



DeepLearning.AI

# Math for Machine Learning

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## Linear algebra - Week 1

Systems of linear equations

Singular and non-singular matrices

Determinants

Rank of a matrix

Row reduction

Null space



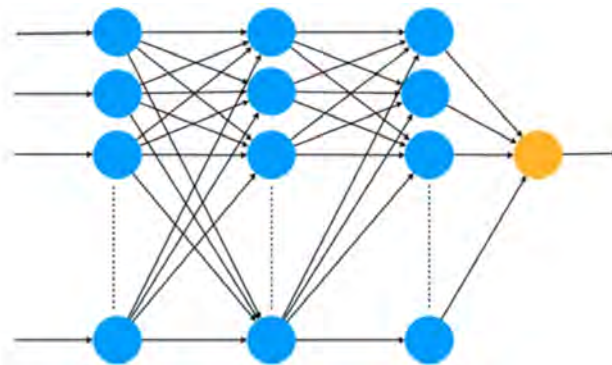
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# System of Linear Equations

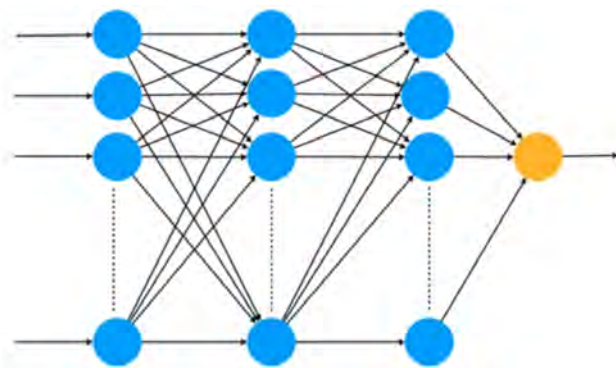
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**Machine learning motivation**

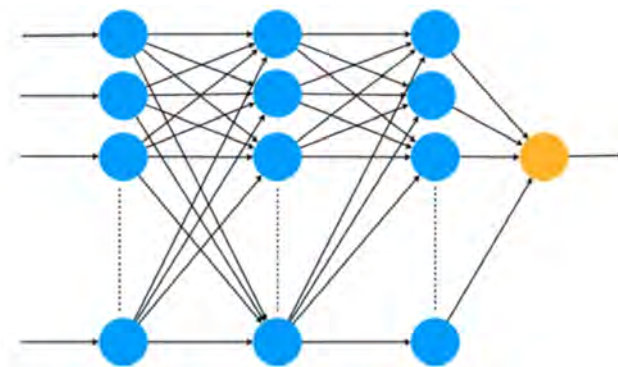
# Neural networks - Matrix operations



# Neural networks - Matrix operations

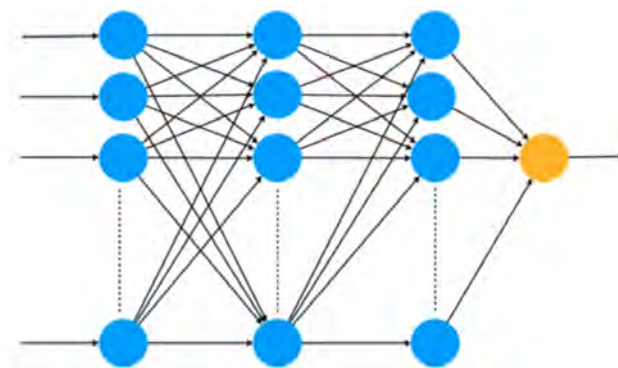


# Neural networks - Matrix operations



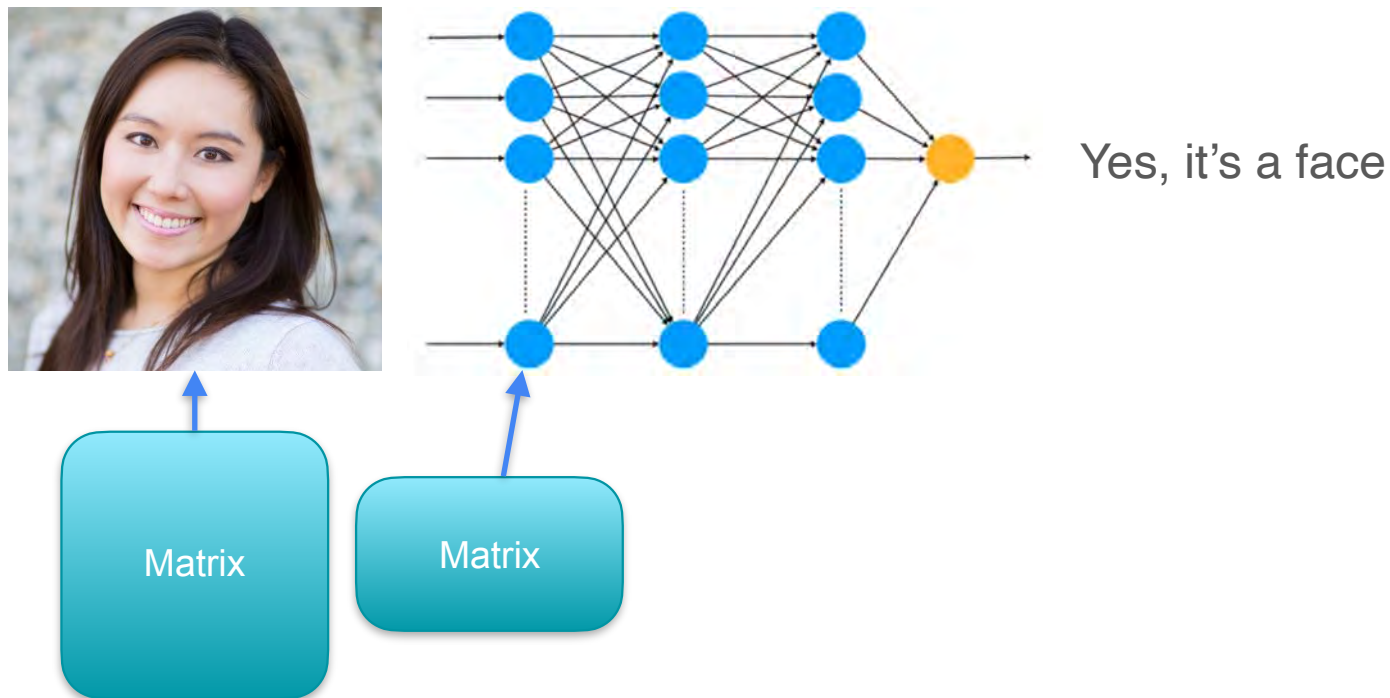
Yes, it's a face

# Neural networks - Matrix operations

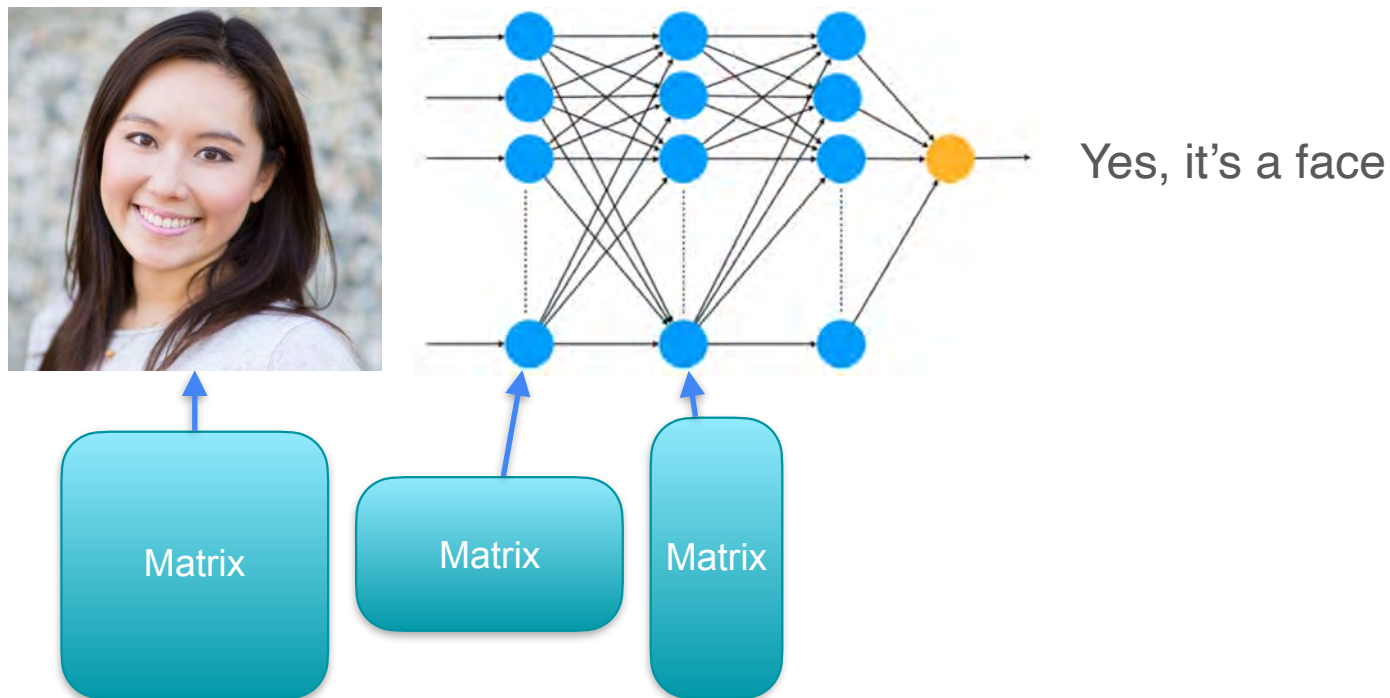


Yes, it's a face

# Neural networks - Matrix operations

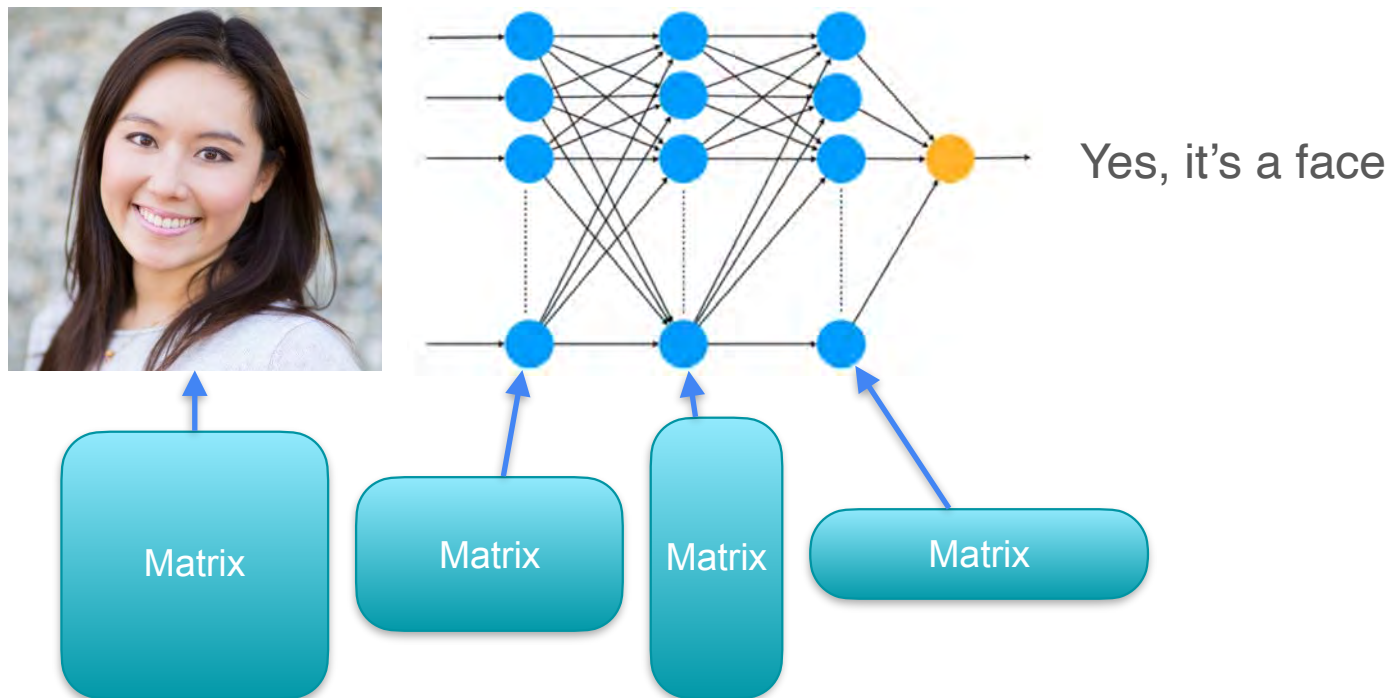


# Neural networks - Matrix operations





# Neural networks - Matrix operations



# Neural networks - image recognition

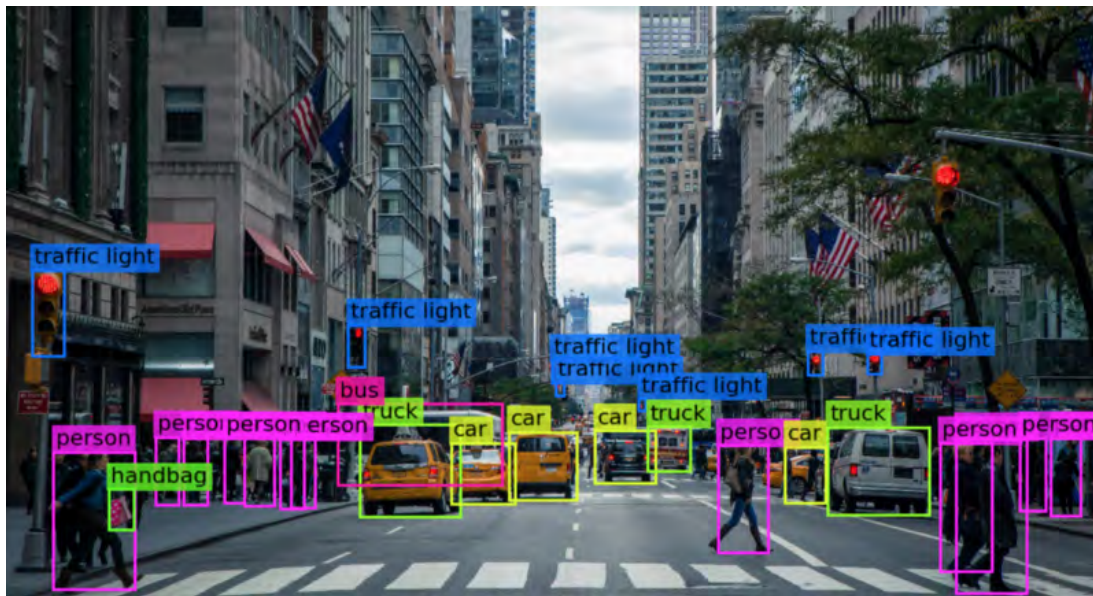


Image recognition in a busy street in New York.

- Image recognition: Getting the computer to see images and recognize what is on them.



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# System of Linear Equations



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**System of sentences**

# Systems of sentences



# Systems of sentences

## System 1



 The dog is **black**  
 The cat is **orange**

# Systems of sentences

## System 1



 The dog is **black**  
 The cat is **orange**

## System 2



 The dog is **black**  
 The dog is **black**

# Systems of sentences



## System 1

 The dog is **black**  
 The cat is **orange**

## System 2



 The dog is **black**  
 The dog is **black**

## System 3



 The dog is **black**  
 The dog is **white**

# Systems of sentences



## System 1

 The dog is **black**  
 The cat is **orange**

## System 2

 The dog is **black**  
 The dog is **black**

## System 3



 The dog is **black**  
 The dog is **white**

Complete





# Systems of sentences

## System 1

 The dog is **black**  
 The cat is **orange**



Complete

## System 2

 The dog is **black**  
 The dog is **black**



Redundant

## System 3

 The dog is **black**  
 The dog is **white**



# Systems of sentences

## System 1

 The dog is **black**  
 The cat is **orange**



Complete

## System 2

 The dog is **black**  
 The dog is **black**

Redundant



## System 3

 The dog is **black**  
 The dog is **white**

Contradictory



# Systems of sentences

## System 1

 The dog is **black**  
 The cat is **orange**

Complete



## System 2

 The dog is **black**  
 The dog is **black**

Redundant

**Singular**

## System 3



 The dog is **black**  
 The dog is **white**

Contradictory

**Singular**

# Systems of sentences



## System 1

 The dog is **black**  
 The cat is **orange**

Complete

**Non-singular**



## System 2

 The dog is **black**  
 The dog is **black**

Redundant

**Singular**

## System 3

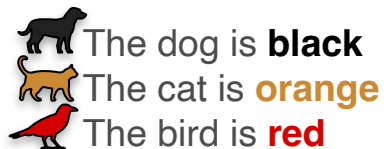
 The dog is **black**  
 The dog is **white**

Contradictory

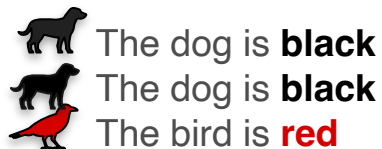
**Singular**

# Systems of sentences

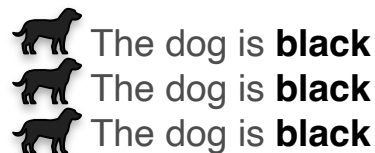
System 1



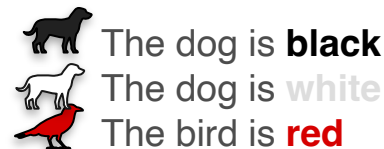
System 2



System 3

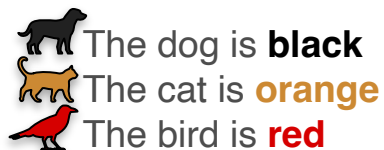


System 4

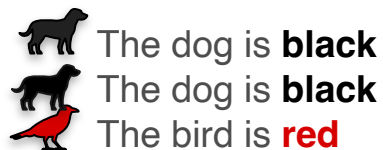


# Systems of sentences

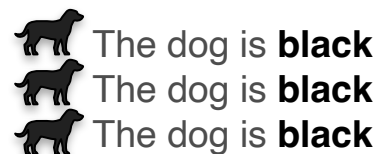
System 1



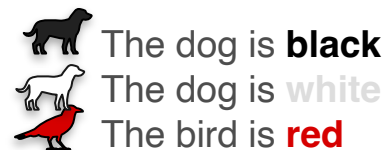
System 2



System 3



System 4

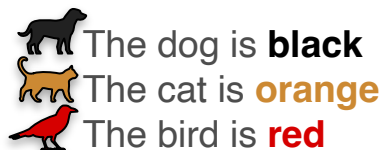


Complete

Non-singular

# Systems of sentences

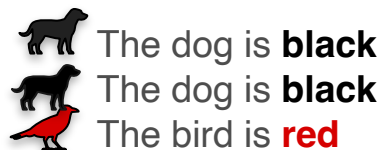
System 1



Complete

Non-singular

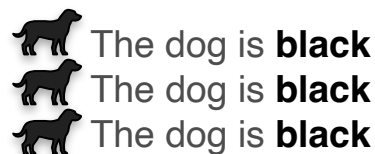
System 2



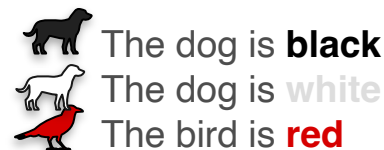
Redundant

Singular

System 3

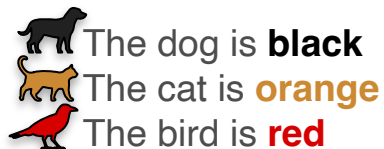


System 4



# Systems of sentences

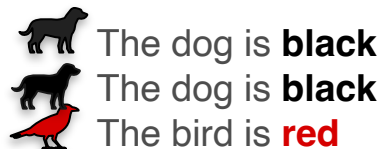
System 1



Complete

Non-singular

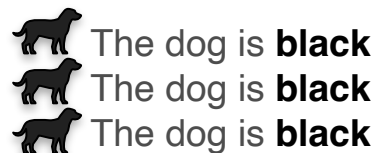
System 2



Redundant

Singular

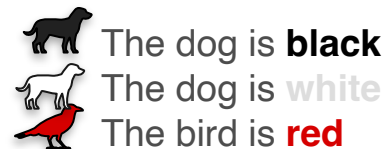
System 3



Redundant

Singular

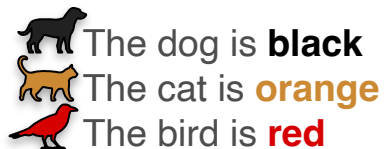
System 4





# Systems of sentences

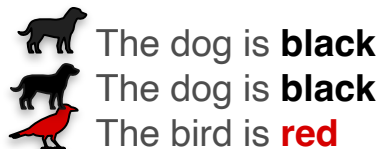
## System 1



Complete

**Non-singular**

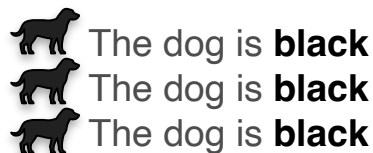
## System 2



Redundant

**Singular**

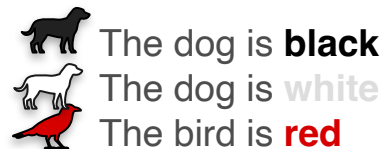
## System 3



Redundant

**Singular**

## System 4



Contradictory

**Singular**

# Quiz: Systems of sentences

Given this system:

- Between the dog, the cat, and the bird, one is red.
- Between the dog and the cat, one is orange.
- The dog is black.

## **Problem 1:**

What color is the bird?

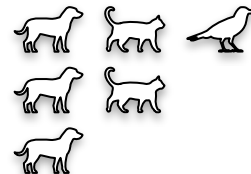
## **Problem 2:**

Is this system singular or non-singular?

# Solution: Systems of information

Given this system:

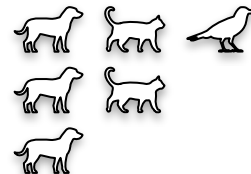
- Between the dog, the cat, and the bird, one is red.
- Between the dog and the cat, one is orange.
- The dog is black.



# Solution: Systems of information

Given this system:

- Between the dog, the cat, and the bird, one is red.
- Between the dog and the cat, one is orange.
- The dog is black.

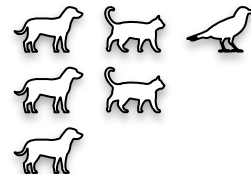


**Solution 1:**

# Solution: Systems of information

Given this system:

- Between the dog, the cat, and the bird, one is red.
- Between the dog and the cat, one is orange.
- The dog is black.



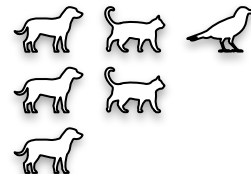
## Solution 1:

The bird is red. 

# Solution: Systems of information

Given this system:

- Between the dog, the cat, and the bird, one is red.
- Between the dog and the cat, one is orange.
- The dog is black.



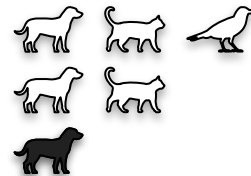
## Solution 1:

The bird is red. 

# Solution: Systems of information

Given this system:

- Between the dog, the cat, and the bird, one is red.
- Between the dog and the cat, one is orange.
- The dog is black.



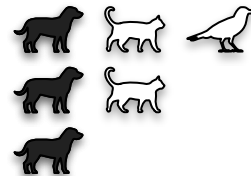
## Solution 1:

The bird is red. 

# Solution: Systems of information

Given this system:

- Between the dog, the cat, and the bird, one is red.
- Between the dog and the cat, one is orange.
- The dog is black.



## Solution 1:

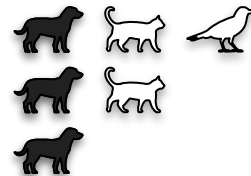
The bird is red. 



# Solution: Systems of information

Given this system:

- Between the dog, the cat, and the bird, one is red.
- Between the dog and the cat, one is orange.
- The dog is black.



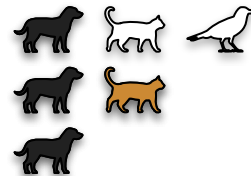
## Solution 1:

The bird is red. 

# Solution: Systems of information

Given this system:

- Between the dog, the cat, and the bird, one is red.
- Between the dog and the cat, one is orange.
- The dog is black.



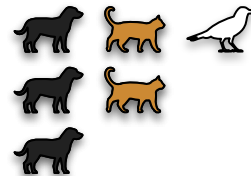
## Solution 1:

The bird is red. 

# Solution: Systems of information

Given this system:

- Between the dog, the cat, and the bird, one is red.
- Between the dog and the cat, one is orange.
- The dog is black.



## Solution 1:

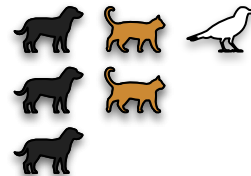
The bird is red. 

# Solution: Systems of information

Given this system:



- Between the dog, the cat, and the bird, one is red.
- Between the dog and the cat, one is orange.
- The dog is black.



## Solution 1:

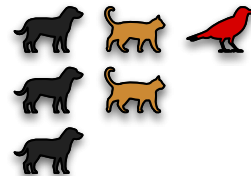
The bird is red. 

# Solution: Systems of information

Given this system:



- Between the dog, the cat, and the bird, one is red.
- Between the dog and the cat, one is orange.
- The dog is black.



## Solution 1:

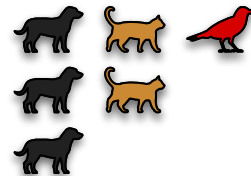
The bird is red. 

# Solution: Systems of information

Given this system:



- Between the dog, the cat, and the bird, one is red.
- Between the dog and the cat, one is orange.
- The dog is black.



## Solution 1:

The bird is red. 

## Solution 2:

It is non-singular.   



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# System of Linear Equations

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**System of equations**

# Sentences $\rightarrow$ Equations

## Sentences

Between the dog and  
the cat, one is black.

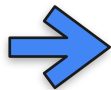




# Sentences $\rightarrow$ Equations

## Sentences

Between the dog and  
the cat, one is black.



## Sentences with numbers

The price of an apple  
and a banana is \$10.



# Sentences → Equations

## Sentences

Between the dog and  
the cat, one is black.



## Sentences with numbers

The price of an apple  
and a banana is \$10.



## Equations

$$a + b = 10$$



# Quiz: Systems of equations 1

You go two days in a row and collect this information:

- **Day 1:** You bought an apple and a banana and they cost \$10.
- **Day 2:** You bought an apple and two bananas and they cost \$12.

**Question:** How much does each fruit cost?

# Solution: Systems of equations 1

- **Day 1:** You bought an apple and a banana and they cost \$10.
- **Day 2:** You bought an apple and two bananas and they cost \$12.
- **Solution:** An apple costs \$8, a banana costs \$2.

# Solution: Systems of equations 1

- **Day 1:** You bought an apple and a banana and they cost \$10.

$$\text{🍏} + \text{🍌} = \$10$$

- **Day 2:** You bought an apple and two bananas and they cost \$12.

- **Solution:** An apple costs \$8, a banana costs \$2.

# Solution: Systems of equations 1

- **Day 1:** You bought an apple and a banana and they cost \$10.

$$\text{🍏} + \text{🍌} = \$10$$

- **Day 2:** You bought an apple and two bananas and they cost \$12.

$$\text{🍏} + \text{🍌} + \text{🍌} = \$12$$

- **Solution:** An apple costs \$8, a banana costs \$2.

# Solution: Systems of equations 1

- **Day 1:** You bought an apple and a banana and they cost \$10.

$$\text{🍏} + \text{🍌} = \$10$$

- **Day 2:** You bought an apple and two bananas and they cost \$12.

$$\text{🍏} + \text{🍌} + \boxed{\text{🍌}} = \$12$$

- **Solution:** An apple costs \$8, a banana costs \$2.

# Solution: Systems of equations 1

- **Day 1:** You bought an apple and a banana and they cost \$10.

$$\text{🍏} + \text{🍌} = \$10$$

- **Day 2:** You bought an apple and two bananas and they cost \$12.

$$\text{🍏} + \text{🍌} + \boxed{\text{🍌}} = \$1\boxed{2}$$

- **Solution:** An apple costs \$8, a banana costs \$2.



# Solution: Systems of equations 1

- **Day 1:** You bought an apple and a banana and they cost \$10.

$$\text{🍏} + \text{🍌} = \$10$$

- **Day 2:** You bought an apple and two bananas and they cost \$12.

$$\text{🍏} + \text{🍌} + \boxed{\text{🍌}} = \$\boxed{12}$$

↙  
\$2

- **Solution:** An apple costs \$8, a banana costs \$2.

# Solution: Systems of equations 1

- **Day 1:** You bought an apple and a banana and they cost \$10.

$$\text{Apple} + \text{Banana} = \$10$$

A blue curved arrow points from the banana icon to the text "\$2" below it.

- **Day 2:** You bought an apple and two bananas and they cost \$12.

$$\text{Apple} + \text{Banana} + \text{Banana} = \$12$$

A blue square box highlights the second banana icon and the "2" in the "\$12". A blue curved arrow points from the bottom of the box to the text "\$2" below it.

- **Solution:** An apple costs \$8, a banana costs \$2.

## Solution: Systems of equations 1

- **Day 1:** You bought an apple and a banana and they cost \$10.

Diagram illustrating the addition: An apple is shown above the number 8, and a banana is shown above the number 2. Arrows point from the numbers 8 and 2 to the apple and banana respectively. The equation is shown as: Apple + Banana = \$10.

- **Day 2:** You bought an apple and two bananas and they cost \$12.

Diagram illustrating the addition of fruit prices:

- 1 apple + 1 banana + 1 banana = \$2
- The second banana and the result '\$2' are highlighted with blue boxes and an arrow, indicating the total cost for the bananas.

- **Solution:** An apple costs \$8, a banana costs \$2.

# Quiz: Systems of equations 2

You go two days in a row and collect this information:

- **Day 1:** You bought an apple and a banana and they cost \$10.
- **Day 2:** You bought two apples and two bananas and they cost \$20.

**Question:** How much does each fruit cost?

# Solution: Systems of equations 2

- **Day 1:** You bought an apple and a banana and they cost \$10.
- **Day 2:** You bought two apples and two bananas and they cost \$20.

# Solution: Systems of equations 2

- **Day 1:** You bought an apple and a banana and they cost \$10.

$$\text{🍏} + \text{🍌} = \$10$$

- **Day 2:** You bought two apples and two bananas and they cost \$20.

# Solution: Systems of equations 2

- **Day 1:** You bought an apple and a banana and they cost \$10.

$$\text{🍏} + \text{🍌} = \$10$$

- **Day 2:** You bought two apples and two bananas and they cost \$20.

$$\text{🍏🍏} + \text{🍌🍌} = \$20$$

# Solution: Systems of equations 2

- **Day 1:** You bought an apple and a banana and they cost \$10.

$$\text{apple} + \text{banana} = \$10$$

- **Day 2:** You bought two apples and two bananas and they cost \$20.

$$2\text{ apples} + 2\text{ bananas} = \$20$$

Same thing!!!



# Solution: Systems of equations 2

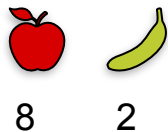
- **Day 1:** You bought an apple and a banana and they cost \$10.

$$\text{apple} + \text{banana} = \$10$$

- **Day 2:** You bought two apples and two bananas and they cost \$20.

$$2 \text{ apples} + 2 \text{ bananas} = \$20$$

Same thing!!!



# Solution: Systems of equations 2

- **Day 1:** You bought an apple and a banana and they cost \$10.

$$\text{apple} + \text{banana} = \$10$$

- **Day 2:** You bought two apples and two bananas and they cost \$20.

$$2 \text{ apples} + 2 \text{ bananas} = \$20$$

Same thing!!!



8

2

5

5

# Solution: Systems of equations 2

- **Day 1:** You bought an apple and a banana and they cost \$10.

$$\text{apple} + \text{banana} = \$10$$

- **Day 2:** You bought two apples and two bananas and they cost \$20.

$$2 \times \text{apple} + 2 \times \text{banana} = \$20$$

Same thing!!!



8      2

5      5

8.3    1.7

# Solution: Systems of equations 2

- **Day 1:** You bought an apple and a banana and they cost \$10.

$$\text{apple} + \text{banana} = \$10$$

- **Day 2:** You bought two apples and two bananas and they cost \$20.

$$2 \times \text{apple} + 2 \times \text{banana} = \$20$$

Same thing!!!



|     |     |
|-----|-----|
| 8   | 2   |
| 5   | 5   |
| 8.3 | 1.7 |
| 0   | 10  |

# Solution: Systems of equations 2

- **Day 1:** You bought an apple and a banana and they cost \$10.

$$\text{apple} + \text{banana} = \$10$$

- **Day 2:** You bought two apples and two bananas and they cost \$20.

$$2 \times \text{apple} + 2 \times \text{banana} = \$20$$

Same thing!!!



8      2

5      5

8.3    1.7

0      10

Infinitely many solutions!

# Quiz: Systems of equations 3

You go two days in a row and collect this information:

- **Day 1:** You bought an apple and a banana and they cost \$10.
- **Day 2:** You bought two apples and two bananas and they cost \$24.

**Question:** How much does each fruit cost?

# Solution: Systems of equations 3

- **Day 1:** You bought an apple and a banana and they cost \$10.
- **Day 2:** You bought two apples and two bananas and they cost \$24.

# Solution: Systems of equations 3

- **Day 1:** You bought an apple and a banana and they cost \$10.

$$\text{🍏} + \text{🍌} = \$10$$

- **Day 2:** You bought two apples and two bananas and they cost \$24.



# Solution: Systems of equations 3

- **Day 1:** You bought an apple and a banana and they cost \$10.

$$\text{🍏} + \text{🍌} = \$10 \quad \Rightarrow \quad \text{🍏🍏} + \text{🍌🍌} = \$20$$

- **Day 2:** You bought two apples and two bananas and they cost \$24.

# Solution: Systems of equations 3

- **Day 1:** You bought an apple and a banana and they cost \$10.

$$\text{🍏} + \text{🍌} = \$10 \quad \Rightarrow \quad \text{🍏🍏} + \text{🍌🍌} = \$20$$

- **Day 2:** You bought two apples and two bananas and they cost \$24.

$$\text{🍏🍏} + \text{🍌🍌} = \$24$$

# Solution: Systems of equations 3

- **Day 1:** You bought an apple and a banana and they cost \$10.

$$\text{🍏} + \text{🍌} = \$10 \quad \Rightarrow \quad \text{🍏🍏} + \text{🍌🍌} = \$20$$

- **Day 2:** You bought two apples and two bananas and they cost \$24.

$$\text{🍏🍏} + \text{🍌🍌} = \$24$$

Contradiction!

# Solution: Systems of equations 3

- **Day 1:** You bought an apple and a banana and they cost \$10.

$$\text{🍏} + \text{🍌} = \$10 \quad \Rightarrow \quad \text{🍏🍏} + \text{🍌🍌} = \$20$$

- **Day 2:** You bought two apples and two bananas and they cost \$24.

$$\text{🍏🍏} + \text{🍌🍌} = \$24$$



Contradiction!



No solutions!

# Systems of equations

# Systems of equations





## System 1

- $a + b = 10$   
 







- $a + 2b = 12$   
 

# Systems of equations

## System 1





- $a + b = 10$   
 
- $a + 2b = 12$   
 

## System 2






- $a + b = 10$   
 
- $2a + 2b = 20$   
   

# Systems of equations






## System 1

- $a + b = 10$   
 
- $a + 2b = 12$   
 

## System 2

- $a + b = 10$   
 
- $2a + 2b = 20$   
  





## System 3

- $a + b = 10$   
 
- $2a + 2b = 24$   
  








# Systems of equations

## System 1






- $a + b = 10$   
 
- $a + 2b = 12$   
 

**Unique solution:**

## System 2





- $a + b = 10$   
 
- $2a + 2b = 20$   
  

## System 3

- $a + b = 10$   
 
- $2a + 2b = 24$   
  

# Systems of equations

## System 1






- $a + b = 10$   
 
- $a + 2b = 12$   
 

Unique solution:






$$\text{🍏 } a = 8$$

$$\text{🍌 } b = 2$$

## System 2





- $a + b = 10$   
 
- $2a + 2b = 20$   
  

## System 3

- $a + b = 10$   
 
- $2a + 2b = 24$   
  

# Systems of equations

## System 1

- $a + b = 10$   
 
- $a + 2b = 12$   
 

Unique solution:

$$\text{🍏 } a = 8$$






$$\text{🍌 } b = 2$$

Complete

## System 2





- $a + b = 10$   
 
- $2a + 2b = 20$   
  

## System 3

- $a + b = 10$   
 
- $2a + 2b = 24$   
  

# Systems of equations

## System 1






- $a + b = 10$   
 
- $a + 2b = 12$   
 

Unique solution:






$$\text{🍏 } a = 8$$

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## System 2

- $a + b = 10$   
 
- $2a + 2b = 20$   
  

## System 3

- $a + b = 10$   
 
- $2a + 2b = 24$   
  

Complete

Non-singular

# Systems of equations

## System 1

- $a + b = 10$   
 
- $a + 2b = 12$   
 

Unique solution:

$$\text{🍏 } a = 8$$

$$\text{🍌 } b = 2$$

Complete






Non-singular

## System 2

- $a + b = 10$   
 
- $2a + 2b = 20$   
  

Infinite solutions

## System 3


- $a + b = 10$   
 
- $2a + 2b = 24$   
  


# Systems of equations

## System 1

- $a + b = 10$   
 
- $a + 2b = 12$   
 

Unique solution:







  $a = 8$

  $b = 2$


Complete


Non-singular

## System 2







- $a + b = 10$   
 
- $2a + 2b = 20$   
   

Infinite solutions

  $a = 8$





  $b = 2$

## System 3

- $a + b = 10$   
 
- $2a + 2b = 24$   
   

# Systems of equations

## System 1

- $a + b = 10$   
 
- $a + 2b = 12$   
 






Unique solution:

$$\begin{aligned} \text{🍏 } a &= 8 \\ \text{🍌 } b &= 2 \end{aligned}$$

Complete

Non-singular






## System 2

- $a + b = 10$   
 
- $2a + 2b = 20$   
  

Infinite solutions





$$\begin{aligned} \text{🍏 } a &= 8, 7 \\ \text{🍌 } b &= 2, 3 \end{aligned}$$

## System 3


- $a + b = 10$   
 
- $2a + 2b = 24$   
  


# Systems of equations

## System 1

- $a + b = 10$   
 
- $a + 2b = 12$   
 

Unique solution:






  $a = 8$

  $b = 2$


Complete

Non-singular

## System 2






- $a + b = 10$   
 
- $2a + 2b = 20$   
  

Infinite solutions

  $a = 8, 7, 6$

  $b = 2, 3, 4$





## System 3

- $a + b = 10$   
 
- $2a + 2b = 24$   
  





# Systems of equations

## System 1






- $a + b = 10$   
 
- $a + 2b = 12$   
 

**Unique solution:**


  $a = 8$

  $b = 2$

## System 2






- $a + b = 10$   
 
- $2a + 2b = 20$   
  

**Infinite solutions**

  $a = 8, 7, 6, \dots$

  $b = 2, 3, 4, \dots$

## System 3





- $a + b = 10$   
 
- $2a + 2b = 24$   
  

**Complete**


**Non-singular**


# Systems of equations

## System 1

- $a + b = 10$   
 
- $a + 2b = 12$   
 

Unique solution:






  $a = 8$

  $b = 2$


Complete

Non-singular

## System 2

- $a + b = 10$   
 
- $2a + 2b = 20$   
  






Infinite solutions

  $a = 8, 7, 6, \dots$

  $b = 2, 3, 4, \dots$





Redundant

## System 3

- $a + b = 10$   
 
- $2a + 2b = 24$   
  

# Systems of equations

## System 1

- $a + b = 10$   
 
- $a + 2b = 12$   
 

Unique solution:

$$\begin{aligned} \text{🍏 } a &= 8 \\ \text{🍌 } b &= 2 \end{aligned}$$

Complete

Non-singular

## System 2

- $a + b = 10$   
 
- $2a + 2b = 20$   
  






Infinite solutions

$$\begin{aligned} \text{🍏 } a &= 8, 7, 6, \dots \\ \text{🍌 } b &= 2, 3, 4, \dots \end{aligned}$$

Redundant





Singular

## System 3

- $a + b = 10$   
 
- $2a + 2b = 24$   
  

# Systems of equations

## System 1

- $a + b = 10$   
 
- $a + 2b = 12$   
 






Unique solution:

$$\begin{aligned} \text{🍏 } a &= 8 \\ \text{🍌 } b &= 2 \end{aligned}$$

Complete

Non-singular

## System 2

- $a + b = 10$   
 
- $2a + 2b = 20$   
  






Infinite solutions

$$\begin{aligned} \text{🍏 } a &= 8, 7, 6, \dots \\ \text{🍌 } b &= 2, 3, 4, \dots \end{aligned}$$

Redundant

Singular

## System 3

- $a + b = 10$   
 
- $2a + 2b = 24$   
  


No solution


# Systems of equations

## System 1

- $a + b = 10$   
 
- $a + 2b = 12$   
 

Unique solution:

  $a = 8$

  $b = 2$


Complete

Non-singular

## System 2

- $a + b = 10$   
 
- $2a + 2b = 20$   
  

Infinite solutions






  $a = 8, 7, 6, \dots$

  $b = 2, 3, 4, \dots$

Redundant

Singular

## System 3

- $a + b = 10$   
 
- $2a + 2b = 24$   
  

No solution

Contradictory

# Systems of equations

## System 1

- $a + b = 10$   
 
- $a + 2b = 12$   
 






Unique solution:

$$\begin{aligned} \text{apple } a &= 8 \\ \text{banana } b &= 2 \end{aligned}$$

Complete

Non-singular

## System 2

- $a + b = 10$   
 
- $2a + 2b = 20$   
  






Infinite solutions

$$\begin{aligned} \text{apple } a &= 8, 7, 6, \dots \\ \text{banana } b &= 2, 3, 4, \dots \end{aligned}$$

Redundant

Singular

## System 3

- $a + b = 10$   
 
- $2a + 2b = 24$   
  

No solution

Contradictory

Singular

# What is a linear equation?

**Linear**

**Non-linear**

# What is a linear equation?

**Linear**

$$a + b = 10$$

**Non-linear**



# What is a linear equation?

**Linear**

$$a + b = 10$$

$$2a + 3b = 15$$

**Non-linear**

# What is a linear equation?

## Linear

$$a + b = 10$$

$$2a + 3b = 15$$

$$3.4a - 48.99b + 2c = 122.5$$

## Non-linear

# What is a linear equation?

**Linear**

$$a + b = 10$$

$$2a + 3b = 15$$

$$3.4a - 48.99b + 2c = 122.5$$

Numbers



**Non-linear**

# What is a linear equation?

## Linear

$$a + b = 10$$

$$2a + 3b = 15$$

$$3.4a - 48.99b + 2c = 122.5$$

Numbers



## Non-linear

$$a^2 + b^2 = 10$$

# What is a linear equation?

## Linear

$$a + b = 10$$

$$2a + 3b = 15$$

$$3.4a - 48.99b + 2c = 122.5$$

Numbers



## Non-linear

$$a^2 + b^2 = 10$$

$$\sin(a) + b^5 = 15$$

# What is a linear equation?

## Linear

$$a + b = 10$$

$$2a + 3b = 15$$

$$3.4a - 48.99b + 2c = 122.5$$

Numbers



## Non-linear

$$a^2 + b^2 = 10$$

$$\sin(a) + b^5 = 15$$

$$2^a - 3^b = 0$$

# What is a linear equation?

## Linear

$$a + b = 10$$

$$2a + 3b = 15$$

$$3.4a - 48.99b + 2c = 122.5$$

Numbers



## Non-linear

$$a^2 + b^2 = 10$$

$$\sin(a) + b^5 = 15$$

$$2^a - 3^b = 0$$

$$ab^2 + \frac{b}{a} - \frac{3}{b} - \log(c) = 4^a$$



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# System of Linear Equations

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**System of equations as lines**

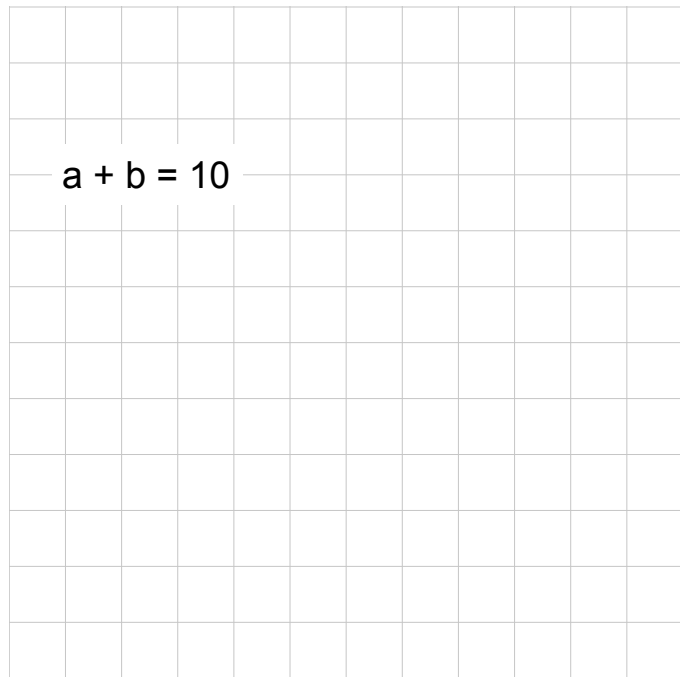


Linear equation  $\rightarrow$  line

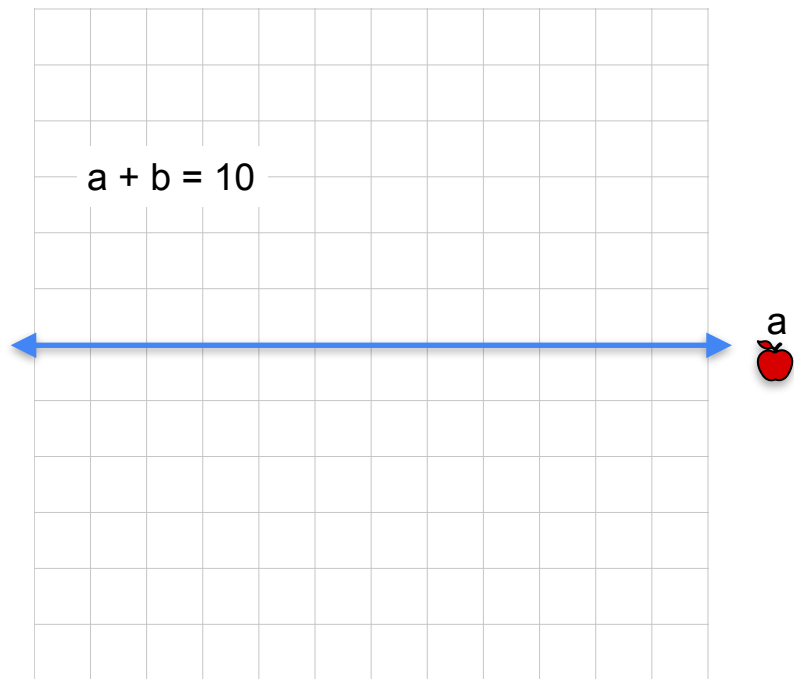
# Linear equation $\rightarrow$ line

$$a + b = 10$$

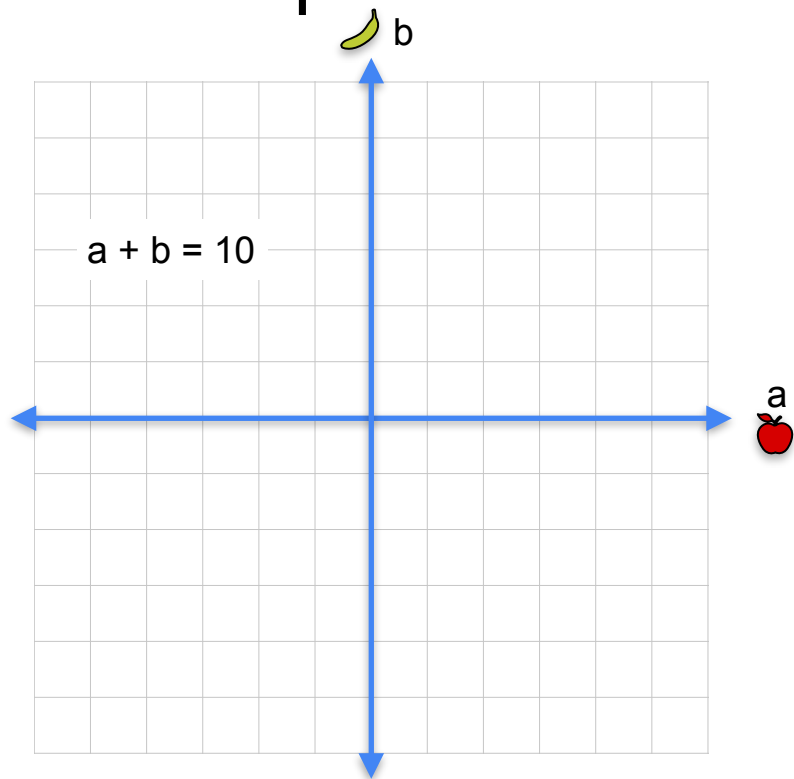
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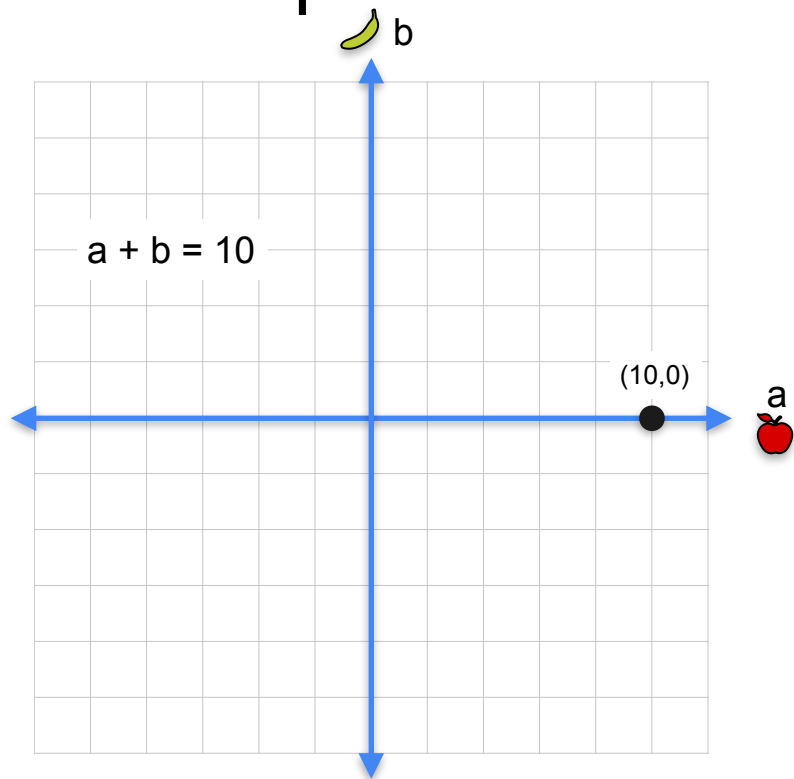
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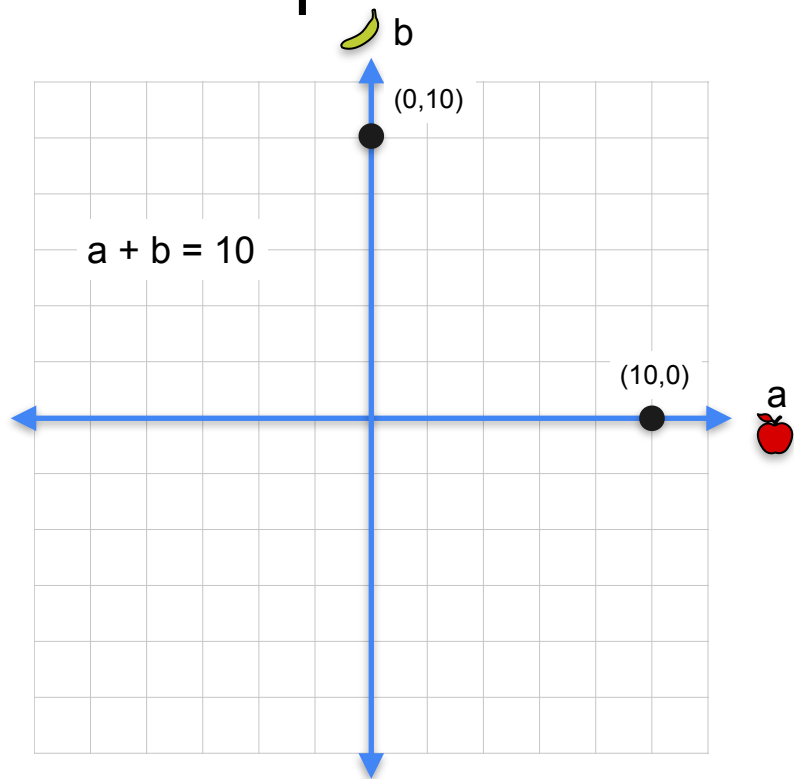
# Linear equation $\rightarrow$ line



