MATLAB and Simulink Model of Active Suspension

A Project Report

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by

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Chapter 1. Abstract

In this project, firstly different kinds of automobile suspension are described, namely independent and solid axle suspensions. They are further classified into active and passive suspension. The basic scope and history of the active suspension are also reviewed. A short introduction to PID controllers and their characteristics is summarized with the effect of the system by changing the respective proportions of the PID controllers. A step-wise methodology is used to create a model of the active suspension i.e. first the problem is defined and different free body diagrams are constructed. Then equations and transfer functions are constructed using the free body diagrams and control theory. Using the equations, a mathematical model is constructed in Simulink for different road profiles and different types of controllers. Passive suspension's response is also analyzing in different road profiles. Finally, all the controllers and passive suspension are compared using graphs. Characteristics of the system are also summarized in form of tables.

Chapter 2. Introduction

2.1 Automobile Suspension and Types

Suspension is a very important set of mechanical linkages, springs, and dampers to control the motion of the car and keep the wheels on the ground. Its main purpose is to improve the ride quality and dampen any road disturbances that are transferred to the car by the tire.

Over the years, due to improvements, different types of suspension have been developed based on the requirements of the car.

Mostly, the main two categories that all suspensions lie are in are either it is an independent suspension or it is a dependent suspension.

Dependent Suspension: These are also called solid axles. In a solid axle suspension, the wheels are mounted either end by a rigid beam so that any movement on one wheel is transmitted to the opposite wheel causing them to steer and camber together [1].

There are different types of solid axles such as Hotchkiss, De Dion, and four-link, etc.

Independent Suspension: Opposite to solid axles, the independent suspension allows the wheel to move vertically without affecting the opposite wheel. Today, most cars and light trucks are equipped with an independent suspension. [1]

Trailing arm suspension, short-long arm suspension, Macpherson strut, and wishbone suspension are examples of some independent suspensions.

Suspension systems can also be divided into either an active or passive suspension depending on whether the suspension can change its stiffness parameters.

2.1.1 Passive Suspension

Passive suspensions are those types of suspensions, in which there is at least one conventional shock absorber and spring. The spring has its linear or non-linear characteristics with force and the damper has non-linear characteristics with the force and relative velocity. Most shocks in cars are hydraulic and work on the principle of fluid friction. When a sudden impact is acted upon a shock absorber, the fluid flows through a small hole in the shock absorber in its piston, and a damping effect is created. The performance of the vehicle is constrained and improvements can be made by optimizing the characteristics of shock absorbers and springs.

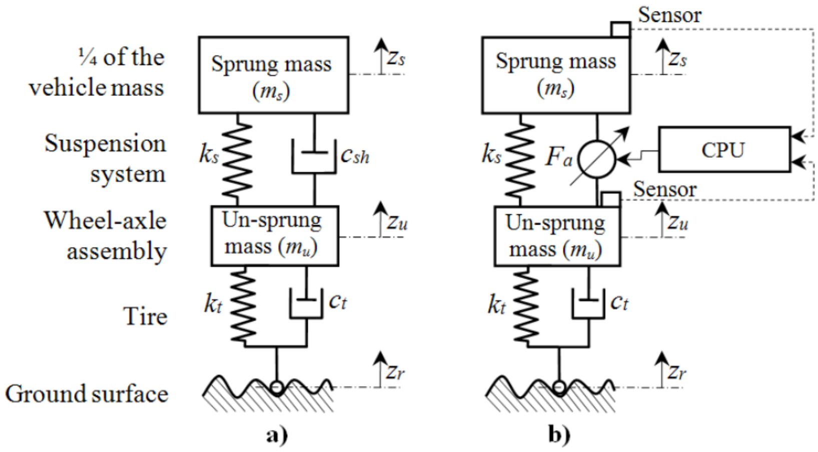

Figure 1 a) Passive Suspension. b) Active Suspension [2]

2.1.2 Active Suspension

The active suspension system, in addition to the already described components, is also comprised of an actuator, sensors, and a control programming unit (CPU). The shock absorber is replaced by an active force actuator. The operational conditions of the vehicle are continuously controlled by sensors that measure the velocity of the sprung and un-sprung masses and lead it to the CPU that ensures correct impulses for the actuator, which creates the desired active damping forces when required. [2]

2.1.2.1 History

Active suspension is one application of a general movement in the latter part of the 20th century to apply the newly emerged control theory body of knowledge to ground vehicles, and more particularly to chassis control systems. Before control theory could be applied, the ground vehicle had to be viewed as a dynamic system. Bill Milliken applied expertise gained in dynamically characterizing World War II aircraft to post-war passenger cars. Later, a member of his team wrote the dynamic equations of motion - the mathematical characterization of the relevant properties of a ground vehicle to control. Lotus was well known for pushing boundaries in automotive engineering. Below highway speeds, aerodynamics is a negligible external force on the vehicle. At racing speeds, they are critical. Under the technical direction of Peter Wright, who began his career as an aerodynamicist, Lotus began looking to computer-controlled suspension for a competitive advantage. In 1979 Lotus asked David Williams, who did flight control on the British military aircraft to do the algorithm work at the Cranfield Institute of Technology. Bill Milliken was a consultant to Lotus but his contributions went beyond the technical aspects. Lotus began to experiment with active suspension using electro-hydraulic actuation. Bill Milliken had long known Bill

Moog, the entrepreneurial founder of the upstate New York hydraulic servo valve company that bears his name. If David Williams' controller was the brain of the early Lotus active suspension, the Moog servo valve was its heart. Bill Milliken introduced Moog and Lotus. Eventually, they formed a joint venture company called Active Control Systems or ACS, in Stuart, Florida where a small team of engineers built on the fundamental pioneering work of Peter Wright and David Williams [3].

2.2 PID Controllers and Feedback System

PID controllers are used to control error from the reference value of the system and with the usage of the controller, the error is rectified and the system stays on the reference value. In a control system, to achieve our desired target settings for the system, usually, a controller is used which is named a PID controller. These controllers have three parts namely "P", "I" and "D". These acronyms relate to Proportional, Integral, and Derivative

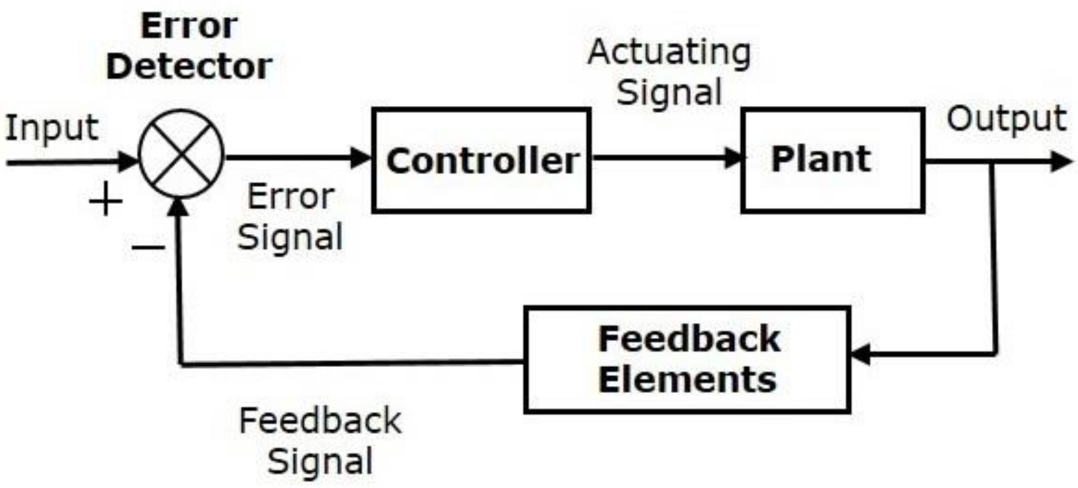

Figure 2 PID Controller and Feedback [4]

Mathematically, the controller can be described as.

$$u(t) = k_P e(t) + k_I \int e(t)dt + k_D \frac{de(t)}{dt} \tag{1}$$

Where the function u(t) denotes the function of the controller in the time domain.

