve. It is computed based on the equation below:

$$\text{RMSE} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2} \tag{10}$$

where:

$$y_i = \ln(-\ln(1 - F(v_i))), \quad \hat{y}_i = ax_i + b, \quad x_i = \ln(v_i). \tag{11}$$

The obtained value for RMSE using Python is equal to 0.1249. The fact that the value is low indicates that there is a small deviation between the observed and the predicted values.

The high value of R^2 and the low value of the RMSE that were obtained show that the fit of the Weibull distribution to our dataset is really good. As a result our dataset is described with both high accuracy and reliability.

2 Part 2: Wind Energy Production

2.1 Selected Turbines and their Power Curves

The WTG 2 IEA 10 MW RWT and WTG 3 IEA 15 MW RWT are selected for the assignment. The suffix 2 $(._2)$ and 3 $(._3)$ will be used to indicate the turbine numbers.

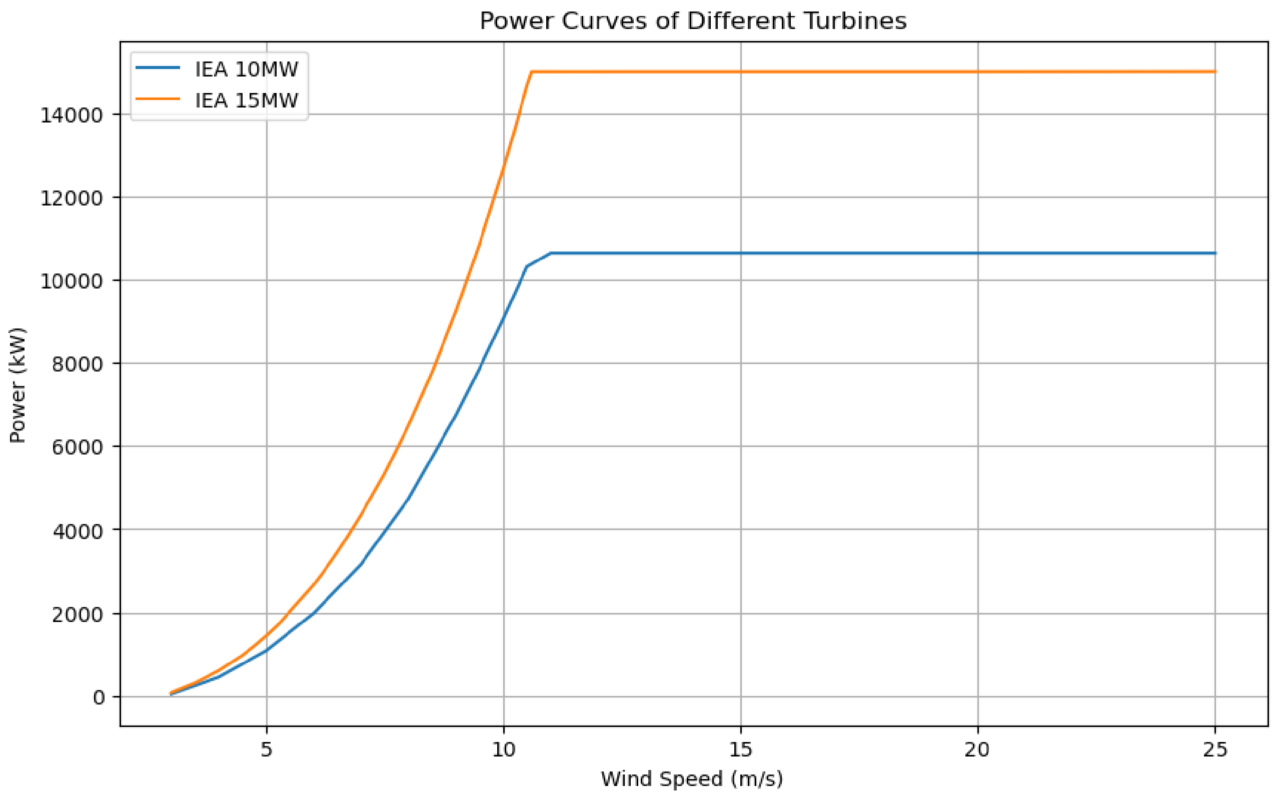

Figure 5: Power curves of the selected WTG

2.2 Hub-Height Adjustment (Log Law)

The data is first filtered to remove any readings where the recorded wind speed is greater at 10 m than that at 50 m. Then, using the following relation, the roughness length z_0 is found for every wind speed.

$$\frac{U_{50}}{U_{10}} = \frac{ln(z_{50}/z_0)}{ln(z_{10}/z_0)} \tag{12}$$

Using the roughness length, the winds speed at the hub heights of 119 m and 150 m are calculated using the same relation.