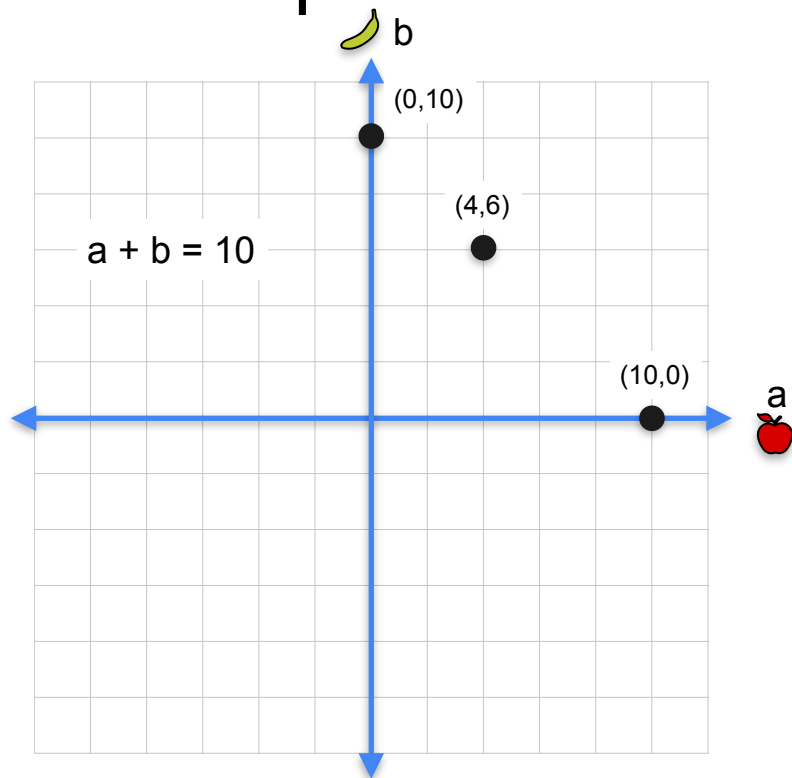
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# Linear equation $\rightarrow$ line

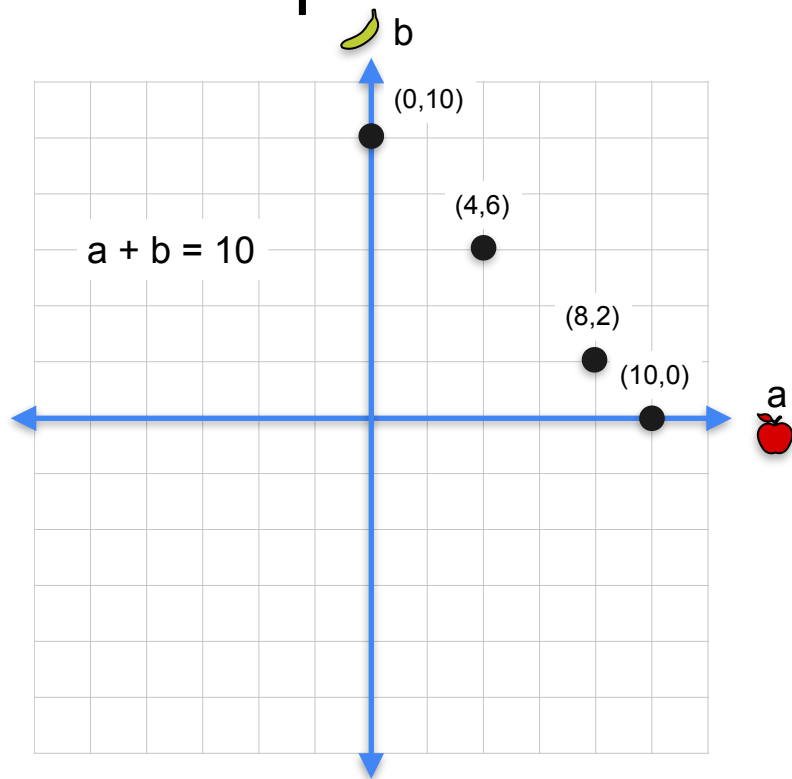


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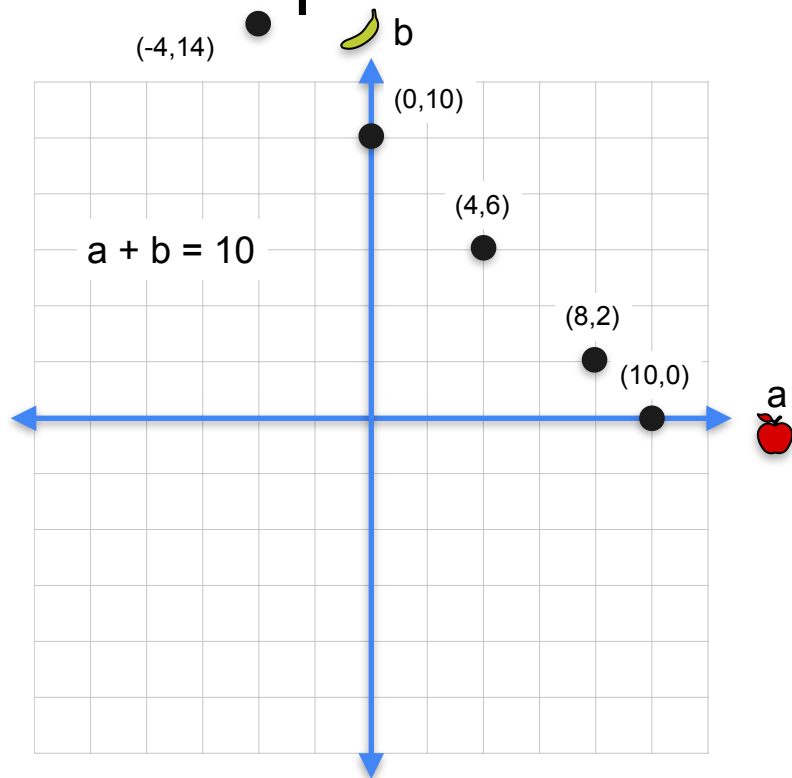




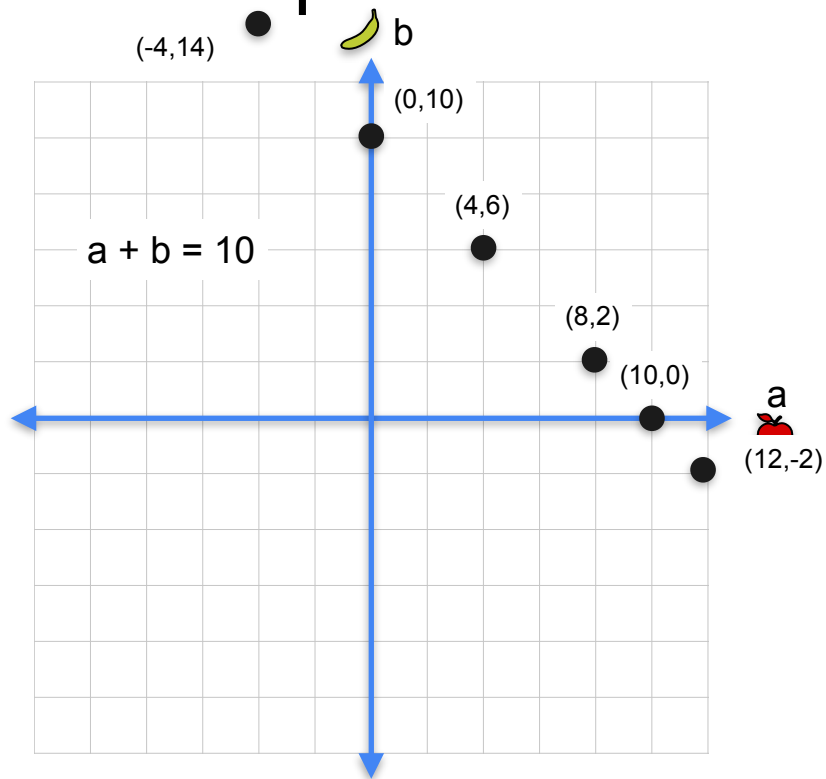
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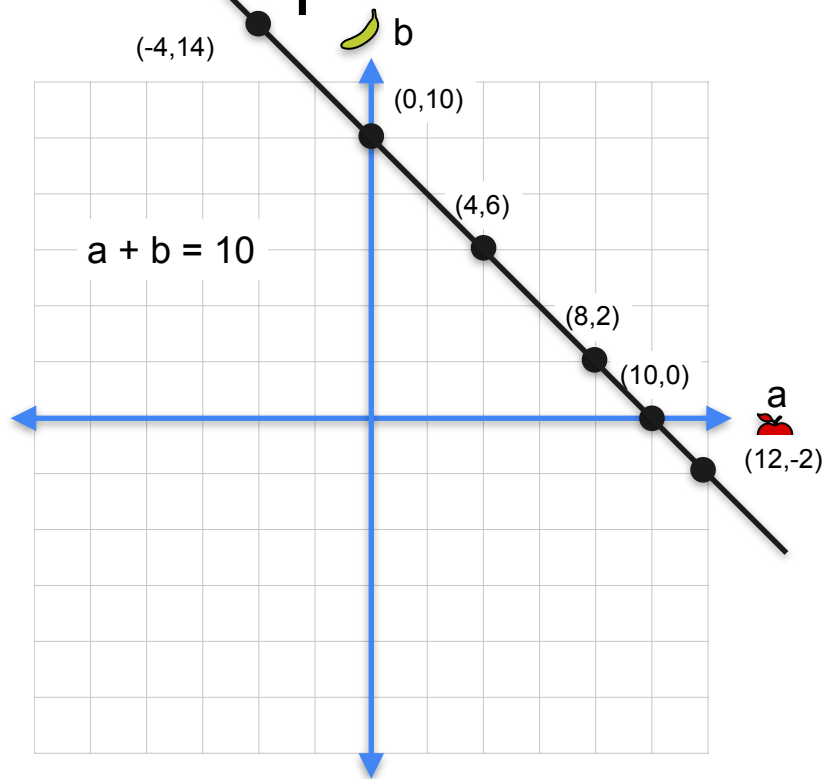
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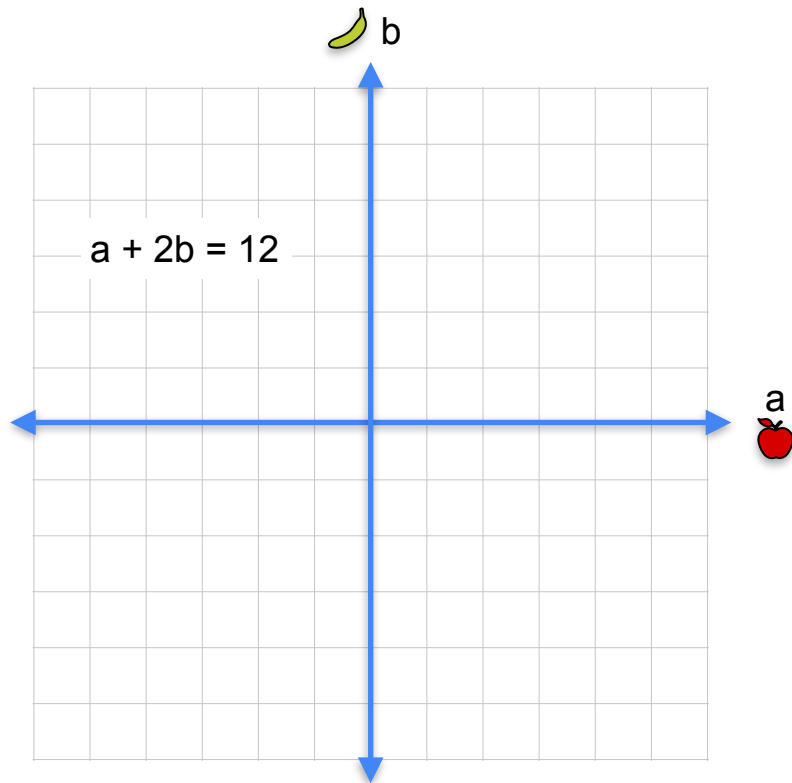
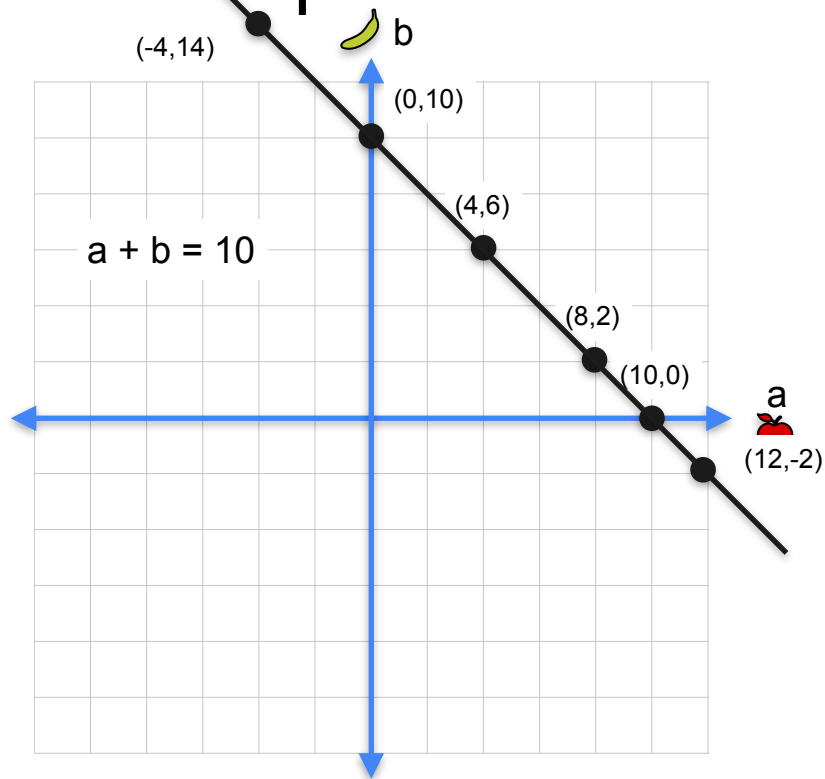
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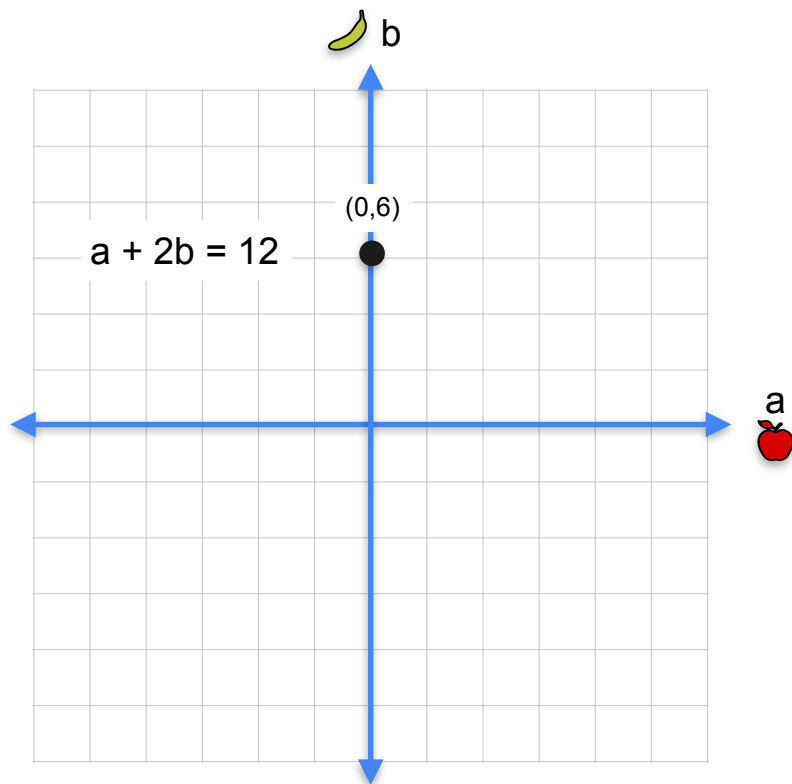
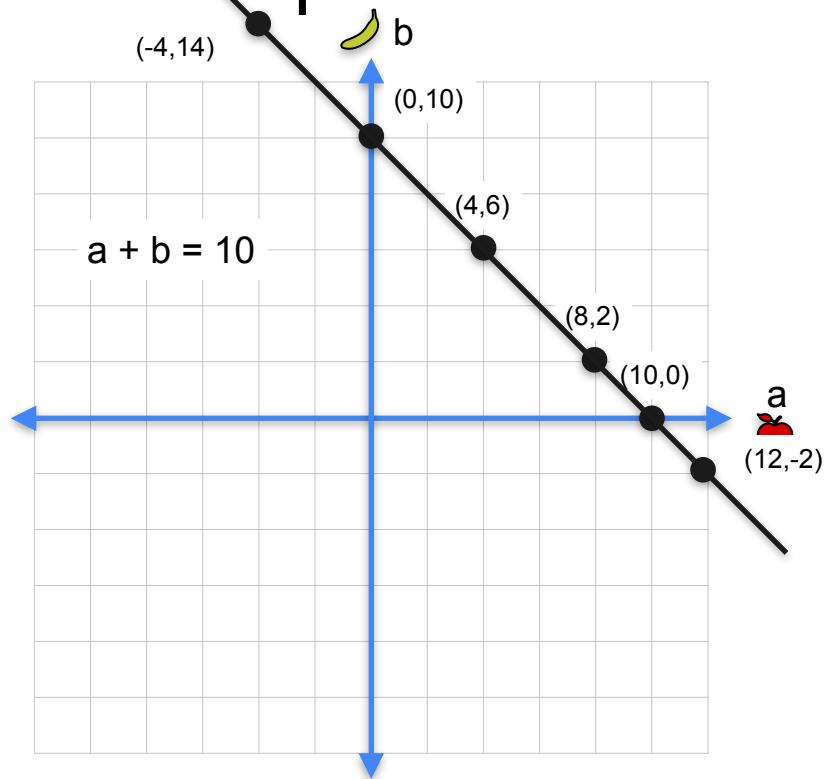
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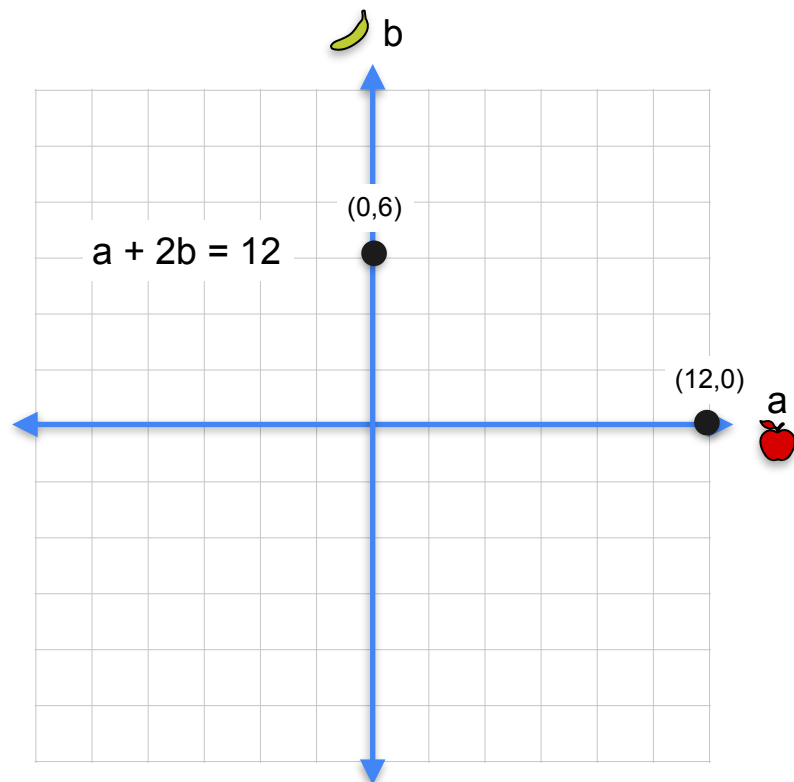
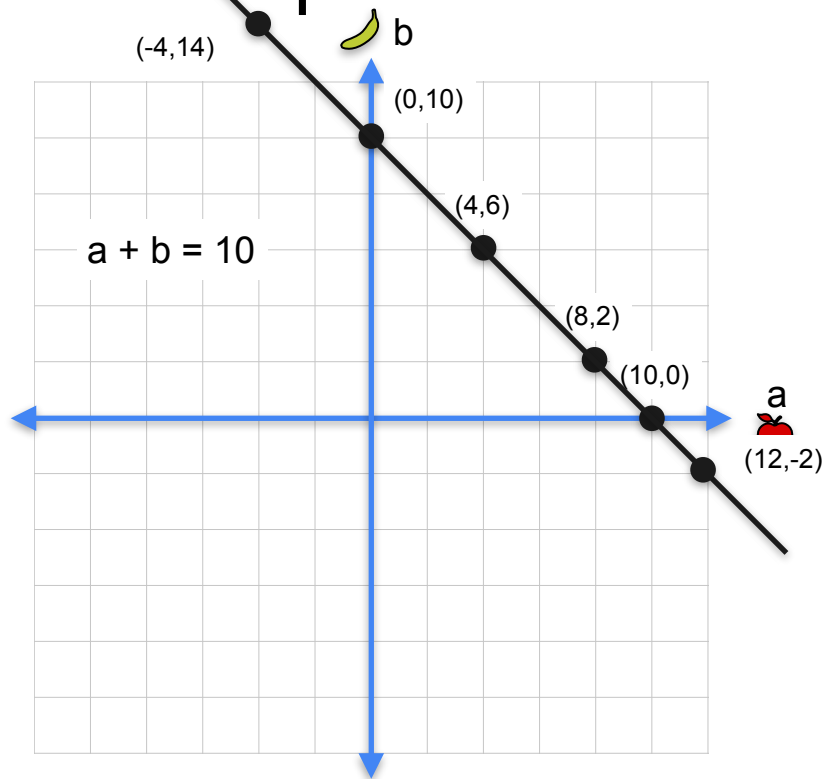
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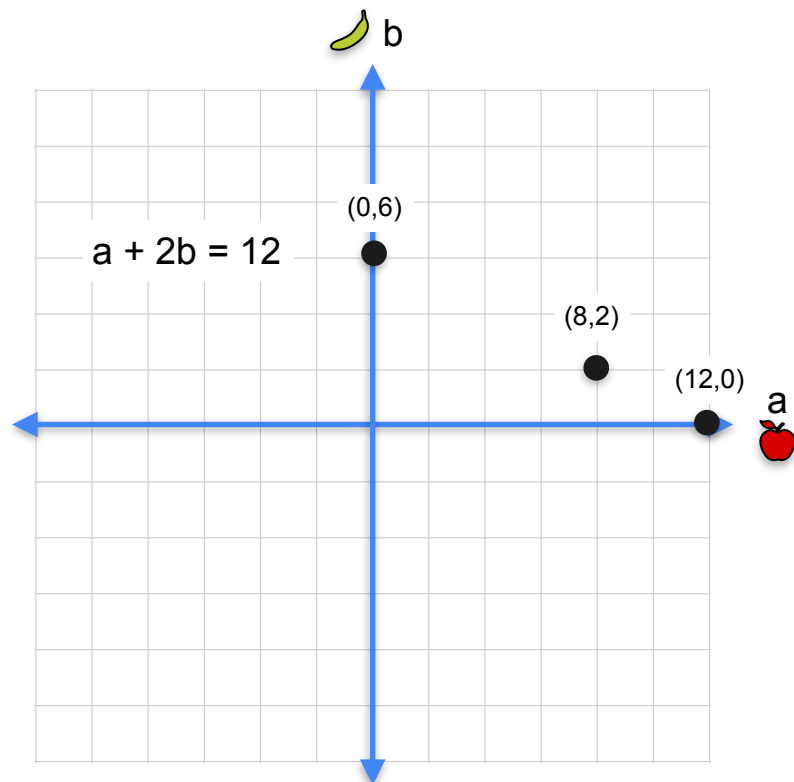
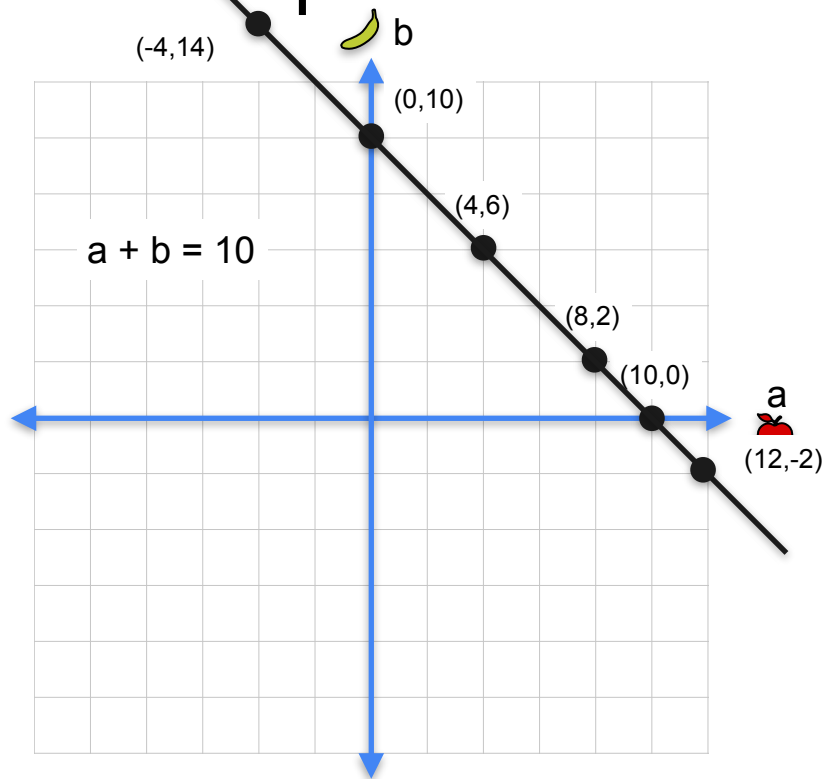
# Linear equation $\rightarrow$ line



# Linear equation $\rightarrow$ line

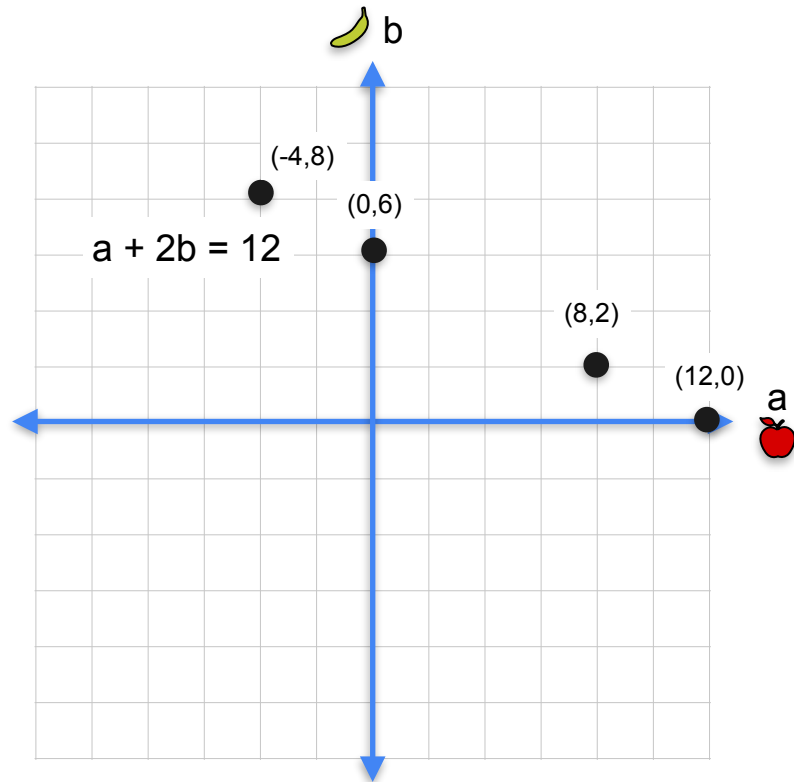
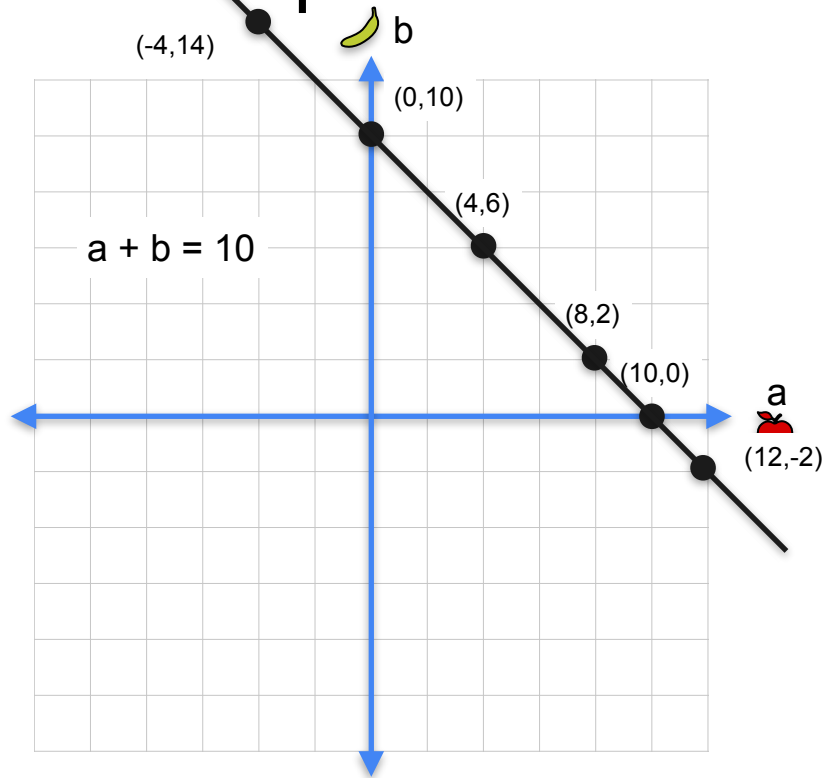


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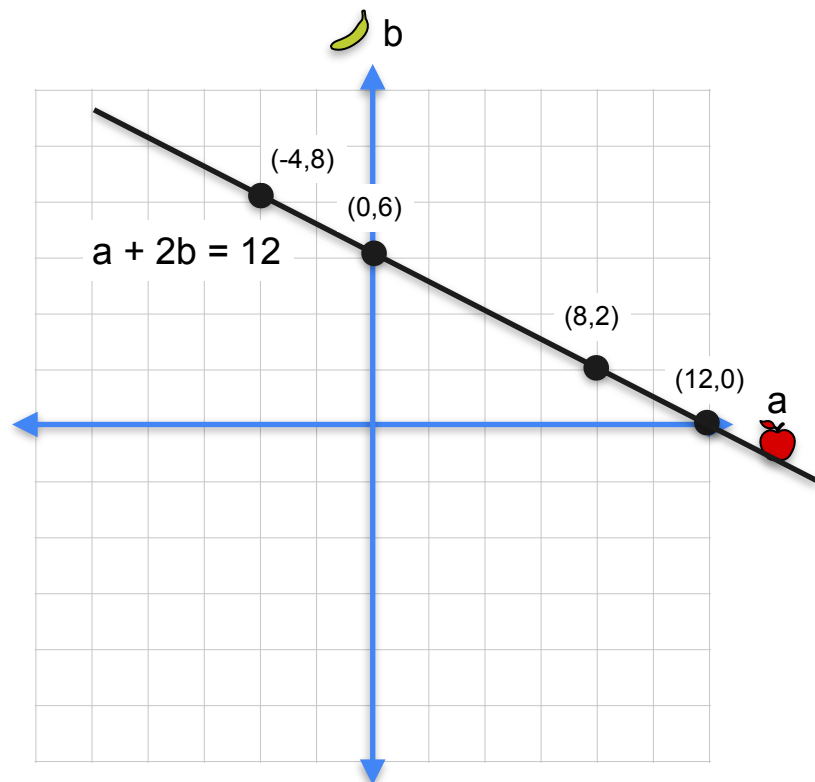
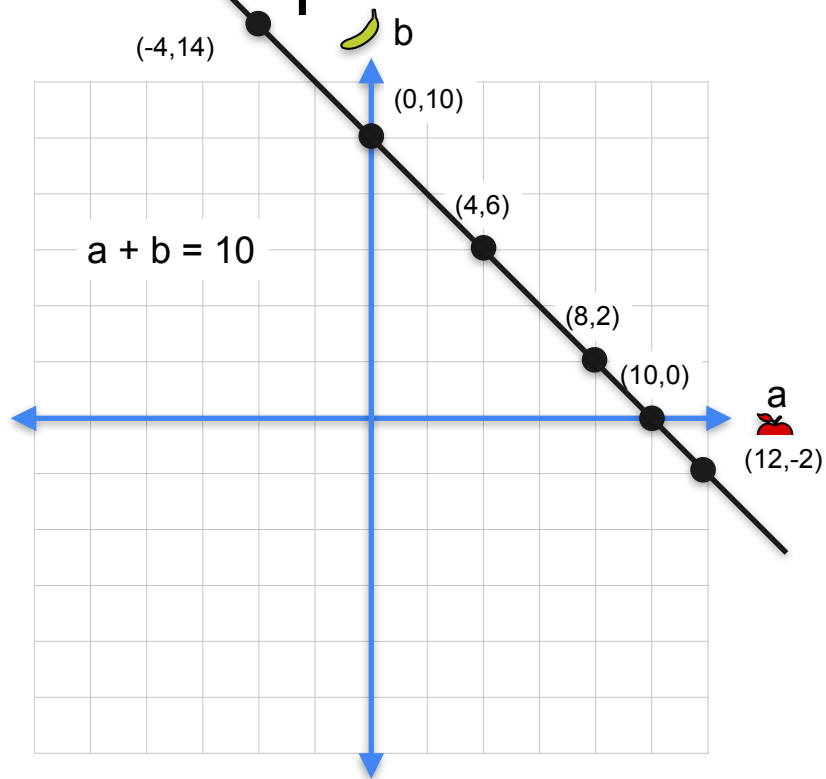




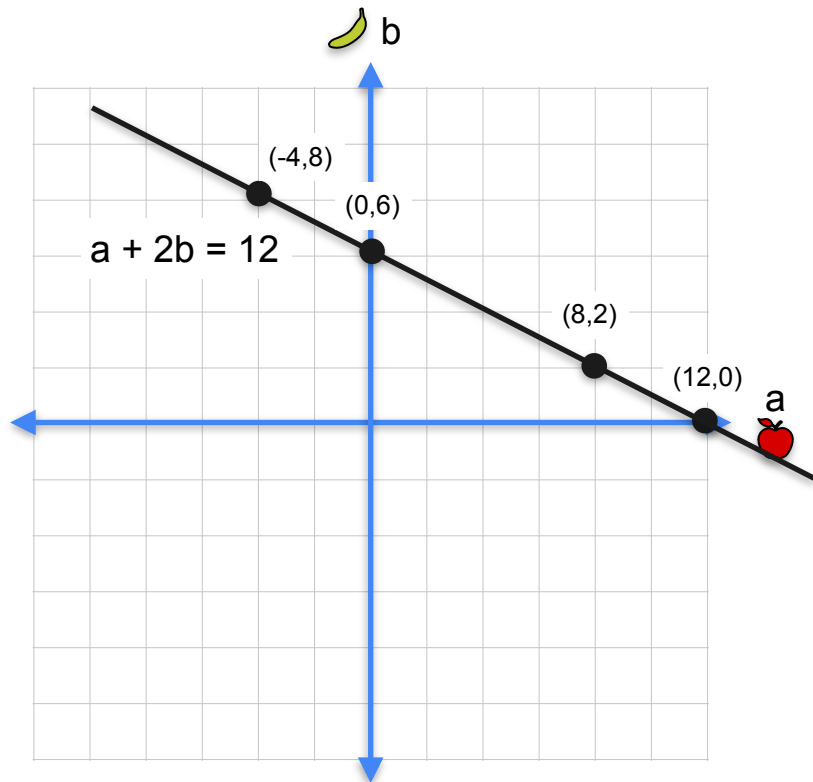
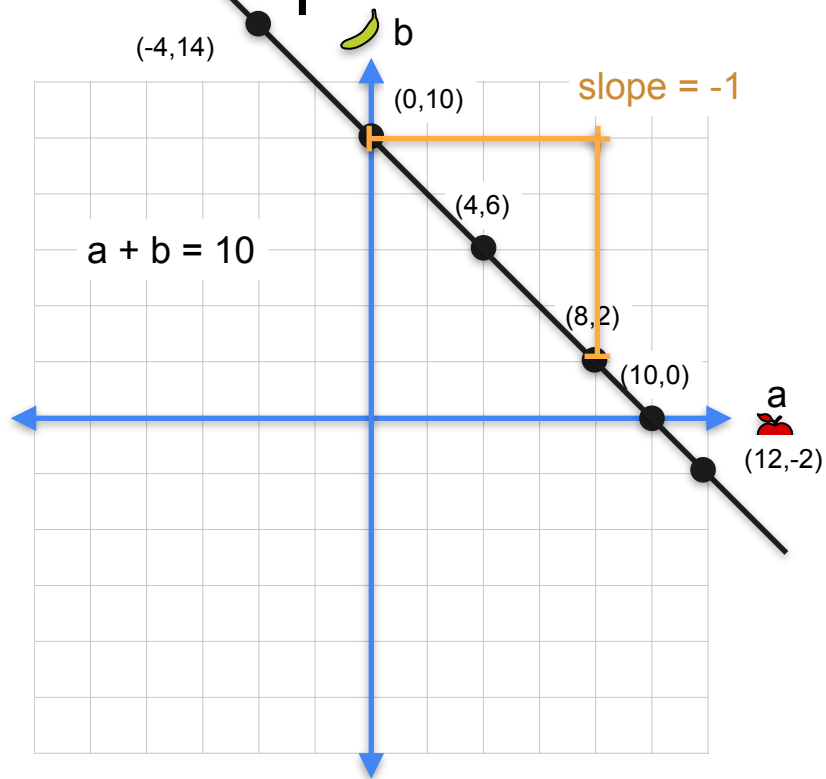
# Linear equation $\rightarrow$ line



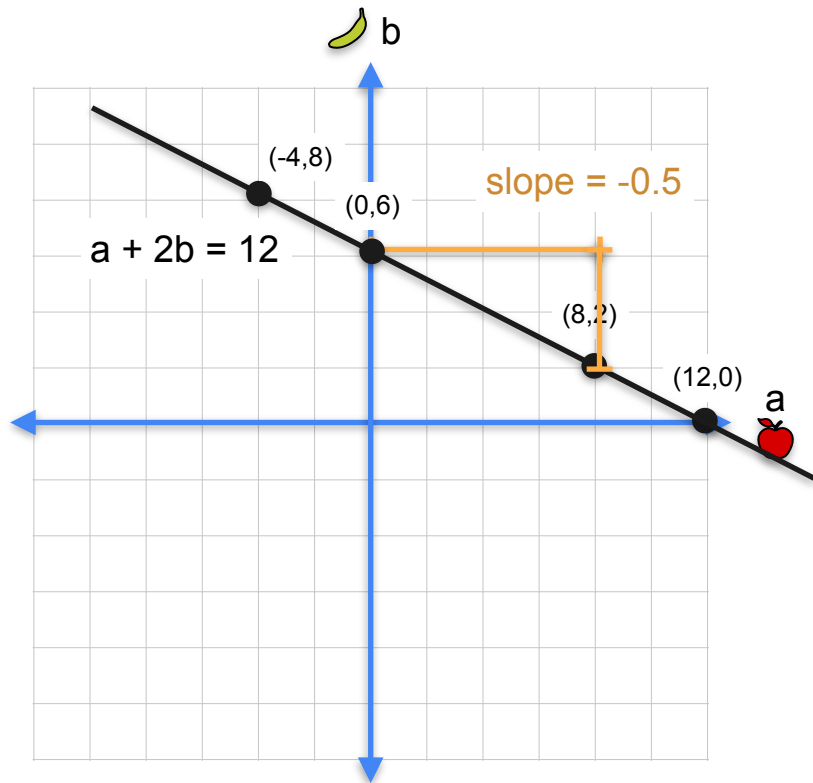
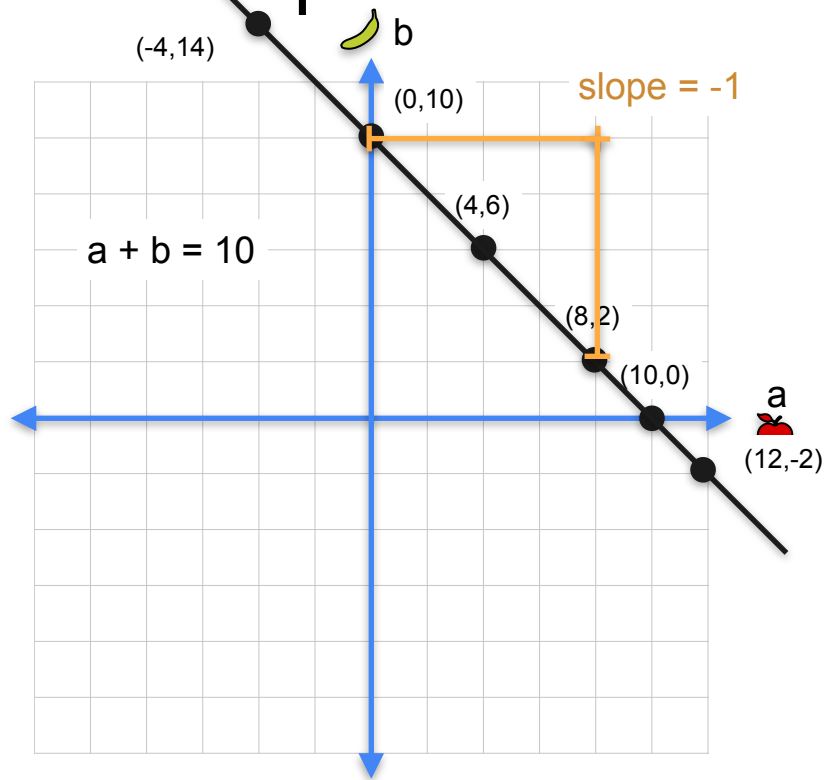
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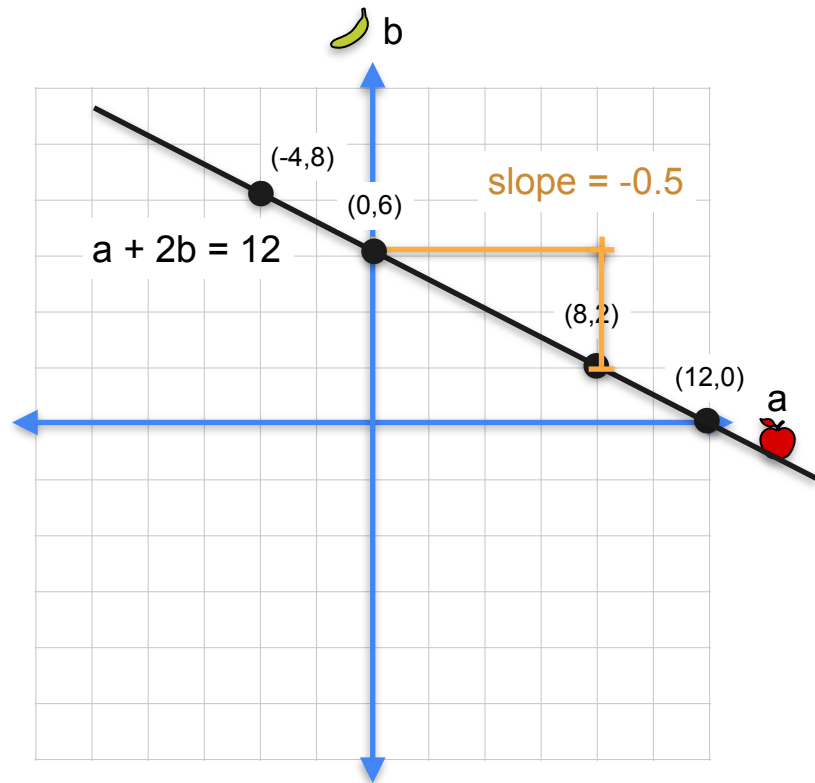
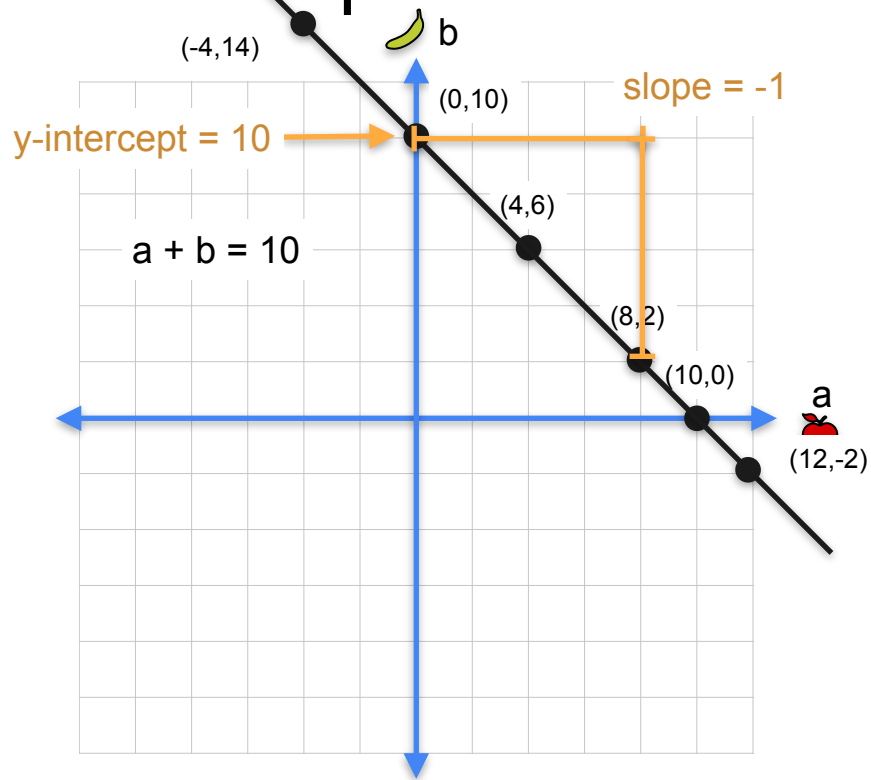
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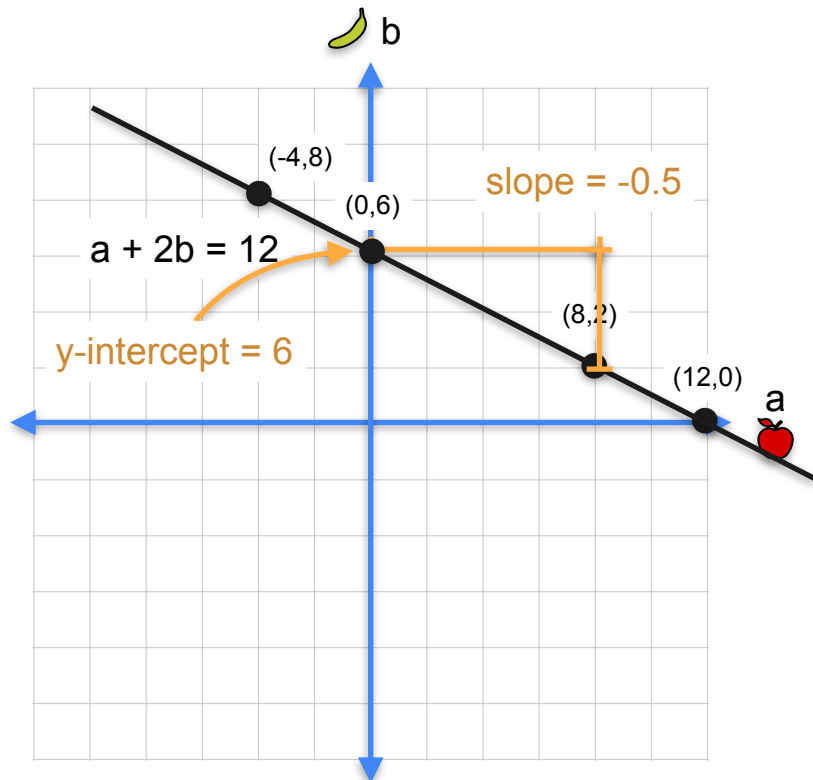
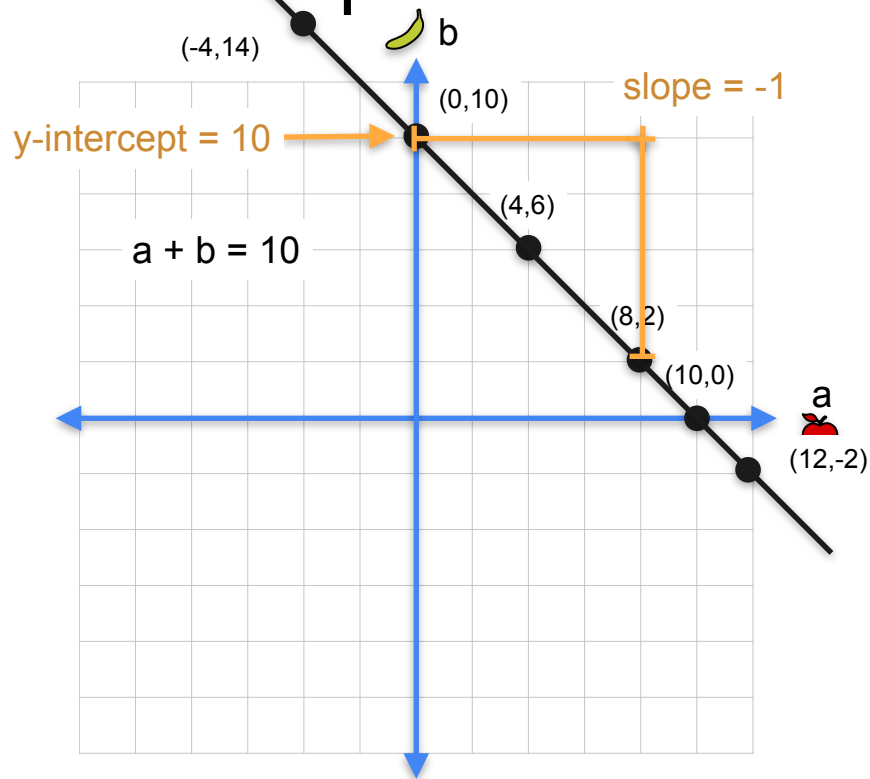
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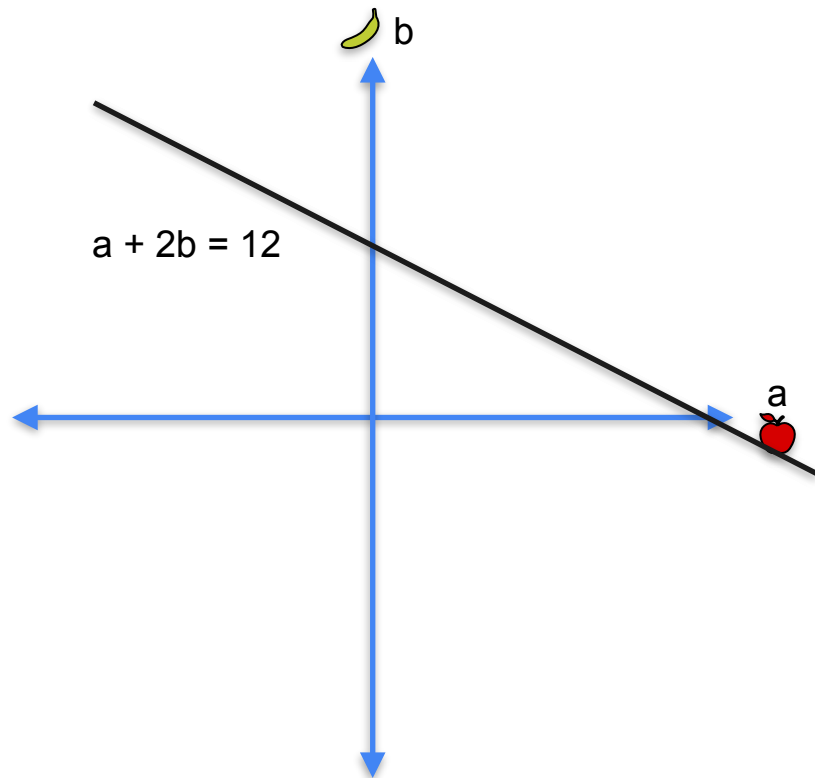
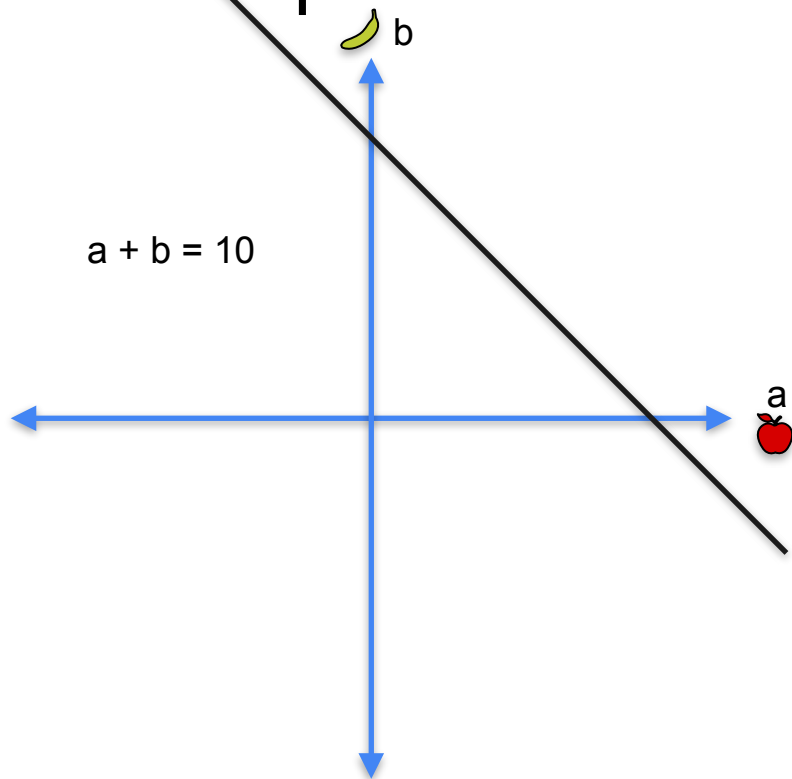
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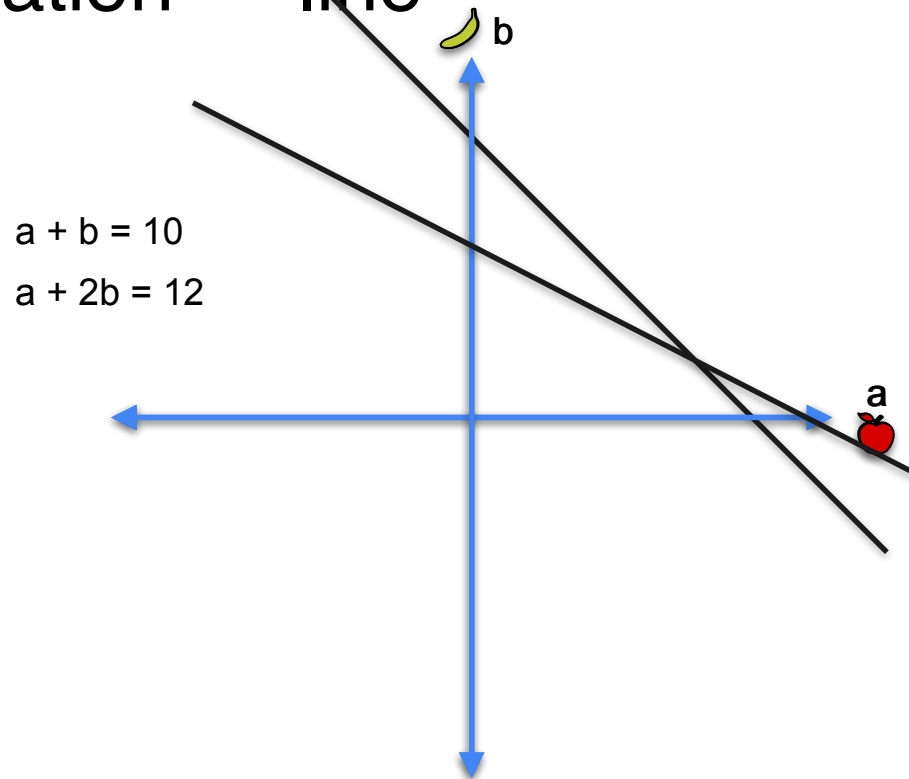
# Linear equation $\rightarrow$ line