For the four different combinations of PIDs, the following table should be kept in mind

Control Response	Rise Time	Overshoot	Settling Time	S-S Error
kp	Decrease	Increase	Small Change	Decrease
ki	Decrease	Increase	Increase	Decrease
kd	Small Change	Decrease	Decrease	No change

Chapter 3. Methodology

Firstly, all the models of are discussed in problem definition and then 2 DOF dynamic model is used for the analysis of active suspension.

Secondly, using Simulink the system is modeled with the integration of different controllers

Finally, all the results are compared with each other to find the best results.

The methodology can be summarized as:

1. Definition of All types of models
2. Understanding of 2 DOF model
3. Equations of Motion for 2 DOF
4. Model of Suspensions with different controllers
5. Comparison of the results obtained

3.1 Problem Definition

A very common approach that is used to model a dynamic system is called the "Lumped Parameter Approach". In this approach, the system under study is divided into discrete lumped parameters which offer a simplified model when compared to a continuous system. An example is shown in Figure 3. The system then can be solved by writing its equations of motions and solving it in either frequency domain or by using state-space methods.

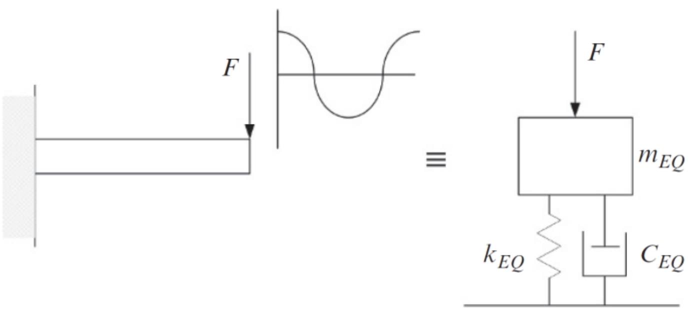

Figure 3 Lumped Parameter Approach to Dynamic Modeling [5]

3.1.1 Modeling Suspension as 1 DOF

A suspension can be considered to be a single degree of freedom with a harmonic or step input. It is assumed that the whole car moves vertically together and there is no roll or pitch of the car. The upright of the car is not modeled and input from the road is assumed to be entirely in heave with all four wheels moving as one. This method yields useful results despite being simplistic. This 1 DOF system is shown in Figure 4.

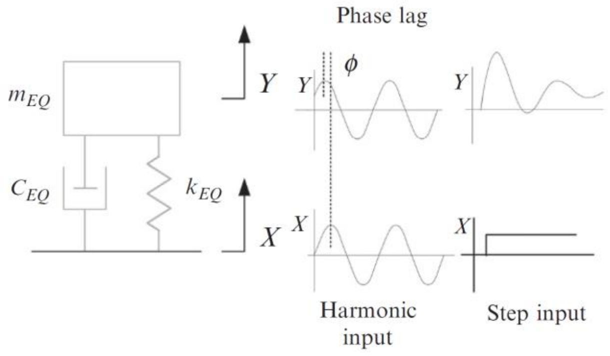

Figure 4 1 DOF Suspension System [5]

3.1.2 Modeling Suspension as 2 DOF

A 2 DOF system is the most popular choice for the analysis of suspension in terms of complexity and desired results. Equations of motions are firstly written for each body with the help of the principle of superposition. Then a transfer function is calculated from the equations by solving matrices using Cramer's rule. The input variable can be chosen as the displacement of the sprung mass or the input force. An example of this type of system is shown in Figure 5.

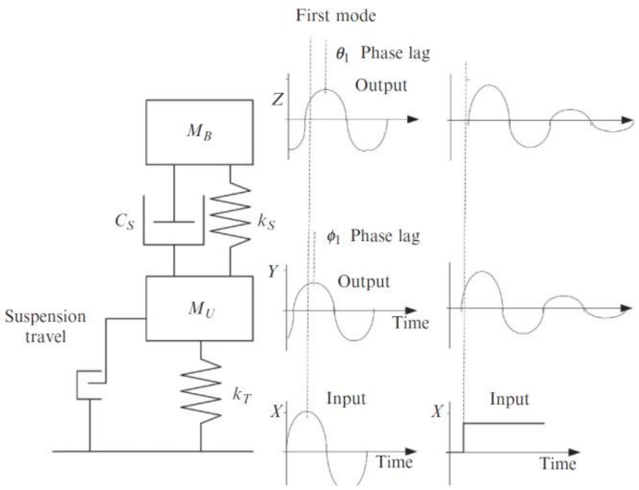

Figure 5 2 DOF Suspension System [5]

3.1.3 Modeling Suspension as 4 DOF

As the degree of freedom increases, the complexity of the model also increases. In this mode, uprights are free to move vertically but now we are dealing with two uprights. The body is also free to move vertically and rotationally about its center of gravity. If the system is modeled from the front view, then car roll can be used as a variable and if the system is modeled from the side view, then the car's pitch can be used as a variable. We can also take road profile as input since two tires are being used in this model. The input consists of a force applied to the body the center of gravity height and a magnitude. An example of this system is shown in Figure 6

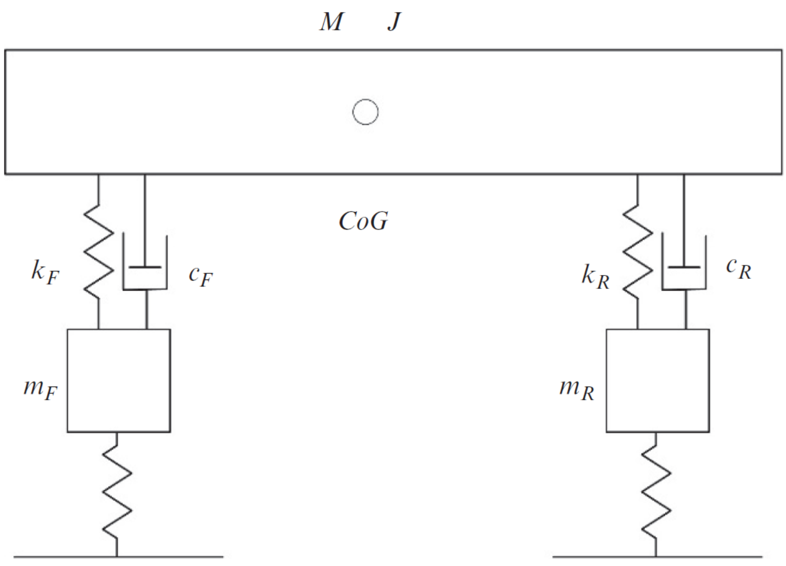

Figure 6 4 DOF Suspension System [5]

3.1.4 Modeling Suspension as 7 DOF

In the 7 DOF freedom system, input from all tires, heave, and pitch is also considered. An example is shown in Figure 7. There are also higher DOF models than 7 DOF up to 14 DOF and as the complexity increases, so does the information obtained from them.

