INF210 Overview

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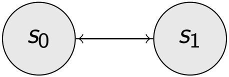
State Transition Systems

- a tuple (S, R)
- $S = \{s_0, s_1 ...\}$
- $R \subseteq S \times S$
- example: switch $= (\{s_0, s_1\}, \{s_0 \times s_1, s_1 \times s_0\})$



Important Properties of STSs

- if a state has no rules going *from* it, it is a **terminal state**.
- if there is a sequence of steps from state a to state b, b is **reachable** from a. Write $R^*(a, b)$.
- if R is a function (in the set theory sense), the STS is **deterministic**.
- if all paths eventually converge (clarify/formalize?) the STS is **confluent**.
- switch is deterministic and confluent, and has no terminal states.

Labeled Transition Systems

Syntax

- extends STSs with labels
- a 3-tuple (S, L, R).
- $R \subseteq S \times L \times S$, often written $s \xrightarrow{a} s'$

Reachability

- consider the STS (S_L, R_L) with:
  - $S_L = S \times L^*$
  - $s \times l \cdot \mathbf{w} \to s' \times \mathbf{w} \in R_L$ iff $s \xrightarrow{l} s' \in R$
- s' is reachable from s by $\mathbf{w}$ if $s' \times \lambda$ is reachable from $s \times \mathbf{w}$.
- write $s \times \mathbf{w} \vdash_R^* s' \times \lambda$

Finite Automata

- $M = (\Sigma, Q, q_0, \Upsilon, F)$
- a finite LTS with initial and final states
- the language accepted by M is
  $L_M = \{\mathbf{w} \mid q_0 \times \mathbf{w} \vdash^*_{\Upsilon} q_i \times \lambda \text{ and } q_i \in F\}$

Kleene's Theorem

Theorem

a language is regular $\iff$ *it is accepted by a finite automata*

$\Rightarrow$

Construct FAs to accept the empty language, $\{\lambda\}$ and singleton languages. Then construct FAs for union, concatenation, and Kleene star.

$\Leftarrow$

Define $R(i, k, j) =$
$\{\mathbf{w} \mid q_j$ is reachable from q_i without visiting q_m with $m \geq k\}$
Then show
$R(i, k+1, j) = R(i, k, j) \cup R(i, k, k) \cdot R(k, k, k)^* \cdot R(k, k, j)$