

For each bus j , the nonlinear dynamical model is as follows [1], [2], [3]:

$$\begin{aligned}
\dot{\theta}_j &= \omega_j \\
M_j \dot{\omega}_j &= r_j + p_j - d_j - D_j \omega_j - \sum_{k:j \rightarrow k} E'_{qj} E'_{qk} B_{jk} \sin(\theta_j - \theta_k) + \sum_{i:i \rightarrow j} E'_{qi} E'_{qj} B_{ij} \sin(\theta_i - \theta_j) \\
0 &= r_j - d_j - D_j \omega_j - \sum_{k:j \rightarrow k} E'_{qj} E'_{qk} B_{jk} \sin(\theta_j - \theta_k) + \sum_{i:i \rightarrow j} E'_{qi} E'_{qj} B_{ij} \sin(\theta_i - \theta_j) \\
0 &= r_j - D_j \omega_j - \sum_{k:j \rightarrow k} E'_{qj} E'_{qk} B_{jk} \sin(\theta_j - \theta_k) + \sum_{i:i \rightarrow j} E'_{qi} E'_{qj} B_{ij} \sin(\theta_i - \theta_j) \\
T'_{dj} \dot{E}'_{qj} &= E_{fj} - [1 - (x_{dj} - x'_{dj}) B_{jj}] E'_{qj} + (x_{dj} - x'_{dj}) \sum_{k \in \mathcal{N}(j)} E'_{qk} B_{jk} \cos(\theta_j - \theta_k)
\end{aligned}$$

where E'_{qj} is the q -axis transient internal voltage, T'_{dj} is the d -axis transient time constant, E_{fj} is the constant excitation voltage set to 1, x_{dj} and x'_{dj} are the d -axis synchronous and transient reactances, respectively, and $\mathcal{N}(j)$ denotes the set of neighbor buses that are connected to bus j .

REFERENCES

- [1] Z. Wang, F. Liu, J. Z. Pang, S. Low, and S. Mei, “Distributed optimal frequency control considering a nonlinear network-preserving model,” *arXiv preprint arXiv:1709.01543*, 2017.
- [2] T. Stegink, C. De Persis, and A. van der Schaft, “A unifying energy-based approach to stability of power grids with market dynamics,” *IEEE Transactions on Automatic Control*, vol. 62, no. 6, pp. 2612–2622, 2017.
- [3] T. W. Stegink, C. De Persis, and A. J. van der Schaft, “Stabilization of structure-preserving power networks with market dynamics,” *arXiv preprint arXiv:1611.04755*, 2016.