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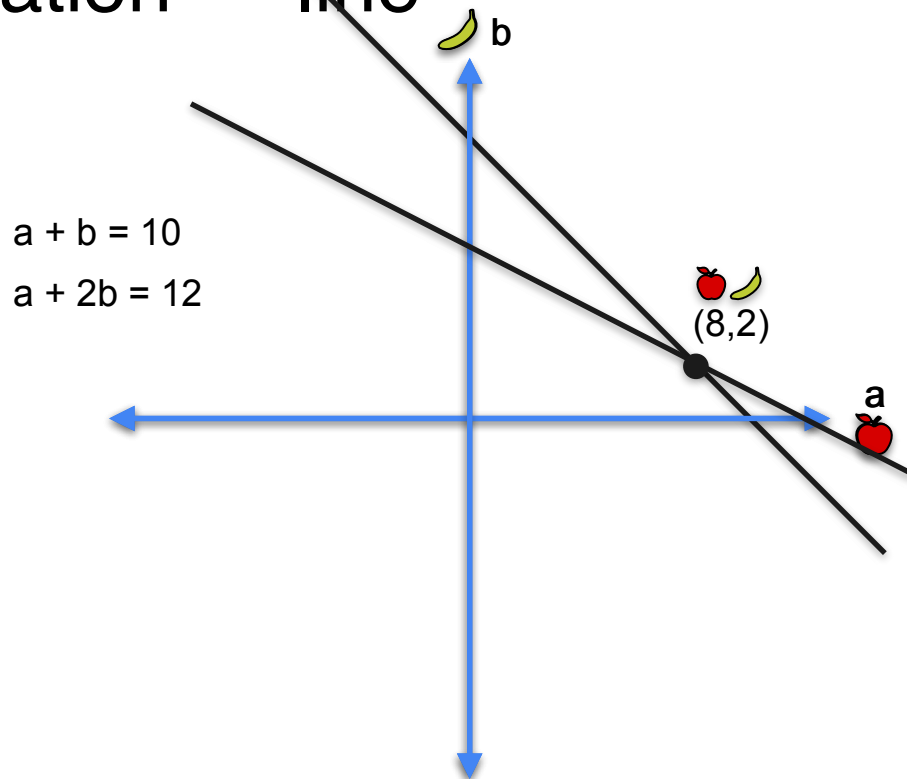


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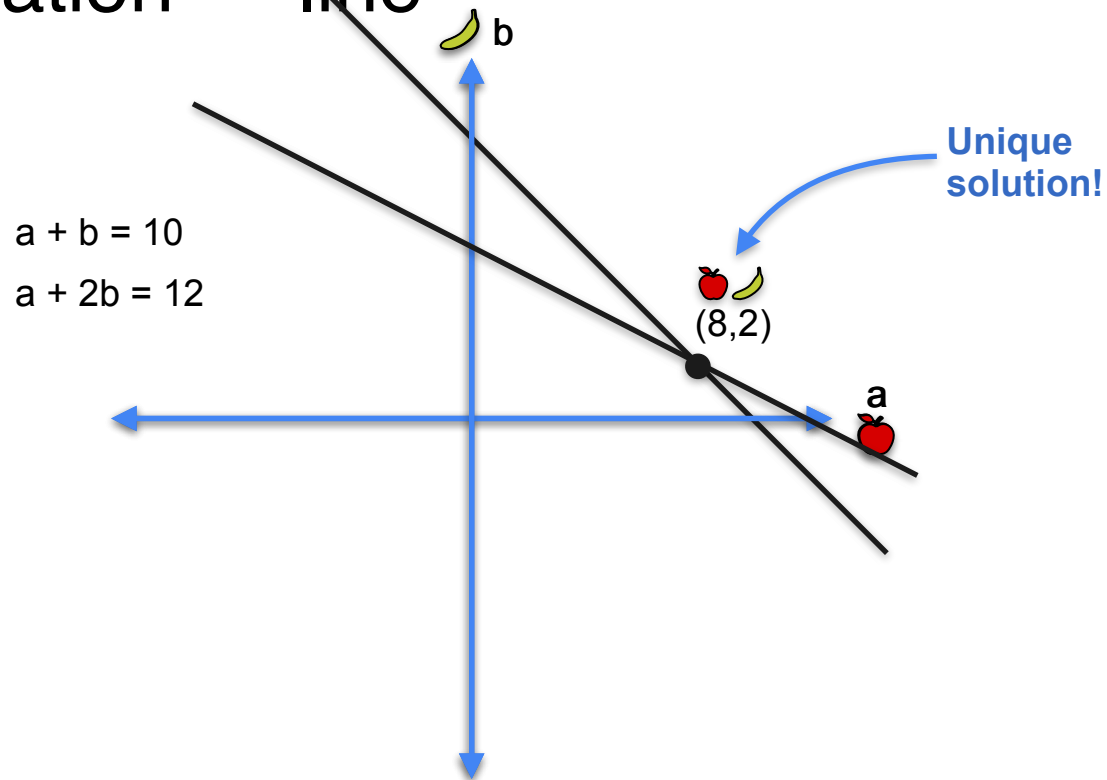




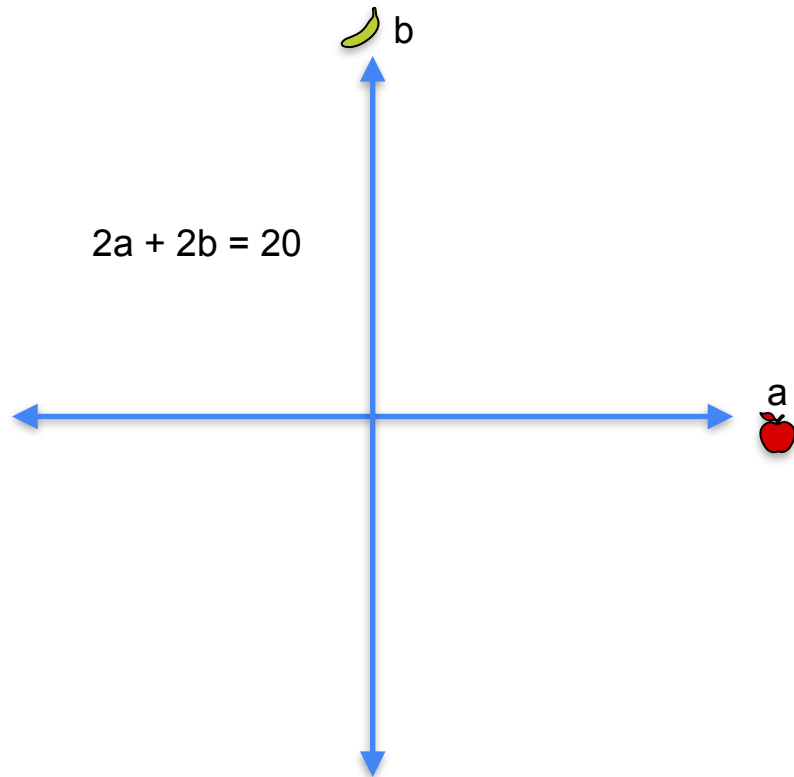
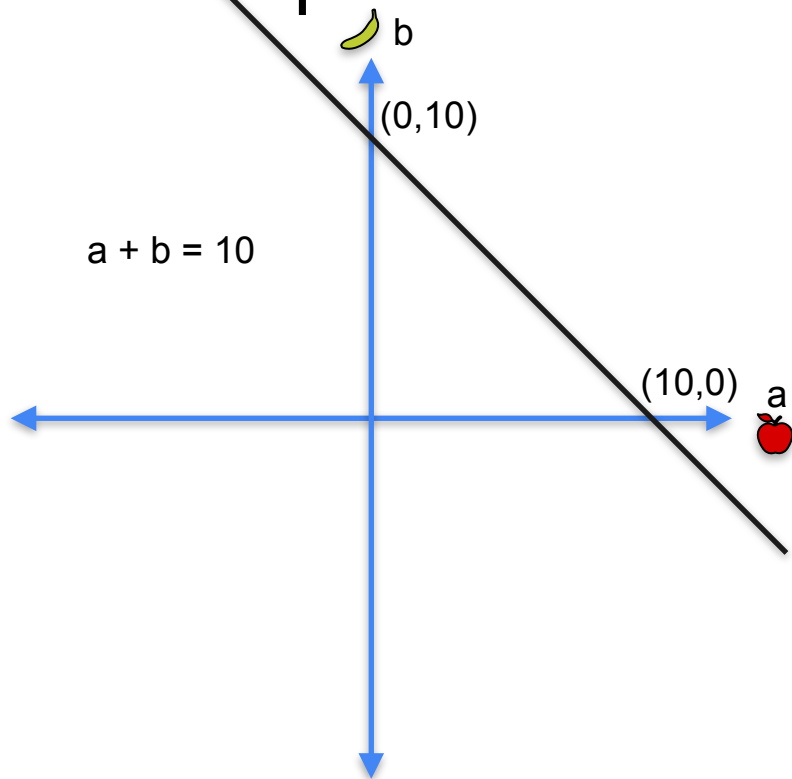
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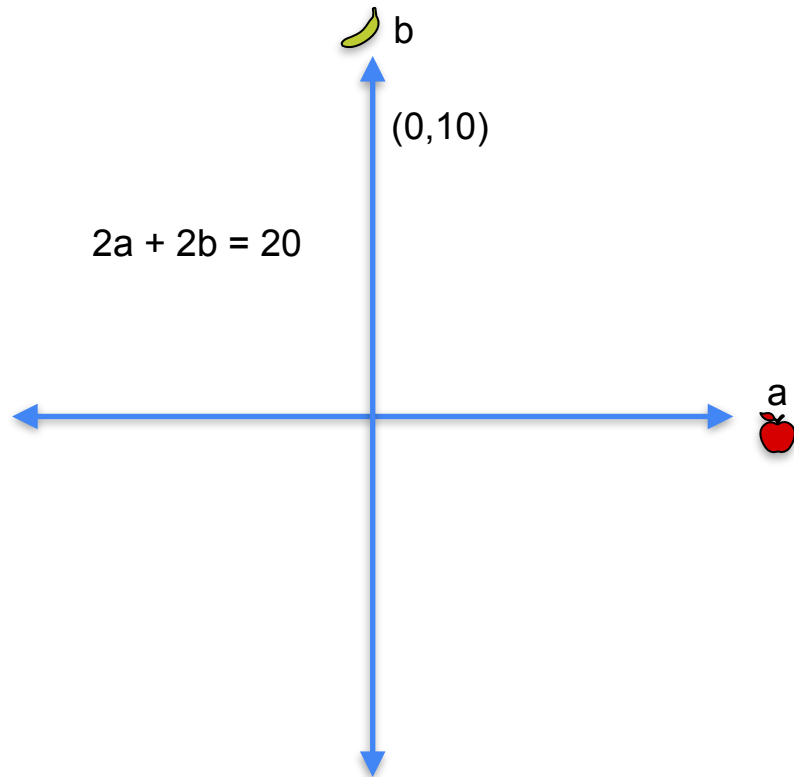
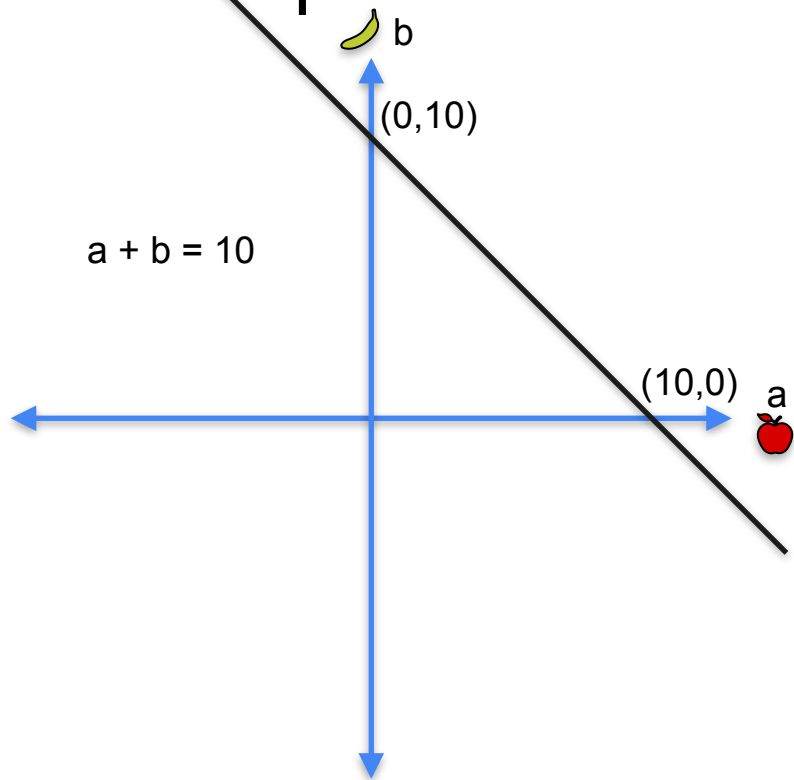
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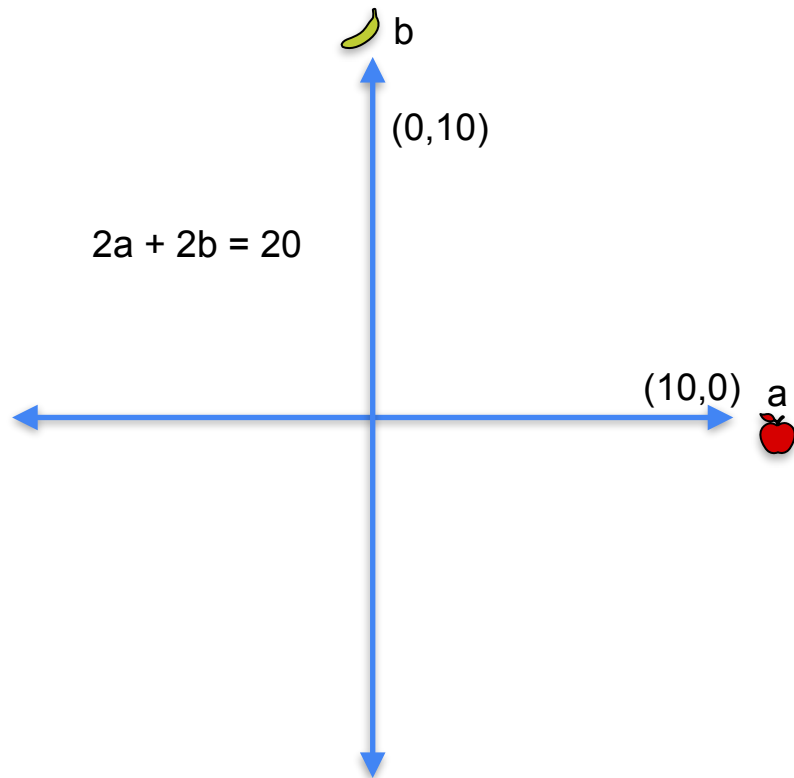
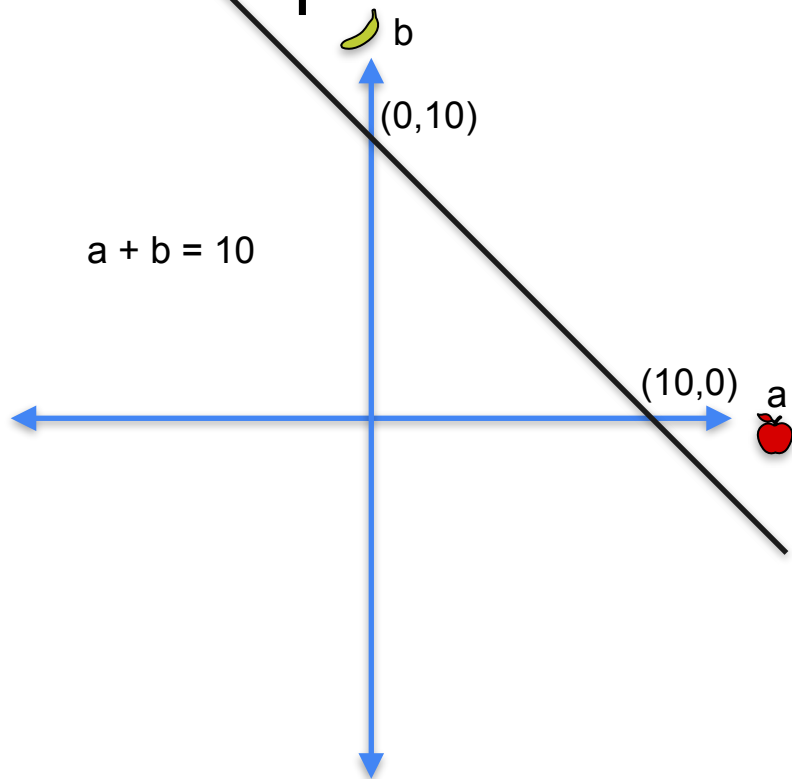
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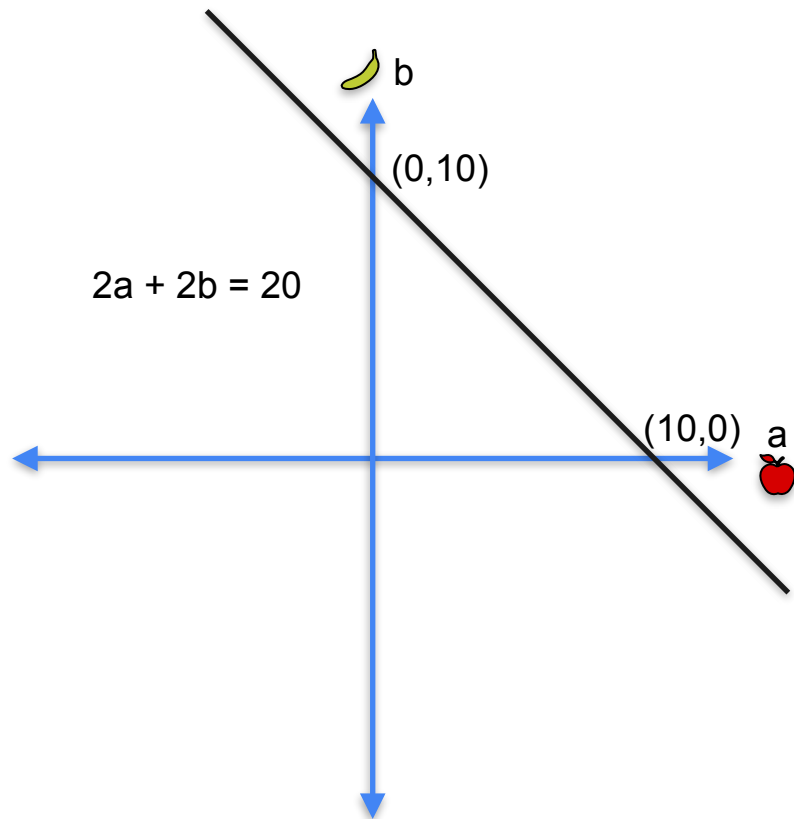
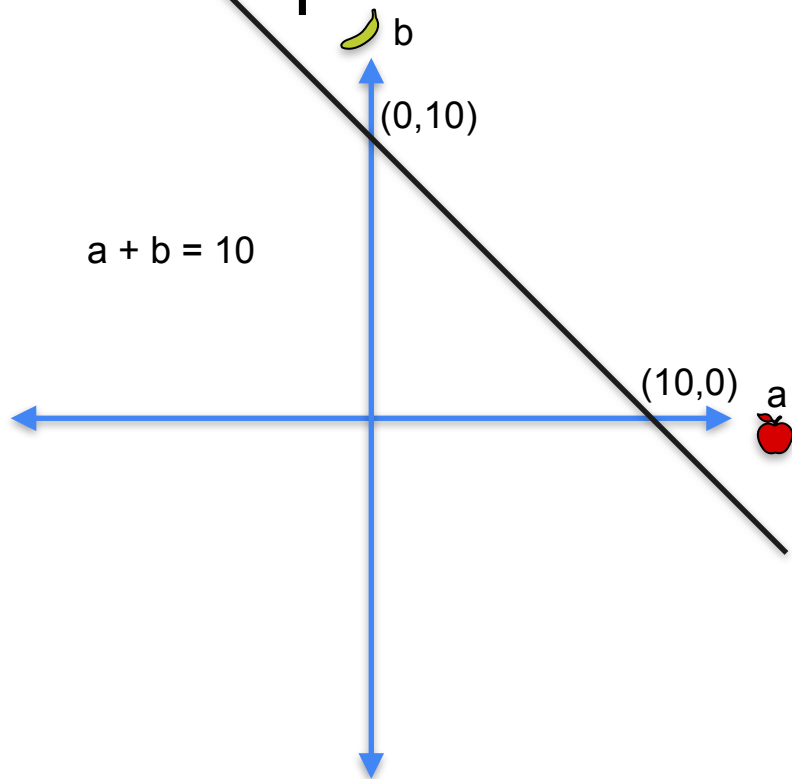
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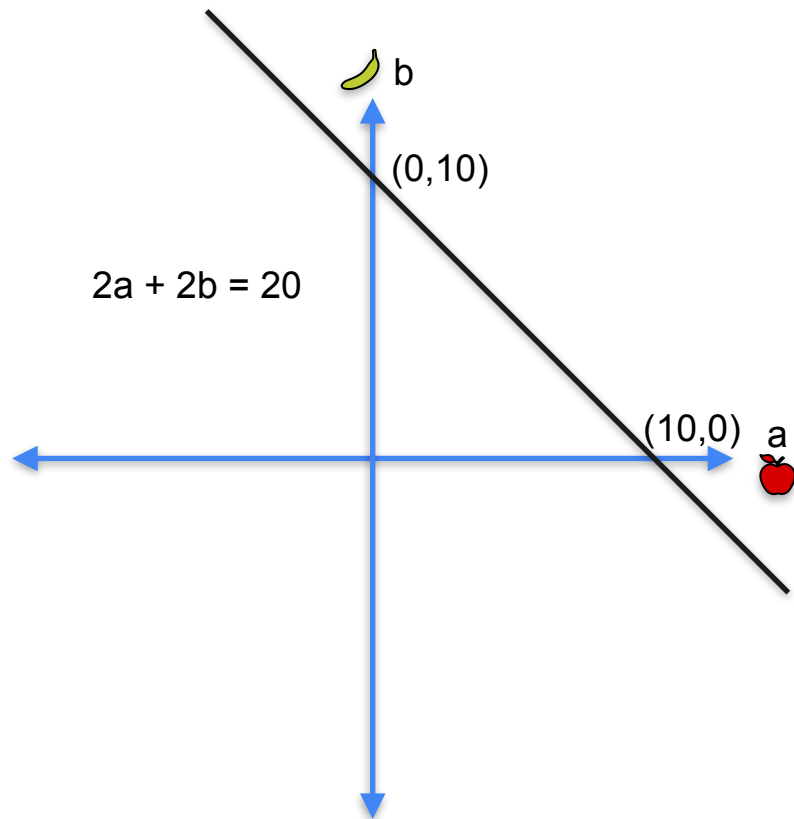
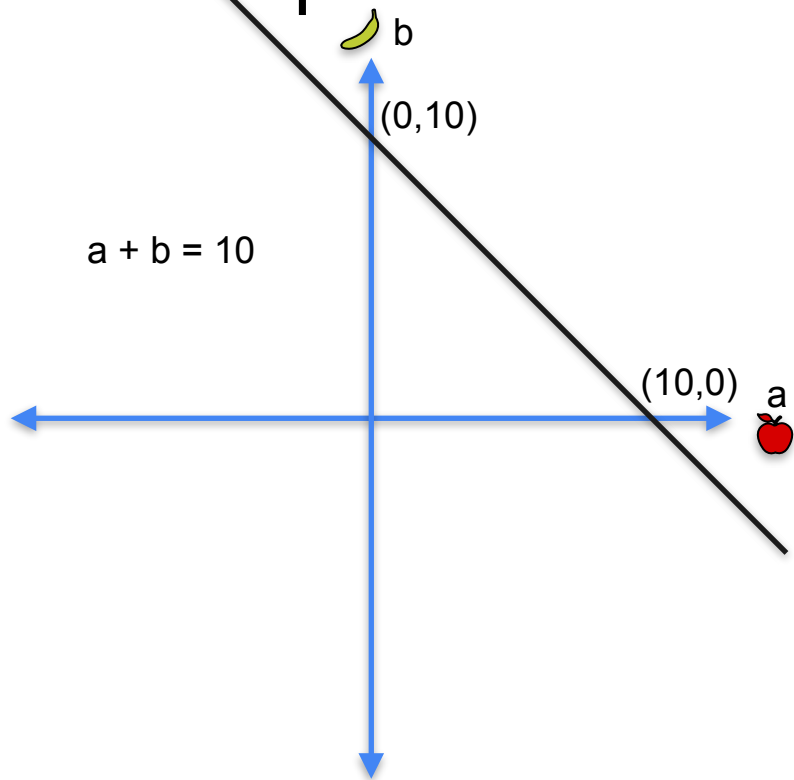
# Linear equation $\rightarrow$ line



# Linear equation $\rightarrow$ line



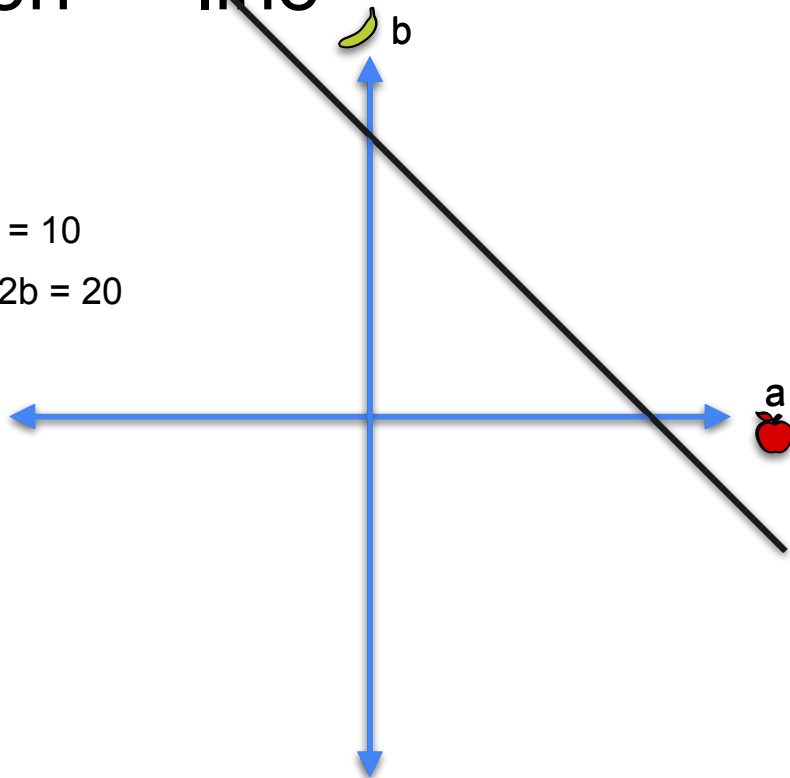
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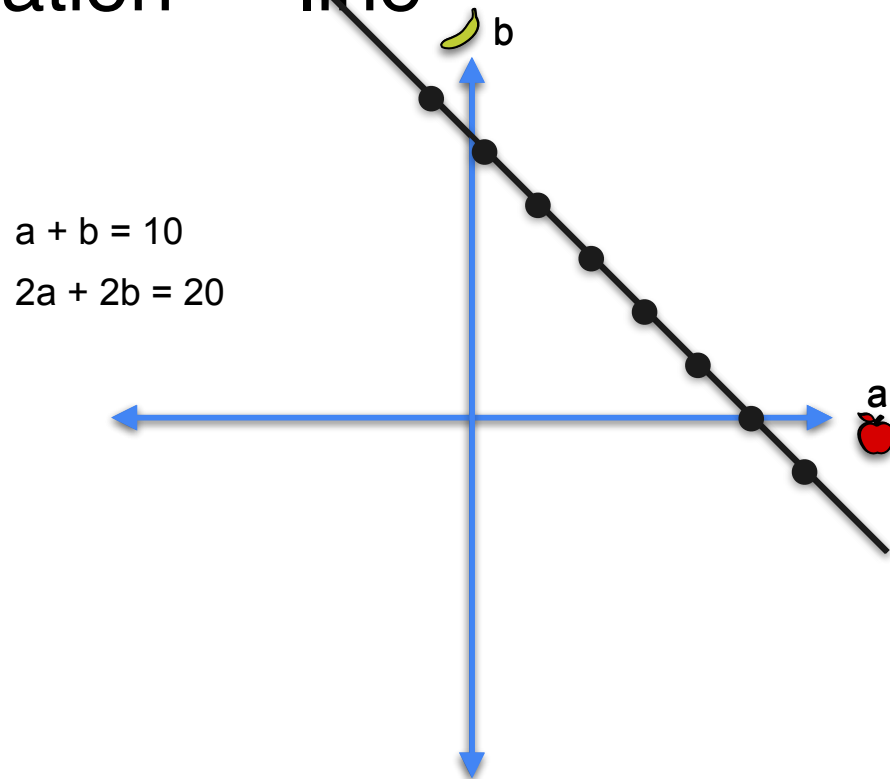
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$$2a + 2b = 20$$

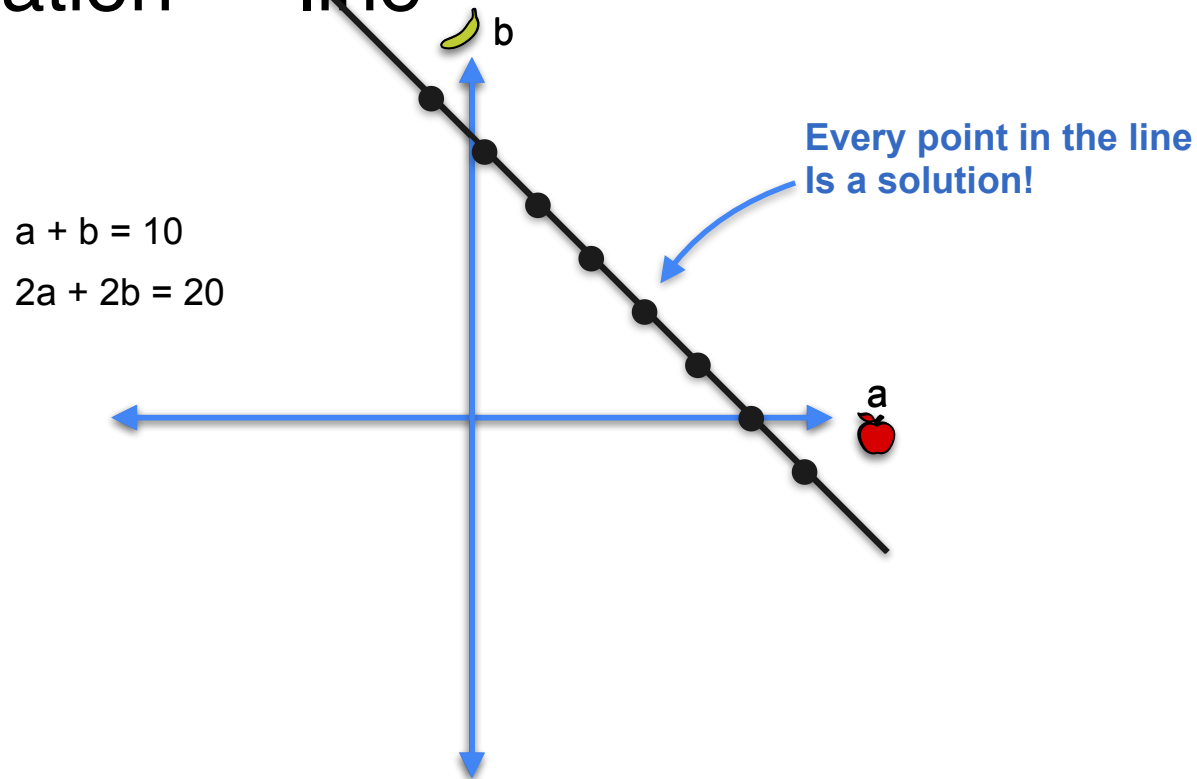




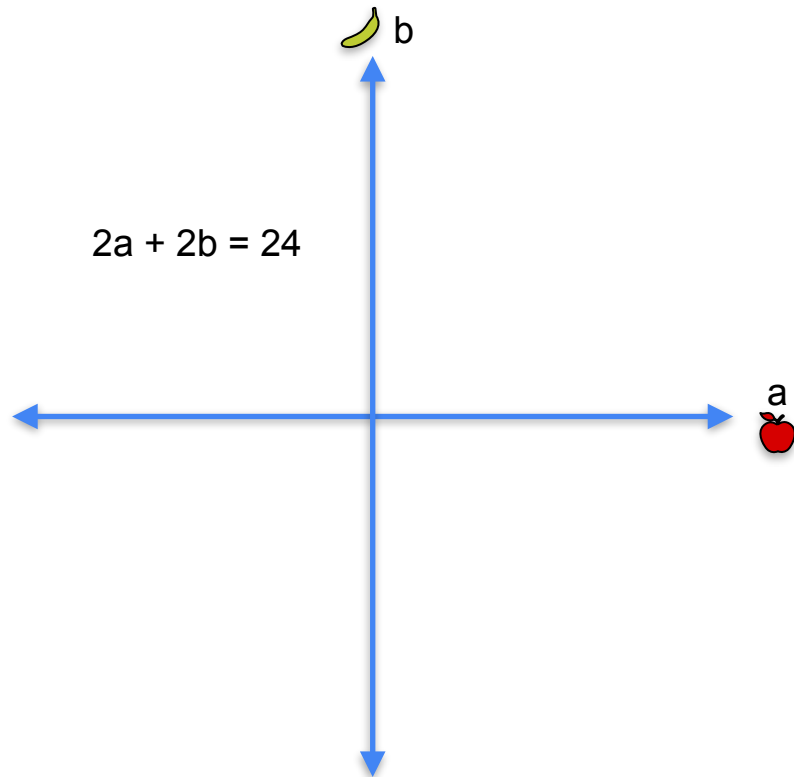
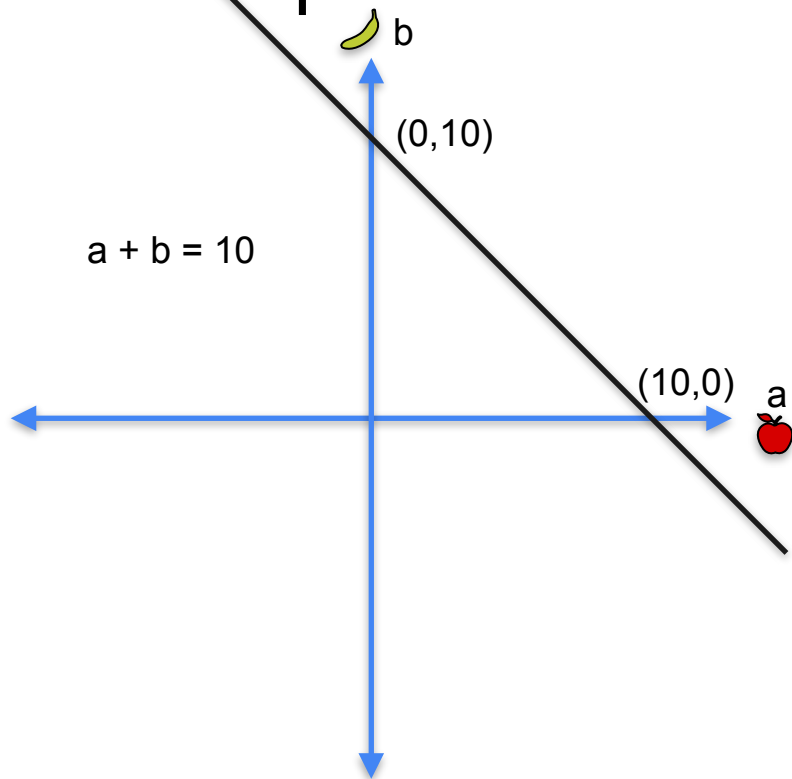
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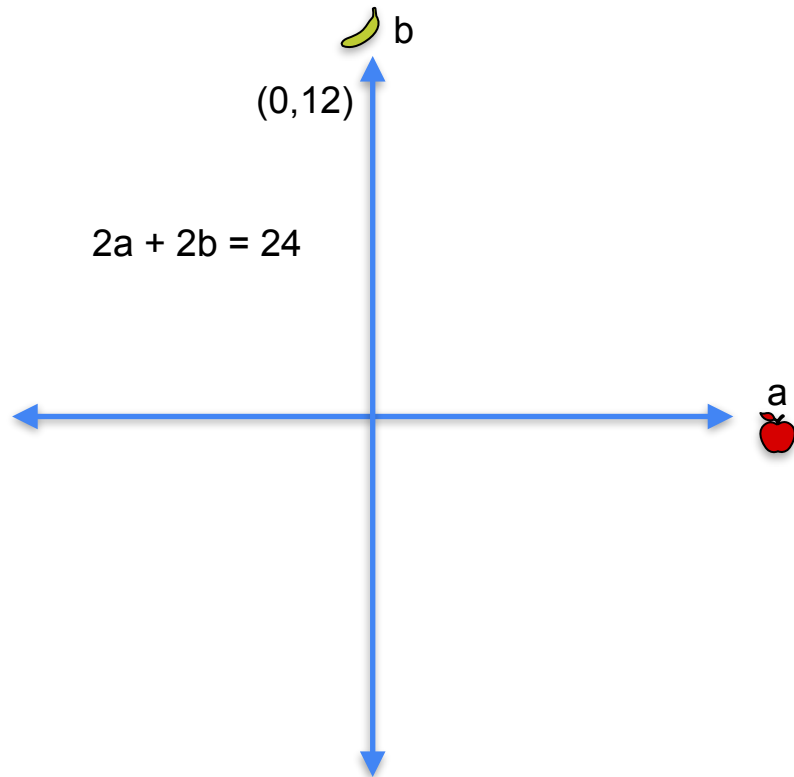
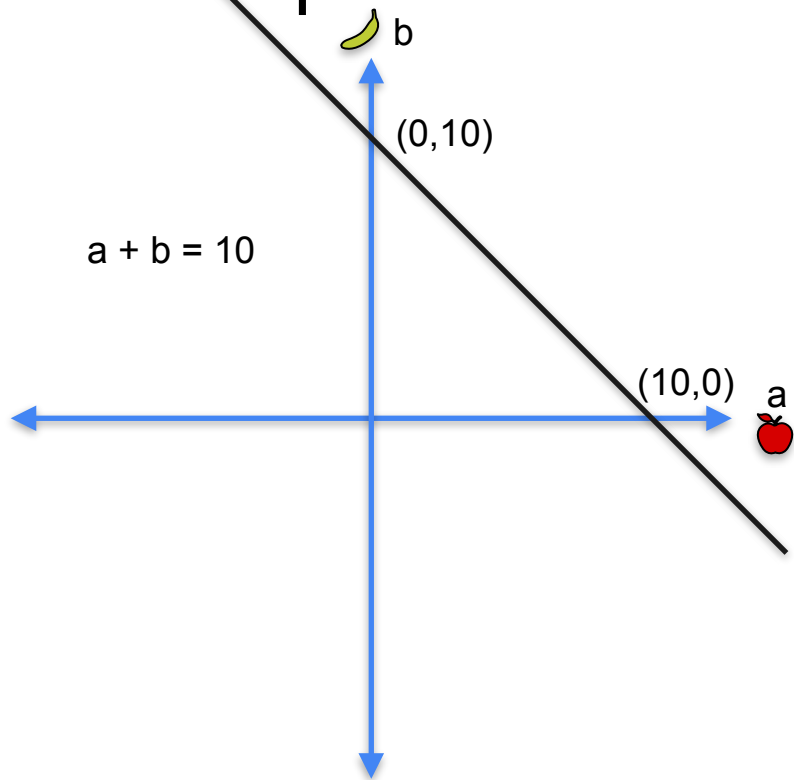
# Linear equation $\rightarrow$ line



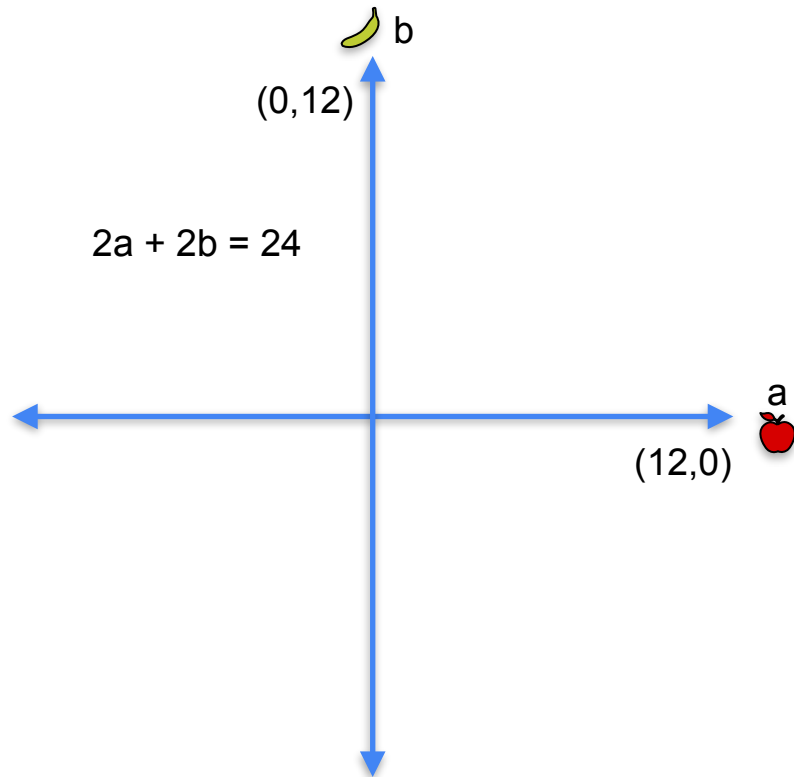
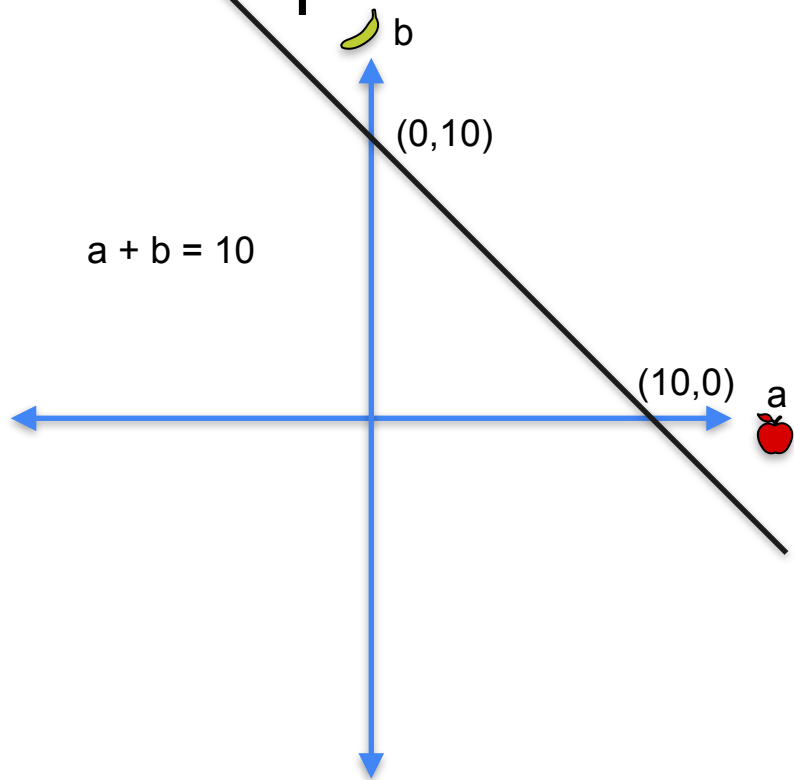
# Linear equation $\rightarrow$ line



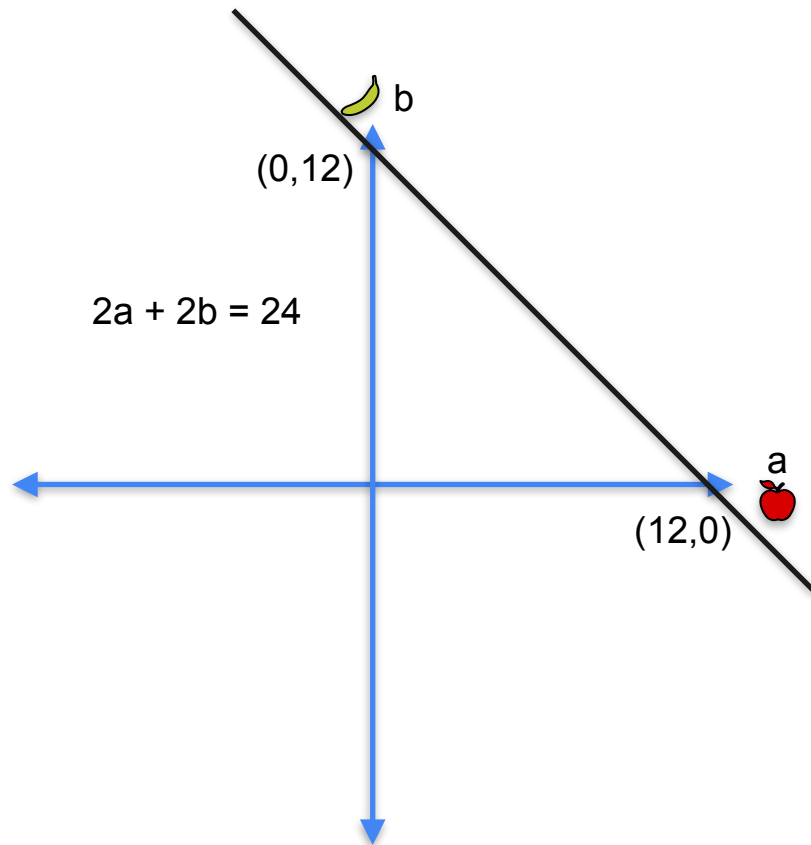
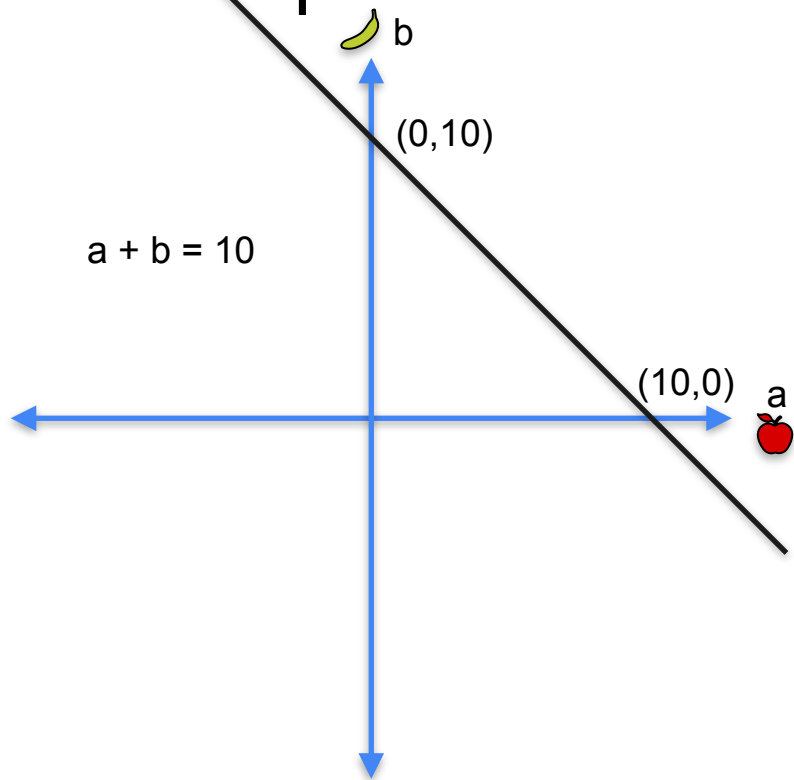
# Linear equation $\rightarrow$ line