$$U_{Hub} = U_{10}\frac{ln(z_{Hub}/z_0)}{ln(z_{10}/z_0)} \tag{13}$$

2.3 Annual Energy Yield and Capacity Factor

For the obtained wind speeds, a Weibull distribution is obtained. To match the power curve points for the integration, the distribution is recreated to match the number of points. Thereafter, using the adjusted wind speeds for the hub heights, the already obtained Weibull parameters ($f(U)$) and the wind-power relation data provided ($P(U)$), the Annual Energy Yield E_y is found using the following relation.

$$E_y = T\int_{U_{ci}}^{Uco} P(U)\cdot f(U)dU \tag{14}$$

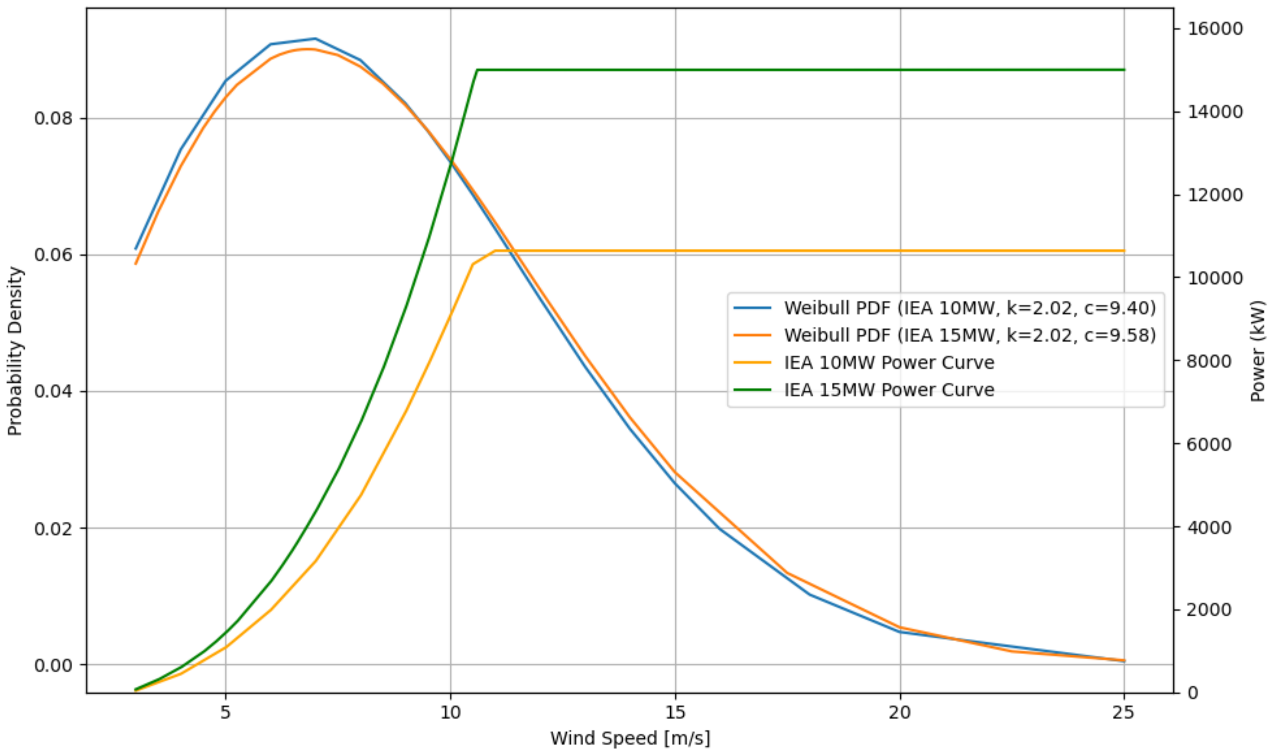

Figure 6: Power curve and the wind probability used for integration

$$E_{y_2} = 46682.49\ MWh, \quad E_{y_3} = 66641.35\ MWh$$

The capacity factor is found using the relation:

$$CF = \frac{E_y}{P_{rated}\cdot T_{year}} \tag{15}$$

$$CF_2 = 0.5329, \quad CF_3 = 0.5072$$

2.4 Recommended Turbine

The WTG 3 AEY is larger than that of WTG 2 but has a slightly lesser CF. The WTG 3, though producing more energy, is larger and thus would cost more in money and energy in order to construct it. Thus, a recovery analysis is needed to ensure that the extra yield is more than that which goes in the installation. If the net energy return is viable, then WTG 3 is the better option.