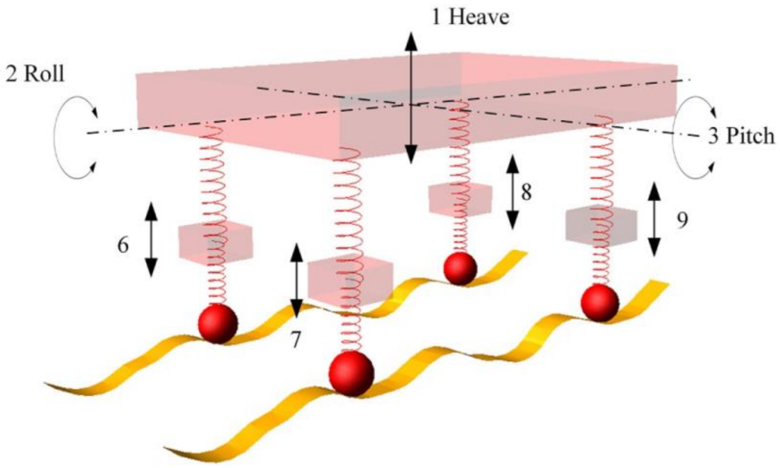

Figure 7 7 DOF Suspension System [5]

3.2 Dynamic Model of 2 DOF

2 DOF model uses equations of motions applied to Figure 5. In the succeeding sections, we will develop a model for 2 DOF for both passive and active suspension.

3.3 Equations of Motions and Free-Body Diagrams of Passive Suspension

Referring to Figure 5 the different terminologies listed in the diagram are described in Table 1 [6]

Table 1 Suspension Parameters and Values

Parameters	Values
Sprung Mass (M_b)	300 kg
Unsprung Mass (M_u)	50 kg
Suspension Spring Stiffness (k_S)	18000 N/m
Suspension Damping Coefficient (C_S)	1200 Ns/m
Tire Stiffness (k_T)	180000 N/m
Tire Damping Coefficient (C_T)	800 Ns/m
Displacement of Sprung Mass (X_1)	-
Displacement of UnSprung Mass (X_2)	-
Tire Profile/Bump (X_3)	-

The free-body diagram of the unsprung and sprung mass is shown in Figure 8 and Figure 9

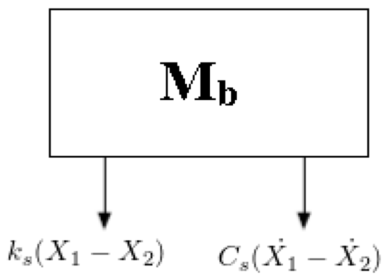

Figure 8 Sprung-Mass Free Body Diagram

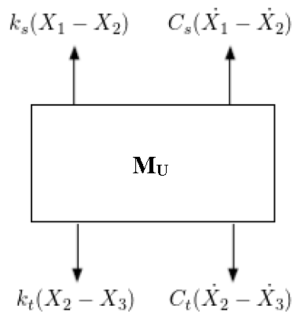

Figure 9 UnSprung-Mass Free Body Diagram

From Figure 8, the equation of motion of the sprung mass is:

$$M_b\ddot{x}_1 + C_s\dot{x}_1 + k_s x_1 - (k_s x_2 + C_s\dot{x}_2) = 0 \tag{2}$$

And from Figure 9, the equation of motion of unsprung mass is:

$$M_u\ddot{x}_2 + (C_s + C_t)\dot{x}_2 + (k_s + k_t)x_2 - (k_s x_1 + C_s\dot{x}_1) - (k_t x_3 + C_t\dot{x}_3) = 0 \tag{3}$$

After rearranging both the equations, we get.

$$M_u\ddot{x}_2 + (x_2 - x_1)k_s + (\dot{x}_2 - \dot{x}_1)C_s + (x_2 - x_3)k_t + (\dot{x}_2 - \dot{x}_3)\mathrm{C}_{\mathrm{b}} = 0 \tag{4}$$

$$M_b\ddot{x}_1 + (x_1 - x_2)k_s + (\dot{x}_1 - \dot{x}_2)C_s = 0 \tag{5}$$

These equations can be converted into the frequency domain by taking Laplace to transform.

$$-(sC_s + k_s)X_1(s) + (M_u s^2 + sC_s + sC_t + k_s + k_t)X_2(s) - (sC_t + k_t)X_3(s) = 0 \tag{6}$$

$$(M_b s^2 + sC_s + k_s)X_1(s) - (sC_s + k_s)X_2(s) = 0 \tag{7}$$

3.4 Equations of Motions and Free-Body Diagrams of Active Suspension

A free-body diagram of the unsprung and sprung mass is shown in Figure 10 and Figure 11

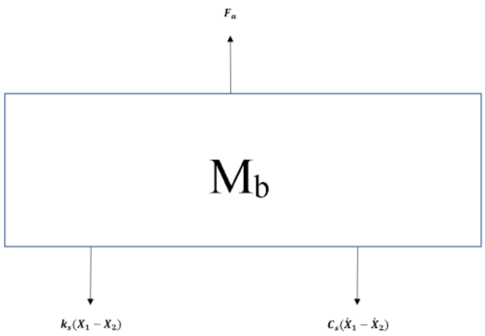

Figure 10 Sprung-Mass Free Body Diagram

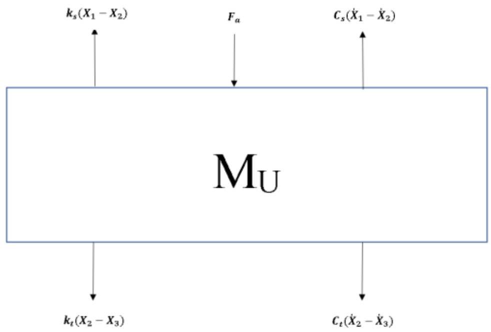

Figure 11 UnSprung-Mass Free Body Diagram

From Figure 10, the equation of motion of sprung mass is:

$$F_a + M_b\ddot{x}_1 + C_s\dot{x}_1 + k_s x_1 - (k_s x_2 + C_s\dot{x}_2) = 0 \tag{8}$$

And from Figure 11, the equation of motion of unsprung mass is:

$$F_a + M_u\ddot{x}_2 + (C_s + C_t)\dot{x}_2 + (k_s + k_t)x_2 - (k_s x_1 + C_s\dot{x}_1) - (k_t x_3 + C_t\dot{x}_3) = 0 \tag{9}$$

After rearranging both the equations, we get.

$$M_u\ddot{x}_2 + (x_2 - x_1)k_s + (\dot{x}_2 - \dot{x}_1)C_s + (x_2 - x_3)k_t + (\dot{x}_2 - \dot{x}_3)C_b - F_a = 0 \tag{10}$$

$$M_b\ddot{x}_1 + (x_1 - x_2)k_s + (\dot{x}_1 - \dot{x}_2)C_s - F_a = 0 \tag{11}$$

These equations can be converted into the frequency domain by taking Laplace transform.

$$-(sC_s + k_s)X_1(s) + (M_u s^2 + sC_s + sC_t + k_s + k_t)X_2(s) - (sC_t + k_t)X_3(s) = F_a \tag{12}$$

$$(M_b s^2 + sC_s + k_s)X_1(s) - (sC_s + k_s)X_2(s) = F_a \tag{13}$$

The transfer function $X_2(s)/F_a(s)$ can be found by using Cramer's rule on Equation (12) and Equation (13) and letting $X_3(s)=0$