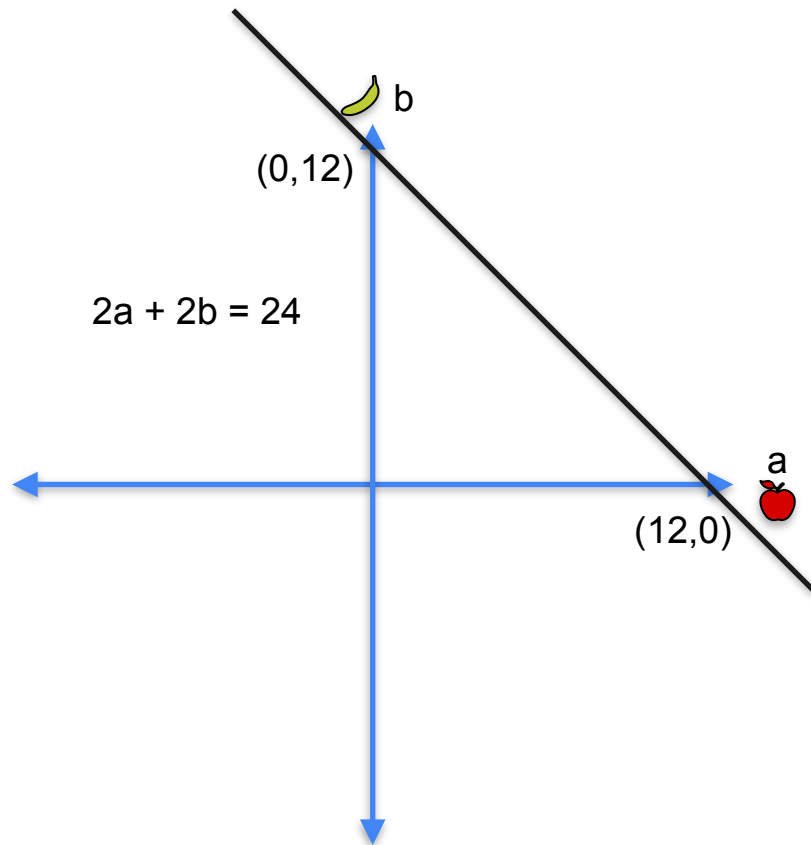
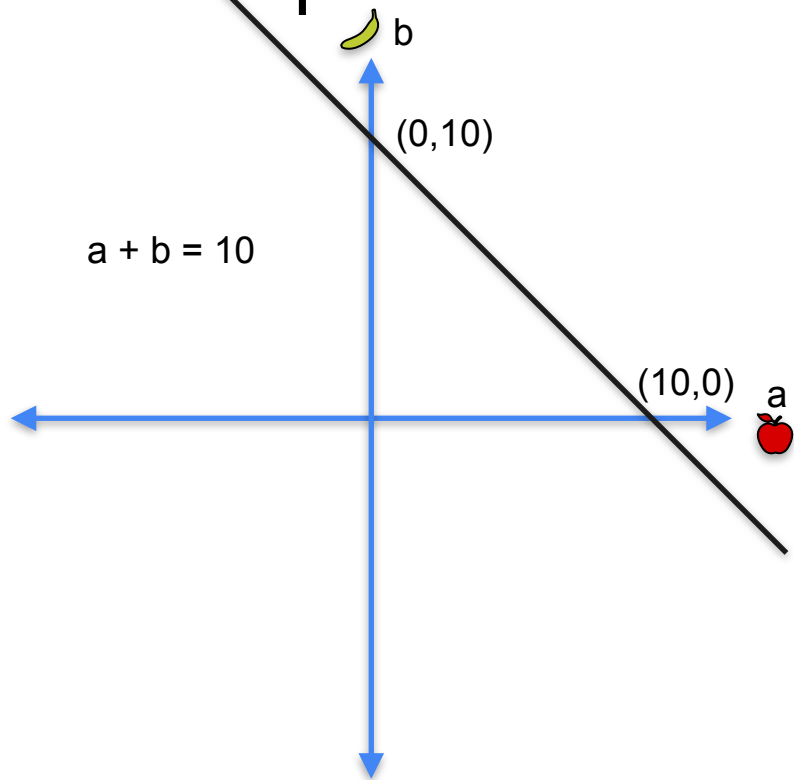
# Linear equation $\rightarrow$ line



# Linear equation $\rightarrow$ line



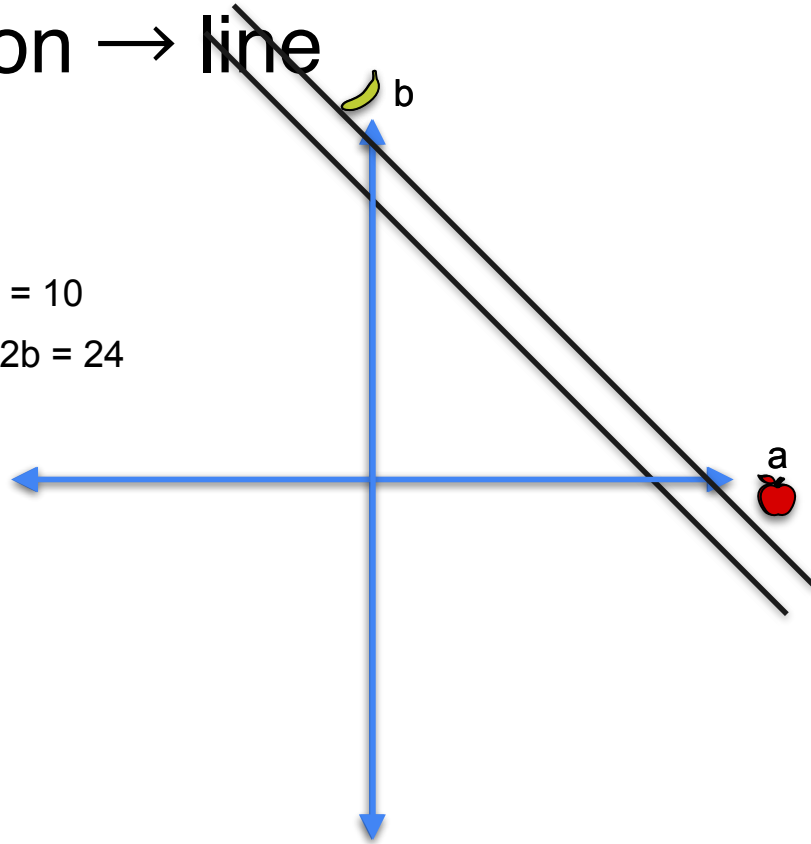
# Linear equation $\rightarrow$ line



# Linear equation → line

$$a + b = 10$$

$$2a + 2b = 24$$

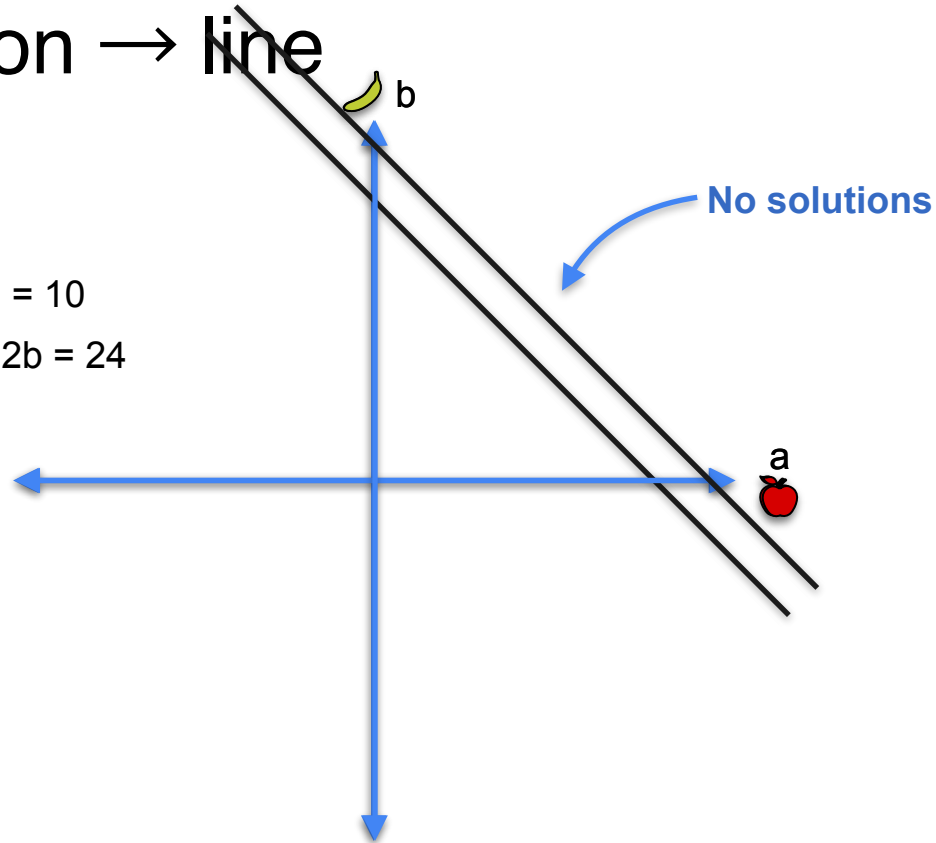




# Linear equation → line

$$a + b = 10$$





$$2a + 2b = 24$$



# Systems of equations as lines



# Systems of equations as lines

## System 1

- $a + b = 10$   
 
- $a + 2b = 12$   
 



# Systems of equations as lines




## System 1

- $a + b = 10$   
 

- $a + 2b = 12$   
 





## System 2

- $a + b = 10$   
 






- $2a + 2b = 20$   
  

# Systems of equations as lines






## System 1

- $a + b = 10$   
 
- $a + 2b = 12$   
 

## System 2

- $a + b = 10$   
 
- $2a + 2b = 20$   
  

## System 3

- $a + b = 10$   
 
- $2a + 2b = 24$   
  

# Systems of equations as lines

System 1

- $a + b = 10$



- $a + 2b = 12$



 b

a 

System 2

- $a + b = 10$



- $2a + 2b = 20$



 b

a 

System 3

- $a + b = 10$



- $2a + 2b = 24$



 b

a 

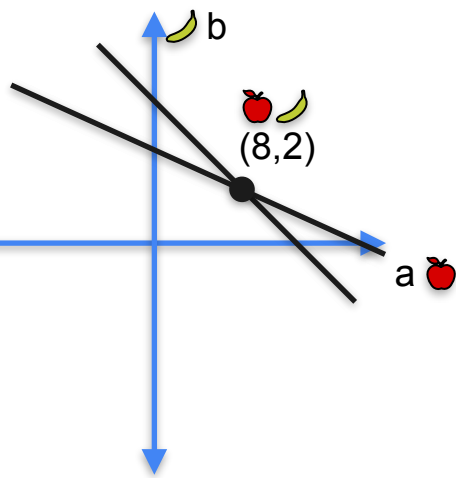
# Systems of equations as lines

System 1

- $a + b = 10$



- $a + 2b = 12$

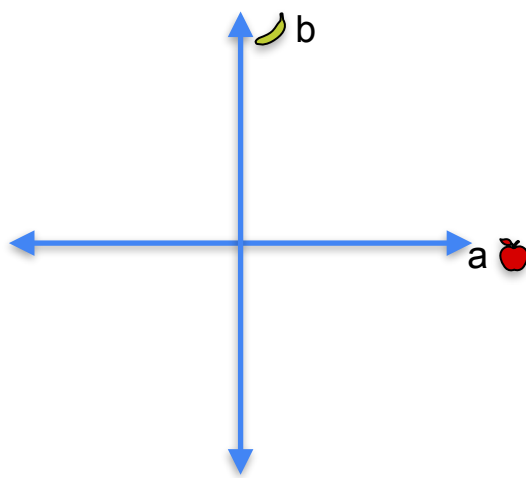


System 2

- $a + b = 10$



- $2a + 2b = 20$

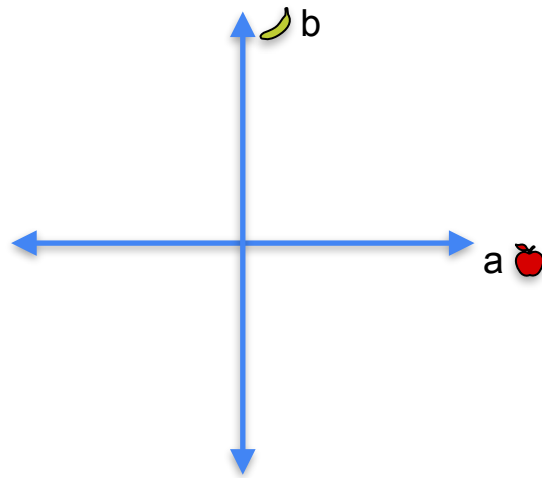


System 3

- $a + b = 10$



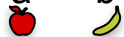
- $2a + 2b = 24$



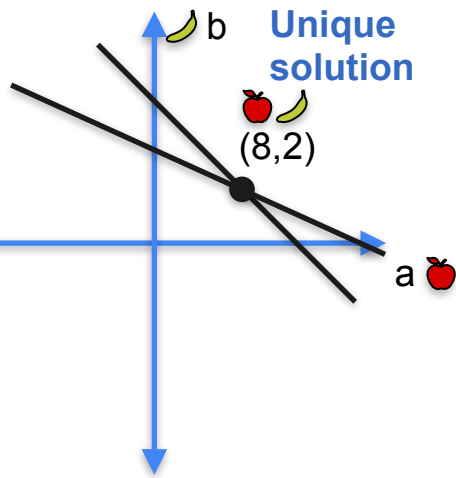
# Systems of equations as lines

System 1

- $a + b = 10$



- $a + 2b = 12$

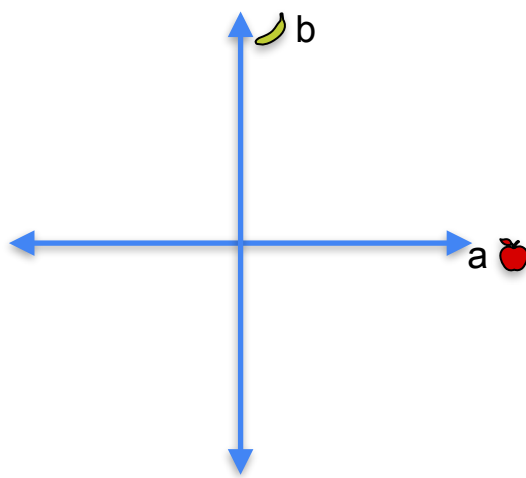


System 2

- $a + b = 10$



- $2a + 2b = 20$

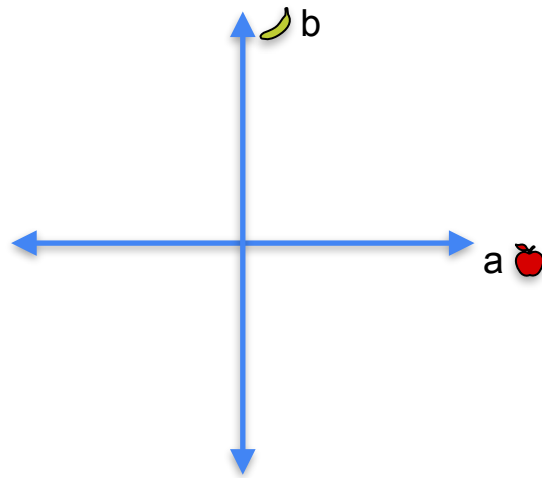


System 3

- $a + b = 10$



- $2a + 2b = 24$

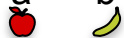




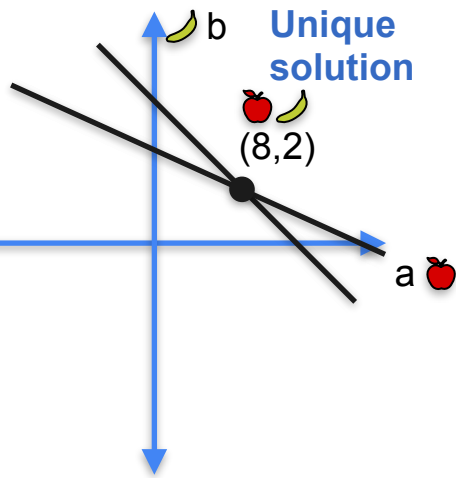
# Systems of equations as lines

System 1

- $a + b = 10$



- $a + 2b = 12$

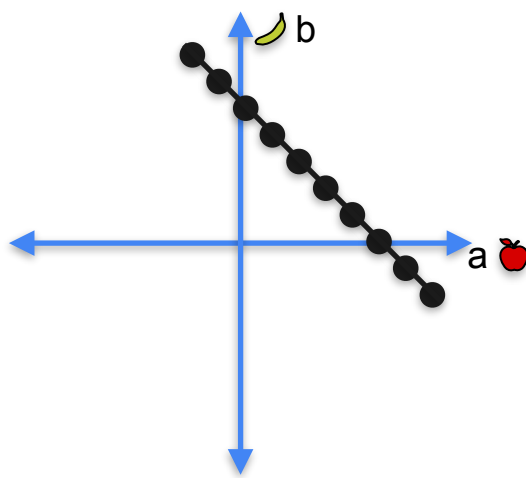


System 2

- $a + b = 10$



- $2a + 2b = 20$

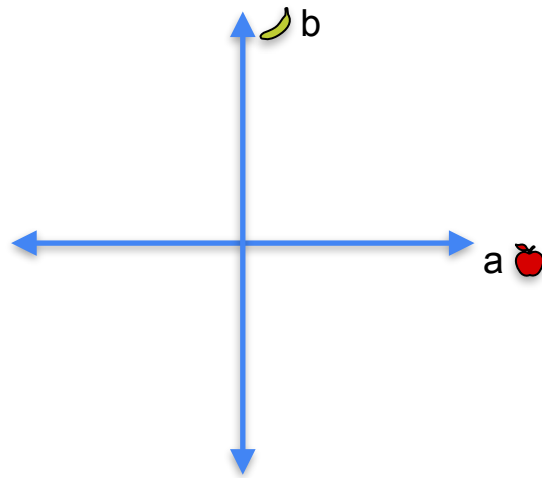


System 3

- $a + b = 10$



- $2a + 2b = 24$



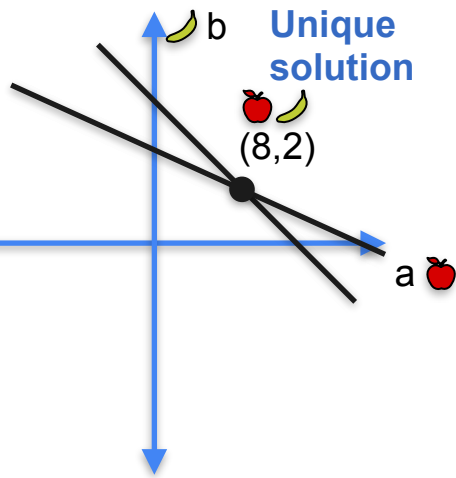
# Systems of equations as lines

System 1

- $a + b = 10$



- $a + 2b = 12$

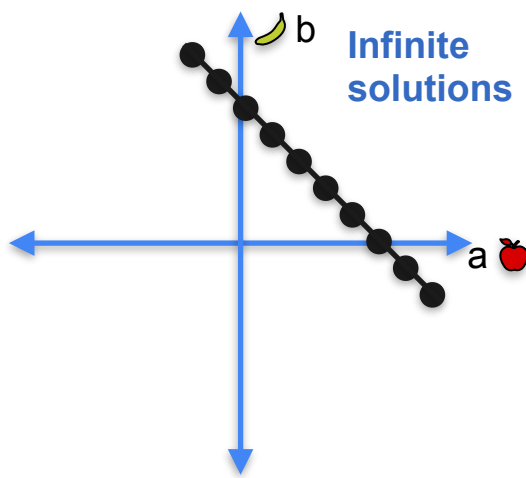


System 2

- $a + b = 10$



- $2a + 2b = 20$

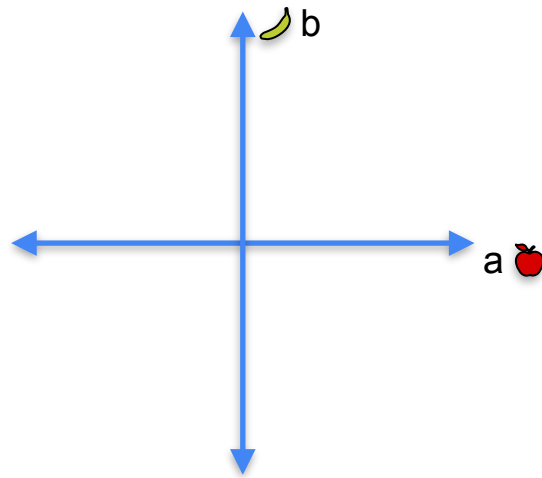


System 3

- $a + b = 10$



- $2a + 2b = 24$



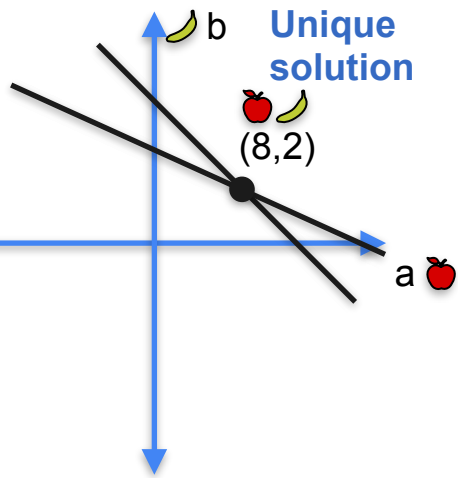
# Systems of equations as lines

System 1

- $a + b = 10$

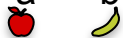


- $a + 2b = 12$

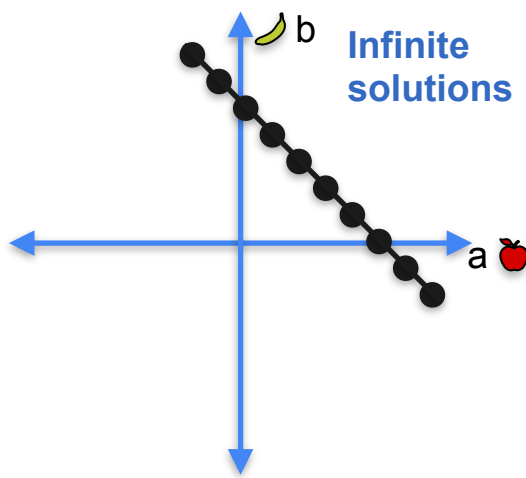


System 2

- $a + b = 10$



- $2a + 2b = 20$

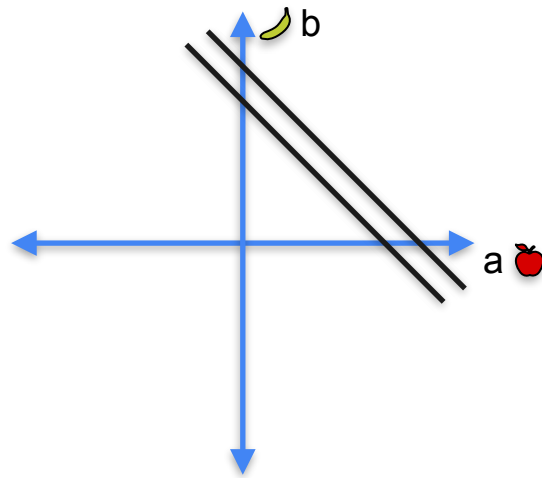


System 3

- $a + b = 10$



- $2a + 2b = 24$



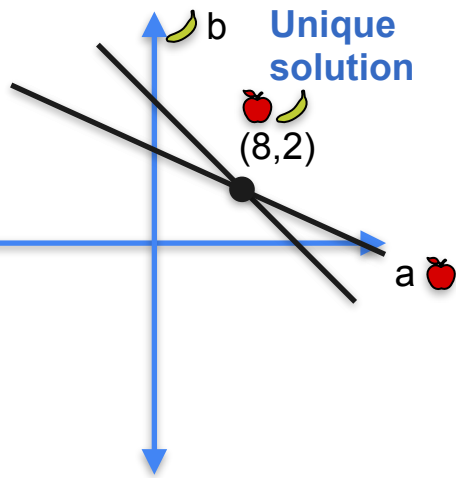
# Systems of equations as lines

System 1

- $a + b = 10$



- $a + 2b = 12$

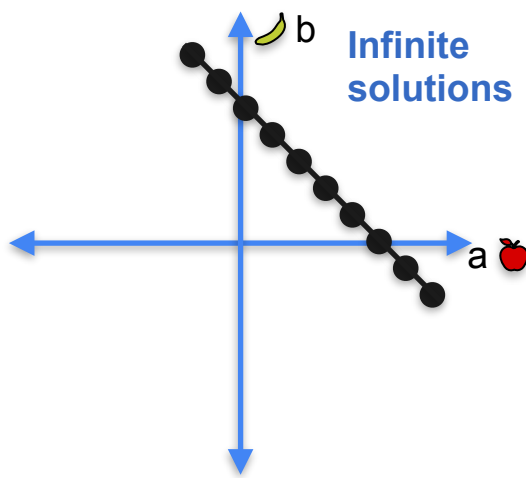


System 2

- $a + b = 10$



- $2a + 2b = 20$

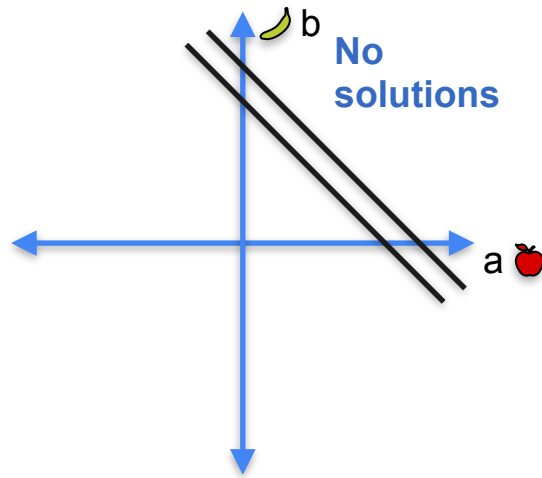


System 3

- $a + b = 10$



- $2a + 2b = 24$



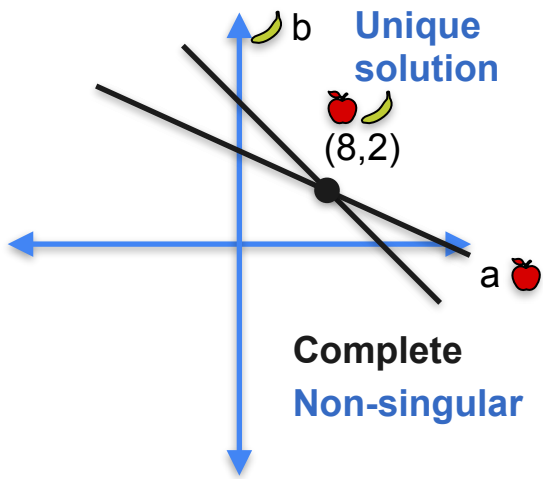
# Systems of equations as lines

System 1

- $a + b = 10$

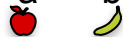


- $a + 2b = 12$

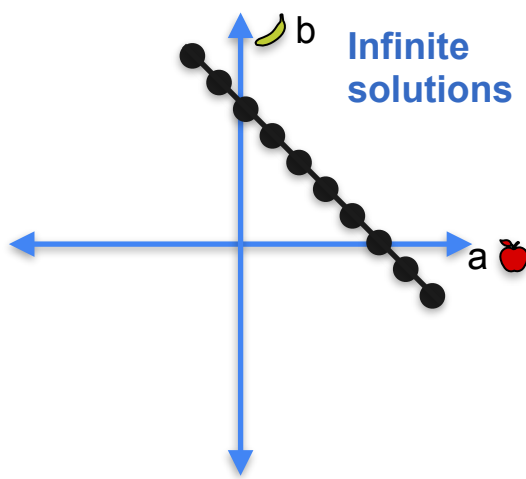


System 2

- $a + b = 10$



- $2a + 2b = 20$

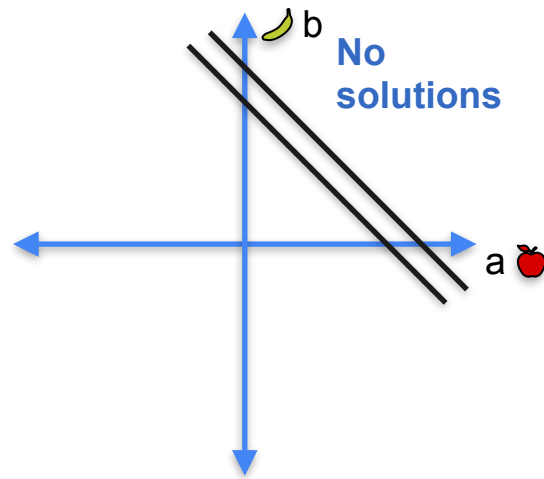


System 3

- $a + b = 10$



- $2a + 2b = 24$



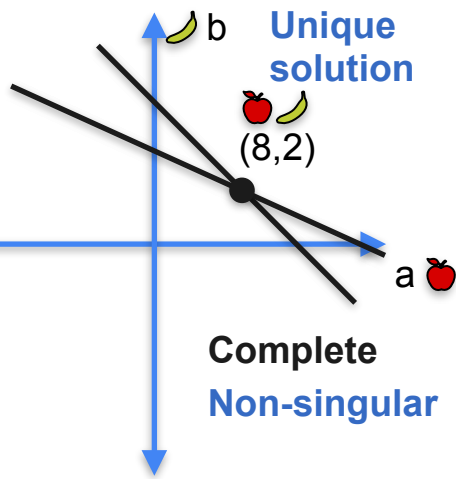
# Systems of equations as lines

System 1

- $a + b = 10$



- $a + 2b = 12$

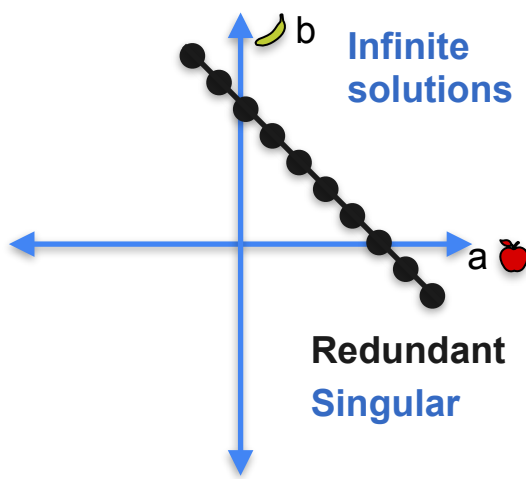


System 2

- $a + b = 10$



- $2a + 2b = 20$

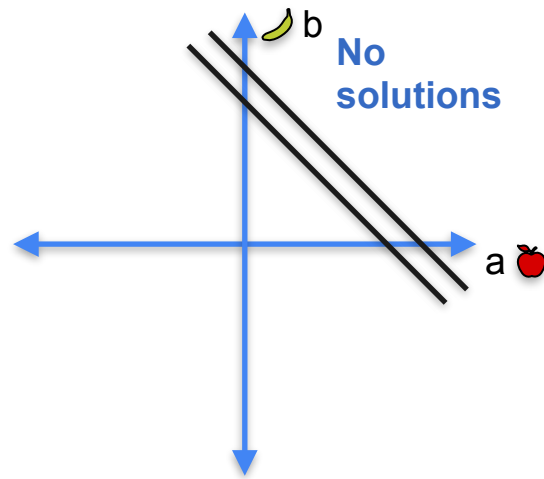


System 3

- $a + b = 10$



- $2a + 2b = 24$



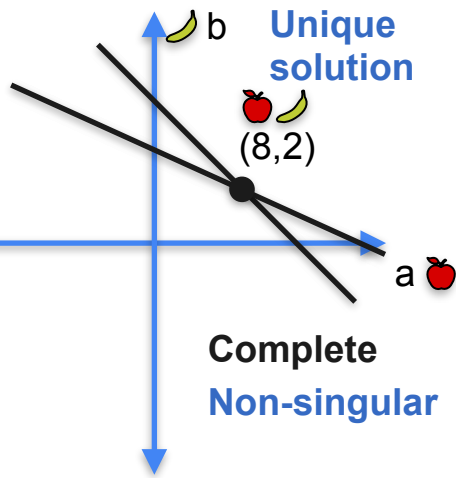
# Systems of equations as lines

System 1

- $a + b = 10$



- $a + 2b = 12$

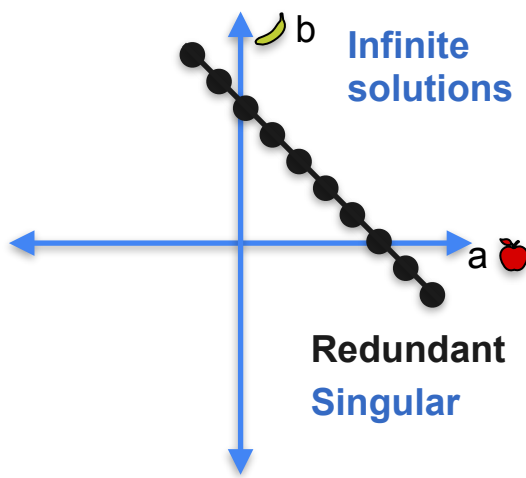


System 2

- $a + b = 10$



- $2a + 2b = 20$

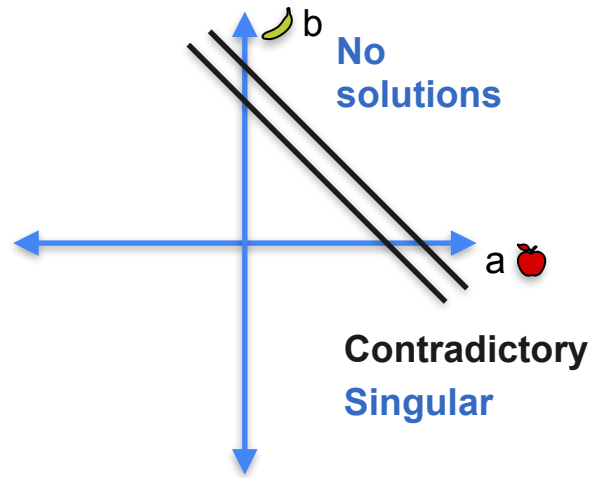


System 3

- $a + b = 10$



- $2a + 2b = 24$

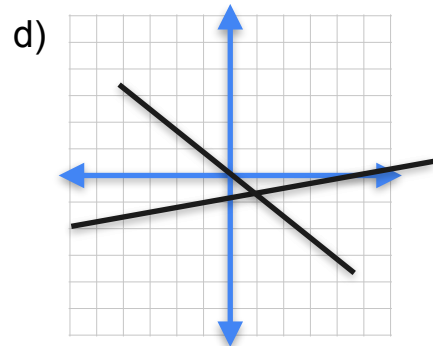
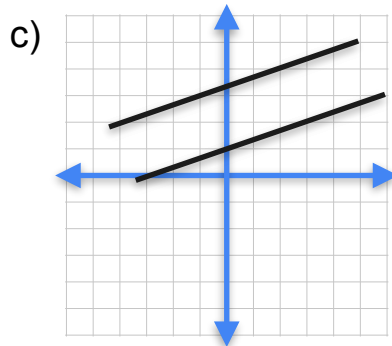
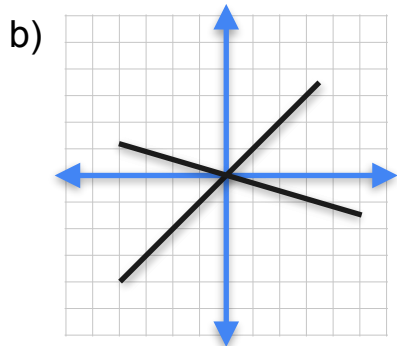
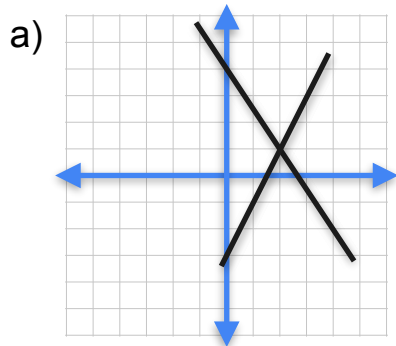


# Quiz

## Problem 1

Which of the following plots corresponds to the system of equations:

- $3a + 2b = 8$
- $2a - b = 3$



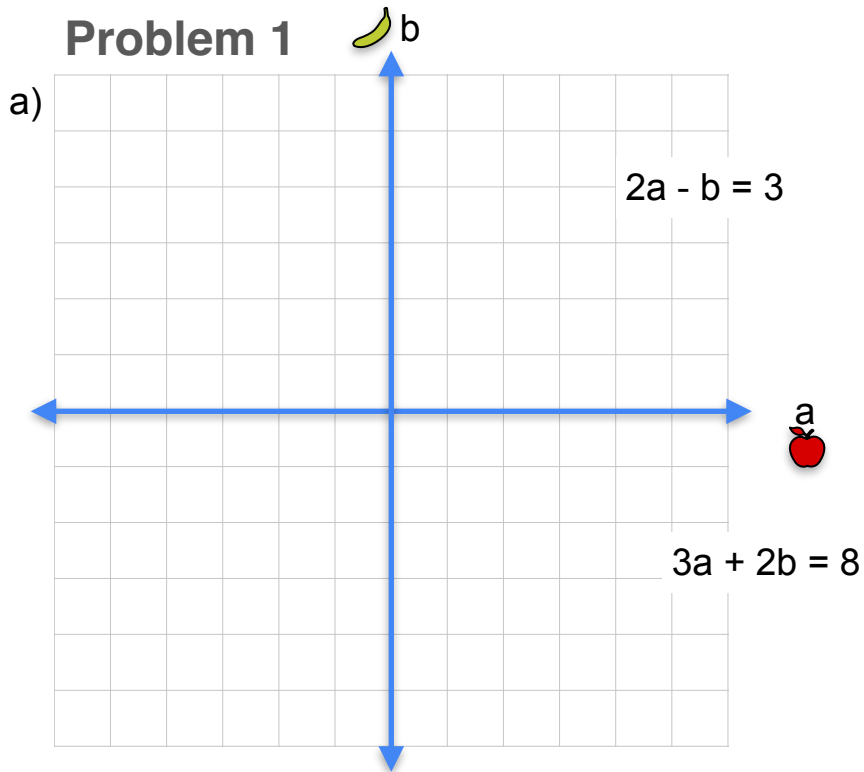
## Problem 2

Is this system singular or non-singular?



# Solution

## Problem 1

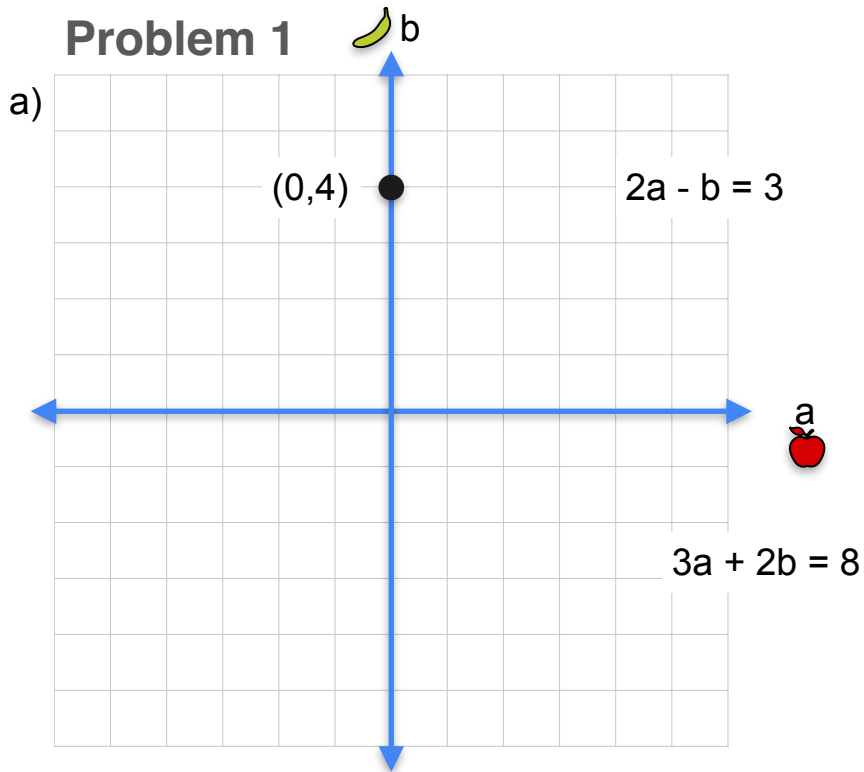


## Problem 2

Since the lines cross at a unique point, the system is non-singular.

# Solution

## Problem 1



## Problem 2

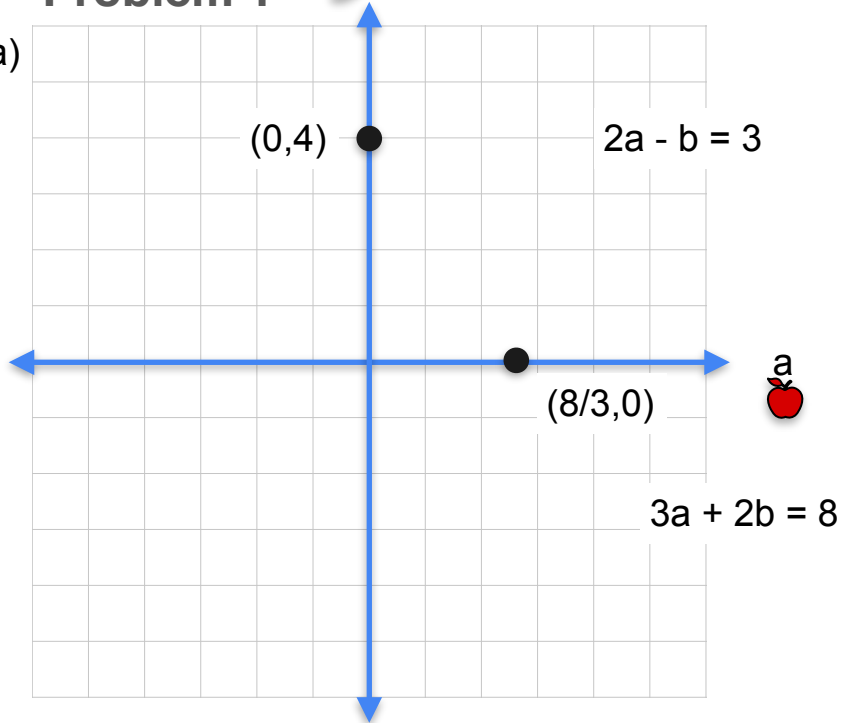
Since the lines cross at a unique point, the system is non-singular.

# Solution

## Problem 1



a)

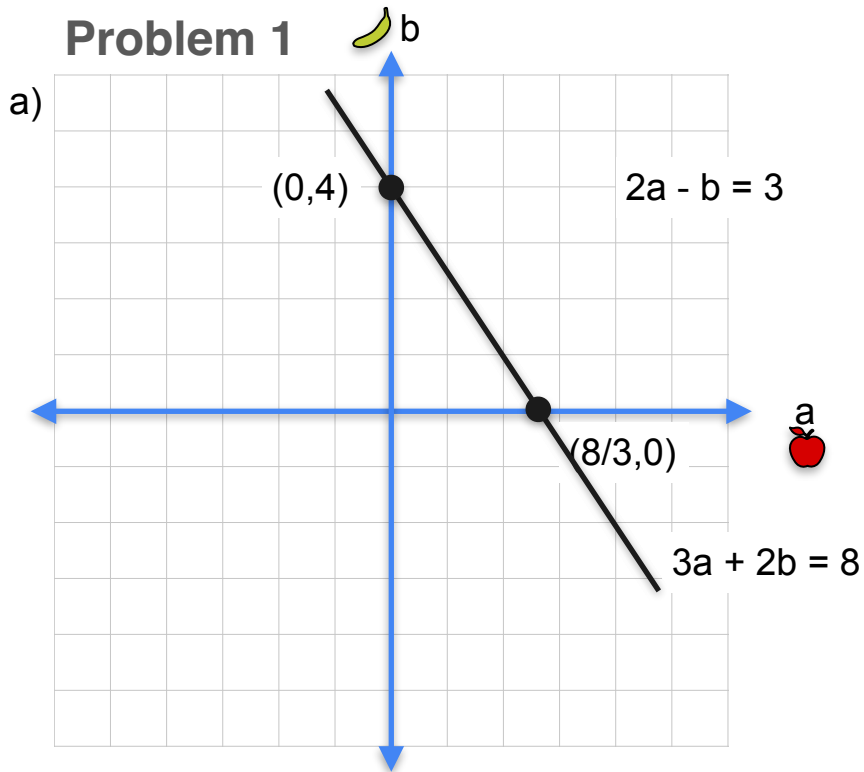


## Problem 2

Since the lines cross at a unique point, the system is non-singular.

# Solution

## Problem 1



## Problem 2

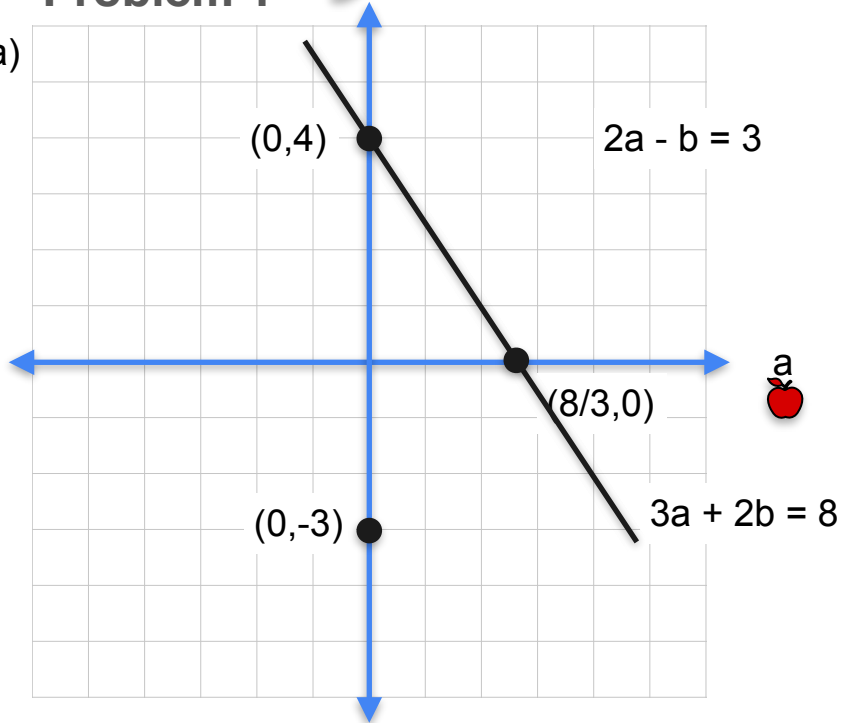
Since the lines cross at a unique point, the system is non-singular.

# Solution

## Problem 1

 b

a)



## Problem 2

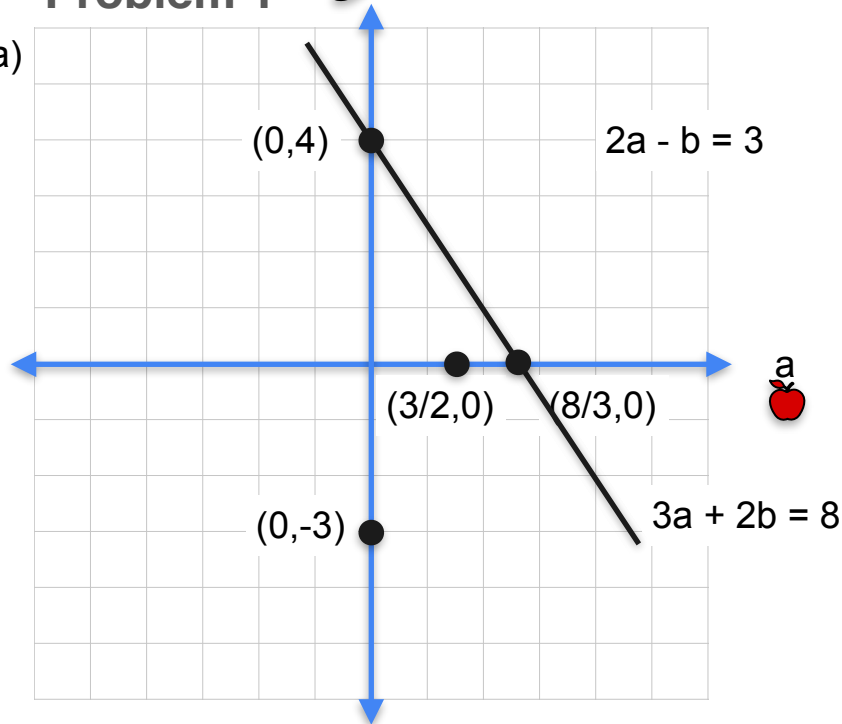
Since the lines cross at a unique point, the system is non-singular.

# Solution

## Problem 1



a)



## Problem 2

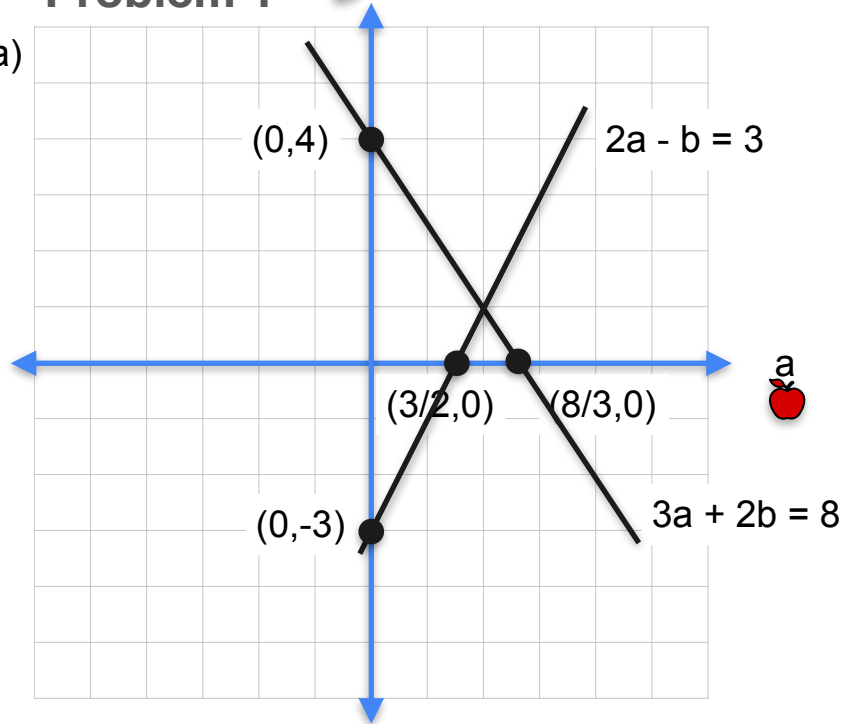
Since the lines cross at a unique point, the system is non-singular.

# Solution

## Problem 1

 b

a)



## Problem 2

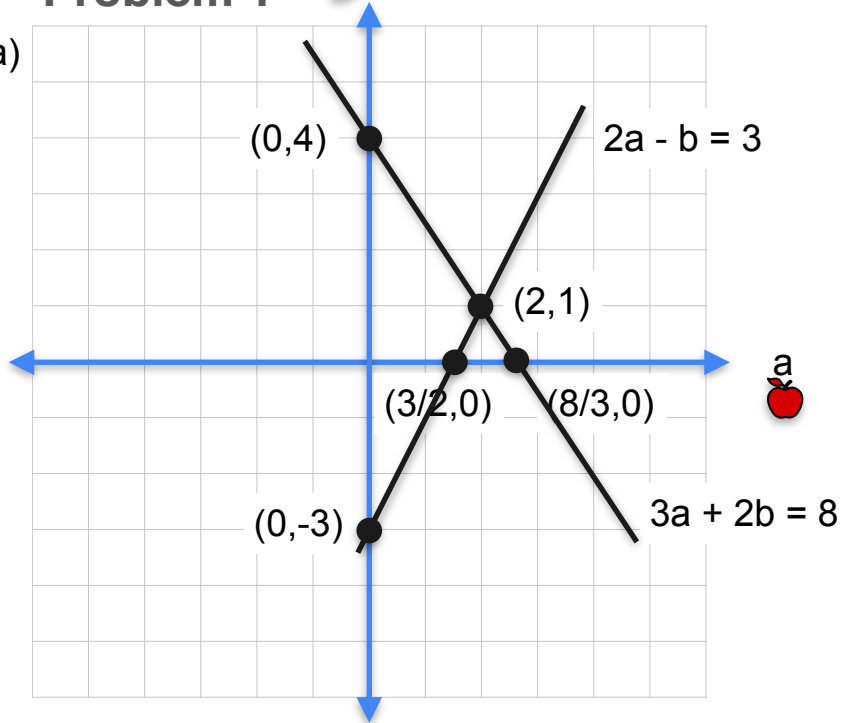
Since the lines cross at a unique point, the system is non-singular.

# Solution

## Problem 1

 b

a)



## Problem 2

Since the lines cross at a unique point, the system is non-singular.