3 Part 3:Wind turbine technology

Wind turbine performance; $D = 220$ m, rotor, $P_{\text{rated}} = 14.1$ MWe

3.1 Power coefficient calculation

The power coefficient computes how efficiently the wind turbine converts its wind energy into mechanical energy. The power coefficient is computed as follows:

$$c_p = 4a(1-a)^2 \tag{16}$$

In order to calculate it, we need to compute the induction factor a, which slows down the flow upstream and downstream the turbine. Given the wind velocity and the downstream velocity we can compute upstream velocity, then a and eventually the C_p as described below. **Constants used:**

$$\rho_{\text{air}} = 1.205\ \mathrm{kg\,m^{-3}}, \quad D = 220\ \mathrm{m}, \quad R = 110\ \mathrm{m}, \quad A = \pi R^2 = \pi(110)^2 = 3.8013 \times 10^4\ \mathrm{m^2}$$

$$U_0 = 10\ \mathrm{m\,s^{-1}}, U_2 = 7\ \mathrm{m\,s^{-1}}$$

$$\begin{aligned}
U_1 &= \frac{U_0 + U_2}{2} = \frac{10+7}{2} = 8.5\ \mathrm{m\,s^{-1}} \\
a &= \frac{U_0 - U_1}{U_0} = \frac{10-8.5}{10} = 0.15 \\
C_p &= 4a(1-a)^2 = 4(0.15)(0.85)^2 = 0.433 \quad (\text{accepted}, \ 0.433 < 0.48)
\end{aligned}$$

3.2 Tip to speed ratio and Power Coefficient given wind speed and angular rotor velocity

The tip to speed ratio is computed using the following equation. First, the angular velocity is converted from rpm to rad/s. Furthermore, to compute the C_p value at 15 m/s, the mechanical power P_{mech} at rated velocity is computed using the electrical power and the efficiency η of the wind turbine.

$$\begin{aligned}
\omega &= \frac{2\pi n}{60} = \frac{2\pi(7.8)}{60} = 0.8168\ \mathrm{rad\,s^{-1}} \\
\lambda &= \frac{\omega R}{U_0} = \frac{0.8168 \times 110}{15} = 5.990 \\
P_{\text{mech}} &= \frac{P_e}{\eta} = \frac{14.1\ \mathrm{MW}}{0.97} = 14.536\ \mathrm{MW} \\
C_p &= \frac{P_{\text{mech}}}{\frac{1}{2}\rho_{\text{air}} A U_0^3} = \frac{14.536 \times 10^6}{\frac{1}{2}(1.205)(3.8013 \times 10^4)(15)^3} = 0.188
\end{aligned}$$

The C_p value at 15 m/s is lower than at the rated power. This makes sense, as the wind velocity is increased whereas the generated power stays constant. Thus, the power extraction ratio reduces.

3.3 Expected Power Coefficient at $U = 9\ m/s$

The wind speed is lower than the rated wind speed. Therefore, the pitch angle $\beta = 0$. Furthermore, the relevant parameters used in the empirical relation for the power coefficient are shown below:

$$\omega = \frac{2\pi \cdot 7.8}{60} = 0.8168\ \mathrm{rad\,s^{-1}}, \qquad \lambda = \frac{0.8168 \cdot 110}{9} = 9.983$$

$$\lambda_i = \left(\frac{1}{\lambda + 0.02\beta} - \frac{0.003}{\beta^3 + 1}\right)^{-1} \stackrel{\beta=0^\circ}{=} \left(\frac{1}{\lambda} - 0.003\right)^{-1} = \left(\frac{1}{9.983} - 0.003\right)^{-1} = 10.292$$

$$\begin{aligned} C_p(\beta, \lambda_i) &= 0.79\left(\frac{151}{\lambda_i} - 0.58\beta - 0.002\beta^{2.14} - 13.2\right) e^{-18.4/\lambda_i} \\ &= 0.79\left(\frac{151}{10.292} - 13.2\right) e^{-18.4/10.292} \end{aligned}$$

$$\boxed{C_p = 0.195}$$

The C_p obtained is greater in comparison to that for the lower velocity in the previous question. This increase can be explained by the fact that this wind speed is still less than the rated wind speed, and therefore the C_p will continue to rise until the rated wind speed is reached.

3.4 Thrust at Rated Wind Speed

To calculate the thrust at the rated wind speed, we need the induction factor a. In order to calculate a, we solve the cubic equation for a $C_p = 4a(1-a)^2$ considering as $C_{p^{rated}} = C_{p^{max}} = 0.48$ according to Appendix 2. Below, the 3 roots for a are presented;

$$4a(1-a)^2 - 0.484 = 0 \quad \Rightarrow \quad a \in \{\boxed{0.179915},\ 0.515532,\ 1.304552\}$$

$a > 1$: Rejected because $U_1 = U_0(1-a) < 0$ (through-disk velocity reverses), which is non-physical for steady forward inflow.
$0.5 < a \leq 1$: Rejected because $U_2 = U_0(1-2a) < 0$ (the wake would go backwards), which cannot occur in steady turbine operation. Choosing the physical actuator-disk root $a = 0.179915$:

$$C_T = 4a(1-a) = 4(0.179915)(1-0.179915) = \boxed{0.590}$$

Thrust (using rated wind speed , $U_0 = 11\ \mathrm{m\,s^{-1}}$):

$$T = \tfrac{1}{2}\rho_{\text{air}} A U_0^2 C_T = \tfrac{1}{2}(1.205)(3.8013 \times 10^4)(11)^2(0.590) = \boxed{1635.552\ \mathrm{kN}}$$