The transfer function $X_2(s)/Fa(s)$ can be found by using Cramer's rule on Equation (12) and Equation (13) and letting $X_3(s)=0$

$$\Delta= \begin{vmatrix} -(sC_s + k_s) & M_u s^2 + sC_s + sC_t + k_s + k_t \\ (M_b s^2 + sC_s + k_s) & -(sC_s + k_s) \end{vmatrix} \tag{14}$$

$$X_1(s) = \frac{\begin{vmatrix} F_a & M_u s^2 + sC_s + sC_t + k_s + k_t \\ F_a & -(sC_s + k_s) \end{vmatrix}}{\Delta} \tag{15}$$

$$X_2(s) = \frac{\begin{vmatrix} -(sC_s + k_s) & F_a \\ (Ms^2 + sb_1 + k_s) & F_a \end{vmatrix}}{\Delta} \tag{16}$$

The transfer function then is:

$$\frac{X_1(s)}{F_a(s)} = \frac{2k_s + k_t + 2C_s s + C_t s + M_u s^2}{(k_s k_t + C_s M_b s^3 + C_t M_b s^3 + C_s M_u s^3 + k_s M_b s^2 + k_t M_b s^2 + k_s{*}M_u s^2 + M_b M_u s^4 + C_s{*}k_t s + C_t k_s s + C_s C_t s^2)} \tag{17}$$

$$\frac{X_2(s)}{F_a(s)} = \frac{M_b s^2 + 2C_s s + 2k_s}{(k_s k_t + C_s M_b s^3 + C_t M_b s^3 + C_s M_u s^3 + k_s M_b s^2 + k_t M_b s^2 + k_s{*}M_u s^2 + M_b M_u s^4 + C_s{*}k_t s + C_t k_s s + C_s C_t s^2)} \tag{18}$$

Using values written in Table 1, the transfer function of both the displacements are:

$$\frac{X_1(s)}{F_a(s)} = \frac{s^2 + 64s + 4320}{300(s^4 + 44s^3 + 4084s^2 + 15360s + 216000)} \tag{19}$$

$$\frac{X_2(s)}{F_a(s)} = \frac{s^2 + 8s + 120}{50(s^4 + 44s^3 + 4084s^2 + 15360s + 216000)} \tag{20}$$

The transfer function has been solved using MATLAB code listed in **Error! Reference source not found.**

The equations for passive and active suspension can be modeled in Simulink with the chosen noise profile.

3.5 Modeling in MATLAB/Simulink

Modeling of the active and passive suspension system was performed in Simulink with the equations derived in the preceding sections.

As seen from Figure 12, the equations have been modeled using the basic blocks available in the Simulink.

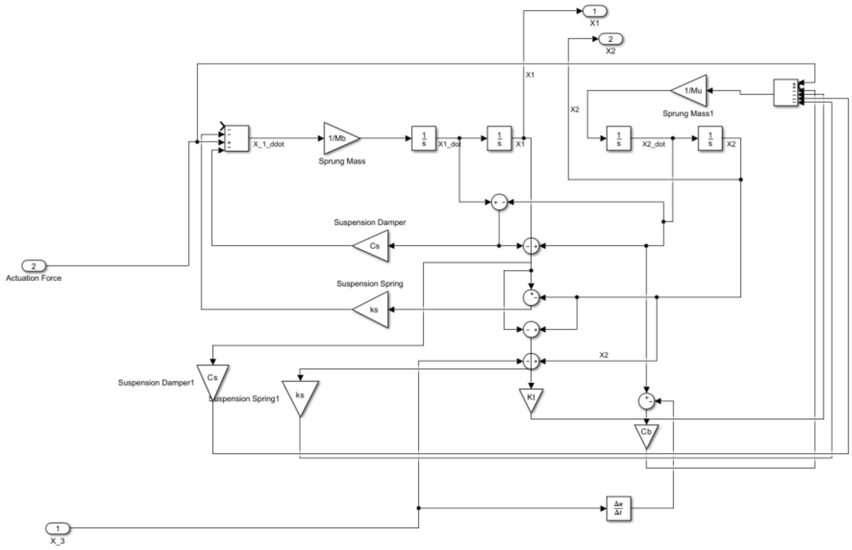

Figure 12 Basic Modeling of Equations Using Simulink

This basic model is also the same for the active suspension but in active suspension, a PID controller is used and is shown in Figure 13 and Figure 14.

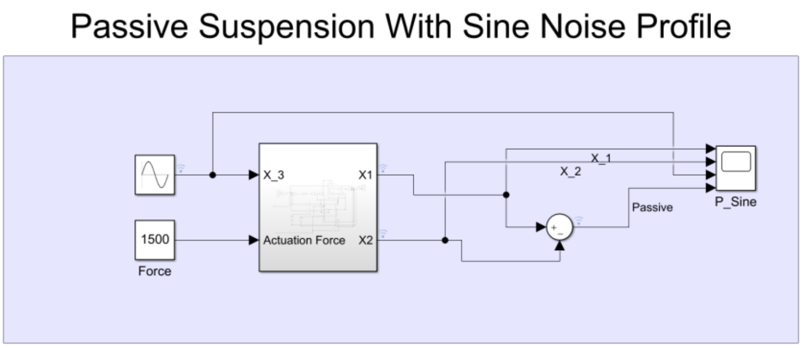

Figure 13 Passive Suspension Model

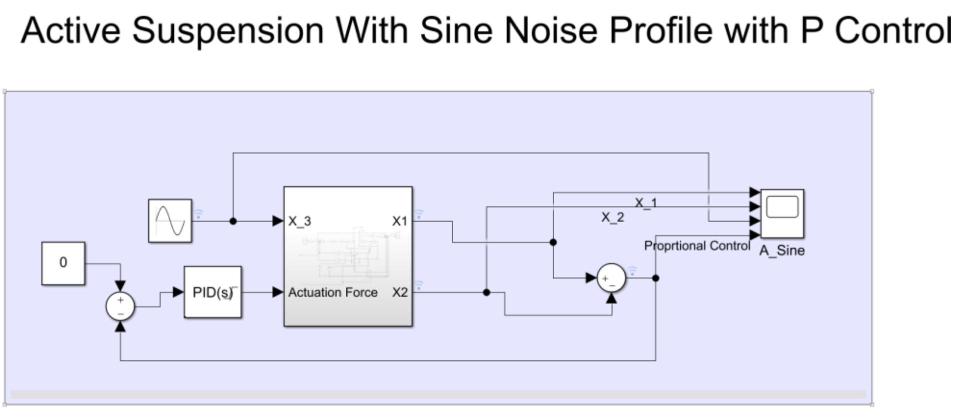

Figure 14 Active Suspension Model

Different noise profiles with different controllers have been used to compare the results obtained by the passive and active suspension in different situations as shown in Figure 15