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# System of Linear Equations

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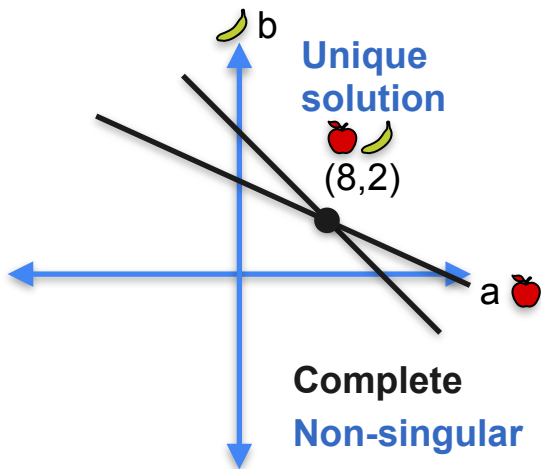
**A geometric notion of  
singularity**

# Systems of equations as lines

System 1

- $a + b = 10$

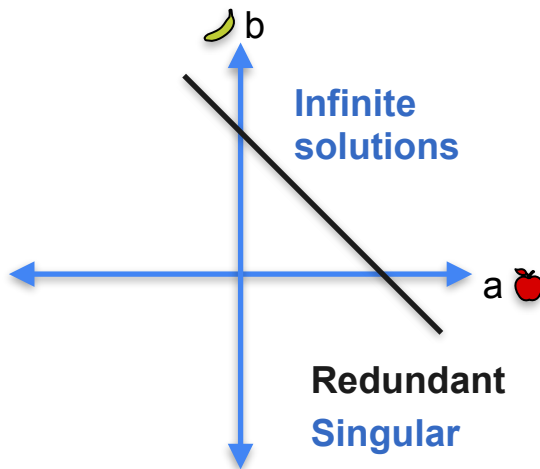
- $a + 2b = 12$



System 2

- $a + b = 10$

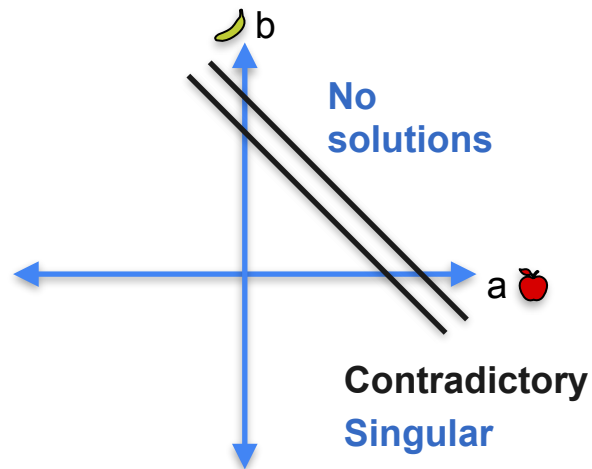
- $2a + 2b = 20$



System 3

- $a + b = 10$

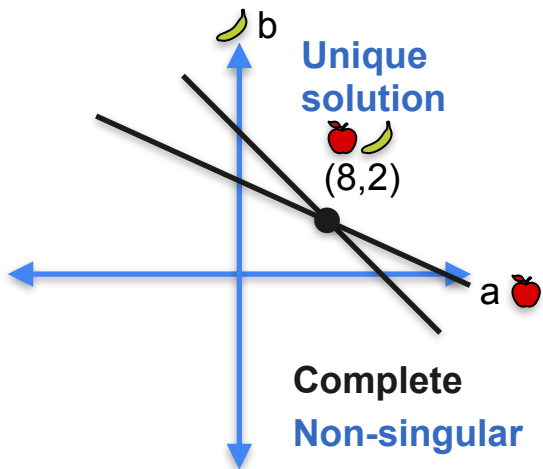
- $2a + 2b = 24$



# Systems of equations as lines

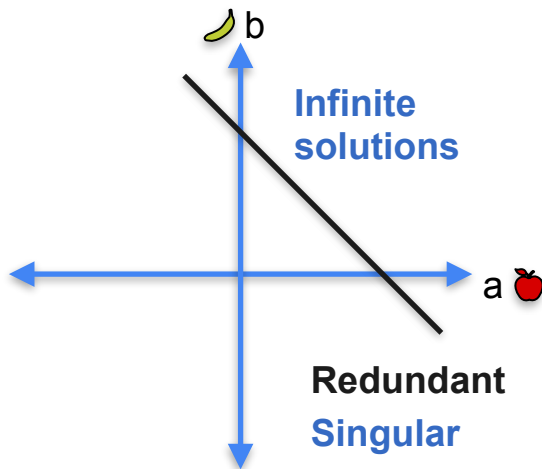
System 1

- $a + b = 10$
- $a + 2b = 12$



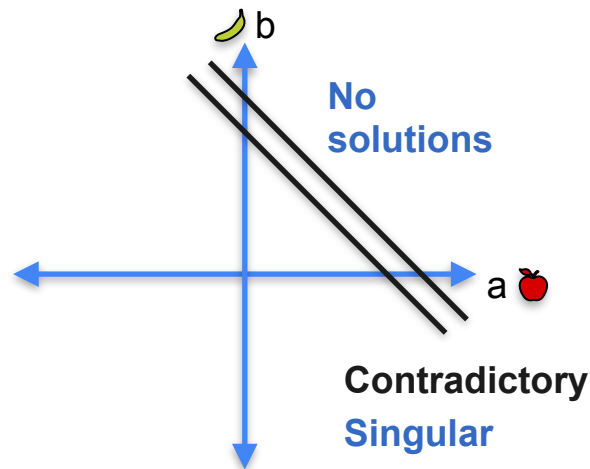
System 2

- $a + b = 10$
- $2a + 2b = 20$



System 3

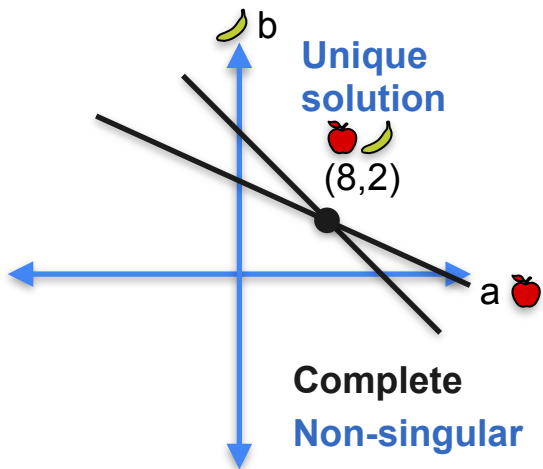
- $a + b = 10$
- $2a + 2b = 24$



# Systems of equations as lines

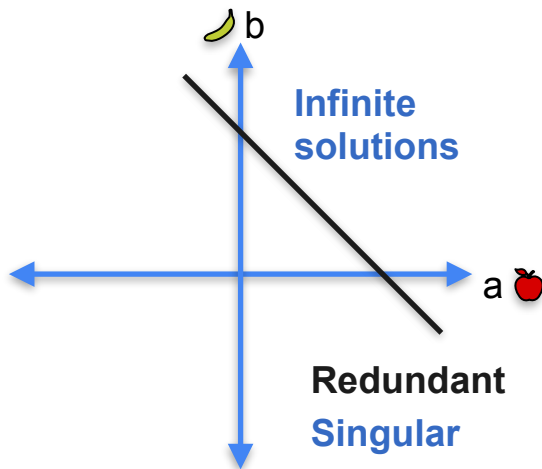
System 1

- $a + b = 0$
- $a + 2b = 0$



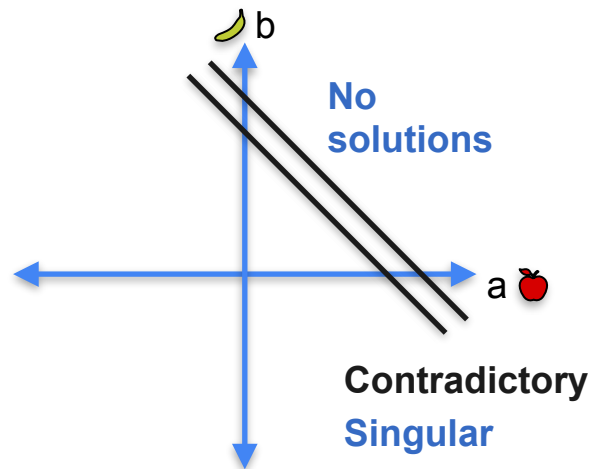
System 2

- $a + b = 10$
- $2a + 2b = 20$



System 3

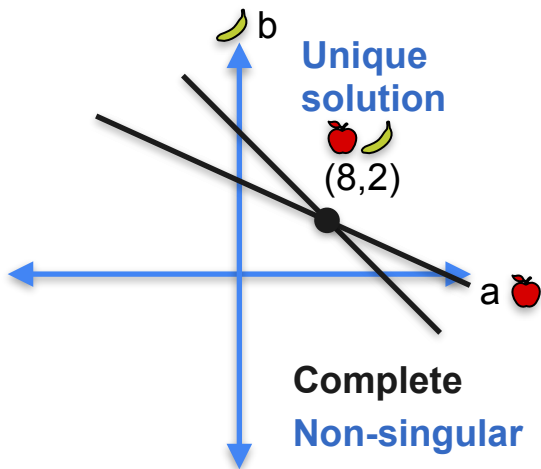
- $a + b = 10$
- $2a + 2b = 24$



# Systems of equations as lines

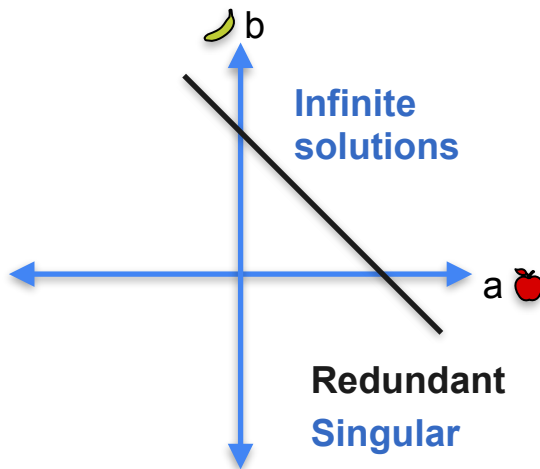
System 1

- $a + b = 0$
- $a + 2b = 0$



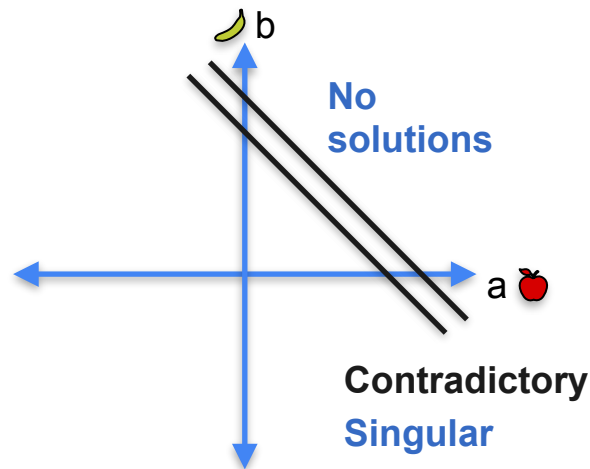
System 2

- $a + b = 0$
- $2a + 2b = 0$



System 3

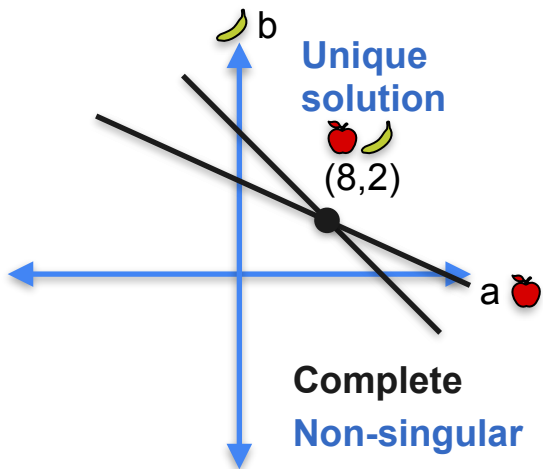
- $a + b = 10$
- $2a + 2b = 24$



# Systems of equations as lines

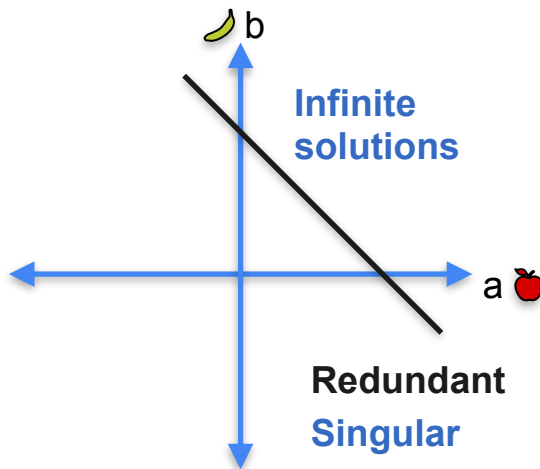
System 1

- $a + b = 0$
- $a + 2b = 0$



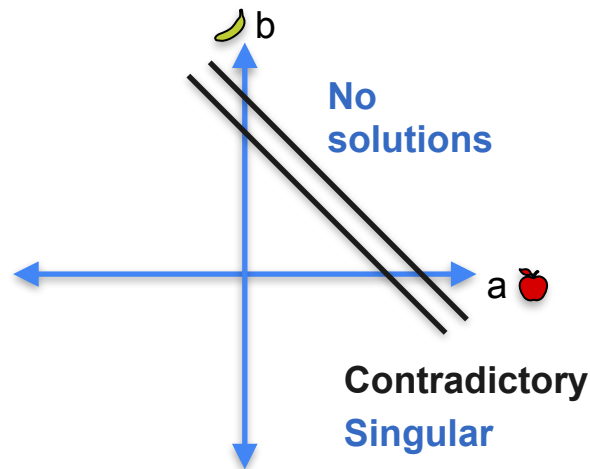
System 2

- $a + b = 0$
- $2a + 2b = 0$



System 3

- $a + b = 0$
- $2a + 2b = 0$



# Systems of equations as lines

System 1

- $a + b = 0$
- $a + 2b = 0$

$b$

Unique  
solution

$a$

Complete  
Non-singular

System 2

- $a + b = 0$
- $2a + 2b = 0$

$b$

Infinite  
solutions

$a$

Redundant  
Singular

System 3

- $a + b = 0$
- $2a + 2b = 0$

$b$

Infinite  
solutions

$a$

Redundant  
Singular



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# System of Linear Equations





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**Singular vs nonsingular  
matrices**








# Systems of equations as matrices

## System 1

- $a + b = 0$   
 
- $a + 2b = 0$   
 



## System 2

- $a + b = 0$   
 
- $2a + 2b = 0$   
  

# Systems of equations as matrices

## System 1

- $a + b = 0$
- $a + 2b = 0$

|  |  |
|---|---|
| 1   | 1   |
| 1   | 2   |



## System 2

- $a + b = 0$
- $2a + 2b = 0$

# Systems of equations as matrices



## System 1

- $a + b = 0$
- $a + 2b = 0$

|  |  |
|---|---|
| 1   | 1   |
| 1   | 2   |

## System 2

- $a + b = 0$
- $2a + 2b = 0$

|  |  |
|---|---|
| 1   | 1   |
| 2   | 2   |

# Systems of equations as matrices



## System 1

- $a + b = 0$

- $a + 2b = 0$

**Non-singular  
system**



**(Unique solution)**

|  |  |
|---|---|
| 1   | 1   |
| 1   | 2   |

## System 2

- $a + b = 0$

- $2a + 2b = 0$



|  |  |
|---|---|
| 1   | 1   |
| 2   | 2   |

# Systems of equations as matrices

## System 1

- $a + b = 0$
- $a + 2b = 0$

Non-singular  
system



|  |  |
|---|---|
| 1   | 1   |
| 1   | 2   |

Non-singular  
matrix

(Unique solution)

## System 2

- $a + b = 0$
- $2a + 2b = 0$



|  |  |
|---|---|
| 1   | 1   |
| 2   | 2   |

# Systems of equations as matrices

## System 1

- $a + b = 0$
- $a + 2b = 0$

Non-singular  
system

|   |   |
|---|---|
|  |  |
| 1   | 1   |
| 1   | 2   |



Non-singular  
matrix

(Unique solution)

## System 2

- $a + b = 0$
- $2a + 2b = 0$

Singular  
system

|   |   |
|---|---|
|  |  |
| 1   | 1   |
| 2   | 2   |



(Infinitely many solutions)

# Systems of equations as matrices

## System 1

- $a + b = 0$
- $a + 2b = 0$

**Non-singular  
system**

|   |   |
|---|---|
|  |  |
| 1   | 1   |
| 1   | 2   |



**Non-singular  
matrix**

**(Unique solution)**

## System 2

- $a + b = 0$
- $2a + 2b = 0$

**Singular  
system**

|   |   |
|---|---|
|  |  |
| 1   | 1   |
| 2   | 2   |

**Singular  
matrix**

**(Infinitely many solutions)**



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# System of Linear Equations

---



**Linear dependence and  
independence**



# Linear dependence between rows



Non-singular

- $a + b = 0$
- $a + 2b = 0$

|   |   |
|---|---|
|  |  |
| 1   | 1   |
| 1   | 2   |

Singular system



- $a + b = 0$
- $2a + 2b = 0$

|   |   |
|---|---|
|  |  |
| 1   | 1   |
| 2   | 2   |

# Linear dependence between rows

Non-singular



- $a + b = 0$
- $a + 2b = 0$

|   |   |
|---|---|
|  |  |
| 1   | 1   |
| 1   | 2   |

Singular system

- $a + b = 0$
- $2a + 2b = 0$





|   |   |
|---|---|
|  |  |
| 1   | 1   |
| 2   | 2   |

Second equation is  
a multiple of the  
first one

# Linear dependence between rows

Non-singular



- $a + b = 0$
- $a + 2b = 0$

|   |   |
|---|---|
|  |  |
| 1   | 1   |
| 1   | 2   |

Singular system

- $a + b = 0$
- $2a + 2b = 0$

Second equation is  
a multiple of the  
first one



|   |   |
|---|---|
|  |  |
| 1   | 1   |
| 2   | 2   |

Second row is a  
multiple of the first  
row

# Linear dependence between rows

Non-singular



- $a + b = 0$
- $a + 2b = 0$

|   |   |
|---|---|
|  |  |
| 1   | 1   |
| 1   | 2   |

Singular system

- $a + b = 0$
- $2a + 2b = 0$

Second equation is  
a multiple of the  
first one

|   |   |
|---|---|
|  |  |
| 1   | 1   |
| 2   | 2   |

Second row is a  
multiple of the first  
row



Rows are  
*linearly dependent*

# Linear dependence between rows

## Non-singular

- $a + b = 0$
- $a + 2b = 0$



No equation is a multiple of the other one

|   |   |
|---|---|
|  |  |
| 1   | 1   |
| 1   | 2   |

## Singular system

- $a + b = 0$
- $2a + 2b = 0$

Second equation is a multiple of the first one

|   |   |
|---|---|
|  |  |
| 1   | 1   |
| 2   | 2   |

Second row is a multiple of the first row



Rows are  
*linearly dependent*

# Linear dependence between rows

## Non-singular

- $a + b = 0$
- $a + 2b = 0$

No equation is a multiple of the other one



|   |   |
|---|---|
|  |  |
| 1   | 1   |
| 1   | 2   |

No row is a multiple of the other one

## Singular system

- $a + b = 0$
- $2a + 2b = 0$

Second equation is a multiple of the first one

|   |   |
|---|---|
|  |  |
| 1   | 1   |
| 2   | 2   |

Second row is a multiple of the first row


Rows are  
*linearly dependent*

# Linear dependence between rows

## Non-singular

- $a + b = 0$
- $a + 2b = 0$

No equation is a multiple of the other one



|   |   |
|---|---|
| 1 | 1 |
| 1 | 2 |

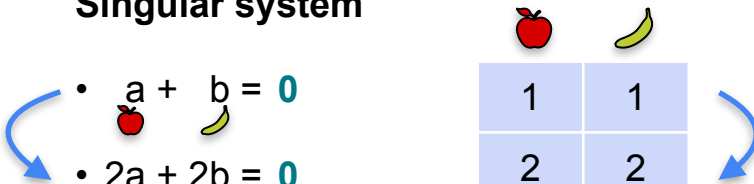
No row is a multiple of the other one

Rows are  
*linearly independent*

## Singular system

- $a + b = 0$
- $2a + 2b = 0$

Second equation is a multiple of the first one



|   |   |
|---|---|
| 1 | 1 |
| 2 | 2 |

Second row is a multiple of the first row

Rows are  
*linearly dependent*



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# System of Linear Equations



---

## **The determinant**





# Linear dependence between rows

Non-singular matrix



|   |   |
|---|---|
|  |  |
| 1   | 1   |
| 1   | 2   |

Singular matrix



|   |   |
|---|---|
|  |  |
| 1   | 1   |
| 2   | 2   |


# Linear dependence between rows

Non-singular matrix

|  |  |
|---|---|
| 1   | 1   |
| 1   | 2   |



Singular matrix

|  |  |
|---|---|
| 1   | 1   |
| 2   | 2   |






# Linear dependence between rows

Non-singular matrix

|  |  |
|---|---|
| 1   | 1   |
| 1   | 2   |

Singular matrix



|  |  |
|---|---|
| 1   | 1   |
| 2   | 2   |



|   |   |
|---|---|
| 1 | 1 |
|---|---|


# Linear dependence between rows

Non-singular matrix

|   |   |
|---|---|
|  |  |
| 1   | 1   |
| 1   | 2   |

Singular matrix

|   |   |
|---|---|
|  |  |
| 1   | 1   |
| 2   | 2   |





|   |   |
|---|---|
| 1 | 1 |
|---|---|



 $\times 2 =$


# Linear dependence between rows

Non-singular matrix

|   |   |
|---|---|
|  |  |
| 1   | 1   |
| 1   | 2   |

Singular matrix



|   |   |
|---|---|
|  |  |
| 1   | 1   |
| 2   | 2   |





|   |   |     |   |   |   |
|---|---|-----|---|---|---|
| 1 | 1 | x 2 | = | 2 | 2 |
|---|---|-----|---|---|---|


# Linear dependence between rows

Non-singular matrix

|   |   |
|---|---|
|  |  |
| 1   | 1   |
| 1   | 2   |

Singular matrix

|   |   |
|---|---|
|  |  |
| 1   | 1   |
| 2   | 2   |






$$\begin{bmatrix} 1 & 1 \end{bmatrix} \times 2 = \begin{bmatrix} 2 & 2 \end{bmatrix}$$

Rows linearly dependent



# Linear dependence between rows


Non-singular matrix

|  |  |
|---|---|
| 1   | 1   |
| 1   | 2   |



Singular matrix

|  |  |
|---|---|
| 1   | 1   |
| 2   | 2   |






|   |   |     |   |   |   |
|---|---|-----|---|---|---|
| 1 | 1 | x 2 | = | 2 | 2 |
|---|---|-----|---|---|---|

Rows linearly dependent

# Linear dependence between rows



Non-singular matrix


|   |   |
|---|---|
|  |  |
| 1   | 1   |
| 1   | 2   |



|   |   |
|---|---|
| 1 | 1 |
|---|---|

Singular matrix

|   |   |
|---|---|
|  |  |
| 1   | 1   |
| 2   | 2   |





|   |   |     |   |   |   |
|---|---|-----|---|---|---|
| 1 | 1 | x 2 | = | 2 | 2 |
|---|---|-----|---|---|---|


Rows linearly dependent



# Linear dependence between rows



Non-singular matrix


|   |   |
|---|---|
|  |  |
| 1   | 1   |
| 1   | 2   |



$$\begin{bmatrix} 1 & 1 \end{bmatrix} \times ? =$$

Singular matrix

|   |   |
|---|---|
|  |  |
| 1   | 1   |
| 2   | 2   |






$$\begin{bmatrix} 1 & 1 \end{bmatrix} \times 2 = \begin{bmatrix} 2 & 2 \end{bmatrix}$$

Rows linearly dependent

# Linear dependence between rows



Non-singular matrix


|   |   |
|---|---|
|  |  |
| 1   | 1   |
| 1   | 2   |



$$\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} \times ? = \begin{bmatrix} 1 & 2 \end{bmatrix}$$

Singular matrix

|   |   |
|---|---|
|  |  |
| 1   | 1   |
| 2   | 2   |






$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} \times 2 = \begin{bmatrix} 2 & 2 \end{bmatrix}$$

Rows linearly dependent

# Linear dependence between rows

Non-singular matrix



|   |   |
|---|---|
|  |  |
| 1   | 1   |
| 1   | 2   |




$$\begin{bmatrix} 1 & 1 \end{bmatrix} \times ? = \begin{bmatrix} 1 & 2 \end{bmatrix}$$

Rows linearly independent

Singular matrix

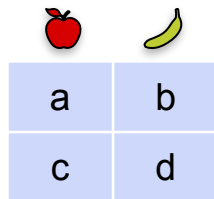
|   |   |
|---|---|
|  |  |
| 1   | 1   |
| 2   | 2   |





$$\begin{bmatrix} 1 & 1 \end{bmatrix} \times 2 = \begin{bmatrix} 2 & 2 \end{bmatrix}$$

Rows linearly dependent

# Determinant

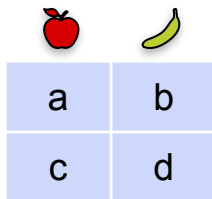


|   |   |
|---|---|
|  |  |
| a   | b   |
| c   | d   |

**Matrix is singular if**

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} * k = \begin{vmatrix} c & d \end{vmatrix}$$

# Determinant



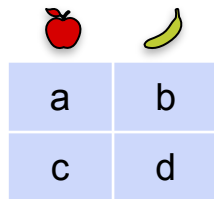
|   |   |
|---|---|
| a | b |
| c | d |

$$ak = c$$

**Matrix is singular if**

|   |   |     |   |   |   |
|---|---|-----|---|---|---|
| a | b | * k | = | c | d |
|---|---|-----|---|---|---|

# Determinant



|   |   |
|---|---|
| a | b |
| c | d |



$$ak = c$$

$$bk = d$$

**Matrix is singular if**

|   |   |     |   |   |   |
|---|---|-----|---|---|---|
| a | b | * k | = | c | d |
|---|---|-----|---|---|---|

# Determinant

|   |   |
|---|---|
|  |  |
| a   | b   |
| c   | d   |

$$ak = c$$



$$bk = d$$

$$\frac{c}{a} = \frac{d}{b} = k$$

**Matrix is singular if**

|   |   |     |   |   |   |
|---|---|-----|---|---|---|
| a | b | * k | = | c | d |
|---|---|-----|---|---|---|

# Determinant

|   |   |
|---|---|
|  |  |
| a   | b   |
| c   | d   |

**Matrix is singular if**

|   |   |     |   |   |   |
|---|---|-----|---|---|---|
| a | b | * k | = | c | d |
|---|---|-----|---|---|---|

$$ak = c$$



$$bk = d$$

$$\frac{c}{a} = \frac{d}{b} = k$$

$$ad = bc$$



# Determinant

|   |   |
|---|---|
|  |  |
| a   | b   |
| c   | d   |

**Matrix is singular if**

|   |   |     |   |   |   |
|---|---|-----|---|---|---|
| a | b | * k | = | c | d |
|---|---|-----|---|---|---|

$$ak = c$$

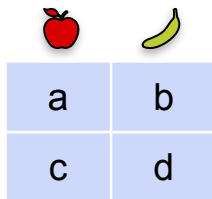
$$bk = d$$

$$\frac{c}{a} = \frac{d}{b} = k$$

$$ad = bc$$

$$ad - bc = 0$$

# Determinant



|   |   |
|---|---|
| a | b |
| c | d |

Matrix is singular if

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} * k = \begin{bmatrix} c & d \end{bmatrix}$$


$$ak = c$$

$$bk = d$$



$$\frac{c}{a} = \frac{d}{b} = k$$

$$ad = bc$$

Determinant


$$ad - bc = 0$$

# Determinant

|   |   |
|---|---|
|  |  |
| a   | b   |
| c   | d   |

$$\text{Determinant} = ad - bc$$

$$ak = c$$

$$bk = d$$

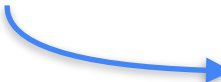
$$\frac{c}{a} = \frac{d}{b} = k$$

Matrix is singular if

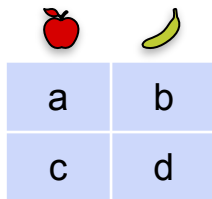
|   |   |     |   |   |   |
|---|---|-----|---|---|---|
| a | b | * k | = | c | d |
|---|---|-----|---|---|---|

Determinant

$$ad = bc$$

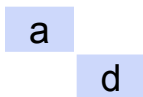

$$ad - bc = 0$$

# Determinant



|   |   |
|---|---|
| a | b |
| c | d |

$$\text{Determinant} = ad - bc$$



$$ak = c$$

$$bk = d$$

$$\frac{c}{a} = \frac{d}{b} = k$$

Matrix is singular if



$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} * k = \begin{bmatrix} c & d \end{bmatrix}$$

Determinant

$$ad = bc$$


$$ad - bc = 0$$

# Determinant

|   |   |
|---|---|
|  |  |
| a   | b   |
| c   | d   |

$$\text{Determinant} = ad - bc$$

$$\begin{array}{c} a \\ - \\ d \end{array}$$

$$ak = c$$

$$bk = d$$

$$\frac{c}{a} = \frac{d}{b} = k$$

Matrix is singular if

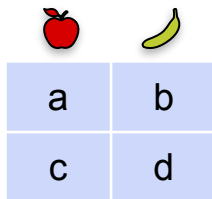

$$\begin{array}{|c|c|} \hline a & b \\ \hline \end{array} * k = \begin{array}{|c|c|} \hline c & d \\ \hline \end{array}$$

Determinant

$$ad = bc$$

$$ad - bc = 0$$

# Determinant

|   |   |
|---|---|
|  |  |
| a   | b   |
| c   | d   |

$$\text{Determinant} = ad - bc$$

$$\begin{array}{c} a \\ d \end{array} - \begin{array}{c} b \\ c \end{array}$$

$$ak = c$$

$$bk = d$$

$$\frac{c}{a} = \frac{d}{b} = k$$

Matrix is singular if

$$\begin{array}{|c|c|} \hline a & b \\ \hline \end{array} * k = \begin{array}{|c|c|} \hline c & d \\ \hline \end{array}$$



Determinant

$$ad = bc$$



$$ad - bc = 0$$

# Determinant

Non-singular matrix



|  |  |
|---|---|
| 1   | 1   |
| 1   | 2   |

Singular matrix

|  |  |
|---|---|
| 1   | 1   |
| 2   | 2   |

# Determinant



Non-singular matrix

|  |  |
|---|---|
| 1   | 1   |
| 1   | 2   |

Determinant

$$\begin{array}{ccccc} 1 & & & & 1 \\ & 2 & & & \\ & & 1 & & \end{array}$$



Singular matrix

|  |  |
|---|---|
| 1   | 1   |
| 2   | 2   |



# Determinant

Non-singular matrix



|  |  |
|---|---|
| 1   | 1   |
| 1   | 2   |

Determinant

|   |   |   |   |   |
|---|---|---|---|---|
| 1 |   | - |   | 1 |
|   | 2 |   | 1 |   |



$$1 \cdot 2 - 1 \cdot 1 = 1$$

Singular matrix

|  |  |
|---|---|
| 1   | 1   |
| 2   | 2   |

# Determinant

Non-singular matrix



|   |   |
|---|---|
|  |  |
| 1   | 1   |
| 1   | 2   |

Determinant

|   |   |   |   |   |
|---|---|---|---|---|
| 1 |   | - |   | 1 |
|   | 2 |   | 1 |   |

$$1 \cdot 2 - 1 \cdot 1 = 1$$

Singular matrix



|   |   |
|---|---|
|  |  |
| 1   | 1   |
| 2   | 2   |

Determinant

|   |   |   |   |   |
|---|---|---|---|---|
| 1 |   | - |   | 1 |
|   | 2 |   | 2 |   |

# Determinant

Non-singular matrix



|   |   |
|---|---|
|  |  |
| 1   | 1   |
| 1   | 2   |

Determinant

|   |   |   |   |   |
|---|---|---|---|---|
| 1 |   | - |   | 1 |
|   | 2 |   | 1 |   |

$$1 \cdot 2 - 1 \cdot 1 = 1$$

Singular matrix

|   |   |
|---|---|
|  |  |
| 1   | 1   |
| 2   | 2   |



Determinant

|   |   |   |   |   |
|---|---|---|---|---|
| 1 |   | - |   | 1 |
|   | 2 |   | 2 |   |

$$1 \cdot 2 - 2 \cdot 1 = 0$$

# Determinant

Non-singular matrix



|   |   |
|---|---|
|  |  |
| 1   | 1   |
| 1   | 2   |

Determinant

|   |   |   |   |   |
|---|---|---|---|---|
| 1 |   | - |   | 1 |
|   | 2 |   | 1 |   |

$$1 \cdot 2 - 1 \cdot 1 = 1$$

Singular matrix

|   |   |
|---|---|
|  |  |
| 1   | 1   |
| 2   | 2   |



Determinant

|   |   |   |   |   |
|---|---|---|---|---|
| 1 |   | - |   | 1 |
|   | 2 |   | 2 |   |

$$1 \cdot 2 - 2 \cdot 1 = 0$$

# Determinant

Non-singular matrix



|   |   |
|---|---|
|  |  |
| 1   | 1   |
| 1   | 2   |

Determinant

$$\begin{array}{ccccc} 1 & & & & 1 \\ & 2 & & & \\ & & 1 & & \end{array}$$

$$1 \cdot 2 - 1 \cdot 1 = 1$$

Singular matrix

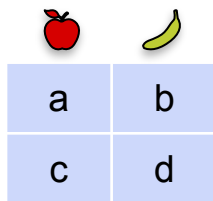

|   |   |
|---|---|
|  |  |
| 1   | 1   |
| 2   | 2   |

Determinant

$$\begin{array}{ccccc} 1 & & & & 1 \\ & 2 & & & \\ & & 2 & & \end{array}$$

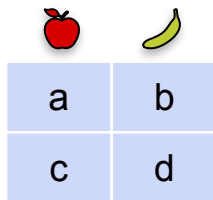
$$1 \cdot 2 - 2 \cdot 1 = 0$$

# Determinant and singularity

|   |   |
|---|---|
|  |  |
| a   | b   |
| c   | d   |

$$ad - bc$$

# Determinant and singularity



|   |   |
|---|---|
| a | b |
| c | d |

**Matrix is singular**



$$ad - bc$$

**Determinant is zero**

# Quiz: Determinant

**Problem 1:** Find the determinant of the following matrices

**Matrix 1**

|    |   |
|----|---|
| 5  | 1 |
| -1 | 3 |

**Matrix 2**

|    |    |
|----|----|
| 2  | -1 |
| -6 | 3  |

**Problem 2:** Are these matrices singular or non-singular?



# Solutions: Determinant

**Matrix 1:**  $\det = 5 \cdot 3 - 1 \cdot (-1) = 15 + 1 = 16$

|    |   |
|----|---|
| 5  | 1 |
| -1 | 3 |

**Non-singular**

**Matrix 2:**  $\det = 2 \cdot 3 - (-1) \cdot (-6) = 6 - 6 = 0$

|    |    |
|----|----|
| 2  | -1 |
| -6 | 3  |

**Singular**



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# System of Linear Equations

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## **System of equations (3x3)**

# Quiz: Systems of equations

**Problem 1:** You're trying to figure out the price of apples, bananas, and cherries at the store. You go three days in a row, and bring this information.

- **Day 1:** You bought an apple, a banana, and a cherry, and paid \$10.
  - **Day 2:** You bought an apple, two bananas, and a cherry, and paid \$15.
  - **Day 3:** You bought an apple, a banana, and two cherries, and paid \$12.
- How much does each fruit cost?

# Solution: Systems of equations

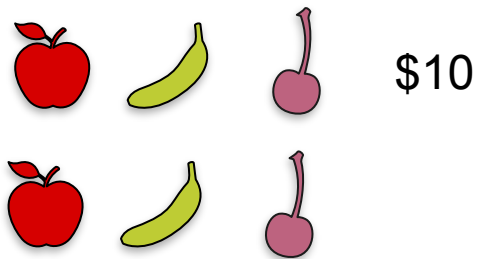
# Solution: Systems of equations



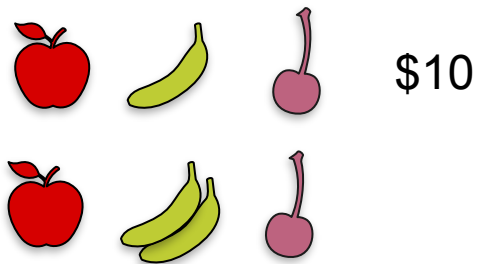
# Solution: Systems of equations



# Solution: Systems of equations

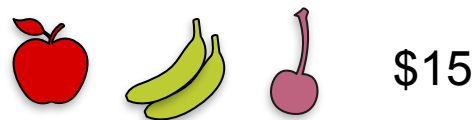


# Solution: Systems of equations

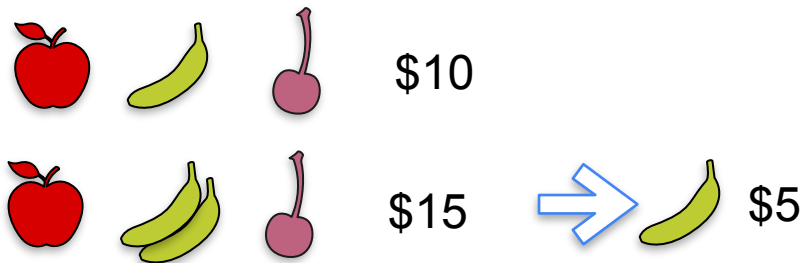




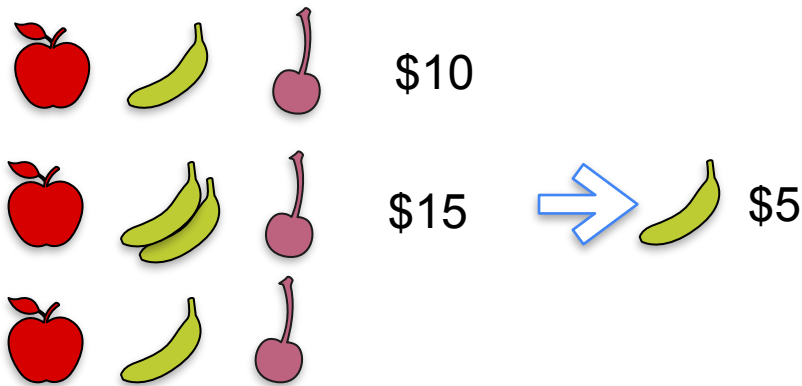
# Solution: Systems of equations



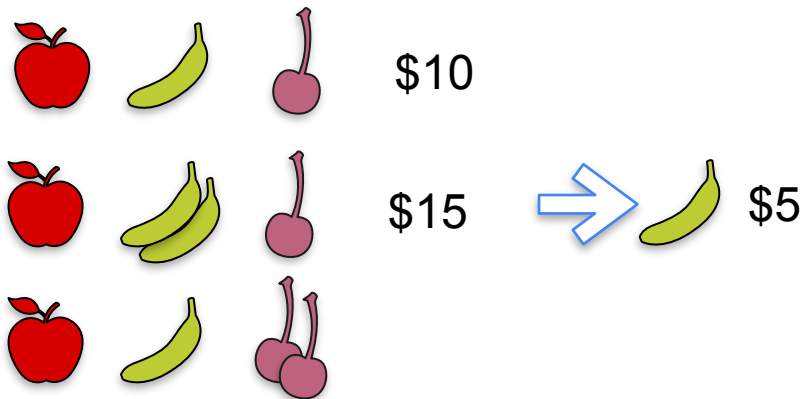
# Solution: Systems of equations



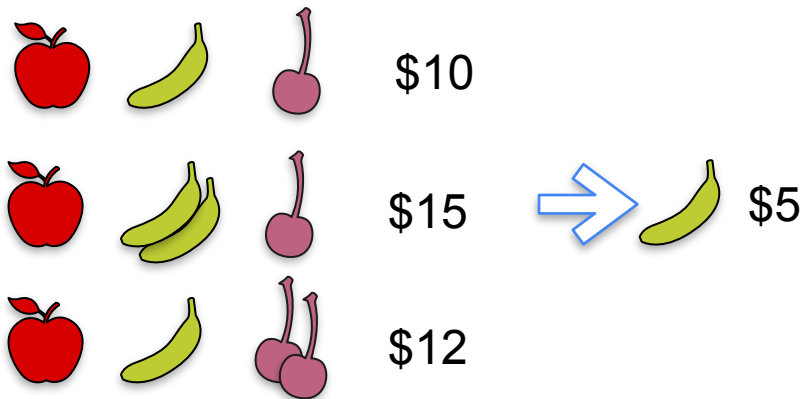
# Solution: Systems of equations



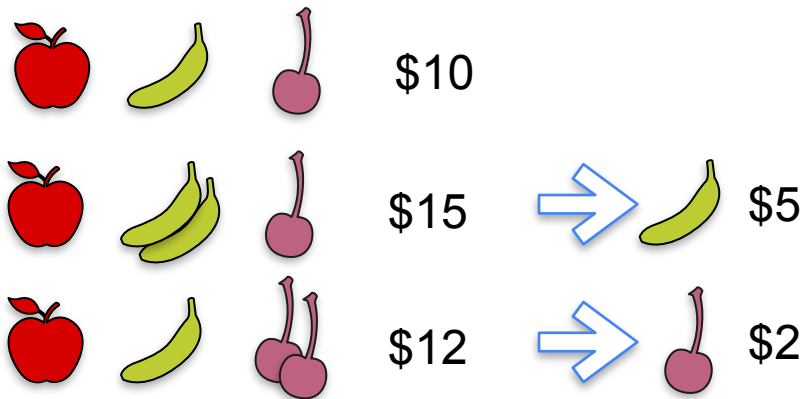
# Solution: Systems of equations



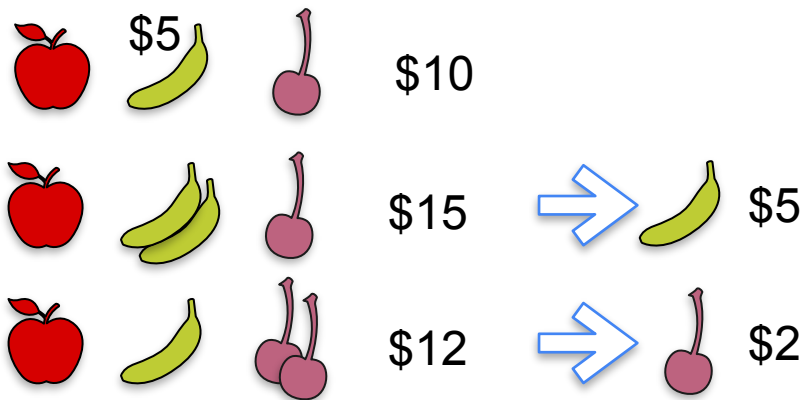
# Solution: Systems of equations



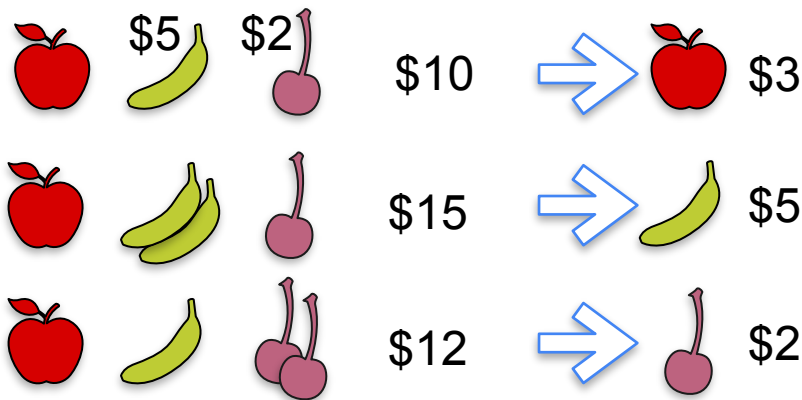
# Solution: Systems of equations



# Solution: Systems of equations

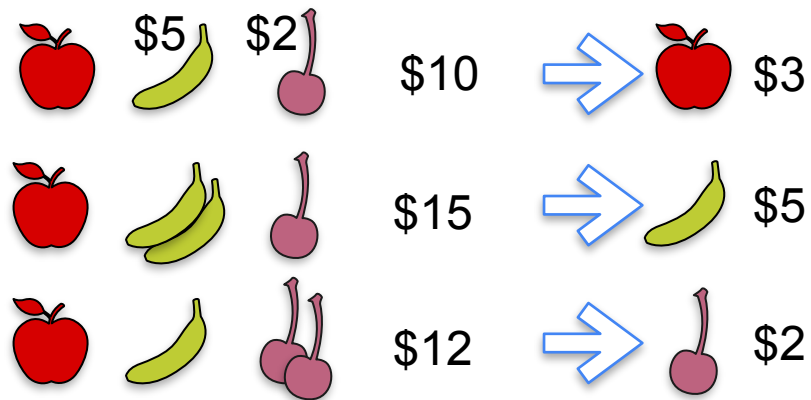


# Solution: Systems of equations





# Solution: Systems of equations



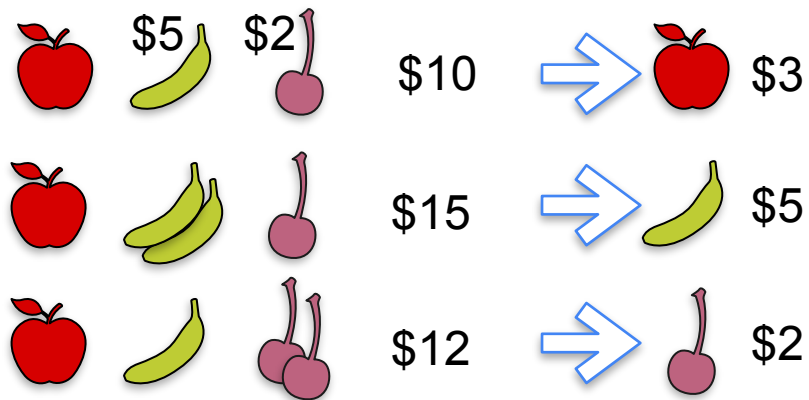
## System of equations 1

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

# Solution: Systems of equations



## System of equations 1

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

## Solution

$$a = 3$$

$$b = 5$$

$$c = 2$$

# Quiz: More systems of equations

## System 2

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 20$$

## System 3

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 18$$

## System 4

$$a + b + c = 10$$

$$2a + 2b + 2c = 20$$

$$3a + 3b + 3c = 30$$

# Solutions: More systems of equations

## System 2

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 20$$

## System 3

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 18$$

## System 4

$$a + b + c = 10$$

$$2a + 2b + 2c = 20$$

$$3a + 3b + 3c = 30$$

# Solutions: More systems of equations

## System 2

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 20$$

## System 3

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 18$$

## System 4

$$a + b + c = 10$$

$$2a + 2b + 2c = 20$$

$$3a + 3b + 3c = 30$$

**Infinitely many sols.**

# Solutions: More systems of equations

## System 2

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 20$$

**Infinitely many sols.**

$$c = 5$$

## System 3

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 18$$

## System 4

$$a + b + c = 10$$

$$2a + 2b + 2c = 20$$

$$3a + 3b + 3c = 30$$

# Solutions: More systems of equations

## System 2

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 20$$

## System 3

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 18$$

## System 4

$$a + b + c = 10$$

$$2a + 2b + 2c = 20$$

$$3a + 3b + 3c = 30$$

**Infinitely many sols.**

$$c = 5$$

$$a + b = 5$$

# Solutions: More systems of equations

## System 2

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 20$$

## System 3

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 18$$

## System 4

$$a + b + c = 10$$

$$2a + 2b + 2c = 20$$

$$3a + 3b + 3c = 30$$

**Infinitely many sols.**

$$c = 5$$

$$a + b = 5$$

$(0,5,5), (1,4,5), (2,3,5), \dots$



# Solutions: More systems of equations

## System 2

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 20$$

**Infinitely many sols.**

$$c = 5$$

$$a + b = 5$$

$(0,5,5), (1,4,5), (2,3,5), \dots$

## System 3

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 18$$

**No solutions**

## System 4

$$a + b + c = 10$$

$$2a + 2b + 2c = 20$$

$$3a + 3b + 3c = 30$$

# Solutions: More systems of equations

## System 2

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 20$$

**Infinitely many sols.**

$$c = 5$$

$$a + b = 5$$

$$(0,5,5), (1,4,5), (2,3,5), \dots$$

## System 3

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 18$$

**No solutions**

From 1st and 2nd:

$$c = 5$$

From 2nd and 3rd:

$$c = 3$$

## System 4

$$a + b + c = 10$$

$$2a + 2b + 2c = 20$$

$$3a + 3b + 3c = 30$$

# Solutions: More systems of equations

## System 2

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 20$$

**Infinitely many sols.**

$$c = 5$$

$$a + b = 5$$

$$(0,5,5), (1,4,5), (2,3,5), \dots$$

## System 3

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 18$$

**No solutions**

From 1st and 2nd:

$$c = 5$$

From 2nd and 3rd:

$$c = 3$$

## System 4

$$a + b + c = 10$$

$$2a + 2b + 2c = 20$$

$$3a + 3b + 3c = 30$$

**Infinitely many solutions**

# Solutions: More systems of equations

## System 2

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 20$$

**Infinitely many sols.**

$$c = 5$$

$$a + b = 5$$

$(0,5,5), (1,4,5), (2,3,5), \dots$

## System 3

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 18$$

**No solutions**

From 1st and 2nd:

$$c = 5$$

From 2nd and 3rd:

$$c = 3$$

## System 4

$$a + b + c = 10$$

$$2a + 2b + 2c = 20$$

$$3a + 3b + 3c = 30$$

**Infinitely many solutions**

Any 3 numbers that add to 10 work.

