On Power Networks Coupled with Market Dynamics

Pengcheng You

Nov 14 2017

Linear model

Linear swing dynamics:

$$\dot{\theta} = \omega \tag{1a}$$

$$M\dot{\omega}_{\mathcal{G}} = r_{\mathcal{G}} + p - d_{\mathcal{G}} - D_{\mathcal{G}}\omega_{\mathcal{G}} - C_{\mathcal{G}}B\bar{C}^T\theta_{\mathcal{N}^+} \tag{1b}$$

$$0 = r_{\mathcal{L}} - d_{\mathcal{L}} - D_{\mathcal{L}}\omega_{\mathcal{L}} - C_{\mathcal{L}}B\bar{C}^T\theta_{\mathcal{N}^+} \tag{1c}$$

$$0 = r_0 - D_0\omega_0 - C_0B\bar{C}^T\theta_{\mathcal{N}^+} \tag{1d}$$

Rational behavior of market participants:

$$T_j^p\dot{p}_j = \lambda - \omega_j + H_j\eta^- - H_j\eta^+ - J_j'(p_j) \tag{1e}$$

$$T_j^d\dot{d}_j = U_j'(d_j) - \lambda + \omega_j - H_j\eta^- + H_j\eta^+ \tag{1f}$$

Price dynamics:

$$\dot{\lambda} = \gamma^{\lambda}\left(-\mathbf{1}_{\mathcal{G}}^T(r_{\mathcal{G}} + p - d_{\mathcal{G}}) - \mathbf{1}_{\mathcal{L}}^T(r_{\mathcal{L}} - d_{\mathcal{L}}) - r_0\right) \tag{2a}$$

$$\dot{\eta}^- = \Gamma^{\eta^-}\left[\underline{F} - H_{\mathcal{G}}^T(r_{\mathcal{G}} + p - d_{\mathcal{G}}) - H_{\mathcal{L}}^T(r_{\mathcal{L}} - d_{\mathcal{L}})\right]_{\eta^-}^+ \tag{2b}$$

$$\dot{\eta}^+ = \Gamma^{\eta^-}\left[H_{\mathcal{G}}^T(r_{\mathcal{G}} + p - d_{\mathcal{G}}) + H_{\mathcal{L}}^T(r_{\mathcal{L}} - d_{\mathcal{L}}) - \overline{F}\right]_{\eta^+}^+ \tag{2c}$$

Nonlinear model

Nonlinear swing dynamics:

$$\dot{x} = f(x) \tag{3}$$

where $x := (\theta, \omega, p, d, E)$

Rational behavior of market participants:

$$T_j^p \dot{p}_j = u_j - J_j'(p_j) \tag{4}$$

$$T_j^d \dot{d}_j = U_j'(d_j) - u_j \tag{5}$$

where u_j is the price at bus j.

Feedback price controller:

$$u = Hx \tag{6}$$

The problem is how to design H to stabilize x, given the rational behavior of market participants.

REFERENCES

[1] C. Zhao, E. Mallada, S. Low, and J. Bialek, "A unified framework for frequency control and congestion management," in *Power Systems Computation Conference (PSCC), 2016*. IEEE, 2016, pp. 1–7.

[2] N. Li, C. Zhao, and L. Chen, "Connecting automatic generation control and economic dispatch from an optimization view," *IEEE Transactions on Control of Network Systems*, vol. 3, no. 3, pp. 254–264, 2016.

[3] C. Zhao, U. Topcu, N. Li, and S. Low, "Design and stability of load-side primary frequency control in power systems," *IEEE Transactions on Automatic Control*, vol. 59, no. 5, pp. 1177–1189, 2014.

[4] E. Mallada, C. Zhao, and S. Low, "Optimal load-side control for frequency regulation in smart grids," *IEEE Transactions on Automatic Control*, 2017.

[5] A. Bacciotti and F. Ceragioli, "Nonpathological lyapunov functions and discontinuous carathéodory systems," *Automatica*, vol. 42, no. 3, pp. 453–458, 2006.