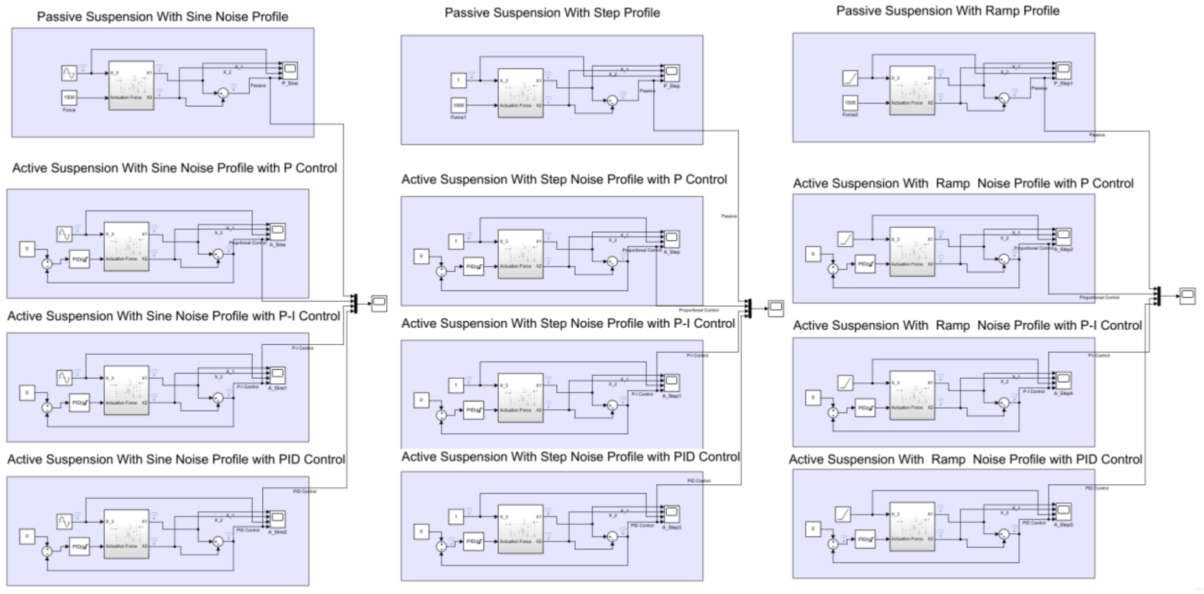

Figure 15 Different Controllers with 3 Different Noise Profiles

3.5.1 Model of Passive Suspension

In the model of passive suspension, instead of a feedback-controlled force, a constant force of 1500 N is applied with various road noise profiles. The passive model behaves differently corresponding to each of the noise profiles. The passive suspension model is shown in Figure 13

3.5.2 Model of Active Suspension

In the model of active suspension, all the basic elements are similar to passive suspension but the actuation force is controlled by the error in the unsprung and sprung mass displacements. The error is controlled by a PID Controller. The model of active suspension is shown in Figure 14.

The basic parameters in the PID controller of Simulink are shown in Figure 16

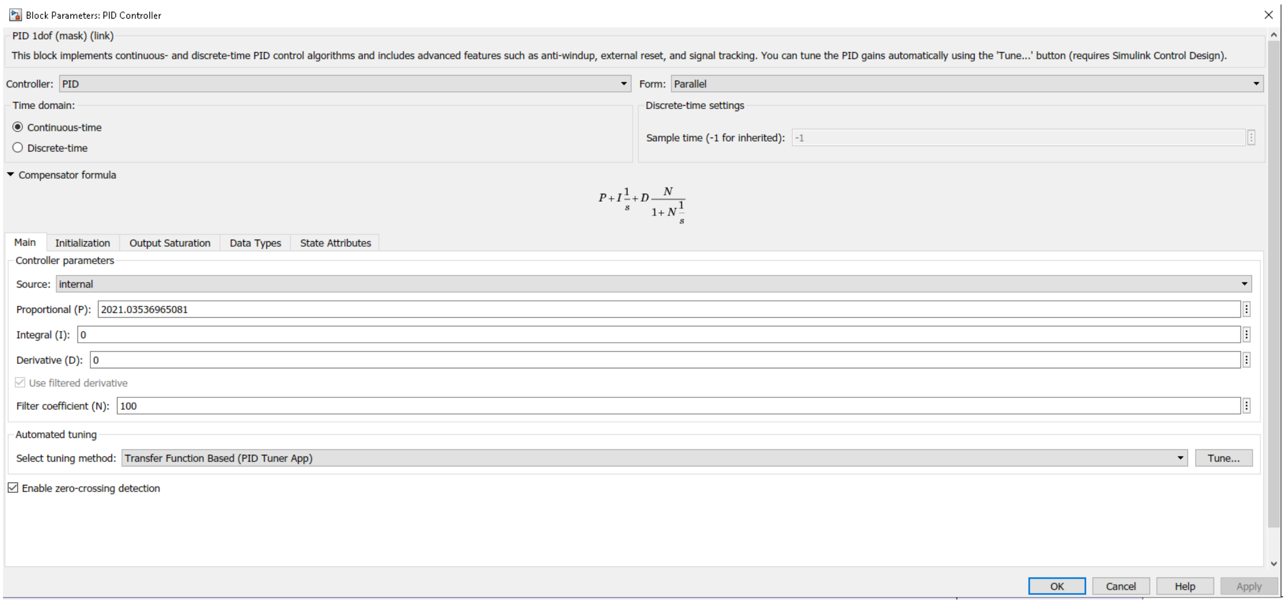

Figure 16 PID Controller Properties in Simulink

The PID can be tuned using the Simulink Tuning Toolbox as shown in Figure 17

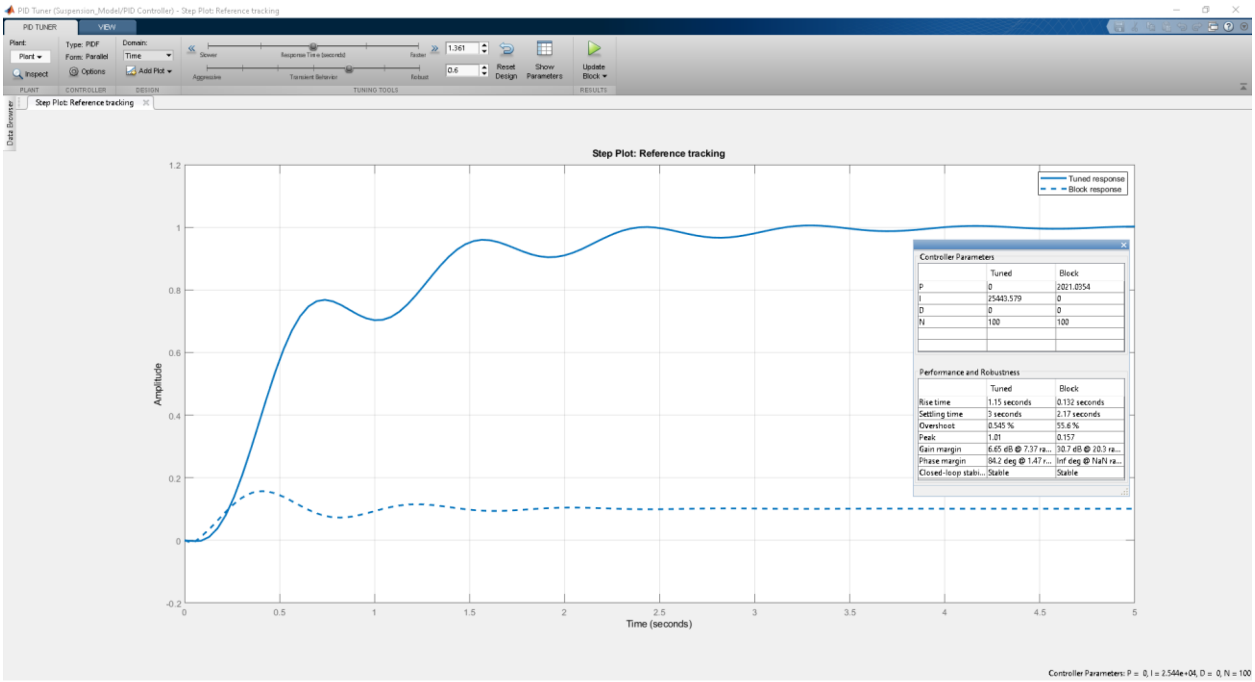

Figure 17 PID Tuner Simulink

The PID Controller can be tuned to give the best settling time and rise for the suspension. This is a manual tool to increase or decrease the values of Ki, Kp, or Kd to vary the different characteristics of the controller. The different information about the system is summarized n the Controller Parameters.