$(0,0,10), (2,7,1), \dots$



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# System of Linear Equations

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**Singular vs non-singular  
matrices**

# Constants don't matter for singularity

## System 1

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

## System 2

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 20$$

## System 3

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 18$$

## System 4

$$a + b + c = 10$$

$$2a + 2b + 2c = 15$$

$$3a + 3b + 3c = 20$$

# Constants don't matter for singularity

## System 1

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

## System 2

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 20$$

## System 3

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 18$$

## System 4

$$a + b + c = 10$$

$$2a + 2b + 2c = 15$$

$$3a + 3b + 3c = 20$$

Unique solution

# Constants don't matter for singularity

## System 1

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

Unique solution

## System 2

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 20$$

Infinite solutions

## System 3

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 18$$

## System 4

$$a + b + c = 10$$

$$2a + 2b + 2c = 15$$

$$3a + 3b + 3c = 20$$



# Constants don't matter for singularity

## System 1

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

Unique solution

## System 2

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 20$$

Infinite solutions

## System 3

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 18$$

No solutions

## System 4

$$a + b + c = 10$$

$$2a + 2b + 2c = 15$$

$$3a + 3b + 3c = 20$$

# Constants don't matter for singularity

## System 1

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

Unique solution

## System 2

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 20$$

Infinite solutions

## System 3

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 18$$

No solutions

## System 4

$$a + b + c = 10$$

$$2a + 2b + 2c = 15$$

$$3a + 3b + 3c = 20$$

Infinite solutions

# Constants don't matter for singularity

## System 1

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

Unique solution

Complete

## System 2

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 20$$

Infinite solutions

## System 3

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 18$$

No solutions

## System 4

$$a + b + c = 10$$

$$2a + 2b + 2c = 15$$

$$3a + 3b + 3c = 20$$

Infinite solutions

# Constants don't matter for singularity

## System 1

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

Unique solution

Complete

## System 2

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 20$$

Infinite solutions

Redundant

## System 3

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 18$$

No solutions

## System 4

$$a + b + c = 10$$

$$2a + 2b + 2c = 15$$

$$3a + 3b + 3c = 20$$

Infinite solutions

# Constants don't matter for singularity

## System 1

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

Unique solution

Complete

## System 2

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 20$$

Infinite solutions

Redundant

## System 3

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 18$$

No solutions

Contradictory

## System 4

$$a + b + c = 10$$

$$2a + 2b + 2c = 15$$

$$3a + 3b + 3c = 20$$

Infinite solutions

# Constants don't matter for singularity

## System 1

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

Unique solution

Complete

## System 2

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 20$$

Infinite solutions

Redundant

## System 3

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 18$$

No solutions

Contradictory

## System 4

$$a + b + c = 10$$

$$2a + 2b + 2c = 15$$

$$3a + 3b + 3c = 20$$

Infinite solutions

Redundant

# Constants don't matter for singularity

## System 1

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

Unique solution

Complete

Non-singular

## System 2

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 20$$

Infinite solutions

Redundant

## System 3

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 18$$

No solutions

Contradictory

## System 4

$$a + b + c = 10$$

$$2a + 2b + 2c = 15$$

$$3a + 3b + 3c = 20$$

Infinite solutions

Redundant

# Constants don't matter for singularity

## System 1

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

Unique solution

Complete

Non-singular

## System 2

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 20$$

Infinite solutions

Redundant

Singular

## System 3

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 18$$

No solutions

Contradictory

## System 4

$$a + b + c = 10$$

$$2a + 2b + 2c = 15$$

$$3a + 3b + 3c = 20$$

Infinite solutions

Redundant



# Constants don't matter for singularity

## System 1

$$\begin{aligned}a + b + c &= 10 \\a + 2b + c &= 15 \\a + b + 2c &= 12\end{aligned}$$

Unique solution

Complete

Non-singular

## System 2

$$\begin{aligned}a + b + c &= 10 \\a + b + 2c &= 15 \\a + b + 3c &= 20\end{aligned}$$

Infinite solutions

Redundant

Singular

## System 3

$$\begin{aligned}a + b + c &= 10 \\a + b + 2c &= 15 \\a + b + 3c &= 18\end{aligned}$$

No solutions

Contradictory

Singular

## System 4

$$\begin{aligned}a + b + c &= 10 \\2a + 2b + 2c &= 15 \\3a + 3b + 3c &= 20\end{aligned}$$

Infinite solutions

Redundant

# Constants don't matter for singularity

## System 1

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

Unique solution

Complete

Non-singular

## System 2

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 20$$

Infinite solutions

Redundant

Singular

## System 3

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 18$$

No solutions

Contradictory

Singular

## System 4

$$a + b + c = 10$$

$$2a + 2b + 2c = 15$$

$$3a + 3b + 3c = 20$$

Infinite solutions

Redundant

Singular

# Constants don't matter for singularity

## System 1

$$\begin{aligned}a + b + c &= 10 \\a + 2b + c &= 15 \\a + b + 2c &= 12\end{aligned}$$



$$\begin{aligned}a + b + c &= 0 \\a + 2b + c &= 0 \\a + b + 2c &= 0\end{aligned}$$

## System 2

$$\begin{aligned}a + b + c &= 10 \\a + b + 2c &= 15 \\a + b + 3c &= 20\end{aligned}$$



$$\begin{aligned}a + b + c &= 0 \\a + b + 2c &= 0 \\a + b + 3c &= 0\end{aligned}$$

## System 3

$$\begin{aligned}a + b + c &= 10 \\a + b + 2c &= 15 \\a + b + 3c &= 18\end{aligned}$$



$$\begin{aligned}a + b + c &= 0 \\a + b + 2c &= 0 \\a + b + 3c &= 0\end{aligned}$$

## System 4

$$\begin{aligned}a + b + c &= 10 \\2a + 2b + 2c &= 20 \\3a + 3b + 3c &= 30\end{aligned}$$



$$\begin{aligned}a + b + c &= 0 \\2a + 2b + 2c &= 0 \\3a + 3b + 3c &= 0\end{aligned}$$

# Constants don't matter for singularity

## System 1

$$a + b + c = 0$$

$$a + 2b + c = 0$$

$$a + b + 2c = 0$$

## System 2

$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

## System 3

$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

## System 4

$$a + b + c = 0$$

$$2a + 2b + 2c = 0$$

$$3a + 3b + 3c = 0$$

# Constants don't matter for singularity

## System 1

$$a + b + c = 0$$

$$a + 2b + c = 0$$

$$a + b + 2c = 0$$

Unique solution:

$$a = 0$$

$$b = 0$$

$$c = 0$$

## System 2

$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

## System 3

$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

## System 4

$$a + b + c = 0$$

$$2a + 2b + 2c = 0$$

$$3a + 3b + 3c = 0$$

# Constants don't matter for singularity

## System 1

$$a + b + c = 0$$

$$a + 2b + c = 0$$

$$a + b + 2c = 0$$

Unique solution:

$$a = 0$$

$$b = 0$$

$$c = 0$$

Complete

Non-singular

## System 2

$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

## System 3

$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

## System 4

$$a + b + c = 0$$

$$2a + 2b + 2c = 0$$

$$3a + 3b + 3c = 0$$

# Constants don't matter for singularity

## System 1

$$a + b + c = 0$$

$$a + 2b + c = 0$$

$$a + b + 2c = 0$$

Unique solution:

$$a = 0$$

$$b = 0$$

$$c = 0$$

Complete

Non-singular

## System 2

$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

Infinite solutions:

$$c = 0$$

$$a + b = 0$$

(i.e.,  $a = -b$ )

## System 3

$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

## System 4

$$a + b + c = 0$$

$$2a + 2b + 2c = 0$$

$$3a + 3b + 3c = 0$$

# Constants don't matter for singularity

## System 1

$$a + b + c = 0$$

$$a + 2b + c = 0$$

$$a + b + 2c = 0$$

Unique solution:

$$a = 0$$

$$b = 0$$

$$c = 0$$

Complete

Non-singular

## System 2

$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

Infinite solutions:

$$c = 0$$

$$a + b = 0$$

(i.e.,  $a = -b$ )

Redundant

Singular

## System 3

$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

## System 4

$$a + b + c = 0$$

$$2a + 2b + 2c = 0$$

$$3a + 3b + 3c = 0$$



# Constants don't matter for singularity

## System 1

$$a + b + c = 0$$

$$a + 2b + c = 0$$

$$a + b + 2c = 0$$

Unique solution:

$$a = 0$$

$$b = 0$$

$$c = 0$$

Complete

Non-singular

## System 2

$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

Infinite solutions:

$$c = 0$$

$$a + b = 0$$

(i.e.,  $a = -b$ )

Redundant

Singular

## System 3

$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

## System 4

$$a + b + c = 0$$

$$2a + 2b + 2c = 0$$

$$3a + 3b + 3c = 0$$

Infinite solutions:

$$a + b + c = 0$$

(i.e.,  $c = -a - b$ )

# Constants don't matter for singularity

## System 1

$$a + b + c = 0$$

$$a + 2b + c = 0$$

$$a + b + 2c = 0$$

Unique solution:

$$a = 0$$

$$b = 0$$

$$c = 0$$

Complete

Non-singular

## System 2

$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

Infinite solutions:

$$c = 0$$

$$a + b = 0$$

$$(i.e., a = -b)$$

Redundant

Singular

## System 3

$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

## System 4

$$a + b + c = 0$$

$$2a + 2b + 2c = 0$$

$$3a + 3b + 3c = 0$$

Infinite solutions:

$$a + b + c = 0$$

$$(i.e., c = -a - b)$$

Redundant

Singular

# Constants don't matter for singularity

## System 1

$$a + b + c = 0$$

$$a + 2b + c = 0$$

$$a + b + 2c = 0$$

## System 2

$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

## System 3

$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

## System 4

$$a + b + c = 0$$

$$2a + 2b + 2c = 0$$

$$3a + 3b + 3c = 0$$

# Constants don't matter for singularity

## System 1

$$a + b + c = 0$$

$$a + 2b + c = 0$$

$$a + b + 2c = 0$$

|   |   |   |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |

## System 2

$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

## System 3

$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

## System 4

$$a + b + c = 0$$

$$2a + 2b + 2c = 0$$

$$3a + 3b + 3c = 0$$

# Constants don't matter for singularity

## System 1

$$a + b + c = 0$$

$$a + 2b + c = 0$$

$$a + b + 2c = 0$$

|   |   |   |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |

## System 2

$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

|   |   |   |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 2 |
| 1 | 1 | 3 |

## System 3

$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

## System 4

$$a + b + c = 0$$

$$2a + 2b + 2c = 0$$

$$3a + 3b + 3c = 0$$

# Constants don't matter for singularity

## System 1

$$a + b + c = 0$$

$$a + 2b + c = 0$$

$$a + b + 2c = 0$$

|   |   |   |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |

## System 2

$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

|   |   |   |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 2 |
| 1 | 1 | 3 |

## System 3

$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

## System 4

$$a + b + c = 0$$

$$2a + 2b + 2c = 0$$

$$3a + 3b + 3c = 0$$

|   |   |   |
|---|---|---|
| 1 | 1 | 1 |
| 2 | 2 | 2 |
| 3 | 3 | 3 |

# Constants don't matter for singularity

## System 1

$$a + b + c = 0$$

$$a + 2b + c = 0$$

$$a + b + 2c = 0$$

|   |   |   |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |

**Non-singular**

## System 2

$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

|   |   |   |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 2 |
| 1 | 1 | 3 |

## System 3

$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

## System 4

$$a + b + c = 0$$

$$2a + 2b + 2c = 0$$

$$3a + 3b + 3c = 0$$

|   |   |   |
|---|---|---|
| 1 | 1 | 1 |
| 2 | 2 | 2 |
| 3 | 3 | 3 |

# Constants don't matter for singularity

## System 1

$$a + b + c = 0$$

$$a + 2b + c = 0$$

$$a + b + 2c = 0$$

|   |   |   |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |

Non-singular

## System 2

$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

|   |   |   |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 2 |
| 1 | 1 | 3 |

Singular

## System 3

$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

## System 4

$$a + b + c = 0$$

$$2a + 2b + 2c = 0$$

$$3a + 3b + 3c = 0$$

|   |   |   |
|---|---|---|
| 1 | 1 | 1 |
| 2 | 2 | 2 |
| 3 | 3 | 3 |



# Constants don't matter for singularity

## System 1

$$\begin{aligned}a + b + c &= 0 \\a + 2b + c &= 0 \\a + b + 2c &= 0\end{aligned}$$

|   |   |   |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |

Non-singular

## System 2

$$\begin{aligned}a + b + c &= 0 \\a + b + 2c &= 0 \\a + b + 3c &= 0\end{aligned}$$

|   |   |   |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 2 |
| 1 | 1 | 3 |

Singular

## System 3

$$\begin{aligned}a + b + c &= 0 \\a + b + 2c &= 0 \\a + b + 3c &= 0\end{aligned}$$

## System 4

$$\begin{aligned}a + b + c &= 0 \\2a + 2b + 2c &= 0 \\3a + 3b + 3c &= 0\end{aligned}$$

|   |   |   |
|---|---|---|
| 1 | 1 | 1 |
| 2 | 2 | 2 |
| 3 | 3 | 3 |

Singular



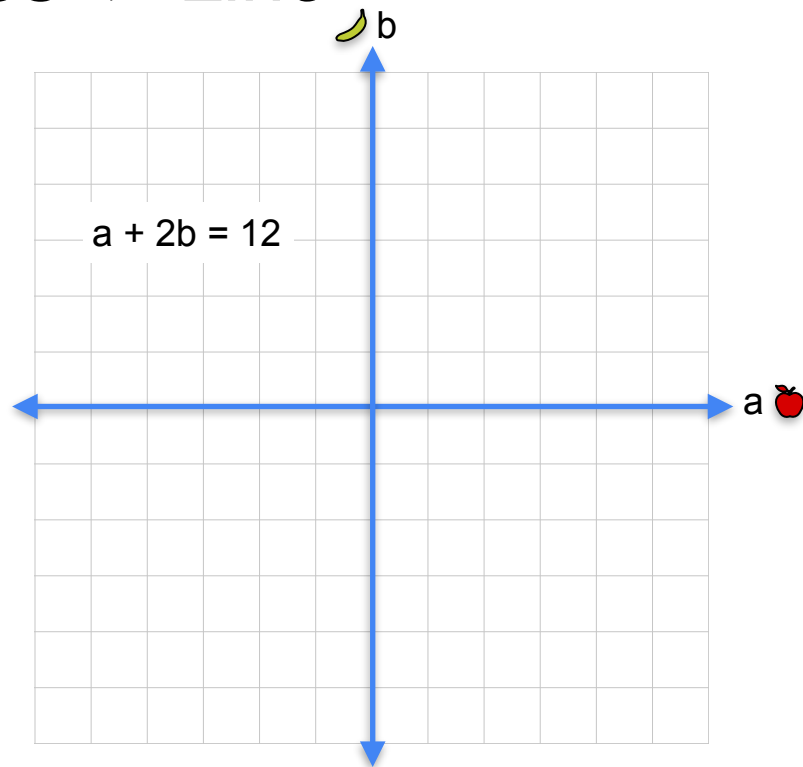
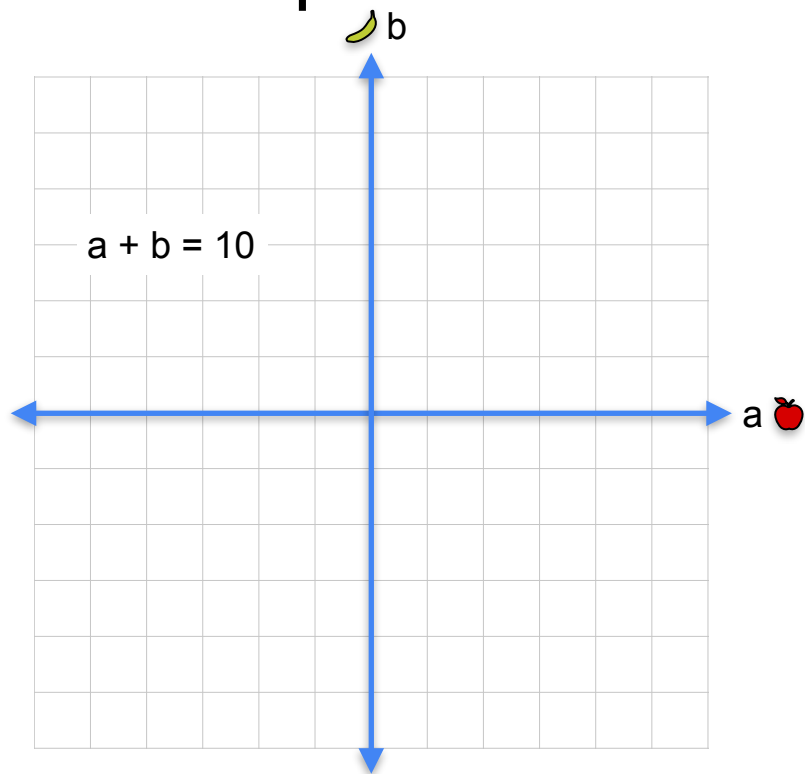
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# System of Linear Equations

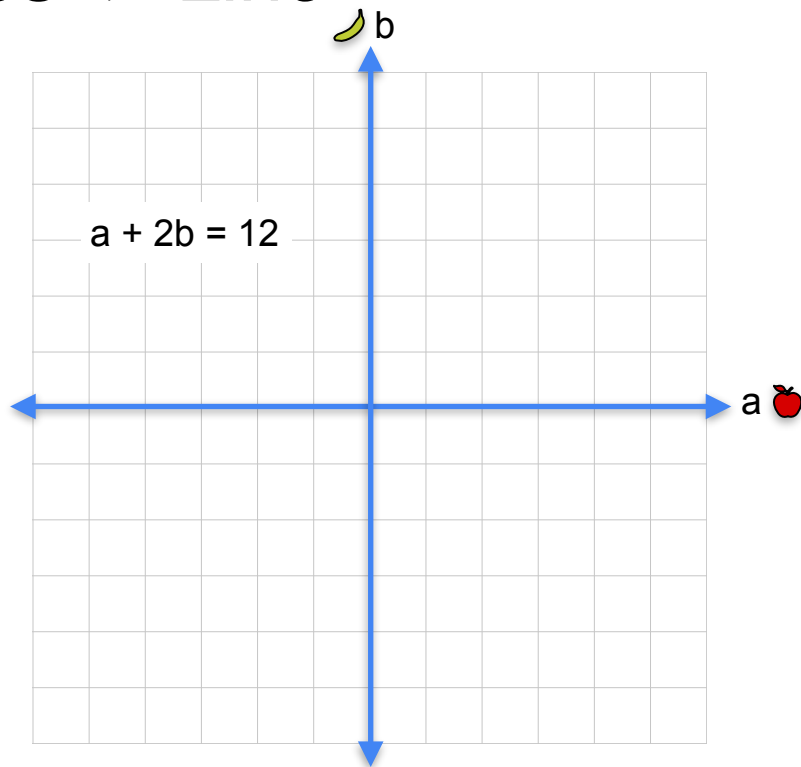
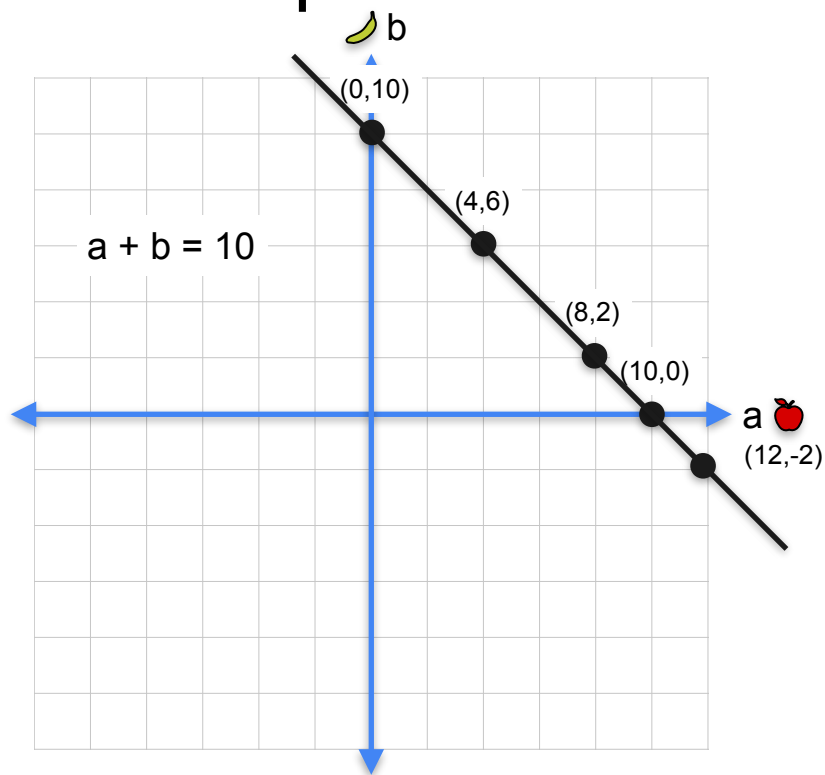
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**System of equations as  
planes (3x3)**

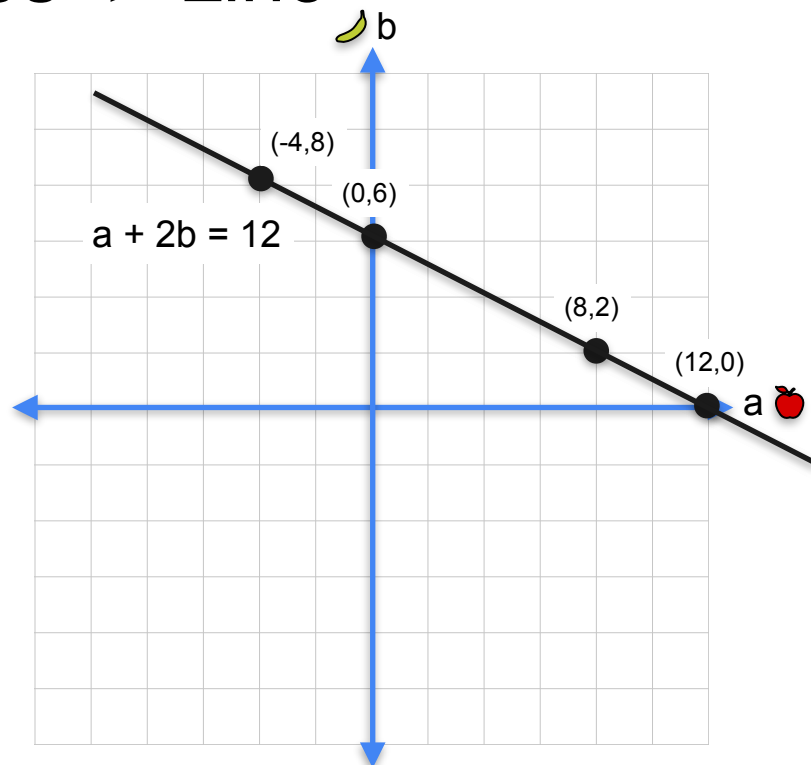
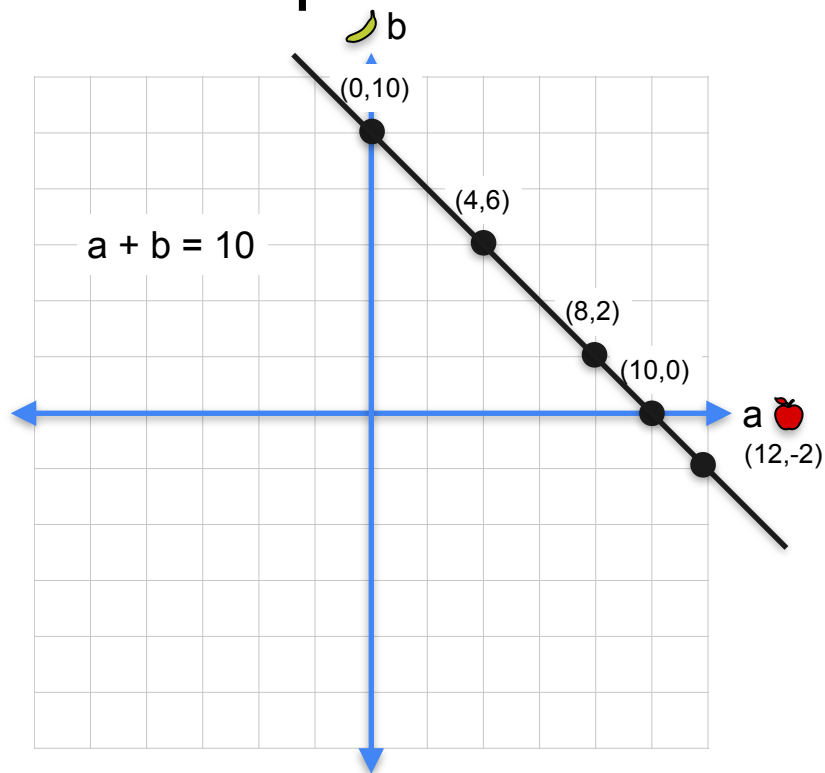
# Linear equation in 2 variables -> Line



# Linear equation in 2 variables -> Line

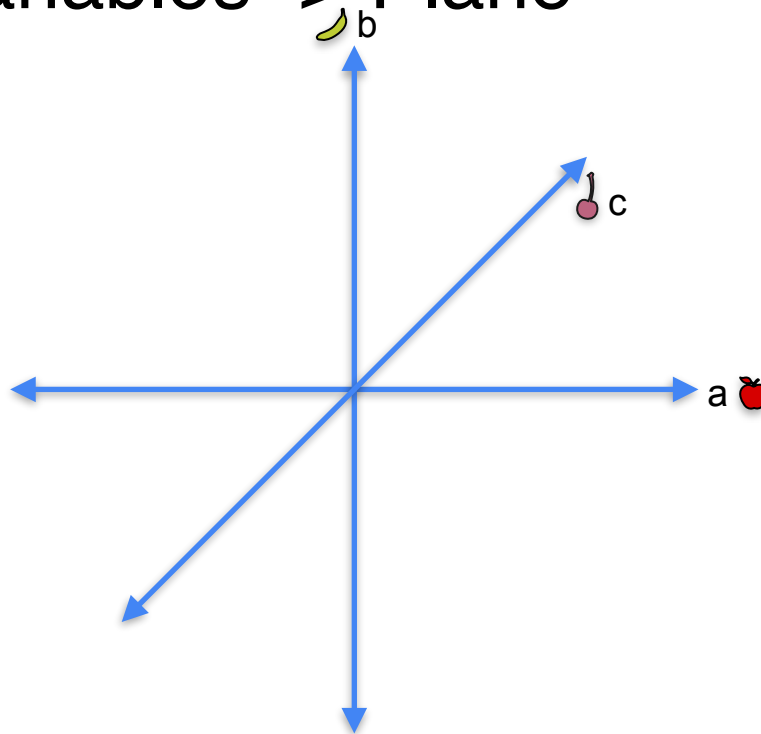


# Linear equation in 2 variables -> Line



# Linear equation in 3 variables -> Plane

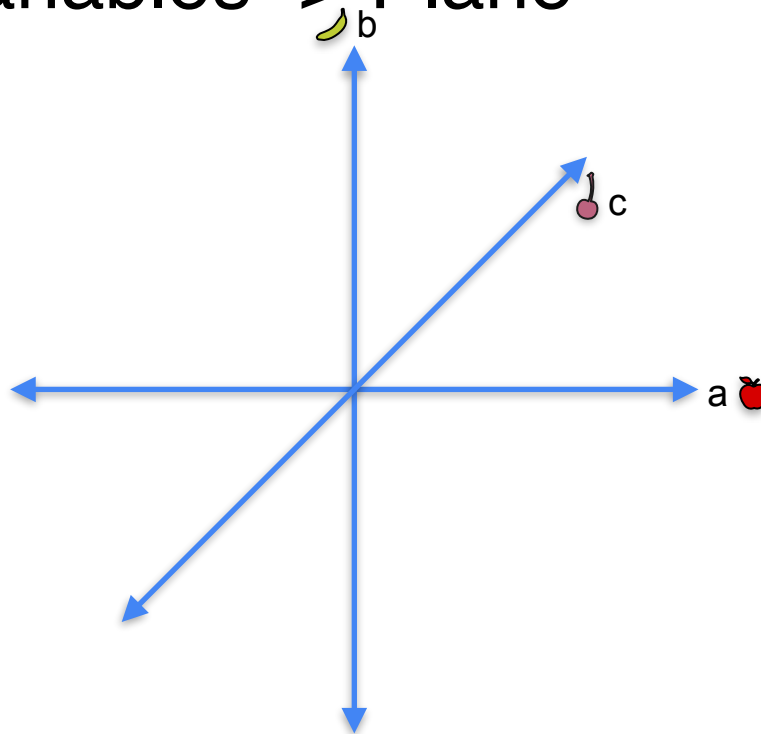
$$a + b + c = 1$$



# Linear equation in 3 variables -> Plane

$$a + b + c = 1$$

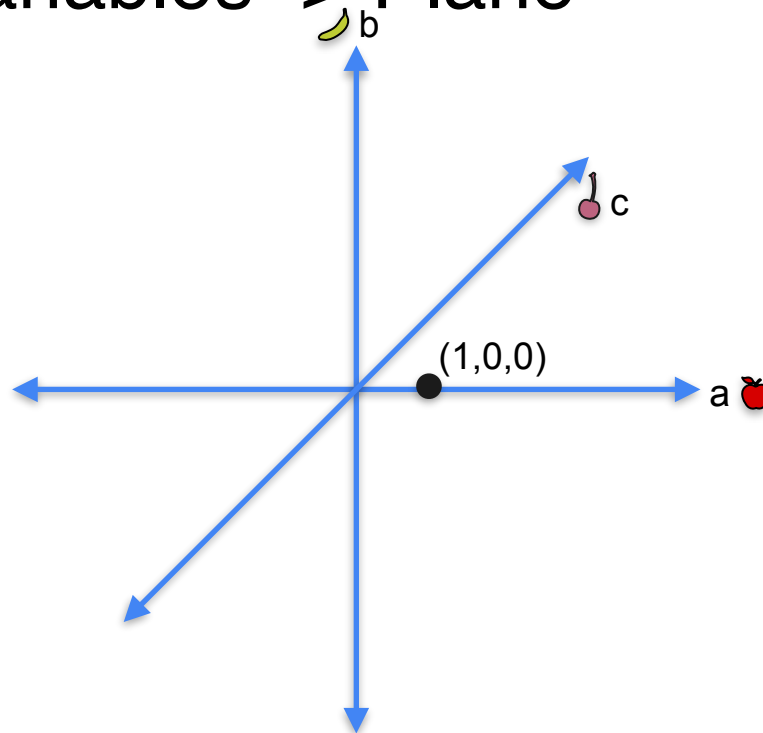
$$1 + 0 + 0 = 1$$



# Linear equation in 3 variables -> Plane

$$a + b + c = 1$$

$$1 + 0 + 0 = 1$$



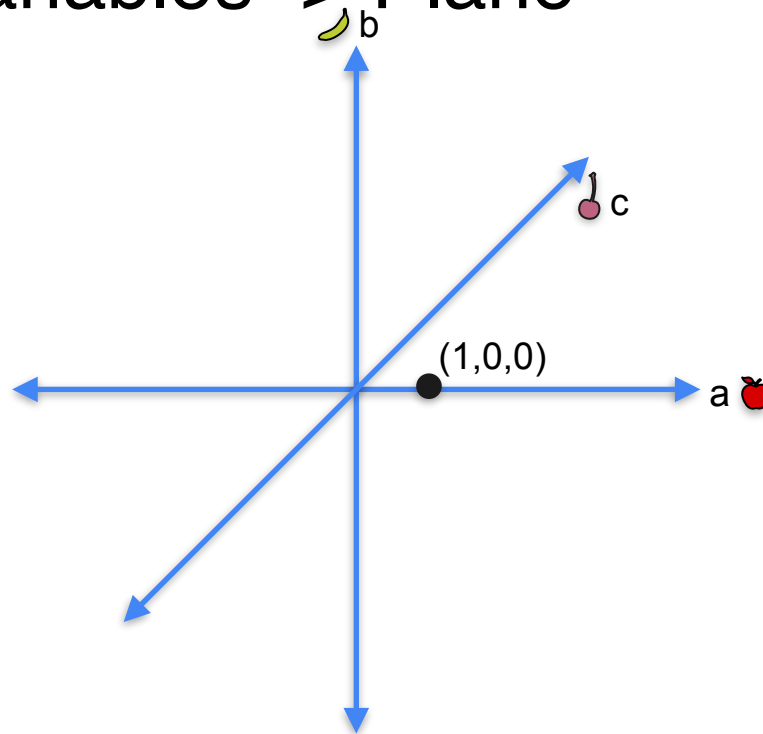


# Linear equation in 3 variables -> Plane

$$a + b + c = 1$$

$$1 + 0 + 0 = 1$$

$$0 + 1 + 0 = 1$$

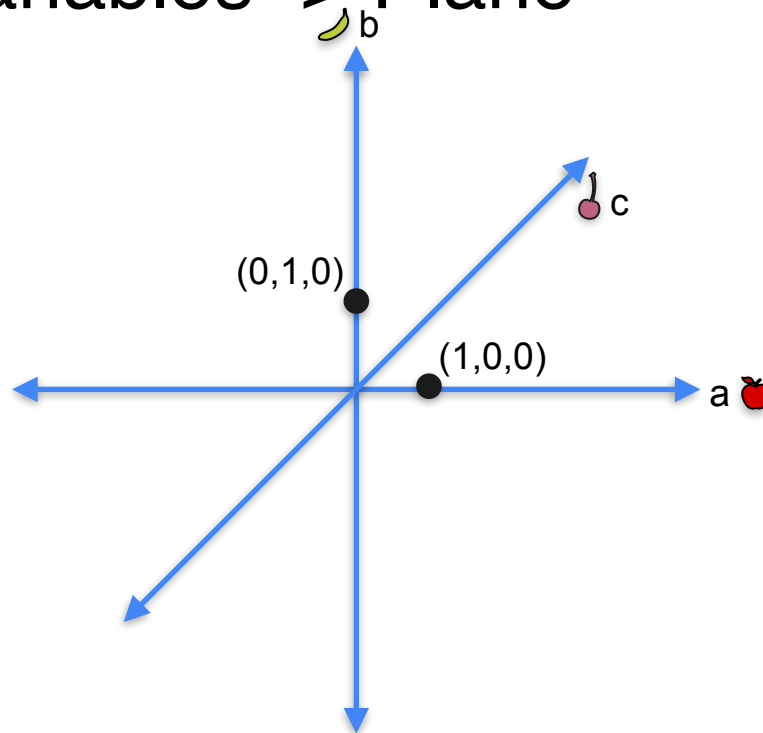


# Linear equation in 3 variables -> Plane

$$a + b + c = 1$$

$$1 + 0 + 0 = 1$$

$$0 + 1 + 0 = 1$$



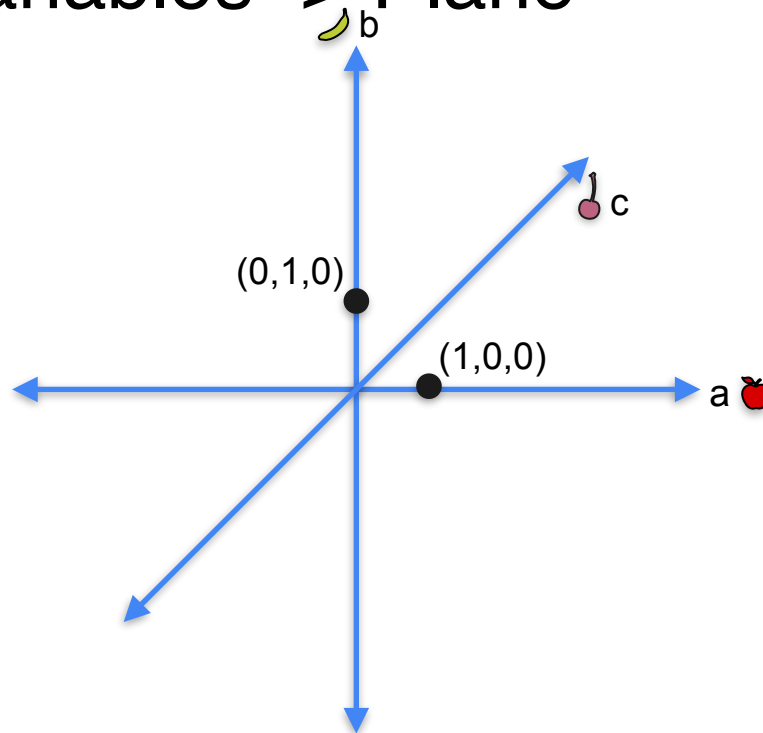
# Linear equation in 3 variables -> Plane

$$a + b + c = 1$$

$$1 + 0 + 0 = 1$$

$$0 + 1 + 0 = 1$$

$$0 + 0 + 1 = 1$$



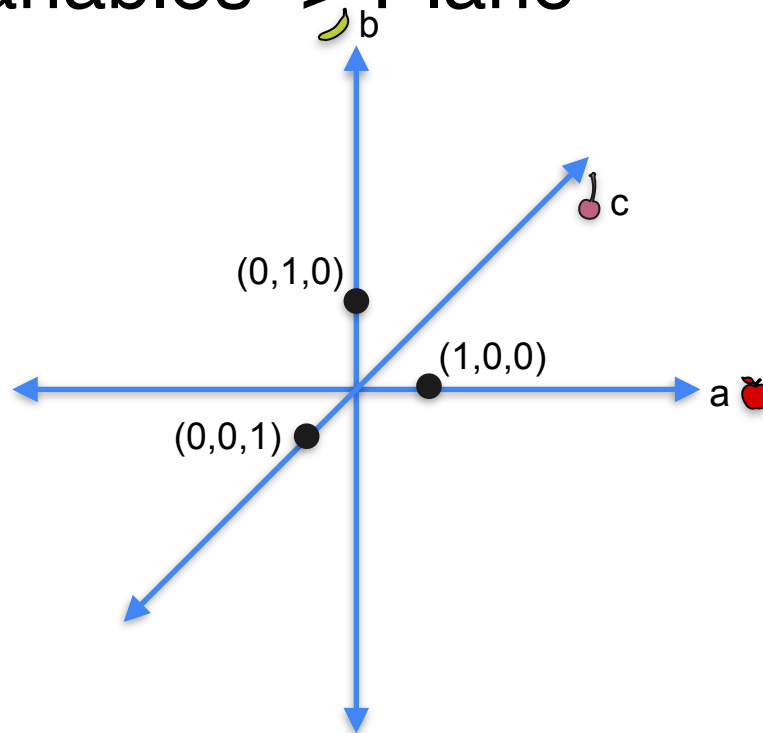
# Linear equation in 3 variables -> Plane

$$a + b + c = 1$$

$$1 + 0 + 0 = 1$$

$$0 + 1 + 0 = 1$$

$$0 + 0 + 1 = 1$$



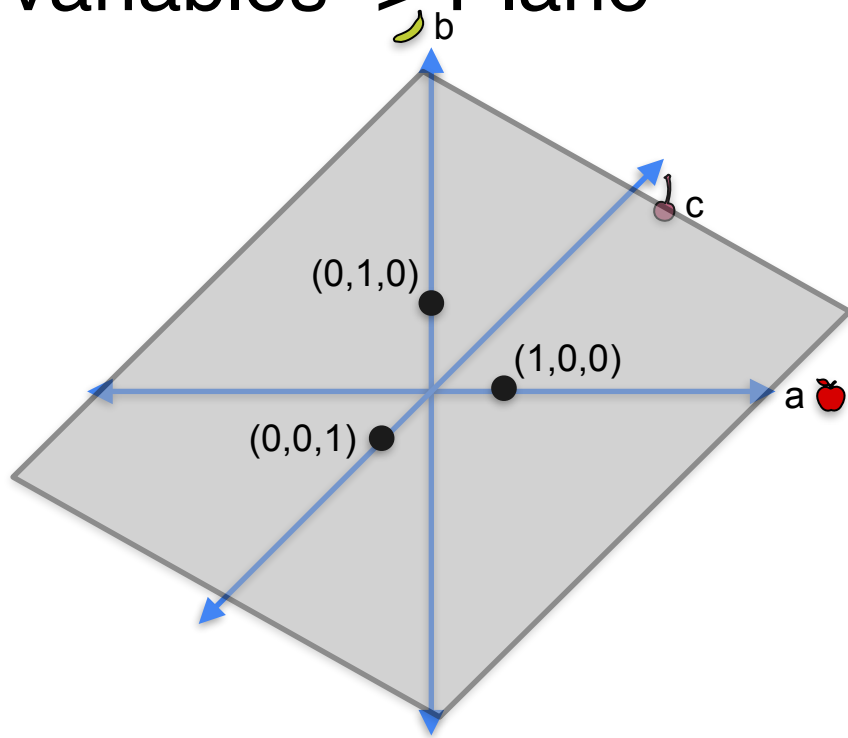
# Linear equation in 3 variables -> Plane

$$a + b + c = 1$$

$$1 + 0 + 0 = 1$$

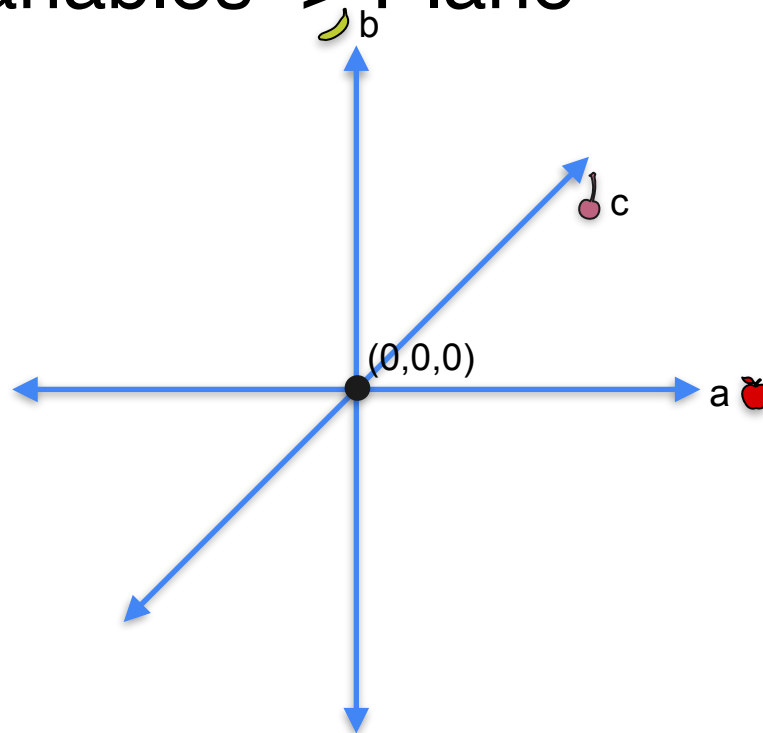
$$0 + 1 + 0 = 1$$

$$0 + 0 + 1 = 1$$



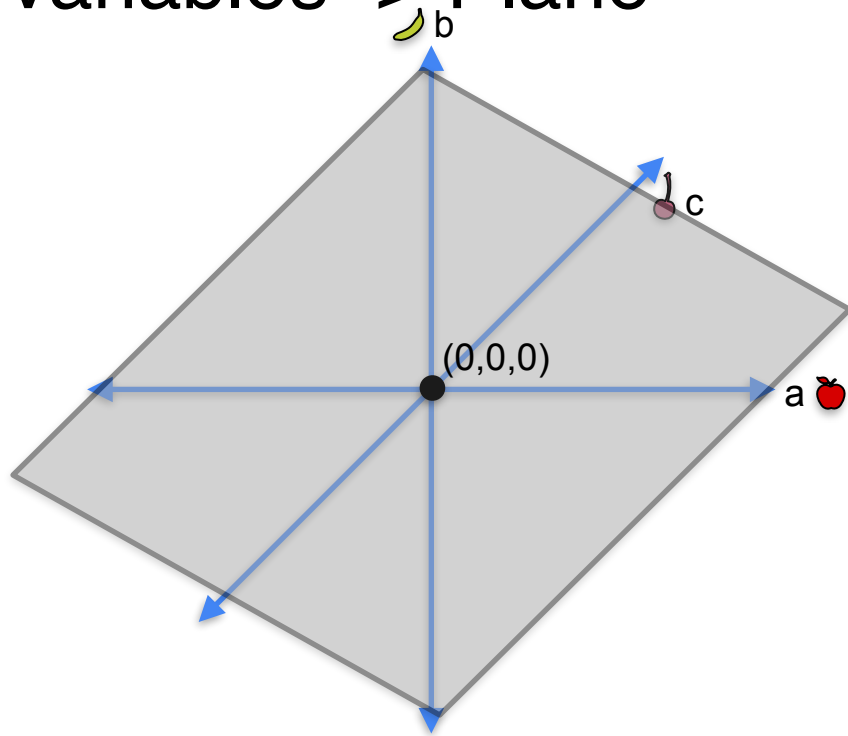
# Linear equation in 3 variables -> Plane

$$3a - 5b + 2c = 0$$



# Linear equation in 3 variables -> Plane

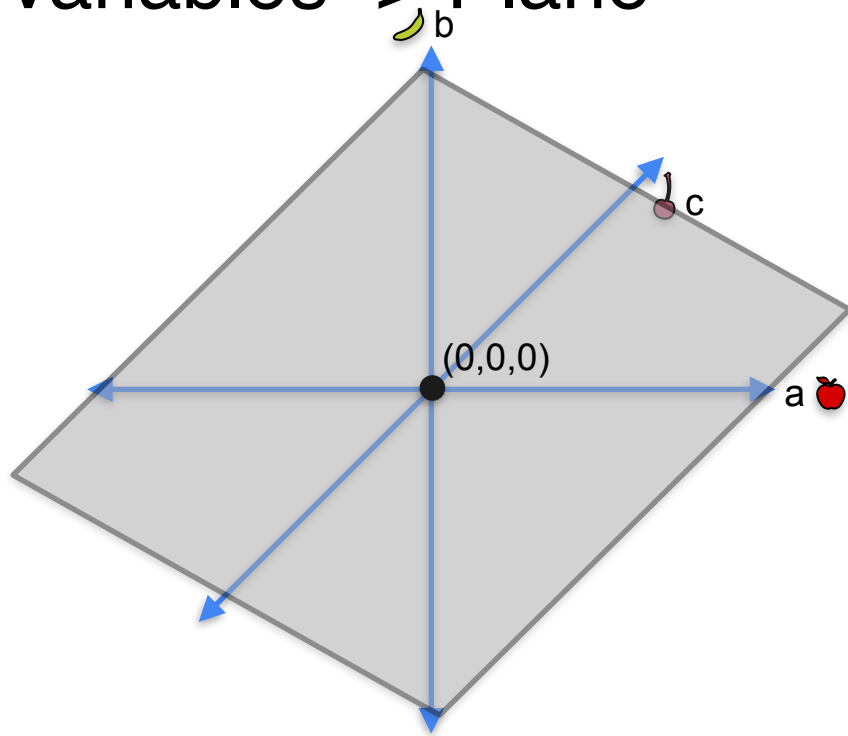
$$3a - 5b + 2c = 0$$



# Linear equation in 3 variables -> Plane

$$3a - 5b + 2c = 0$$

$$3(0) + 5(0) + 2(0) = 0$$

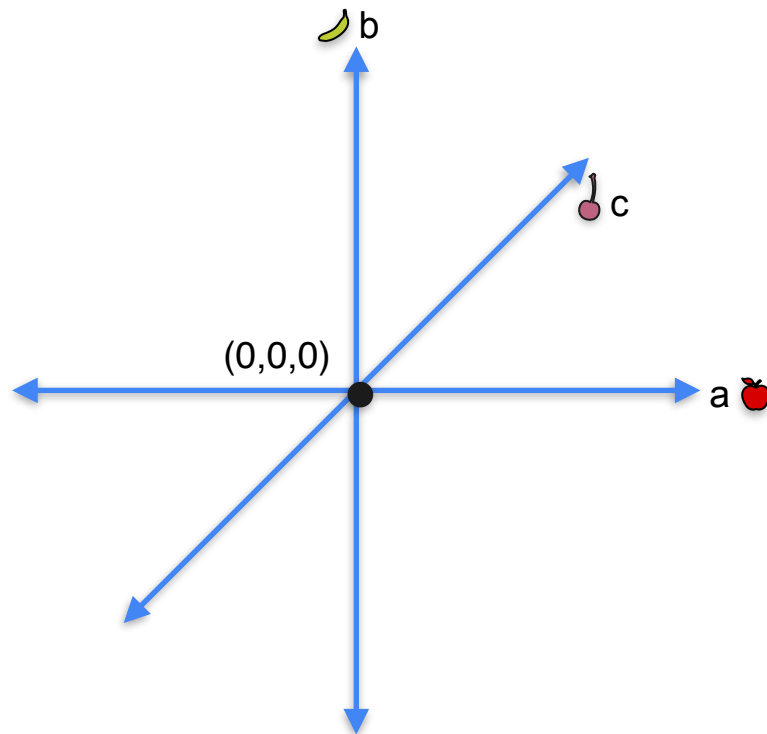




# System 1

## System 1

- $a + b + c = 0$
- $a + 2b + c = 0$
- $a + b + 2c = 0$



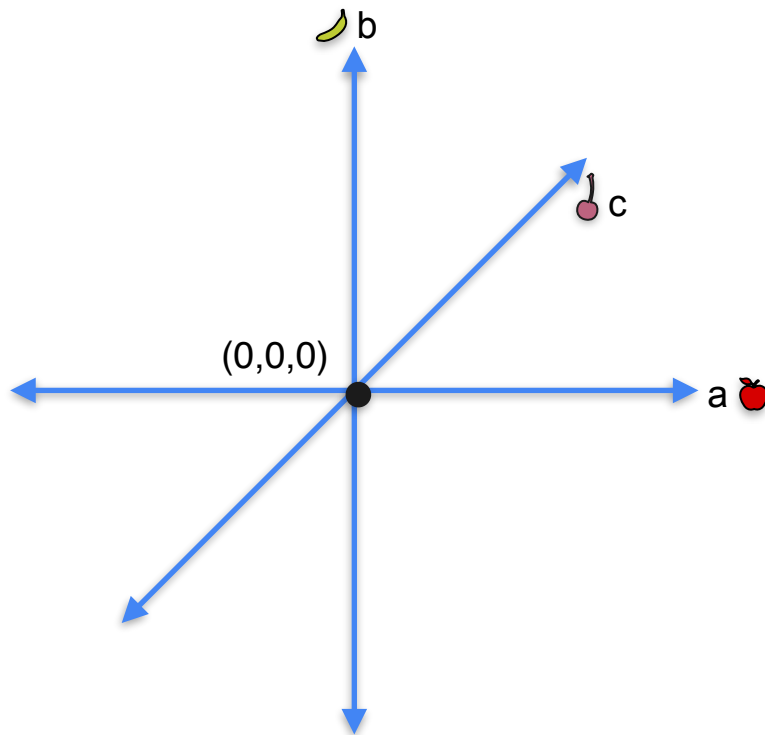
# System 1

## System 1

- $a + b + c = 0$

- $a + 2b + c = 0$

- $a + b + 2c = 0$



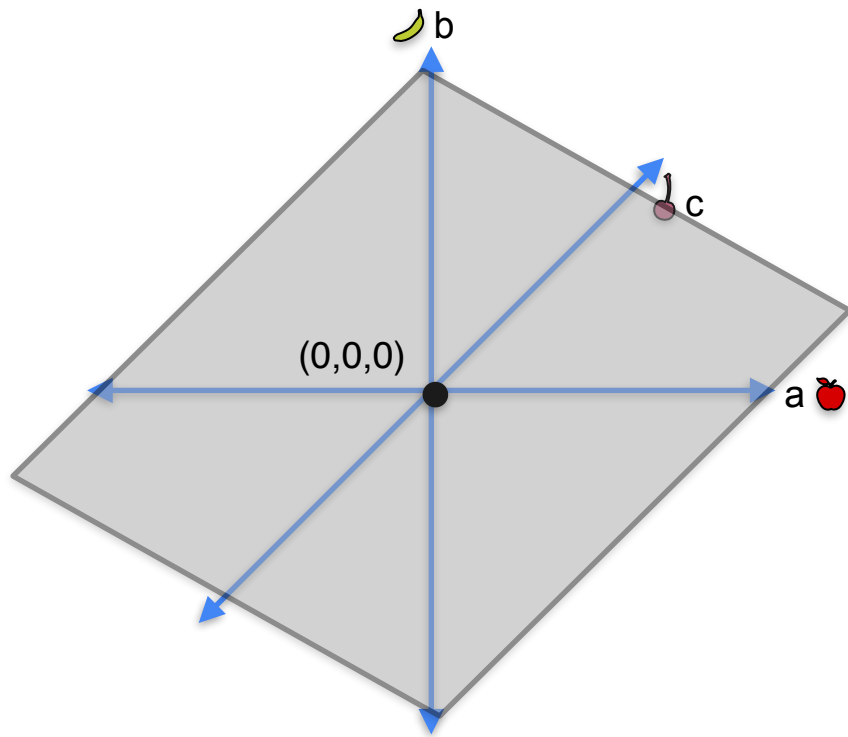
# System 1

## System 1

- $a + b + c = 0$

- $a + 2b + c = 0$

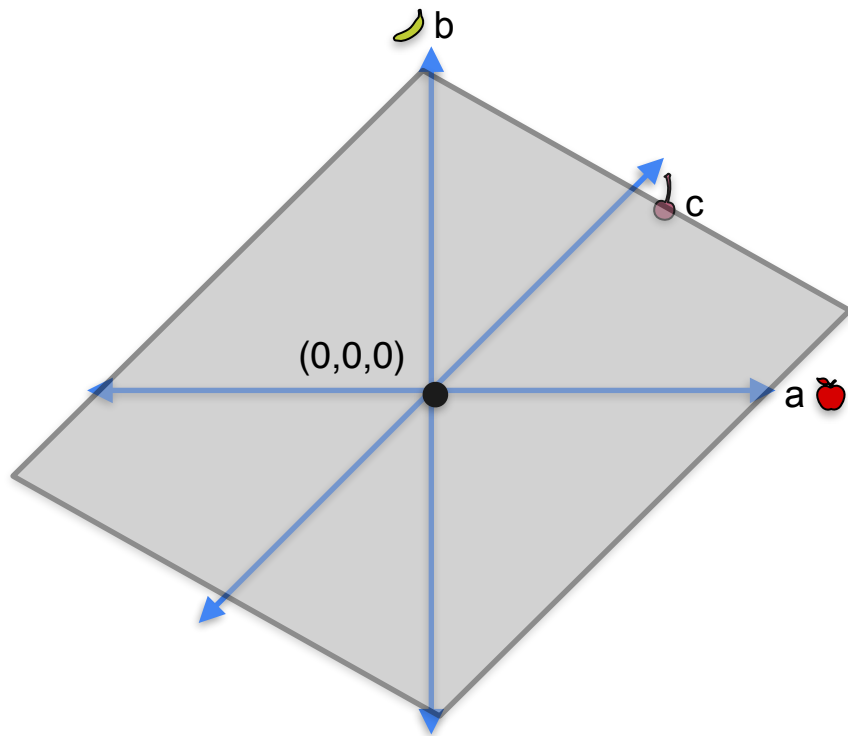
- $a + b + 2c = 0$



# System 1

## System 1

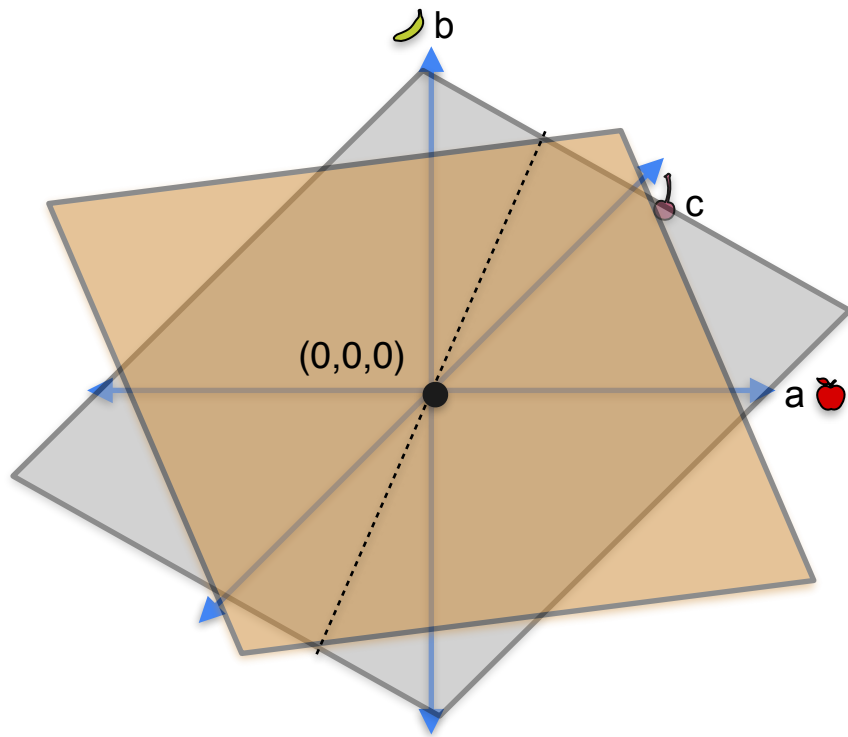
- $a + b + c = 0$
- $a + 2b + c = 0$
- $a + b + 2c = 0$