Chapter 4. Results and Discussion

4.1 Comparison of Different Results

Three road profiles step, sine, and ramp have been used to compare the different controllers of active suspension and the passive suspension.

For the sine profile, an amplitude of 0.05 is used and for the ramp profile, an average bump height of 7.5 cm is used for the analysis. The results are shown in

4.2 Ramp-Step-Ramp Noise Profile

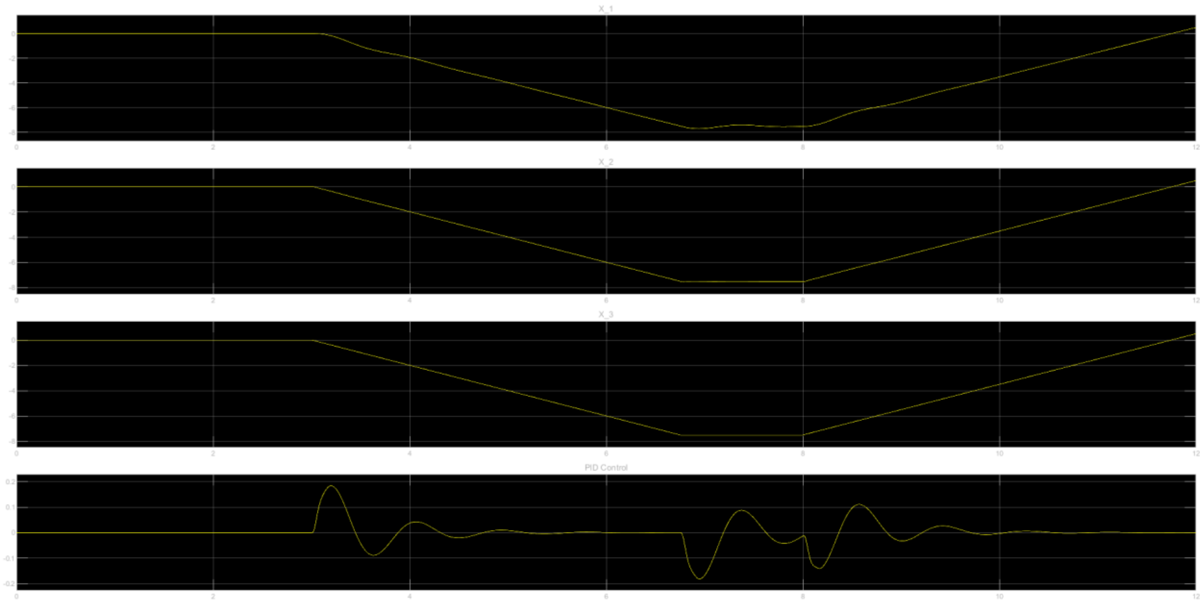

Figure 18 PID Control Response of Ramp-Step-Ramp Profile

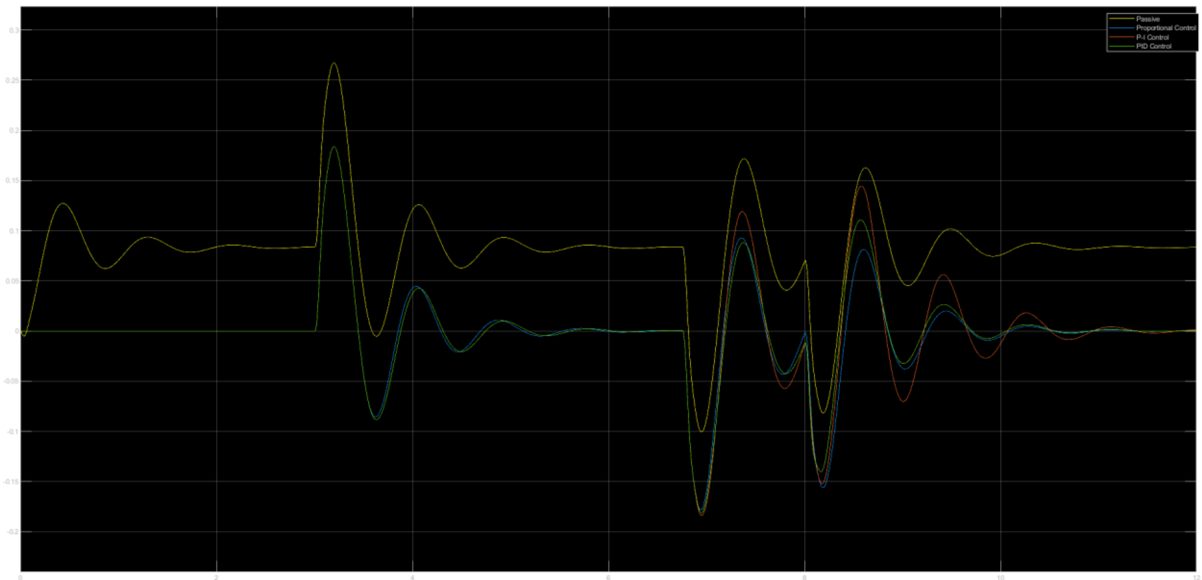

Figure 19 Comparison of Different Ramp-Step-Ramp Responses

In Figure 19, a comparison of different controllers with the passive suspension has been made. The first thing to notice is the steady-state error of the passive suspension and the proportional control. They do not come to rest which is the zero-amplitude condition. P-I control has a slightly larger settling time and also has a larger overshoot percentage.

4.2.1 Characteristics of the System:

4.2.1.1 Proportional Controller

Rise Time	0.132
Settling Time	2.17
Percentage Overshoot	55.4%
P	2044
I	0
D	0

4.2.1.2 P-I Controller

Rise Time	1.07
Settling Time	2.78
Percentage Overshoot	0.944%
P	2044
I	29116
D	0

4.2.1.3 PID Controller

Rise Time	1.35
Settling Time	2.42
Percentage Overshoot	0%
P	2044
I	29116
D	500

4.3 Sinusoidal Noise Profile

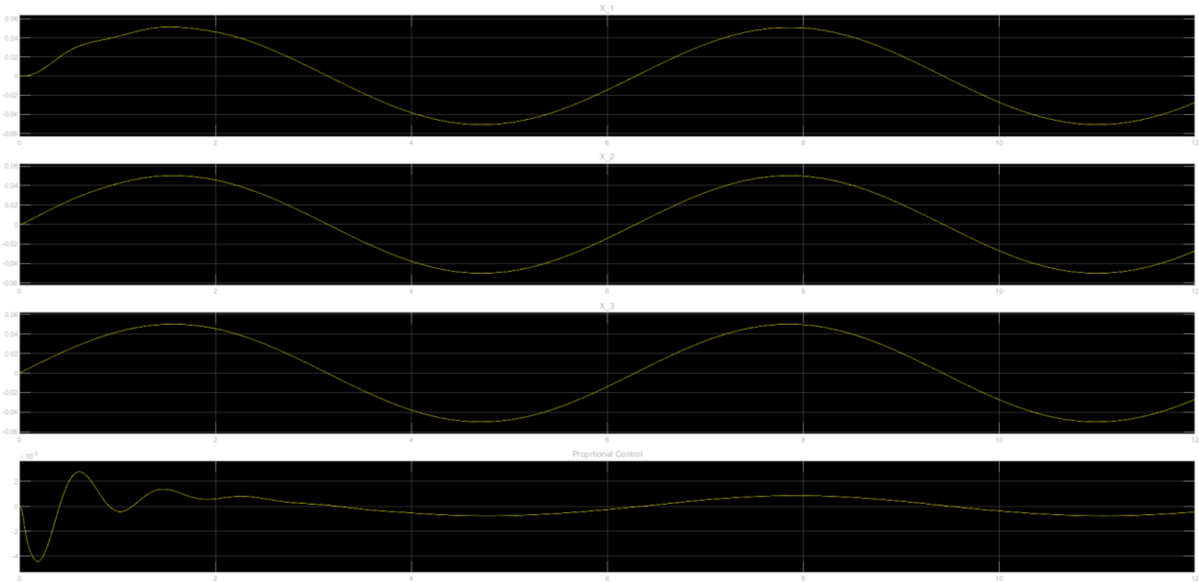

Figure 20 PID Control Response of Sine Profile

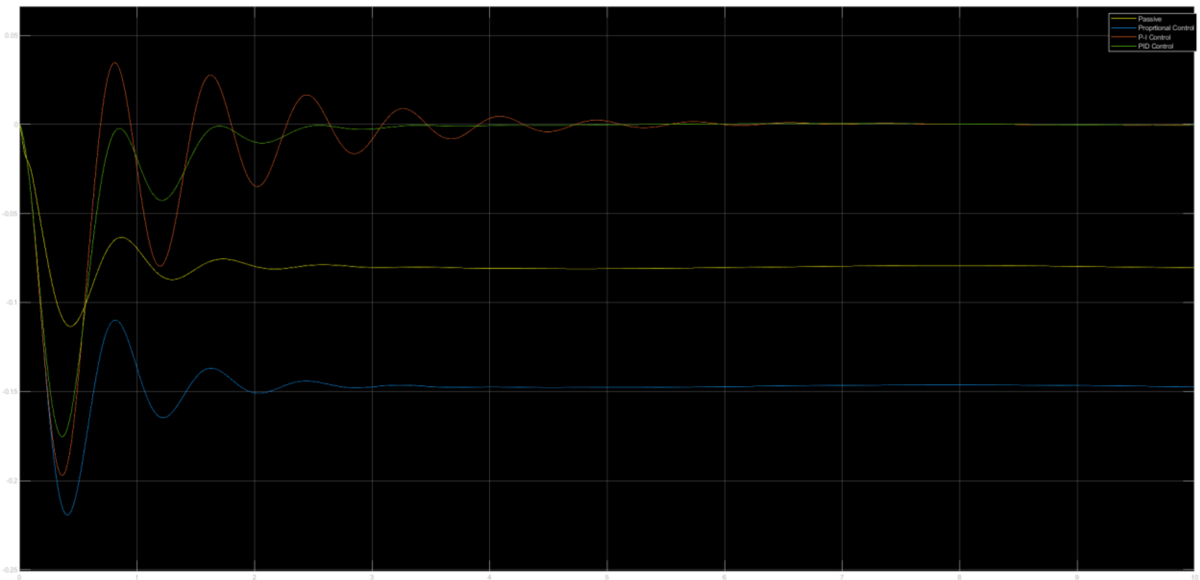

Figure 21 Comparison of Different Sine Responses

During a sinusoidal excitation, the proportional controller and passive suspension have high amounts of steady-state error. P-I controller has a high settling time as compared to other controllers while PID controller settles with little overshoot and rising time.

4.3.1 Characteristics of the System:

4.3.1.1 Proportional Controller

Rise Time	0.132 s
Settling Time	2.17 s
Percentage Overshoot	55.4%
P	2021
I	0
D	0

4.3.1.2 P-I Controller

Rise Time	1.06 s
Settling Time	2.78 s
Percentage Overshoot	1.12%
P	2021
I	29600
D	0

4.3.1.3 PID Controller

Rise Time	1.32 s
Settling Time	2.36 s
Percentage Overshoot	0%
P	2021
I	29600
D	500