# System 1

## System 1

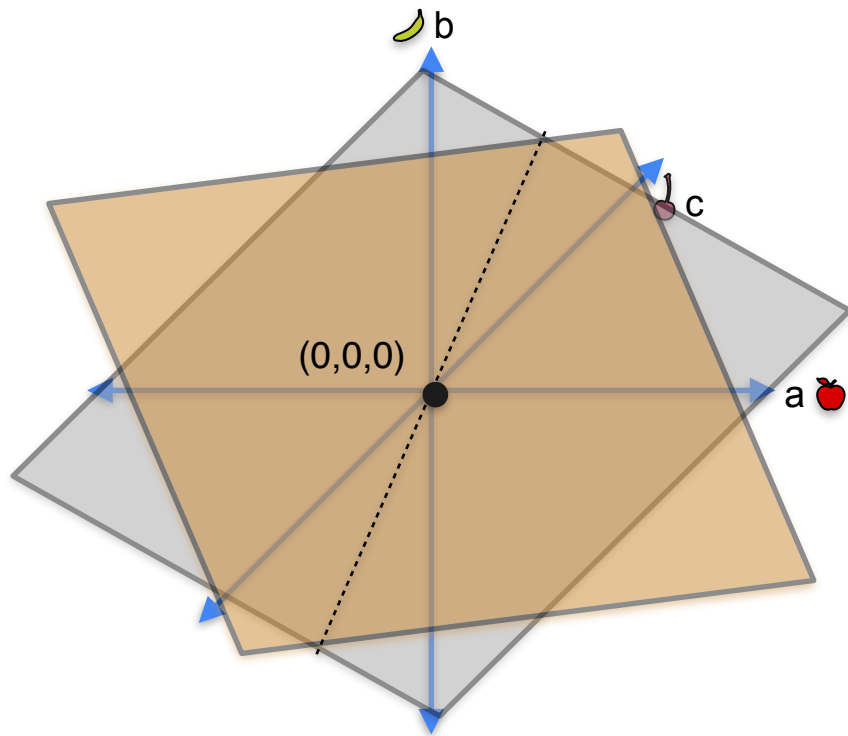
- $a + b + c = 0$
- $a + 2b + c = 0$
- $a + b + 2c = 0$



# System 1

## System 1

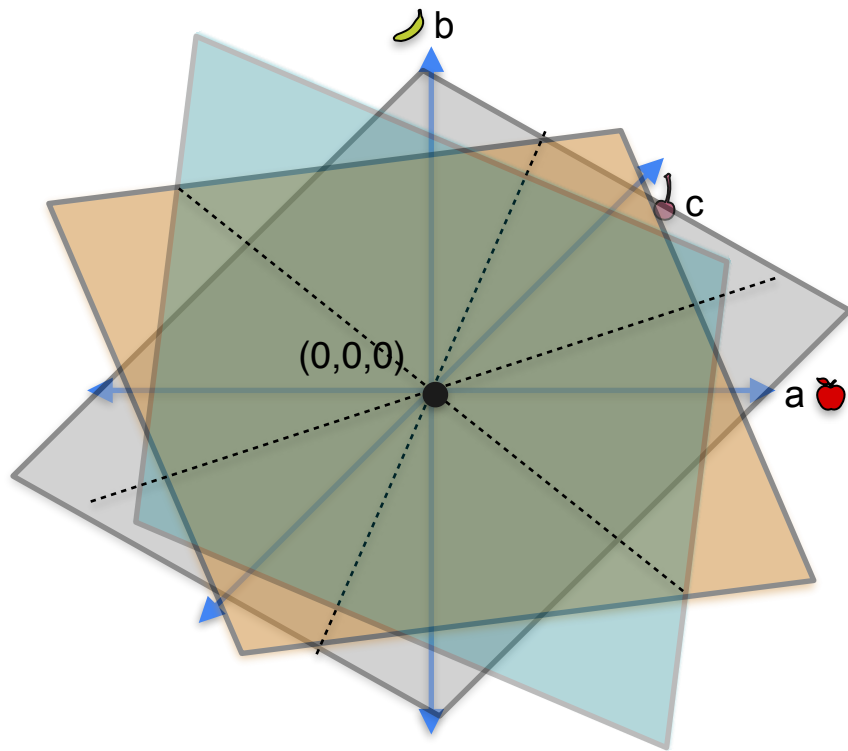
- $a + b + c = 0$
- $a + 2b + c = 0$
- $a + b + 2c = 0$



# System 1

## System 1

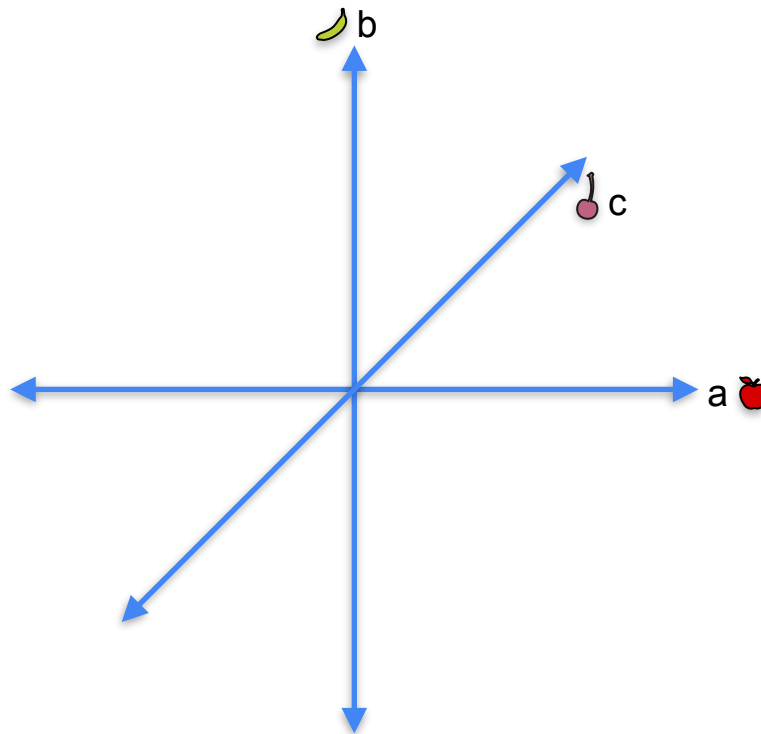
- $a + b + c = 0$
- $a + 2b + c = 0$
- $a + b + 2c = 0$



# System 2

## System 2

- $a + b + c = 0$
- $a + b + 2c = 0$
- $a + b + 3c = 0$





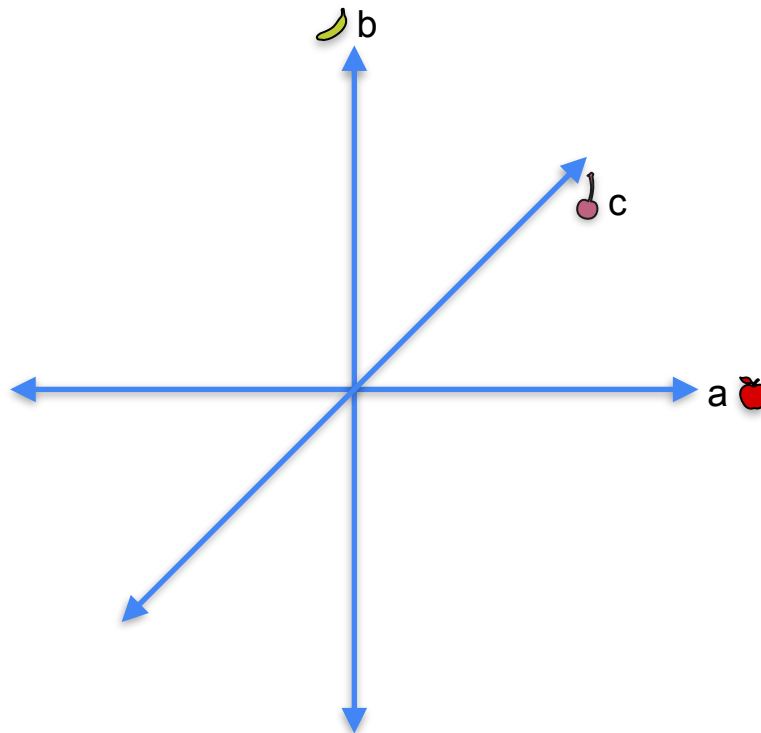
# System 2

## System 2

- $a + b + c = 0$

- $a + b + 2c = 0$

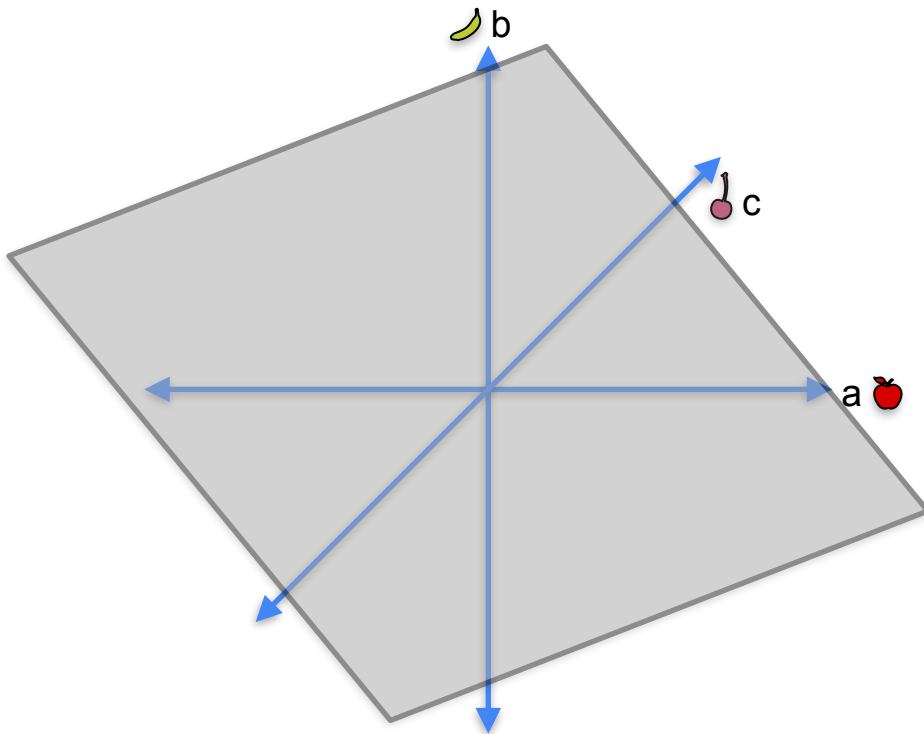
- $a + b + 3c = 0$



# System 2

## System 2

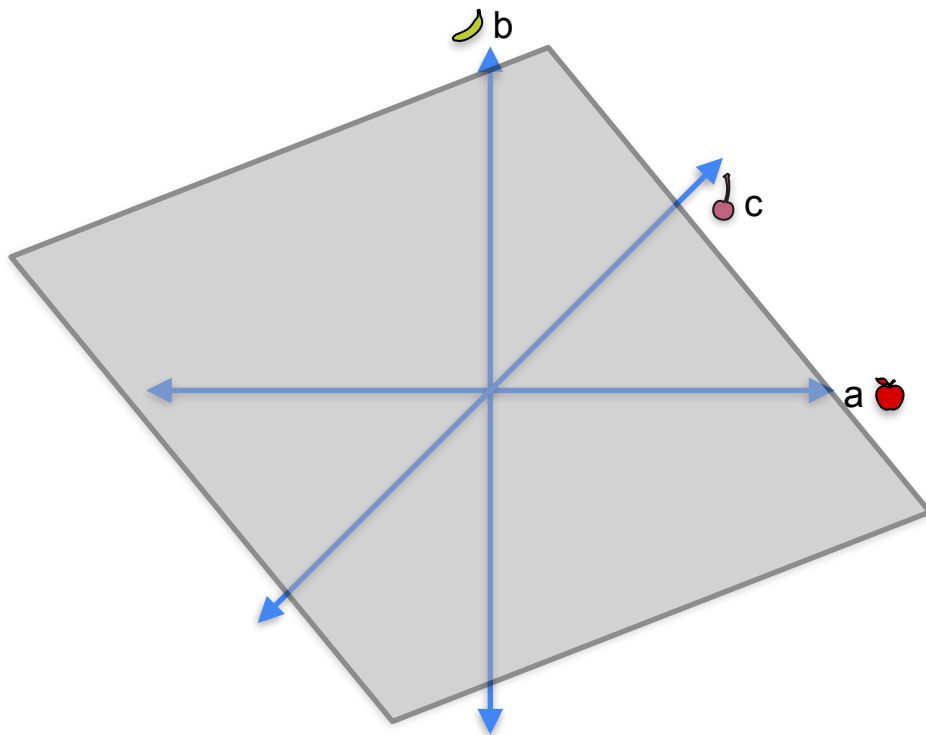
- $a + b + c = 0$
- $a + b + 2c = 0$
- $a + b + 3c = 0$



# System 2

## System 2

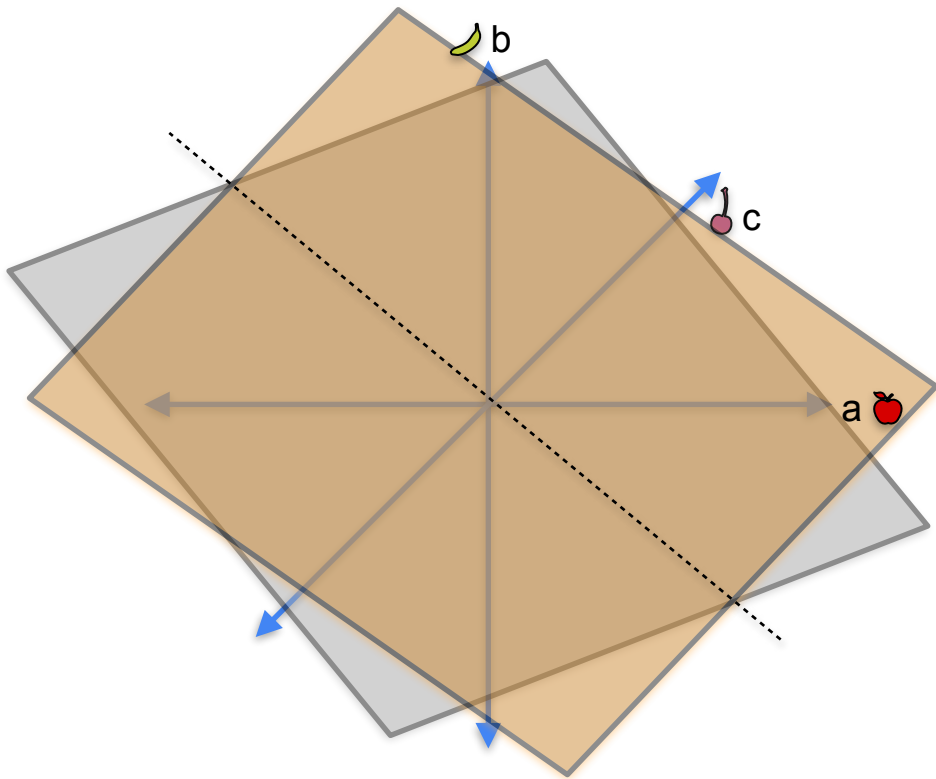
- $a + b + c = 0$
- $a + b + 2c = 0$
- $a + b + 3c = 0$



# System 2

## System 2

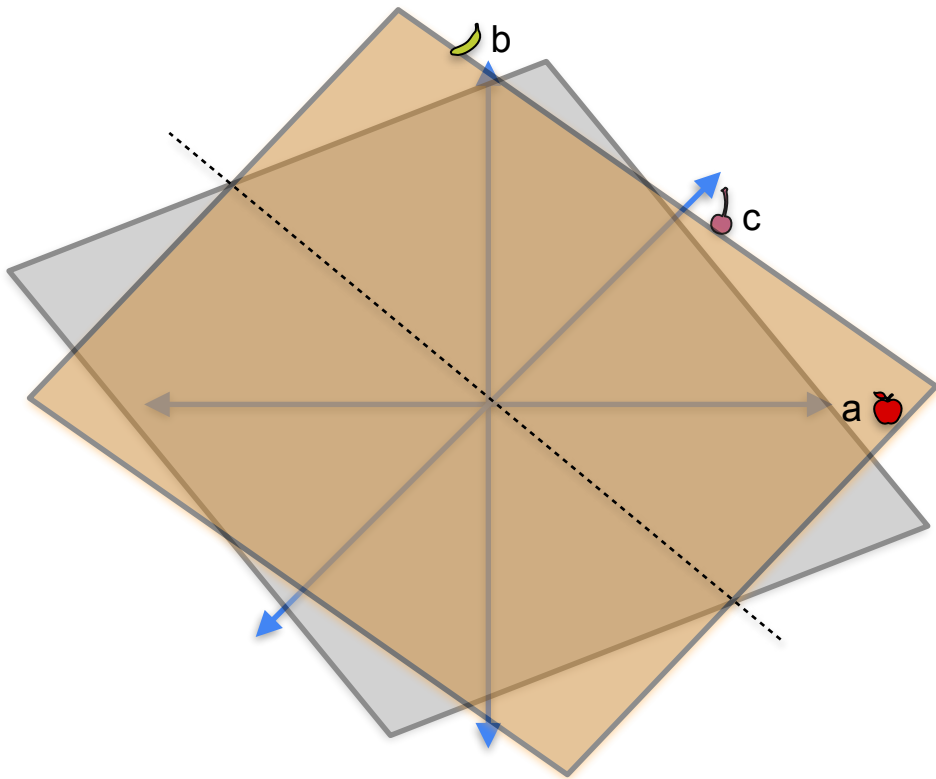
- $a + b + c = 0$
- $a + b + 2c = 0$
- $a + b + 3c = 0$



# System 2

## System 2

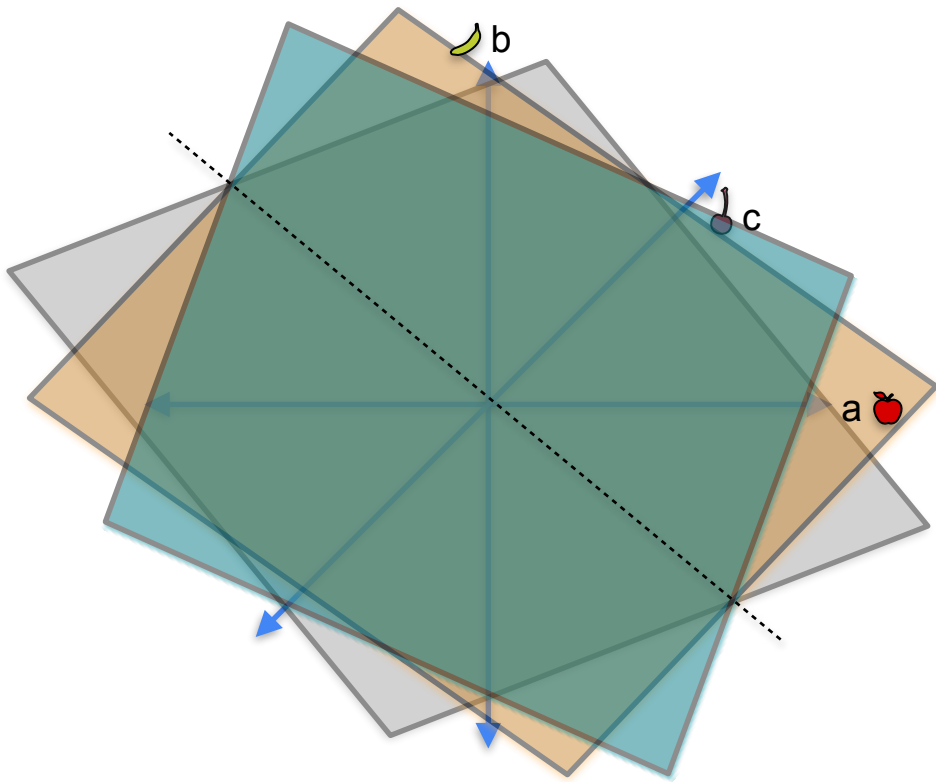
- $a + b + c = 0$
- $a + b + 2c = 0$
- $a + b + 3c = 0$



# System 2

## System 2

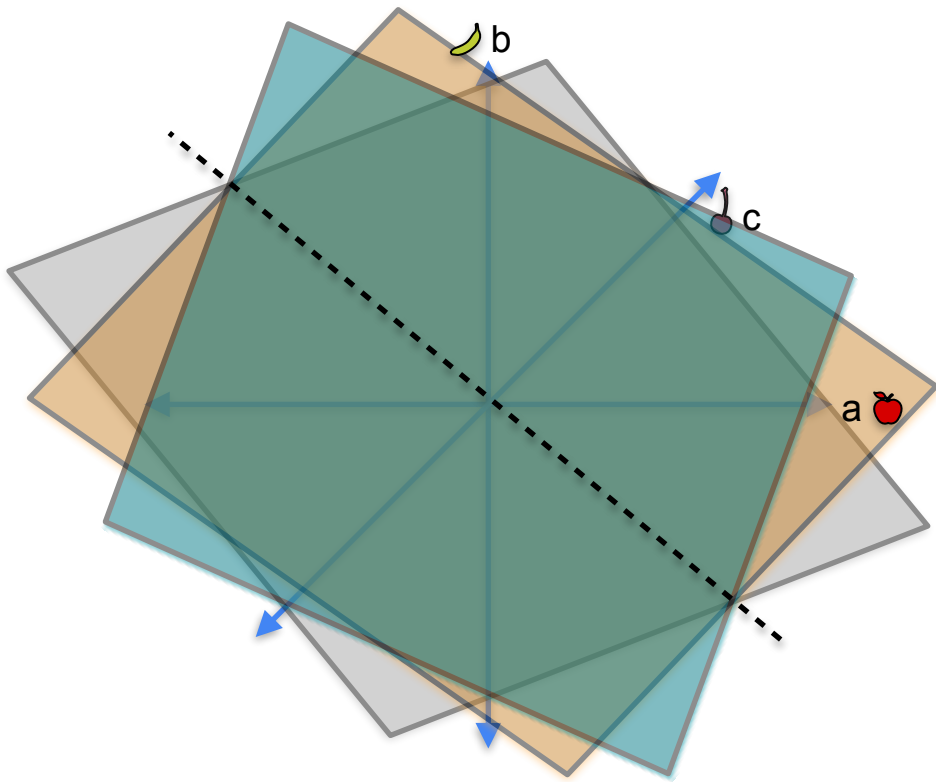
- $a + b + c = 0$
- $a + b + 2c = 0$
- $a + b + 3c = 0$



# System 2

## System 2

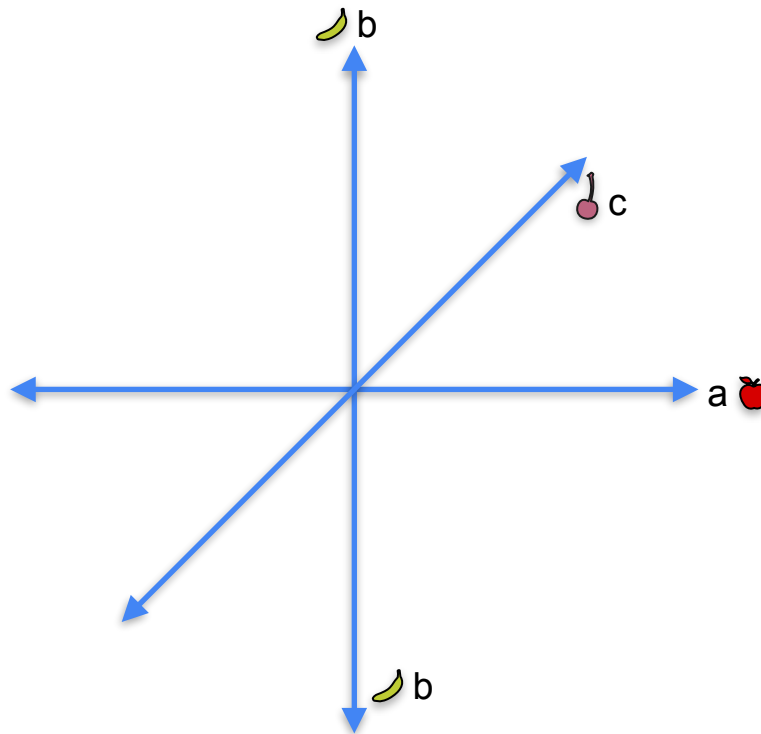
- $a + b + c = 0$
- $a + b + 2c = 0$
- $a + b + 3c = 0$



# System 3

## System 3

- $a + b + c = 0$
- $2a + 2b + 2c = 0$
- $3a + 3b + 3c = 0$

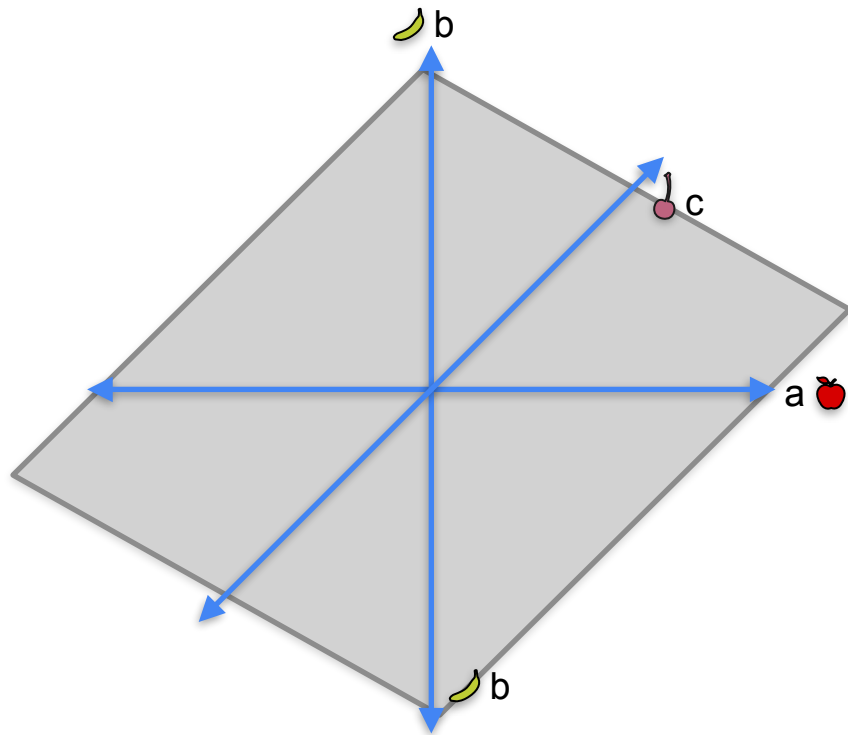




# System 3

## System 3

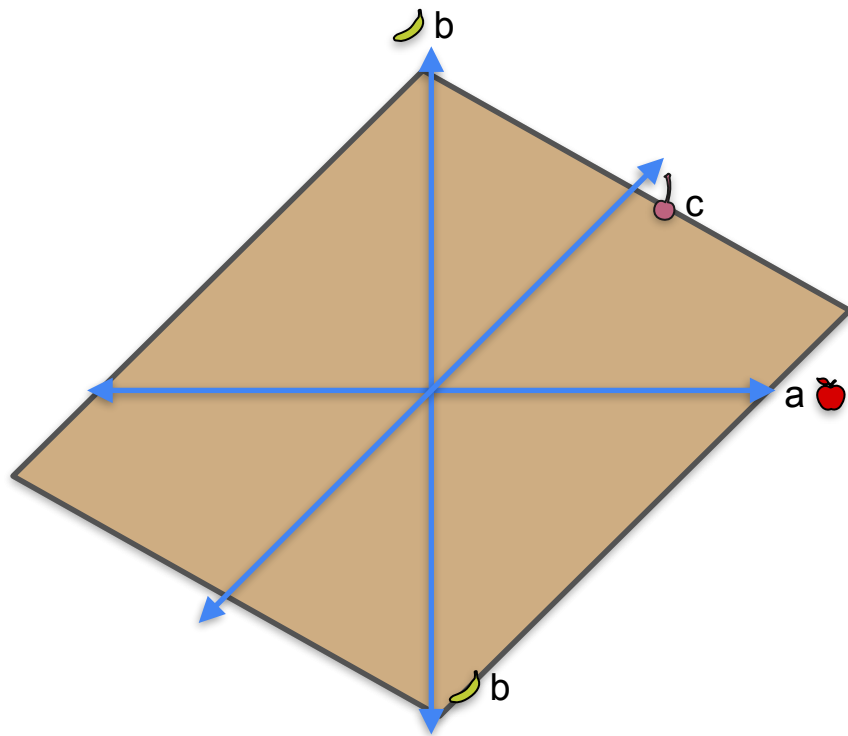
- $a + b + c = 0$
- $2a + 2b + 2c = 0$
- $3a + 3b + 3c = 0$



# System 3

## System 3

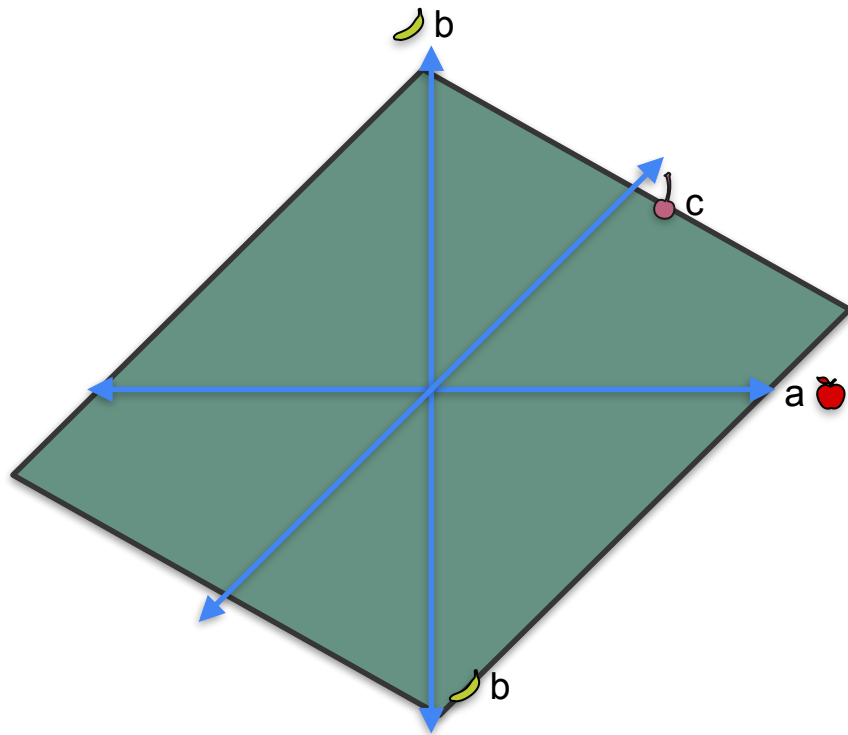
- $a + b + c = 0$
- $2a + 2b + 2c = 0$  ←
- $3a + 3b + 3c = 0$



# System 3

## System 3

- $a + b + c = 0$
- $2a + 2b + 2c = 0$
- $3a + 3b + 3c = 0$





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# System of Linear Equations

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**Linear dependence and  
independence (3x3)**

# Linear dependence and independence

$$a = 1$$

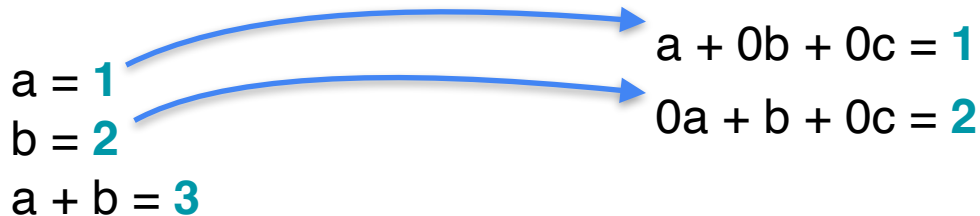
$$b = 2$$

$$a + b = 3$$

# Linear dependence and independence


$$\begin{aligned} a &= 1 \\ b &= 2 \\ a + b &= 3 \end{aligned} \qquad a + 0b + 0c = 1$$

# Linear dependence and independence



The diagram illustrates the substitution of values for variables  $a$  and  $b$  into linear equations. On the left, the values are listed:  $a = 1$ ,  $b = 2$ , and  $a + b = 3$ . On the right, two equations are shown:  $a + 0b + 0c = 1$  and  $0a + b + 0c = 2$ . Two blue curved arrows originate from the right side of the first two equations on the left and point to the right side of the corresponding equations on the right, indicating the substitution of the values of  $a$  and  $b$  into the equations.

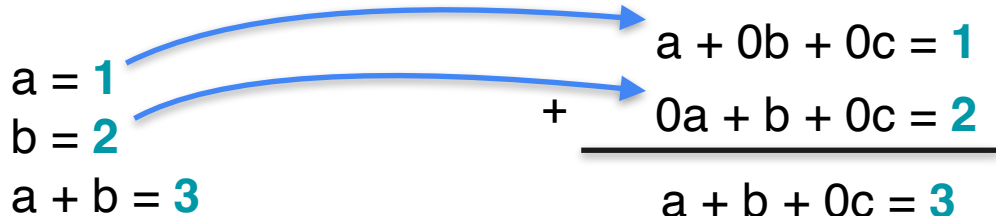
$$\begin{array}{l} a = 1 \\ b = 2 \\ a + b = 3 \end{array} \quad \begin{array}{l} a + 0b + 0c = 1 \\ 0a + b + 0c = 2 \end{array}$$

# Linear dependence and independence

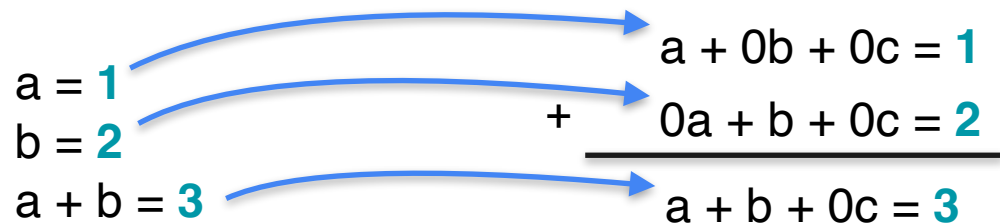
$$\begin{array}{l} a = 1 \\ b = 2 \\ a + b = 3 \end{array} \quad \begin{array}{l} \xrightarrow{\hspace{1.5cm}} \\ \xrightarrow{\hspace{1.5cm}} \end{array} \quad \begin{array}{l} a + 0b + 0c = 1 \\ 0a + b + 0c = 2 \\ \hline \end{array}$$



# Linear dependence and independence


$$\begin{array}{l} a = 1 \\ b = 2 \\ a + b = 3 \end{array} \qquad \begin{array}{r} a + 0b + 0c = 1 \\ + \quad 0a + b + 0c = 2 \\ \hline a + b + 0c = 3 \end{array}$$

# Linear dependence and independence



The diagram illustrates the process of substituting known values into a linear equation. On the left, two equations are listed:  $a = 1$  and  $b = 2$ . A third equation,  $a + b = 3$ , is shown below them. On the right, two equations are stacked and added:  $a + 0b + 0c = 1$  and  $0a + b + 0c = 2$ . A horizontal line is drawn under the second equation. Below the line, the result of the addition is shown:  $a + b + 0c = 3$ . Blue curved arrows indicate the substitution of  $a = 1$  into the first equation on the right,  $b = 2$  into the second equation on the right, and the sum of the two equations into the third equation on the right.

$$\begin{array}{l} a = 1 \\ b = 2 \\ a + b = 3 \end{array} \quad \begin{array}{r} \xrightarrow{\quad} a + 0b + 0c = 1 \\ \xrightarrow{\quad} + \quad 0a + b + 0c = 2 \\ \hline \xrightarrow{\quad} a + b + 0c = 3 \end{array}$$

# Linear dependence and independence

$$\begin{array}{lcl} a = 1 & \xrightarrow{\quad} & a + 0b + 0c = 1 \\ b = 2 & \xrightarrow{\quad} & 0a + b + 0c = 2 \\ a + b = 3 & \xrightarrow{\quad} & \hline a + b + 0c = 3 \end{array}$$

|   |   |   |
|---|---|---|
| 1 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 1 | 0 |

# Linear dependence and independence

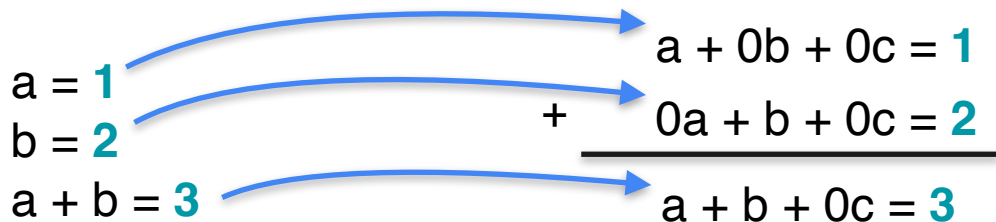


Diagram illustrating the derivation of a third equation from two others:

$$\begin{array}{rcl} a = 1 & \xrightarrow{\quad} & a + 0b + 0c = 1 \\ b = 2 & \xrightarrow{\quad} & 0a + b + 0c = 2 \\ a + b = 3 & \xrightarrow{\quad} & a + b + 0c = 3 \end{array}$$

The third equation is the sum of the first two, indicated by the '+' sign and the blue arrows.

|   |   |   |
|---|---|---|
| 1 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 1 | 0 |

Row 1 + Row 2 = Row 3

# Linear dependence and independence

$$\begin{array}{rcl} a = 1 & \rightarrow & a + 0b + 0c = 1 \\ b = 2 & \rightarrow & 0a + b + 0c = 2 \\ \hline a + b = 3 & \rightarrow & a + b + 0c = 3 \end{array}$$

|   |   |   |
|---|---|---|
| 1 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 1 | 0 |

Row 1 + Row 2 = Row 3

Row 3 **depends** on rows 1 and 2

# Linear dependence and independence

$$\begin{array}{rcl} a = 1 & \rightarrow & a + 0b + 0c = 1 \\ b = 2 & \rightarrow & 0a + b + 0c = 2 \\ \hline a + b = 3 & \rightarrow & a + b + 0c = 3 \end{array}$$

|   |   |   |
|---|---|---|
| 1 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 1 | 0 |

Row 1 + Row 2 = Row 3

Row 3 **depends** on rows 1 and 2

Rows are **linearly dependent**

# Linear dependence and independence

$$\begin{array}{rcl} a = 1 & \xrightarrow{\quad} & a + 0b + 0c = 1 \\ b = 2 & \xrightarrow{\quad} & 0a + b + 0c = 2 \\ a + b = 3 & \xrightarrow{\quad} & \hline & & a + b + 0c = 3 \end{array}$$

|   |   |   |
|---|---|---|
| 1 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 1 | 0 |

Row 1 + Row 2 = Row 3

Row 3 **depends** on rows 1 and 2

Rows are **linearly dependent**

# Linear dependence and independence

$$\begin{array}{rcl} a = 1 & \rightarrow & a + 0b + 0c = 1 \\ b = 2 & \rightarrow & 0a + b + 0c = 2 \\ \hline a + b = 3 & \rightarrow & a + b + 0c = 3 \end{array}$$

|   |   |   |
|---|---|---|
| 1 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 1 | 0 |

Row 1 + Row 2 = Row 3

Row 3 **depends** on rows 1 and 2

Rows are **linearly dependent**



# Linear dependence and independence

$$\begin{array}{rcl} a = 1 & \rightarrow & a + 0b + 0c = 1 \\ b = 2 & \rightarrow & 0a + b + 0c = 2 \\ \hline a + b = 3 & \rightarrow & a + b + 0c = 3 \end{array}$$

|   |   |   |
|---|---|---|
| 1 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 1 | 0 |

Row 1 + Row 2 = Row 3

Row 3 **depends** on rows 1 and 2

Rows are **linearly dependent**

# Linear dependence and independence


$$a + b + c = 0$$

$$2a + 2b + 2c = 0$$

$$3a + 3b + 3c = 0$$

|   |   |   |
|---|---|---|
| 1 | 1 | 1 |
| 2 | 2 | 2 |
| 3 | 3 | 3 |

# Linear dependence and independence

$$\begin{aligned}a + b + c &= 0 \\2a + 2b + 2c &= 0 \\3a + 3b + 3c &= 0\end{aligned}$$

$$a + b + c = 0$$

|   |   |   |
|---|---|---|
| 1 | 1 | 1 |
| 2 | 2 | 2 |
| 3 | 3 | 3 |

# Linear dependence and independence

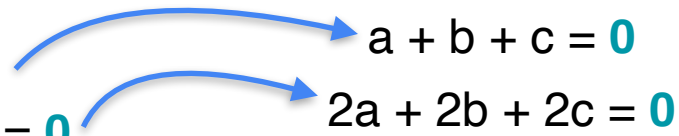
$$\begin{array}{l} a + b + c = 0 \\ 2a + 2b + 2c = 0 \\ 3a + 3b + 3c = 0 \end{array}$$


Diagram illustrating linear dependence. The first two equations are shown on the left, and the third equation is shown on the right. Blue arrows indicate that the first two equations are linearly dependent on the third equation.

$$\begin{array}{l} a + b + c = 0 \\ 2a + 2b + 2c = 0 \end{array}$$

|   |   |   |
|---|---|---|
| 1 | 1 | 1 |
| 2 | 2 | 2 |
| 3 | 3 | 3 |

# Linear dependence and independence

$$\begin{array}{rcl} a + b + c & = & 0 \\ 2a + 2b + 2c & = & 0 \\ 3a + 3b + 3c & = & 0 \end{array} \quad \begin{array}{l} \xrightarrow{\quad} \\ + \end{array} \quad \begin{array}{rcl} a + b + c & = & 0 \\ \hline 2a + 2b + 2c & = & 0 \end{array}$$

|   |   |   |
|---|---|---|
| 1 | 1 | 1 |
| 2 | 2 | 2 |
| 3 | 3 | 3 |

# Linear dependence and independence

$$\begin{array}{rcl} a + b + c = 0 & \xrightarrow{\quad} & a + b + c = 0 \\ 2a + 2b + 2c = 0 & \xrightarrow{\quad} & + \quad 2a + 2b + 2c = 0 \\ 3a + 3b + 3c = 0 & & \hline & & 3a + 3b + 3c = 0 \end{array}$$

|   |   |   |
|---|---|---|
| 1 | 1 | 1 |
| 2 | 2 | 2 |
| 3 | 3 | 3 |

# Linear dependence and independence

$$\begin{array}{rcl} a + b + c = 0 & \xrightarrow{\quad} & a + b + c = 0 \\ 2a + 2b + 2c = 0 & \xrightarrow{\quad + \quad} & 2a + 2b + 2c = 0 \\ 3a + 3b + 3c = 0 & \xrightarrow{\quad} & 3a + 3b + 3c = 0 \end{array}$$

---

|   |   |   |
|---|---|---|
| 1 | 1 | 1 |
| 2 | 2 | 2 |
| 3 | 3 | 3 |

# Linear dependence and independence

$$\begin{array}{rcl} a + b + c = 0 & \xrightarrow{\quad} & a + b + c = 0 \\ 2a + 2b + 2c = 0 & \xrightarrow{\quad} & + \quad 2a + 2b + 2c = 0 \\ 3a + 3b + 3c = 0 & \xrightarrow{\quad} & \hline & & 3a + 3b + 3c = 0 \end{array}$$

|   |   |   |
|---|---|---|
| 1 | 1 | 1 |
| 2 | 2 | 2 |
| 3 | 3 | 3 |

Row 1 + Row 2 = Row 3



# Linear dependence and independence

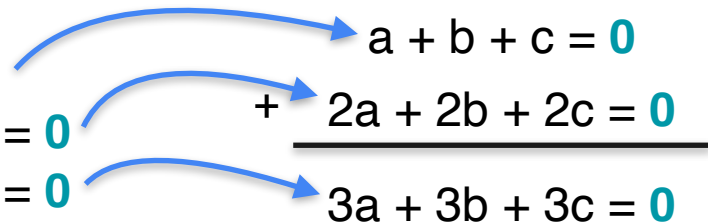
$$\begin{array}{rcl} a + b + c = 0 & \xrightarrow{\quad} & a + b + c = 0 \\ 2a + 2b + 2c = 0 & \xrightarrow{\quad + \quad} & 2a + 2b + 2c = 0 \\ 3a + 3b + 3c = 0 & \xrightarrow{\quad} & 3a + 3b + 3c = 0 \end{array}$$

|   |   |   |
|---|---|---|
| 1 | 1 | 1 |
| 2 | 2 | 2 |
| 3 | 3 | 3 |

Row 1 + Row 2 = Row 3

Row 3 **depends** on rows 1 and 2

# Linear dependence and independence

$$\begin{array}{rcl} a + b + c = 0 & \xrightarrow{\quad} & a + b + c = 0 \\ 2a + 2b + 2c = 0 & \xrightarrow{\quad + \quad} & 2a + 2b + 2c = 0 \\ 3a + 3b + 3c = 0 & \xrightarrow{\quad} & 3a + 3b + 3c = 0 \end{array}$$


|   |   |   |
|---|---|---|
| 1 | 1 | 1 |
| 2 | 2 | 2 |
| 3 | 3 | 3 |

Row 1 + Row 2 = Row 3

Row 3 **depends** on rows 1 and 2

Rows are **linearly dependent**

# Linear dependence and independence

$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

|   |   |   |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 2 |
| 1 | 1 | 3 |

# Linear dependence and independence

$$\begin{aligned}a + b + c &= 0 \\a + b + 2c &= 0 \\a + b + 3c &= 0\end{aligned}$$

$$a + b + c = 0$$

|   |   |   |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 2 |
| 1 | 1 | 3 |

# Linear dependence and independence

$$\begin{array}{l} a + b + c = 0 \\ a + b + 2c = 0 \\ a + b + 3c = 0 \end{array} \quad \begin{array}{l} \xrightarrow{\quad} a + b + c = 0 \\ \xrightarrow{\quad} a + b + 3c = 0 \end{array}$$

|   |   |   |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 2 |
| 1 | 1 | 3 |

# Linear dependence and independence

$$\begin{array}{l} a + b + c = 0 \\ a + b + 2c = 0 \\ a + b + 3c = 0 \end{array} \quad \begin{array}{l} \xrightarrow{\quad} \\ + \xrightarrow{\quad} \end{array} \quad \begin{array}{l} a + b + c = 0 \\ \hline a + b + 3c = 0 \end{array}$$

|   |   |   |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 2 |
| 1 | 1 | 3 |

# Linear dependence and independence

$$\begin{array}{l} a + b + c = 0 \\ a + b + 2c = 0 \\ a + b + 3c = 0 \end{array} \quad \begin{array}{l} \xrightarrow{\quad} \\ + \xrightarrow{\quad} \\ \hline \end{array} \quad \begin{array}{l} a + b + c = 0 \\ a + b + 3c = 0 \\ \hline 2a + 2b + 4c = 0 \end{array}$$

|   |   |   |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 2 |
| 1 | 1 | 3 |

# Linear dependence and independence

$$\begin{array}{rcl} a + b + c = 0 & \xrightarrow{\quad} & a + b + c = 0 \\ a + b + 2c = 0 & + & a + b + 3c = 0 \\ a + b + 3c = 0 & \xrightarrow{\quad} & \hline 2a + 2b + 4c = 0 \end{array}$$

$\downarrow \div 2$

|   |   |   |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 2 |
| 1 | 1 | 3 |



# Linear dependence and independence

$$\begin{array}{l} a + b + c = 0 \\ a + b + 2c = 0 \\ a + b + 3c = 0 \end{array}$$
$$\begin{array}{r} a + b + c = 0 \\ + \quad a + b + 2c = 0 \\ \hline 2a + 2b + 3c = 0 \\ \downarrow \div 2 \\ a + b + 1.5c = 0 \end{array}$$

|   |   |   |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 2 |
| 1 | 1 | 3 |

# Linear dependence and independence

Diagram illustrating the elimination of a variable from a system of linear equations:

$$\begin{array}{rcl} a + b + c & = & 0 \\ a + b + 2c & = & 0 \\ a + b + 3c & = & 0 \end{array}$$

Operations:

$$\begin{array}{rcl} & & a + b + c = 0 \\ & + & a + b + 3c = 0 \\ \hline & & 2a + 2b + 4c = 0 \\ & \downarrow \div 2 & \\ & & a + b + 2c = 0 \end{array}$$

Corresponding coefficient matrix:

|   |   |   |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 2 |
| 1 | 1 | 3 |

# Linear dependence and independence

$$\begin{array}{l} a + b + c = 0 \\ a + b + 2c = 0 \\ a + b + 3c = 0 \end{array}$$
$$\begin{array}{r} a + b + c = 0 \\ + \quad a + b + 3c = 0 \\ \hline 2a + 2b + 4c = 0 \\ \downarrow \div 2 \\ a + b + 2c = 0 \end{array}$$

|   |   |   |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 2 |
| 1 | 1 | 3 |

Average of Row 1 and Row 3 is Row 2  
Row 2 **depends** on rows 1 and 3

# Linear dependence and independence

$$\begin{array}{rcl} a + b + c & = & 0 \\ a + b + 2c & = & 0 \\ a + b + 3c & = & 0 \end{array}$$
$$\begin{array}{rcl} & & + \\ & & \hline a + b + c & = & 0 \\ a + b + 3c & = & 0 \\ \hline 2a + 2b + 4c & = & 0 \\ \downarrow \div 2 & & \\ a + b + 2c & = & 0 \end{array}$$

|   |   |   |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 2 |
| 1 | 1 | 3 |