4.4 Step Profile

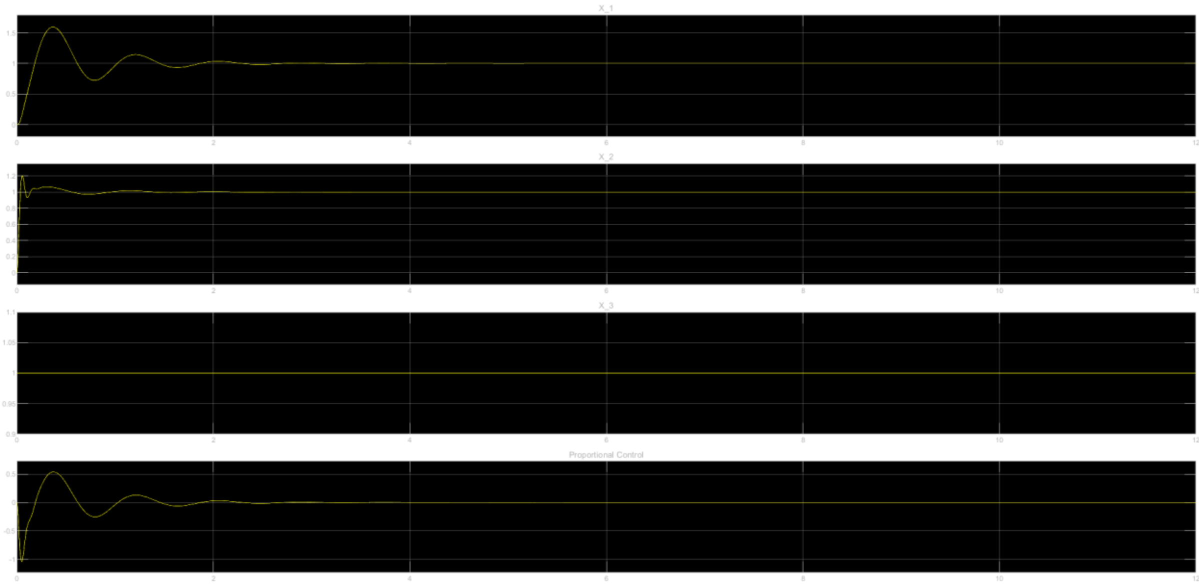

Figure 22 PID Control Response of Step Profile

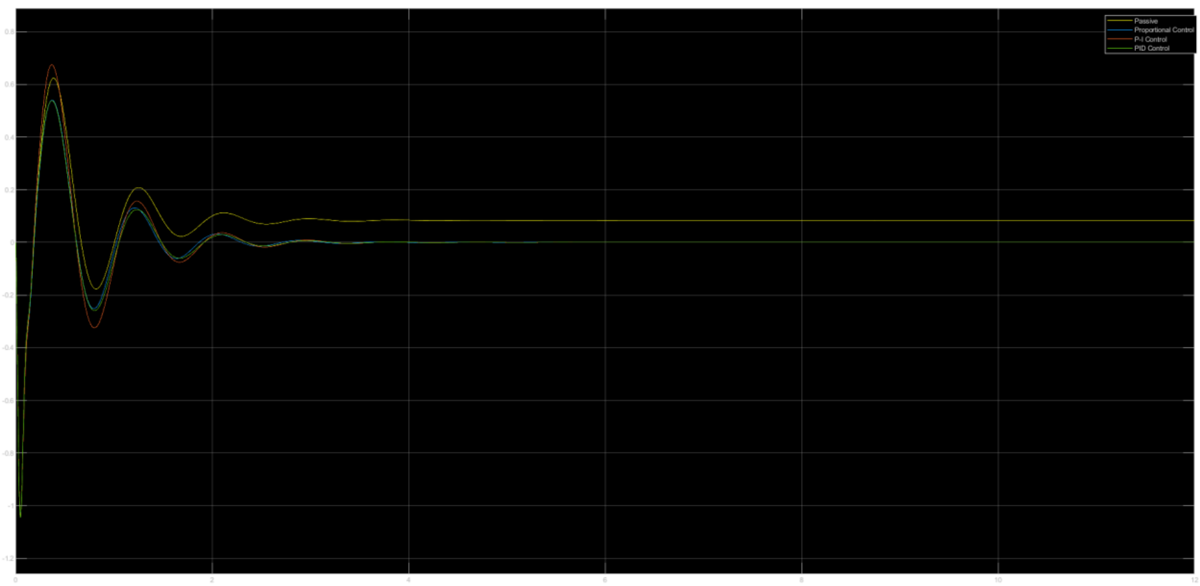

Figure 23 Comparison of Different Step Responses

The step response of the controller is almost the same except for the steady-state error of the passive suspension and percentage overshoot.

The percentage overshoot of proportional controller in every road profile is very high but its settling time is very low which is an advantage

Overall PID controller has the most controlled response with no percentage overshoot and reasonable

4.4.1 Characteristics of the System:

4.4.1.1 Proportional Controller

Rise Time	0.132 s
Settling Time	2.17 s
Percentage Overshoot	55.4%
P	2044
I	0
D	0

4.4.1.2 P-I Controller

Rise Time	1.07
Settling Time	2.78
Percentage Overshoot	0.944%
P	2044
I	29116
D	0

4.4.1.3 PID Controller

Rise Time	1.35
Settling Time	2.42
Percentage Overshoot	0%
P	2044
I	29116
D	500

4.5 Changing Tires and Stiffness:

Now to compare the changes in the system, we assume that the tires are changed and mass of the car increases by 40 kg and stiffness decreases by 10000 N/m, and damping coefficient increases by 100 Ns/m

Parameters	Old Values	New Values
Sprung Mass (M_b)	300 kg	300 kg
Unsprung Mass (M_u)	50 kg	90 kg
Suspension Spring Stiffness (k_S)	18000 N/m	1800 N/m
Suspension Damping Coefficient (C_S)	1200 Ns/m	1200 Ns/m
Tire Stiffness (k_T)	180000 N/m	170000 N/m
Tire Damping Coefficient (C_T)	800 Ns/m	900 Ns/m
Displacement of Sprung Mass (X_1)	-	-
Displacement of UnSprung Mass (X_2)	-	-
Tire Profile/Bump (X_3)	-	

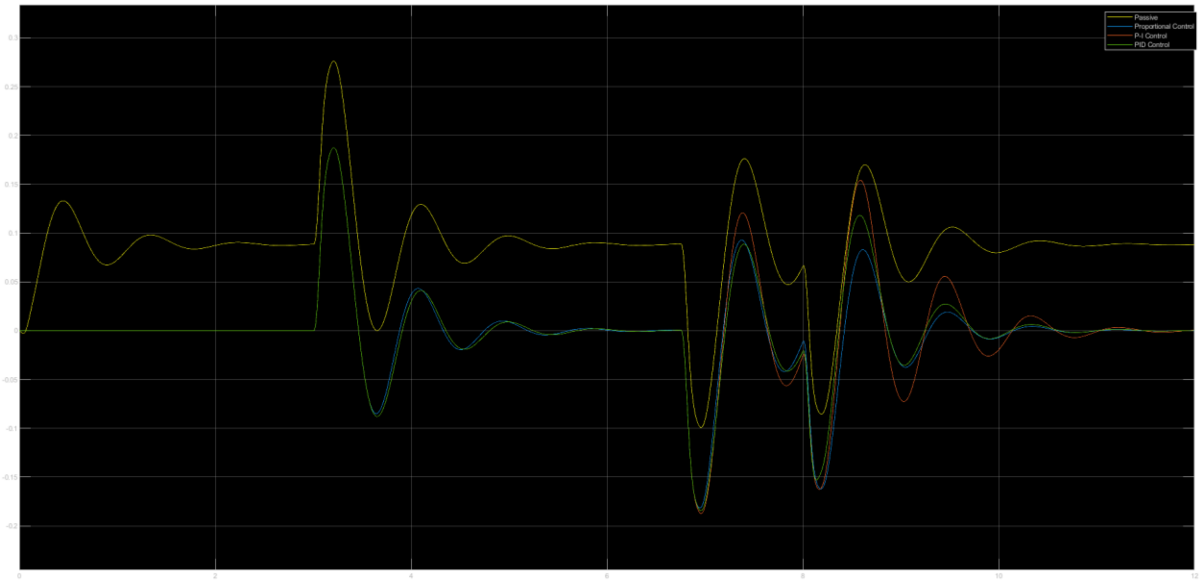

Figure 24 New Response to Ramp-Step-Ramp Profile

After changing the system parameters, the new response is shown in Figure 24.

Parameter	P Controller	P-I Controller	PID Controller
Rise Time	0.135s	1.07s	1.24s
Settling Time	2.21s	2.83s	2.25s
Percentage Overshoot	53.7%	1.71%	0.998%
P	2044	2044	2044
I	0	29116	29116
D	0	0	500

The new characteristics of the system are shown in the table above.

The response of the system has become better by changing the tires i.e the rise time of PID Controller has changed from 1.35 s to 1.24 s and settling time has changed from 2.42 s to 2.25 s which shows that with the further tuning of the system, PID controller can be used in all situations. The percentage overshoot has slightly increased but it is a negligible change.

Chapter 5. Conclusion

From the data obtained in the preceding sections, we conclude that active suspension performs much better than conventional passive suspension. The magnitude of the vibrations and their settling is actively controlled by the system's CPU and ensures maximum ride comfortability. The control system can be further improved by considering models of higher degrees of freedom and tuning the Ki, Kp, and Kd parameters of the controller to get the required characteristics of the system

Chapter 6. Bibliography

[1] Thomas D. Gillespie, Fundamentals of Vehicle Dynamics, 1st ed., Micihgan: SAE, 1992.

[2] Shpetim Lajqi and Stanislav Pehan, "Designs and Optimizations of Active and Semi-Active Non-linear Suspension Systems for a Terrain Vehicle," *Journal of Mechanical Engineering*, vol. 58, no. 12, pp. 732-743, 2012.

[3] Daniel Eugene Williams, "Active Suspension: Future Lessons From the Past," *SAE International*, vol. 2, no. 2, pp. 147-165, 2018.

[4] "CISE-302, Linear-Control Systems Lab Manual," [Online]. Available: kfupm.edu.sa/departments/se/SiteCollectionDocuments/CISE-302-Linear-Control-Systems-Lab-Manual.pdf.

[5] Balkwill, James, Performance Vehicle Dynamics: Engineering and Applications, Elsevier/Butterworth-Heinemann, 2019.

[6] D. C. Tran, "Quarter Car Modelling," [Online]. Available: https://www.mathworks.com/matlabcentral/fileexchange/46316-quarter-car-modelling-zip.