Average of Row 1 and Row 3 is Row 2

Row 2 **depends** on rows 1 and 3

Rows are **linearly dependent**

# Linear dependence and independence

$$a + b + c = 0$$

$$a + 2b + c = 0$$

$$a + b + 2c = 0$$

|   |   |   |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |

# Linear dependence and independence

$$a + b + c = 0$$

$$a + 2b + c = 0 \longrightarrow \text{No relations between equations}$$

$$a + b + 2c = 0$$

|   |   |   |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |

# Linear dependence and independence

$$a + b + c = 0$$

$$a + 2b + c = 0 \longrightarrow \text{No relations between equations}$$

$$a + b + 2c = 0$$

|   |   |   |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |

No relations between rows

# Linear dependence and independence

$$a + b + c = 0$$

$$a + 2b + c = 0 \longrightarrow \text{No relations between equations}$$

$$a + b + 2c = 0$$

|   |   |   |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |

No relations between rows

Rows are **linearly independent**



# Quiz: Linear dependence and independence

**Problem:** Determine if the following matrices have linearly dependent or independent rows

|   |   |   |
|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 3 | 2 | 3 |

|   |   |    |
|---|---|----|
| 1 | 1 | 1  |
| 1 | 1 | 2  |
| 0 | 0 | -1 |

|   |   |   |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 2 | 2 |
| 0 | 0 | 3 |

|   |   |    |
|---|---|----|
| 1 | 2 | 5  |
| 0 | 3 | -2 |
| 2 | 4 | 10 |

# Solution: Linear dependence and independence

**Problem:** Determine if the following matrices have linear dependent or independent rows

|   |   |   |
|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 3 | 2 | 3 |

|   |   |    |
|---|---|----|
| 1 | 1 | 1  |
| 1 | 1 | 2  |
| 0 | 0 | -1 |

|   |   |   |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 2 | 2 |
| 0 | 0 | 3 |

|   |   |    |
|---|---|----|
| 1 | 2 | 5  |
| 0 | 3 | -2 |
| 2 | 4 | 10 |

# Solution: Linear dependence and independence

**Problem:** Determine if the following matrices have linear dependent or independent rows

|   |   |   |
|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 3 | 2 | 3 |

|   |   |    |
|---|---|----|
| 1 | 1 | 1  |
| 1 | 1 | 2  |
| 0 | 0 | -1 |

|   |   |   |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 2 | 2 |
| 0 | 0 | 3 |

|   |   |    |
|---|---|----|
| 1 | 2 | 5  |
| 0 | 3 | -2 |
| 2 | 4 | 10 |

$$3\text{Row1} + 2\text{Row2} = \text{Row3}$$

**Dependent (singular)**

# Solution: Linear dependence and independence

**Problem:** Determine if the following matrices have linear dependent or independent rows

|   |   |   |
|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 3 | 2 | 3 |

$$3\text{Row1} + 2\text{Row2} = \text{Row3}$$

**Dependent (singular)**

|   |   |    |
|---|---|----|
| 1 | 1 | 1  |
| 1 | 1 | 2  |
| 0 | 0 | -1 |

$$\text{Row1} - \text{Row2} = \text{Row3}$$

**Dependent (singular)**

|   |   |   |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 2 | 2 |
| 0 | 0 | 3 |

|   |   |    |
|---|---|----|
| 1 | 2 | 5  |
| 0 | 3 | -2 |
| 2 | 4 | 10 |

# Solution: Linear dependence and independence

**Problem:** Determine if the following matrices have linear dependent or independent rows

|   |   |   |
|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 3 | 2 | 3 |

$$3\text{Row1} + 2\text{Row2} = \text{Row3}$$

**Dependent (singular)**

|   |   |    |
|---|---|----|
| 1 | 1 | 1  |
| 1 | 1 | 2  |
| 0 | 0 | -1 |

$$\text{Row1} - \text{Row2} = \text{Row3}$$

**Dependent (singular)**

|   |   |   |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 2 | 2 |
| 0 | 0 | 3 |

No relations

**Independent  
(Non-singular)**

|   |   |    |
|---|---|----|
| 1 | 2 | 5  |
| 0 | 3 | -2 |
| 2 | 4 | 10 |

# Solution: Linear dependence and independence

**Problem:** Determine if the following matrices have linear dependent or independent rows

|   |   |   |
|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 3 | 2 | 3 |

$$3\text{Row1} + 2\text{Row2} = \text{Row3}$$

**Dependent (singular)**

|   |   |    |
|---|---|----|
| 1 | 1 | 1  |
| 1 | 1 | 2  |
| 0 | 0 | -1 |

$$\text{Row1} - \text{Row2} = \text{Row3}$$

**Dependent (singular)**

|   |   |   |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 2 | 2 |
| 0 | 0 | 3 |

No relations

**Independent  
(Non-singular)**

|   |   |    |
|---|---|----|
| 1 | 2 | 5  |
| 0 | 3 | -2 |
| 2 | 4 | 10 |

$$2\text{Row1} = \text{Row3}$$

**Dependent (singular)**



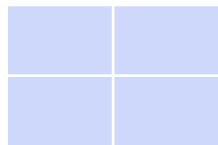
DeepLearning.AI

# System of Linear Equations

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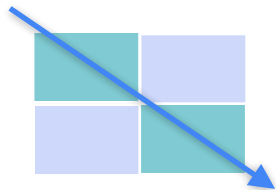
## **The determinant (3x3)**

# Diagonals in a 3x3 matrix





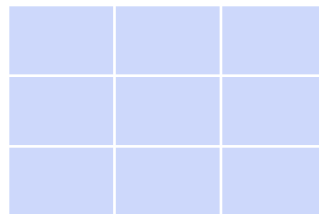
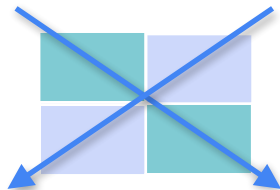
# Diagonals in a 3x3 matrix



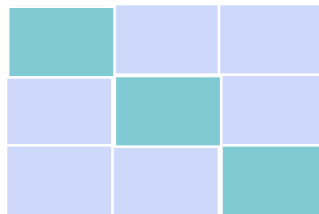
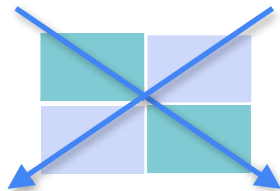
# Diagonals in a 3x3 matrix



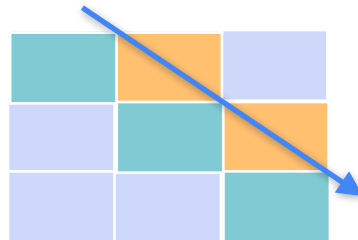
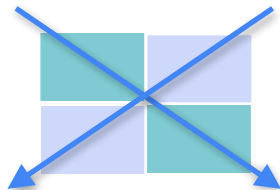
# Diagonals in a 3x3 matrix



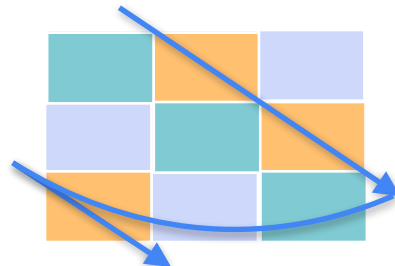
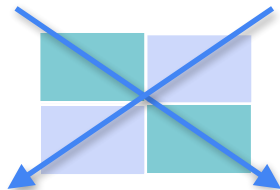
# Diagonals in a 3x3 matrix



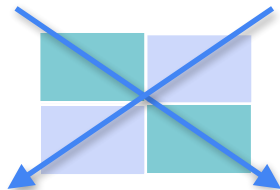
# Diagonals in a 3x3 matrix



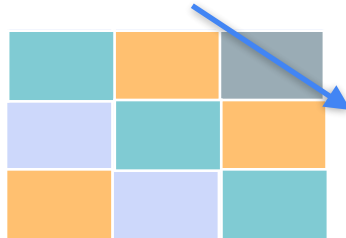
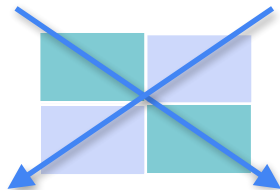
# Diagonals in a 3x3 matrix



# Diagonals in a 3x3 matrix

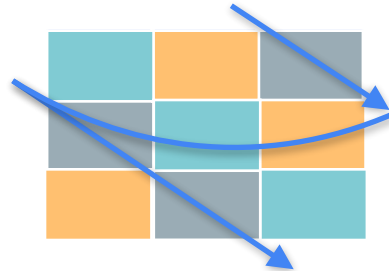
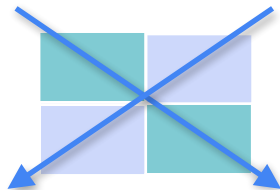


# Diagonals in a 3x3 matrix





# Diagonals in a 3x3 matrix



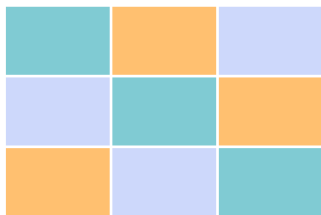
# Diagonals in a 3x3 matrix



# Determinant

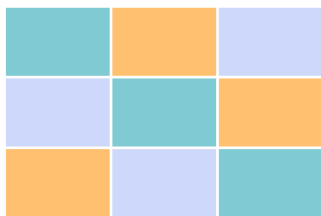
# Determinant

Add

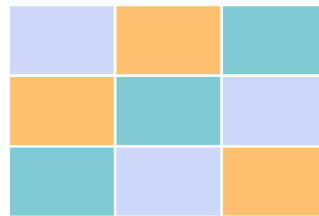


# Determinant

Add



Subtract



# The determinant

|   |   |   |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |

# The determinant

|   |   |   |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |

# The determinant

|   |   |   |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |

|   |   |   |
|---|---|---|
| 1 |   |   |
|   | 2 |   |
|   |   | 2 |

$$+ 1 \cdot 2 \cdot 2$$



# The determinant

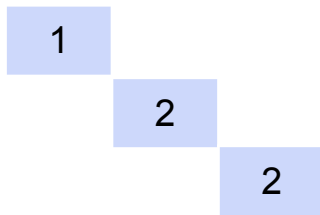
|   |   |   |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |

|   |   |   |
|---|---|---|
| 1 |   |   |
|   | 2 |   |
|   |   | 2 |

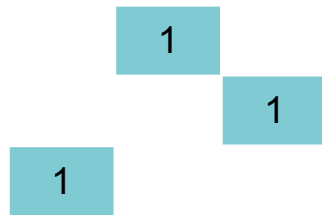
$$+ 1 \cdot 2 \cdot 2$$

# The determinant

|   |   |   |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |



$$+ 1 \cdot 2 \cdot 2$$



$$+ 1 \cdot 1 \cdot 1$$

# The determinant

|   |   |   |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |

|   |   |   |  |
|---|---|---|--|
| 1 |   |   |  |
|   | 2 |   |  |
|   |   | 2 |  |

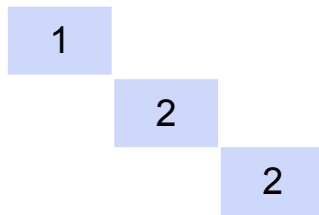
$$+ 1 \cdot 2 \cdot 2$$

|   |  |   |   |
|---|--|---|---|
|   |  | 1 |   |
|   |  |   | 1 |
| 1 |  |   |   |

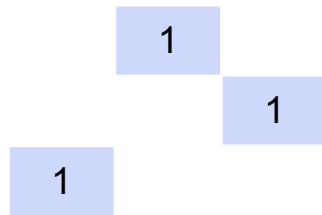
$$+ 1 \cdot 1 \cdot 1$$

# The determinant

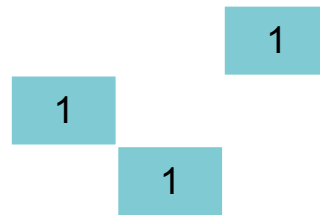
|   |   |   |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |



$$+ 1 \cdot 2 \cdot 2$$



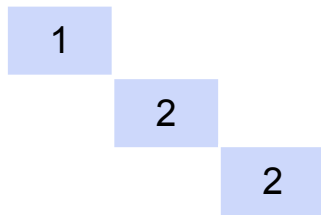
$$+ 1 \cdot 1 \cdot 1$$



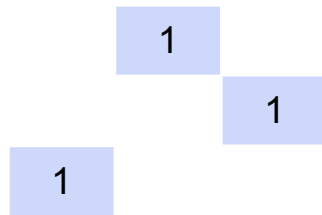
$$+ 1 \cdot 1 \cdot 1$$

# The determinant

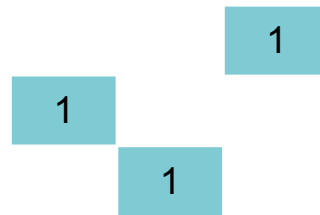
|   |   |   |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |



$$+ 1 \cdot 2 \cdot 2$$



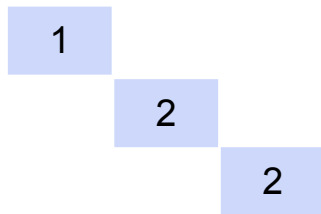
$$+ 1 \cdot 1 \cdot 1$$



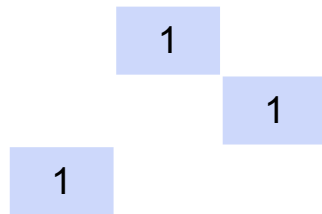
$$+ 1 \cdot 1 \cdot 1$$

# The determinant

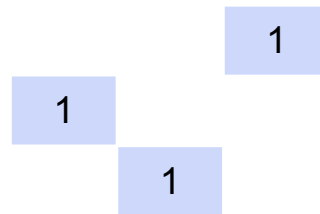
|   |   |   |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |



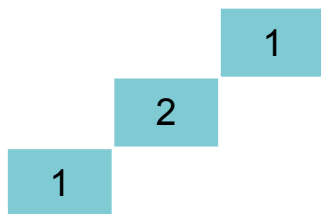
$$+ 1 \cdot 2 \cdot 2$$



$$+ 1 \cdot 1 \cdot 1$$



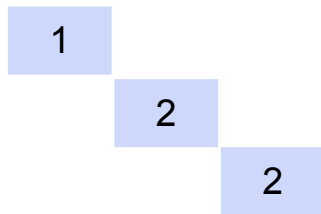
$$+ 1 \cdot 1 \cdot 1$$



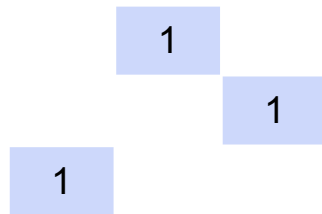
$$- 1 \cdot 2 \cdot 1$$

# The determinant

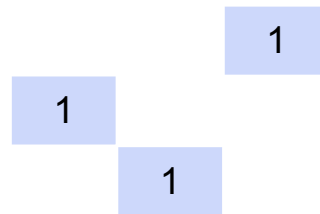
|   |   |   |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |



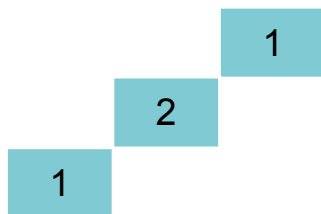
$$+ 1 \cdot 2 \cdot 2$$



$$+ 1 \cdot 1 \cdot 1$$



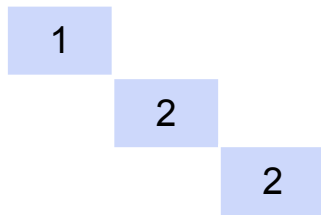
$$+ 1 \cdot 1 \cdot 1$$



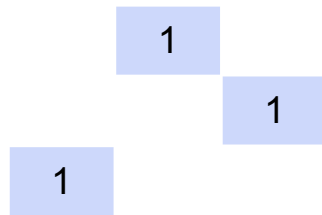
$$- 1 \cdot 2 \cdot 1$$

# The determinant

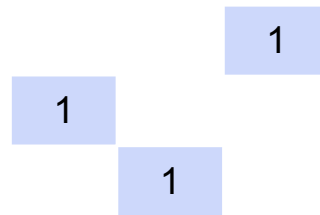
|   |   |   |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |



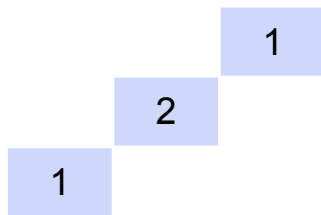
$$+ 1 \cdot 2 \cdot 2$$



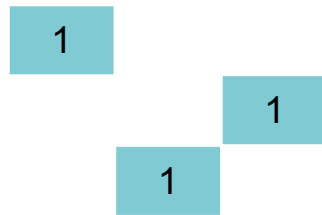
$$+ 1 \cdot 1 \cdot 1$$



$$+ 1 \cdot 1 \cdot 1$$



$$- 1 \cdot 2 \cdot 1$$

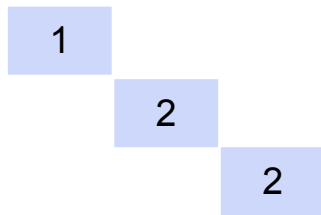


$$- 1 \cdot 1 \cdot 1$$

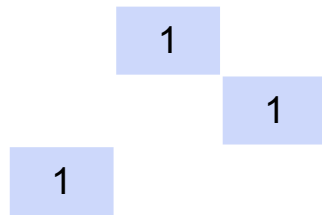


# The determinant

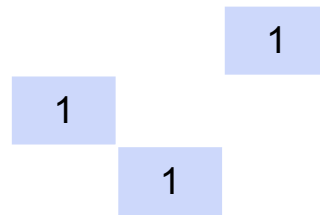
|   |   |   |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |



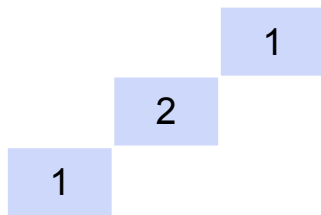
$$+ 1 \cdot 2 \cdot 2$$



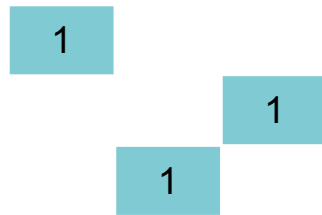
$$+ 1 \cdot 1 \cdot 1$$



$$+ 1 \cdot 1 \cdot 1$$



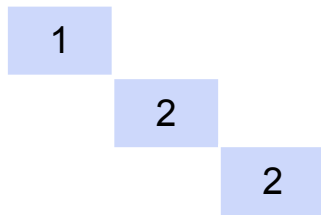
$$- 1 \cdot 2 \cdot 1$$



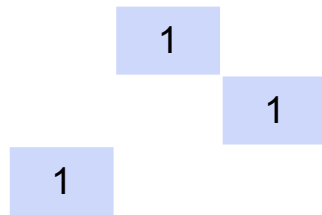
$$- 1 \cdot 1 \cdot 1$$

# The determinant

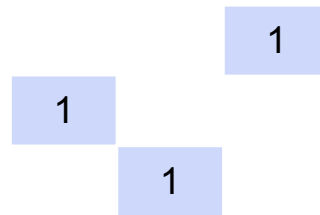
|   |   |   |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |



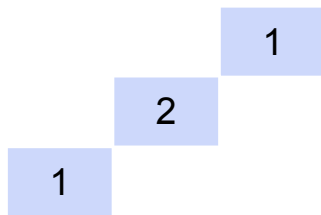
$$+ 1 \cdot 2 \cdot 2$$



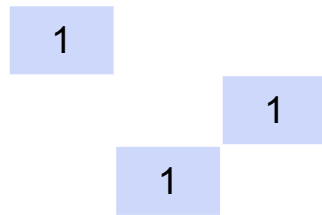
$$+ 1 \cdot 1 \cdot 1$$



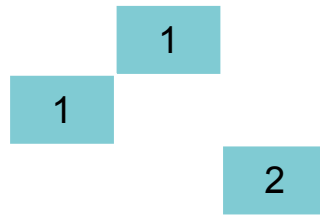
$$+ 1 \cdot 1 \cdot 1$$



$$- 1 \cdot 2 \cdot 1$$



$$- 1 \cdot 1 \cdot 1$$



$$- 1 \cdot 1 \cdot 2$$

# The determinant

|   |   |   |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |

$+ 1 \cdot 2 \cdot 2$

$+ 1 \cdot 1 \cdot 1$

$+ 1 \cdot 1 \cdot 1$

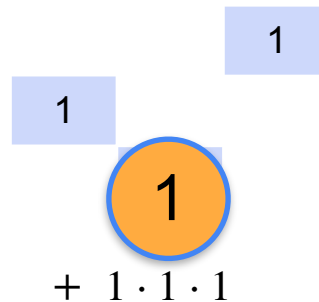
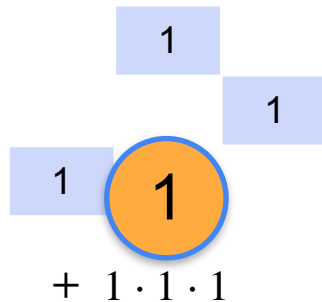
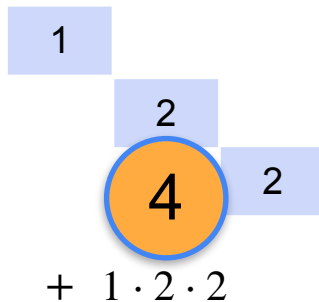
$- 1 \cdot 2 \cdot 1$

$- 1 \cdot 1 \cdot 1$

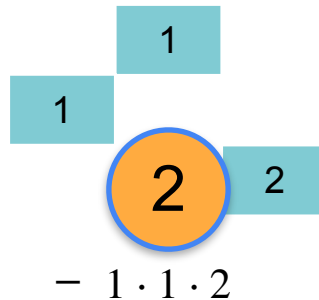
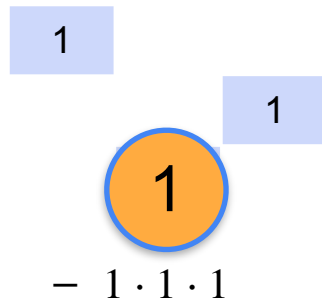
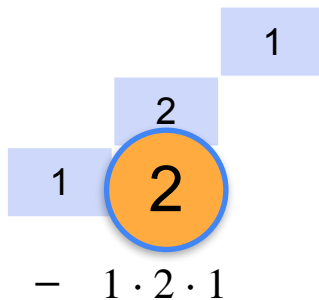
$- 1 \cdot 1 \cdot 2$

# The determinant

|   |   |   |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |

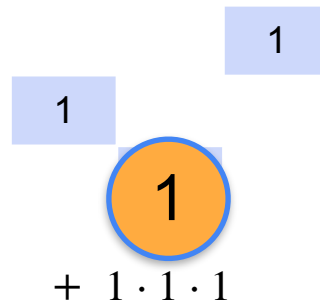
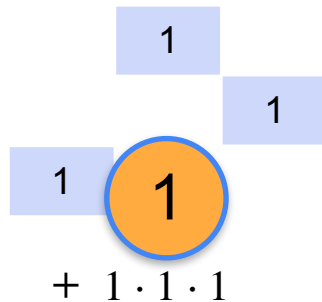
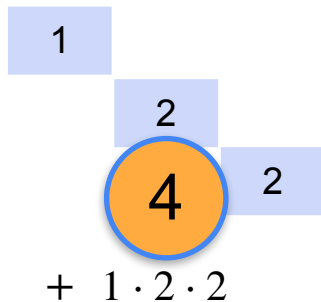


$$\text{Det} = 4 + 1 + 1 - 2 - 1 - 2$$

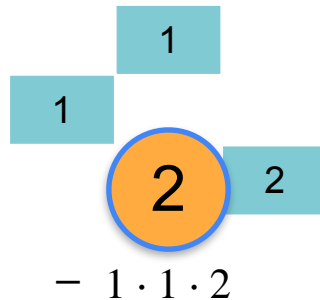
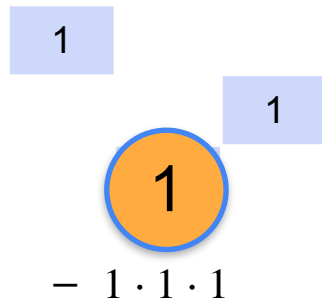
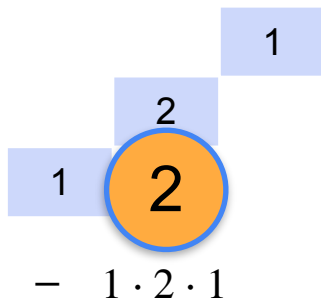


# The determinant

|   |   |   |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 2 | 1 |
| 1 | 1 | 2 |



$$\begin{aligned}\text{Det} &= 4 + 1 + 1 \\ &\quad - 2 - 1 - 2 \\ &= 1\end{aligned}$$



# Quiz: Determinants

**Problem:** Find the determinant of the following matrices (from the previous quiz).  
Verify that those with determinant 0 are precisely the singular matrices.

|   |   |   |
|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 3 | 3 | 3 |

|   |   |    |
|---|---|----|
| 1 | 1 | 1  |
| 1 | 1 | 2  |
| 0 | 0 | -1 |

|   |   |   |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 2 | 2 |
| 0 | 0 | 3 |

|   |   |    |
|---|---|----|
| 1 | 2 | 5  |
| 0 | 3 | -2 |
| 2 | 4 | 10 |

# Solution: Determinants

**Problem:** Find the determinant of the following matrices (from the previous quiz). Verify that those with determinant 0 are precisely the singular matrices.

|   |   |   |
|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 3 | 3 | 3 |

|   |   |    |
|---|---|----|
| 1 | 1 | 1  |
| 1 | 1 | 2  |
| 0 | 0 | -1 |

|   |   |   |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 2 | 2 |
| 0 | 0 | 3 |

|   |   |    |
|---|---|----|
| 1 | 2 | 5  |
| 0 | 3 | -2 |
| 2 | 4 | 10 |

# Solution: Determinants

**Problem:** Find the determinant of the following matrices (from the previous quiz).  
Verify that those with determinant 0 are precisely the singular matrices.

|   |   |   |
|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 3 | 3 | 3 |

|   |   |    |
|---|---|----|
| 1 | 1 | 1  |
| 1 | 1 | 2  |
| 0 | 0 | -1 |

|   |   |   |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 2 | 2 |
| 0 | 0 | 3 |

|   |   |    |
|---|---|----|
| 1 | 2 | 5  |
| 0 | 3 | -2 |
| 2 | 4 | 10 |

Determinant = 0

**Singular**



# Solution: Determinants

**Problem:** Find the determinant of the following matrices (from the previous quiz).  
Verify that those with determinant 0 are precisely the singular matrices.

|   |   |   |
|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 3 | 3 | 3 |

Determinant = 0

**Singular**

|   |   |    |
|---|---|----|
| 1 | 1 | 1  |
| 1 | 1 | 2  |
| 0 | 0 | -1 |

Determinant = 0

**Singular**

|   |   |   |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 2 | 2 |
| 0 | 0 | 3 |

|   |   |    |
|---|---|----|
| 1 | 2 | 5  |
| 0 | 3 | -2 |
| 2 | 4 | 10 |

# Solution: Determinants

**Problem:** Find the determinant of the following matrices (from the previous quiz).  
Verify that those with determinant 0 are precisely the singular matrices.

|   |   |   |
|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 3 | 3 | 3 |

Determinant = 0

**Singular**

|   |   |    |
|---|---|----|
| 1 | 1 | 1  |
| 1 | 1 | 2  |
| 0 | 0 | -1 |

Determinant = 0

**Singular**

|   |   |   |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 2 | 2 |
| 0 | 0 | 3 |

Determinant = 6

**Non-singular**

|   |   |    |
|---|---|----|
| 1 | 2 | 5  |
| 0 | 3 | -2 |
| 2 | 4 | 10 |

# Solution: Determinants

**Problem:** Find the determinant of the following matrices (from the previous quiz). Verify that those with determinant 0 are precisely the singular matrices.

|   |   |   |
|---|---|---|
| 1 | 0 | 1 |
| 0 | 1 | 0 |
| 3 | 3 | 3 |

Determinant = 0

**Singular**

|   |   |    |
|---|---|----|
| 1 | 1 | 1  |
| 1 | 1 | 2  |
| 0 | 0 | -1 |

Determinant = 0

**Singular**

|   |   |   |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 2 | 2 |
| 0 | 0 | 3 |

Determinant = 6

**Non-singular**

|   |   |    |
|---|---|----|
| 1 | 2 | 5  |
| 0 | 3 | -2 |
| 2 | 4 | 10 |

Determinant = 0

**Singular**

# The determinant

|   |   |   |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 2 | 2 |
| 0 | 0 | 3 |

$$\begin{aligned}\text{Det} &= 6+0+0-0-0-0 \\ &= 6\end{aligned}$$

# The determinant

|   |   |   |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 2 | 2 |
| 0 | 0 | 3 |

|   |   |   |
|---|---|---|
| 1 |   |   |
|   | 2 |   |
|   |   | 3 |

$$+ 1 \cdot 2 \cdot 3$$

$$\text{Det} = 6 + 0 + 0 - 0 - 0 - 0$$

$$= 6$$

# The determinant

|   |   |   |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 2 | 2 |
| 0 | 0 | 3 |

|   |   |   |
|---|---|---|
| 1 |   |   |
|   | 2 |   |
|   |   | 3 |

$$+ 1 \cdot 2 \cdot 3$$

|   |   |   |
|---|---|---|
|   | 1 |   |
|   |   | 2 |
| 0 |   |   |

$$+ 1 \cdot 2 \cdot 0$$

$$\text{Det} = 6 + 0 + 0 - 0 - 0 - 0$$

$$= 6$$

# The determinant

|   |   |   |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 2 | 2 |
| 0 | 0 | 3 |

|   |   |   |
|---|---|---|
| 1 |   |   |
|   | 2 |   |
|   |   | 3 |

$$+ 1 \cdot 2 \cdot 3$$

|   |   |   |
|---|---|---|
|   | 1 |   |
|   |   | 2 |
| 0 |   |   |

$$+ 1 \cdot 2 \cdot 0$$

|   |   |   |
|---|---|---|
|   |   | 1 |
| 0 |   |   |
|   | 0 |   |

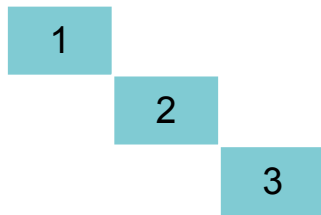
$$+ 1 \cdot 0 \cdot 0$$

$$\text{Det} = 6 + 0 + 0 - 0 - 0 - 0$$

$$= 6$$

# The determinant

|   |   |   |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 2 | 2 |
| 0 | 0 | 3 |



$$+ 1 \cdot 2 \cdot 3$$



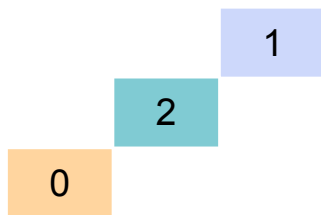
$$+ 1 \cdot 2 \cdot 0$$



$$+ 1 \cdot 0 \cdot 0$$

$$\text{Det} = 6 + 0 + 0 - 0 - 0 - 0$$

$$= 6$$

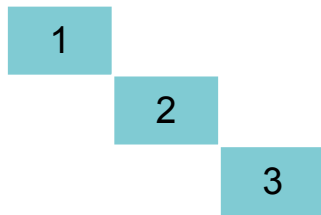


$$- 1 \cdot 2 \cdot 0$$



# The determinant

|   |   |   |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 2 | 2 |
| 0 | 0 | 3 |



$$+ 1 \cdot 2 \cdot 3$$



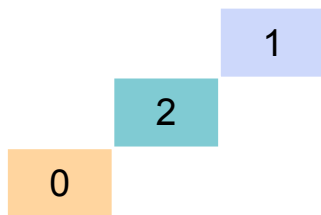
$$+ 1 \cdot 2 \cdot 0$$



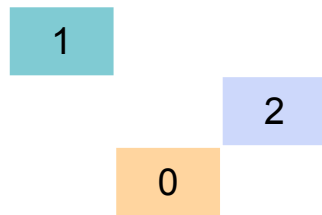
$$+ 1 \cdot 0 \cdot 0$$

$$\text{Det} = 6 + 0 + 0 - 0 - 0 - 0$$

$$= 6$$



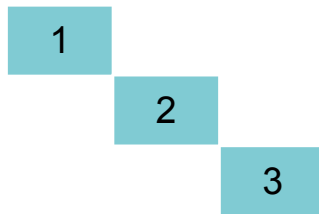
$$- 1 \cdot 2 \cdot 0$$



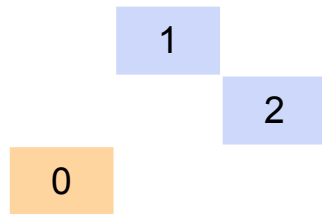
$$- 1 \cdot 2 \cdot 0$$

# The determinant

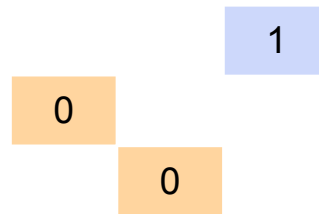
|   |   |   |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 2 | 2 |
| 0 | 0 | 3 |



$$+ 1 \cdot 2 \cdot 3$$



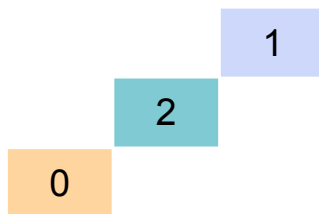
$$+ 1 \cdot 2 \cdot 0$$



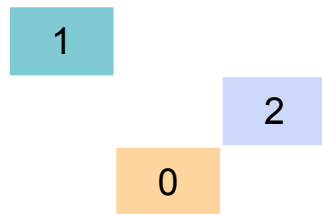
$$+ 1 \cdot 0 \cdot 0$$

$$\text{Det} = 6 + 0 + 0 - 0 - 0 - 0$$

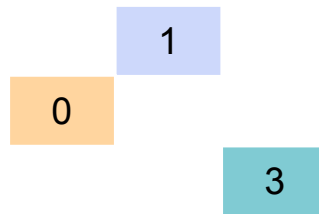
$$= 6$$



$$- 1 \cdot 2 \cdot 0$$



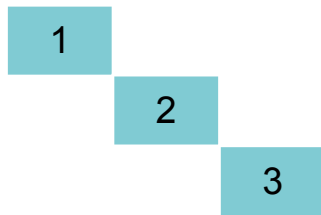
$$- 1 \cdot 2 \cdot 0$$



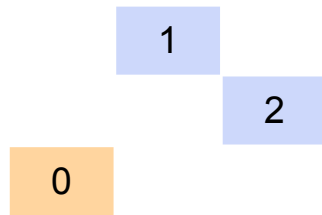
$$- 1 \cdot 0 \cdot 3$$

# The determinant

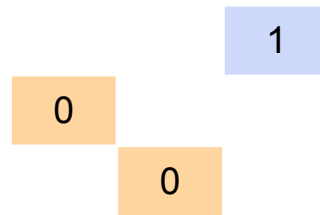
|   |   |   |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 2 | 2 |
| 0 | 0 | 3 |



$$+ 1 \cdot 2 \cdot 3$$



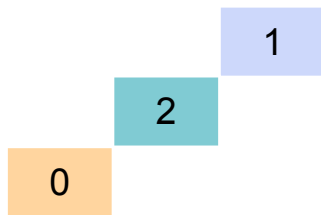
$$+ 1 \cdot 2 \cdot 0$$



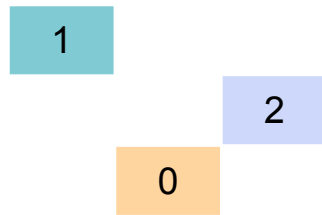
$$+ 1 \cdot 0 \cdot 0$$

$$\text{Det} = 6 + 0 + 0 - 0 - 0 - 0$$

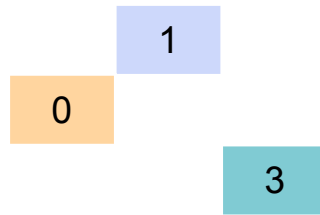
$$= 6$$



$$- 1 \cdot 2 \cdot 0$$



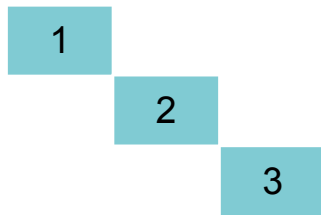
$$- 1 \cdot 2 \cdot 0$$



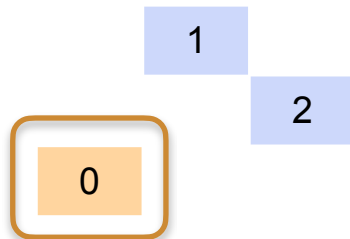
$$- 1 \cdot 0 \cdot 3$$

# The determinant

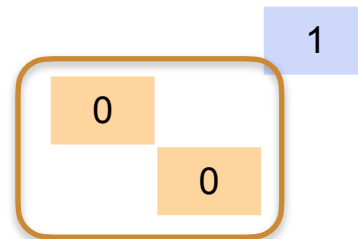
|   |   |   |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 2 | 2 |
| 0 | 0 | 3 |



$$+ 1 \cdot 2 \cdot 3$$



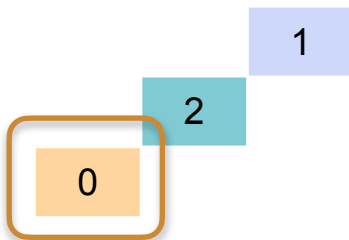
$$+ 1 \cdot 2 \cdot 0$$



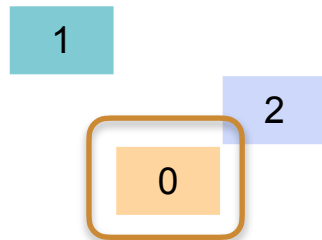
$$+ 1 \cdot 0 \cdot 0$$

$$\text{Det} = 6 + 0 + 0 - 0 - 0 - 0$$

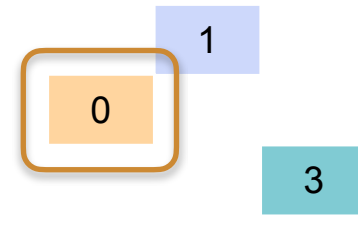
$$= 6$$



$$- 1 \cdot 2 \cdot 0$$



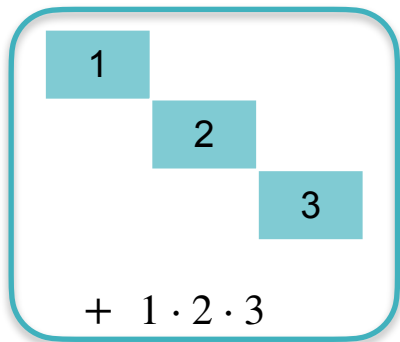
$$- 1 \cdot 2 \cdot 0$$



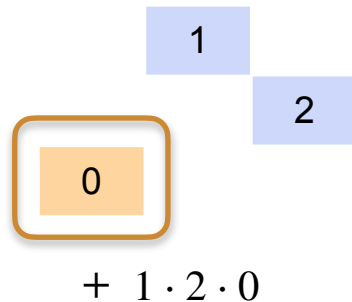
$$- 1 \cdot 0 \cdot 3$$

# The determinant

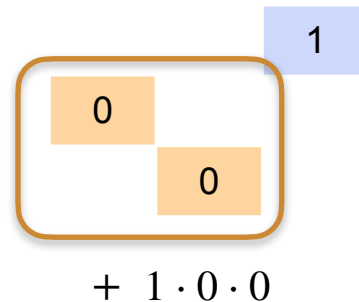
|   |   |   |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 2 | 2 |
| 0 | 0 | 3 |



$$+ 1 \cdot 2 \cdot 3$$



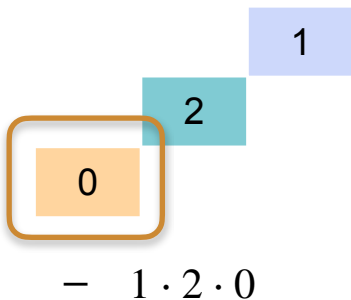
$$+ 1 \cdot 2 \cdot 0$$



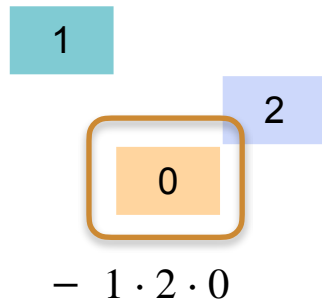
$$+ 1 \cdot 0 \cdot 0$$

$$\text{Det} = 6 + 0 + 0 - 0 - 0 - 0$$

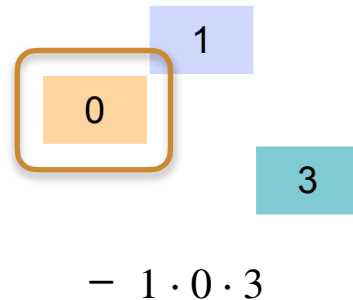
$$= 6$$



$$- 1 \cdot 2 \cdot 0$$



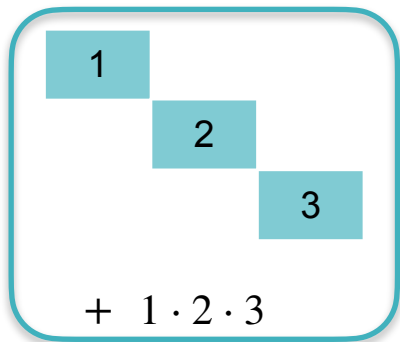
$$- 1 \cdot 2 \cdot 0$$

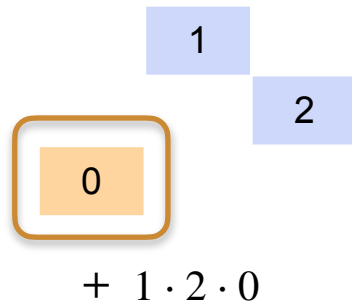


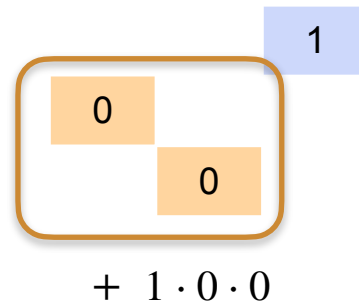
$$- 1 \cdot 0 \cdot 3$$

# The determinant

|   |   |   |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 2 | 2 |
| 0 | 0 | 3 |

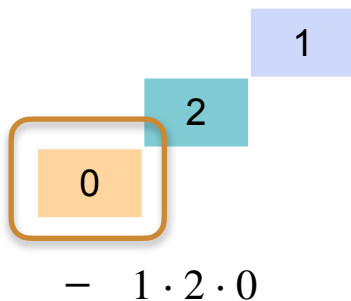

$$+ 1 \cdot 2 \cdot 3$$

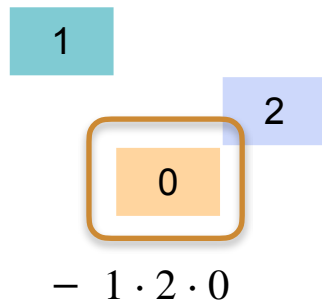

$$+ 1 \cdot 2 \cdot 0$$

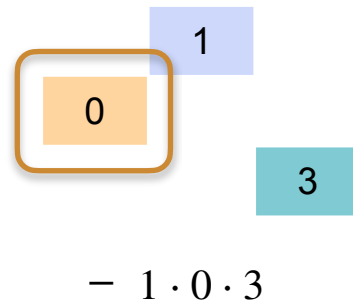

$$+ 1 \cdot 0 \cdot 0$$

$$\text{Det} = 6 + 0 + 0 - 0 - 0 - 0$$

$$= 6$$


$$- 1 \cdot 2 \cdot 0$$


$$- 1 \cdot 2 \cdot 0$$

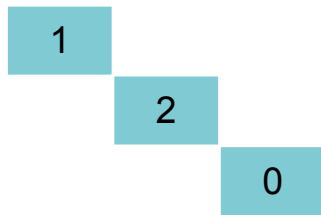

$$- 1 \cdot 0 \cdot 3$$

# The determinant

|   |   |   |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 2 | 2 |
| 0 | 0 | 0 |

# The determinant

|   |   |   |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 2 | 2 |
| 0 | 0 | 0 |



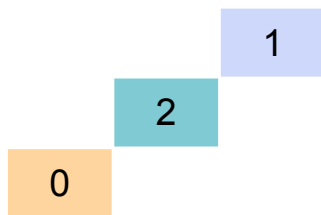
$$+ 1 \cdot 2 \cdot 0$$



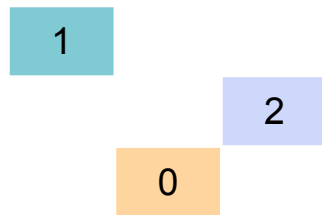
$$+ 1 \cdot 2 \cdot 0$$



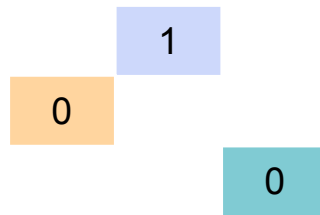
$$+ 1 \cdot 0 \cdot 0$$



$$- 1 \cdot 2 \cdot 0$$



$$- 1 \cdot 2 \cdot 0$$

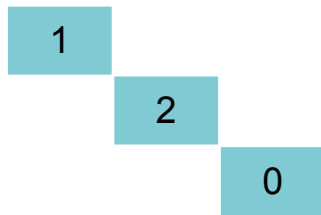


$$- 1 \cdot 0 \cdot 0$$



# The determinant

|   |   |   |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 2 | 2 |
| 0 | 0 | 0 |



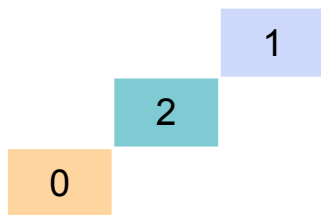
$$+ 1 \cdot 2 \cdot 0$$



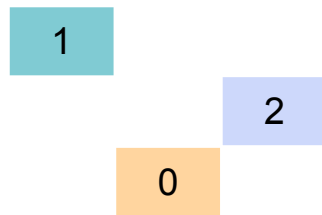
$$+ 1 \cdot 2 \cdot 0$$



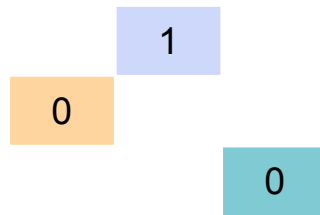
$$+ 1 \cdot 0 \cdot 0$$



$$- 1 \cdot 2 \cdot 0$$



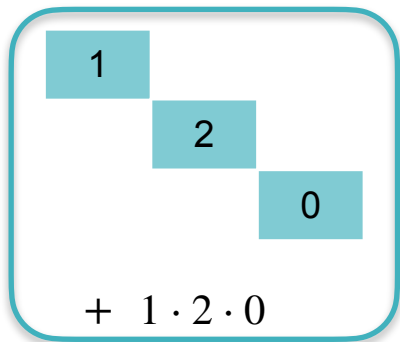
$$- 1 \cdot 2 \cdot 0$$

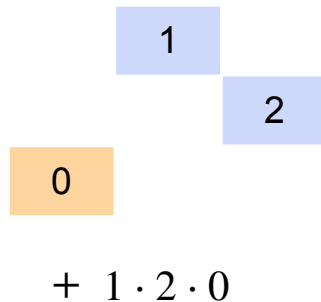


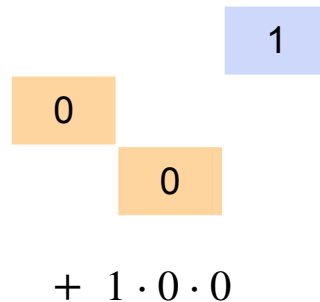
$$- 1 \cdot 0 \cdot 0$$

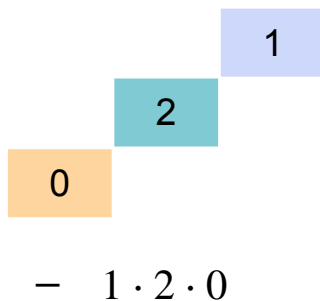
# The determinant

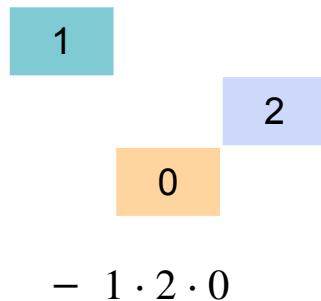
|   |   |   |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 2 | 2 |
| 0 | 0 | 0 |

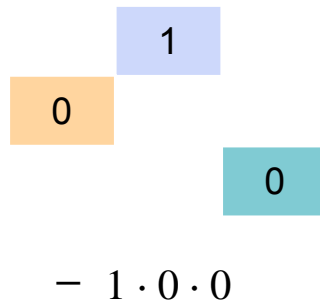

$$+ 1 \cdot 2 \cdot 0$$


$$+ 1 \cdot 2 \cdot 0$$


$$+ 1 \cdot 0 \cdot 0$$

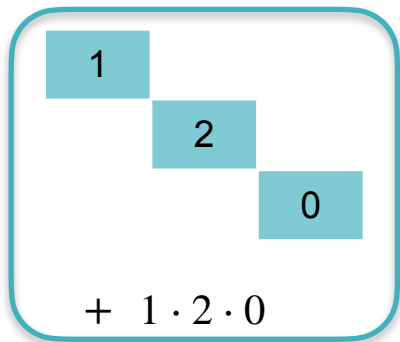

$$- 1 \cdot 2 \cdot 0$$


$$- 1 \cdot 2 \cdot 0$$


$$- 1 \cdot 0 \cdot 0$$

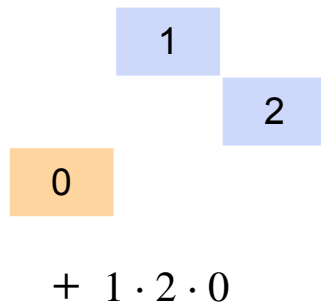
# The determinant

|   |   |   |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 2 | 2 |
| 0 | 0 | 0 |

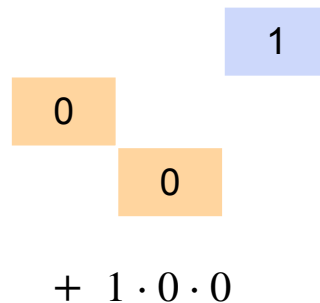


$$+ 1 \cdot 2 \cdot 0$$

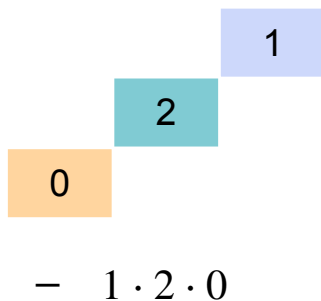
$$\text{Det} = 0+0+0-0-0-0$$



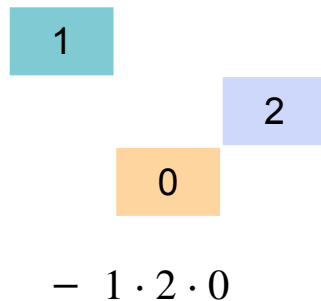
$$+ 1 \cdot 2 \cdot 0$$



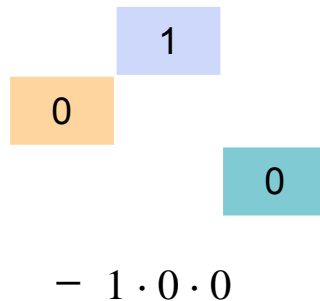
$$+ 1 \cdot 0 \cdot 0$$



$$- 1 \cdot 2 \cdot 0$$



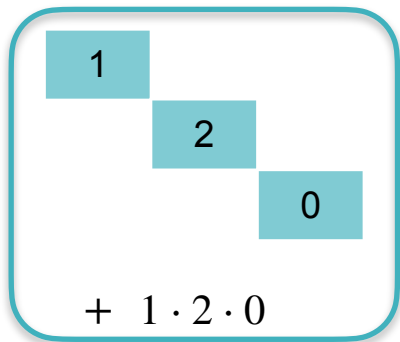
$$- 1 \cdot 2 \cdot 0$$



$$- 1 \cdot 0 \cdot 0$$

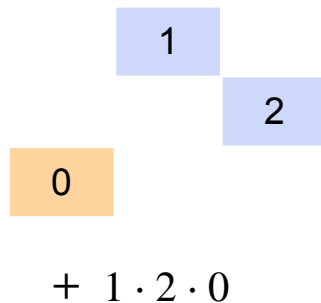
# The determinant

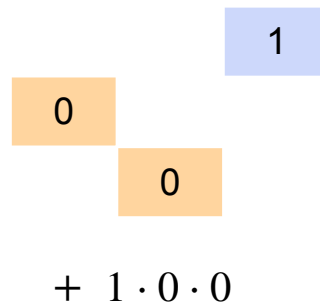
|   |   |   |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 2 | 2 |
| 0 | 0 | 0 |

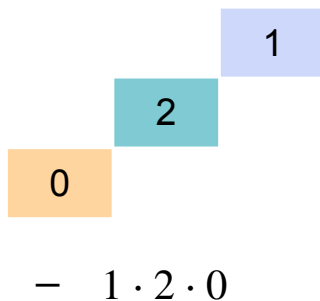

$$+ 1 \cdot 2 \cdot 0$$

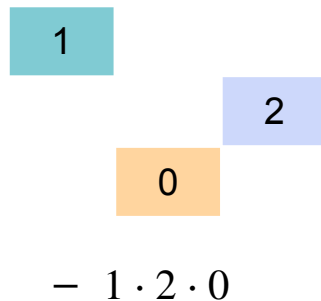
$$\text{Det} = 0+0+0-0-0-0$$

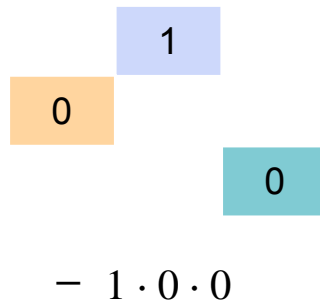
$$= 0$$


$$+ 1 \cdot 2 \cdot 0$$


$$+ 1 \cdot 0 \cdot 0$$


$$- 1 \cdot 2 \cdot 0$$


$$- 1 \cdot 2 \cdot 0$$


$$- 1 \cdot 0 \cdot 0$$



DeepLearning.AI

# System of Linear Equations

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## **Conclusion**