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A Very Good Title

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Abstract

Lorem ipsum dolor sit amet, his veri singulis necessitatibus ad. Nec insolens periculis ex. Te pro purto eros error, nec alia graeci placerat cu. Hinc volutpat similique no qui, ad labitur mentitum democritum sea. Sale inimicus te eum.

No eros nemore impedit his, per at salutandi eloquentiam, ea semper euismod meliore sea. Mutat scaevola cotidieque cu mel. Eum an convenire tractatos, ei duo nulla molestie, quis hendrerit et vix. In aliquam intellegam philosophia sea. At quo bonorum adipisci. Eros labitur deleniti ius in, sonet congue ius at, pro suas meis habeo no.

Acknowledgements

Est suavitate gubergren referrentur an, ex mea dolor eloquentiam, novum ludus suscipit in nec. Ea mea essent prompta constituam, has ut novum prodesset vulputate. Ad noster electram pri, nec sint accusamus dissentias at. Est ad laoreet fierent invidunt, ut per assueverit conclusionemque. An electram efficiendi mea.

`Åsmund Aqissiaq Arild Kløvstad`
Thursday 31st March, 2022

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Chapter 1

Introduction

1.1 Related works

In this section we summarize some key papers and their significance to the project.

1.1.1 Patch Theory (Darcs)

Here we discuss several proposed formalisms for a the patch theory employed by Darcs [1]. [7, 13, 5] all attempt to describe Darcs' patch theory. (focus on Lynagh, I think)

Lynagh [7] proposes an "algebra of patches" as a theoretical basis for the Darcs [1] version control system.

In this model a repository state is a set of updates (called *patches*, but we want to avoid that ambiguity) and a patch is a change to this set. For example pulling the repository $\{c\}$ into the repository $\{a, b\}$ results in a new repository $\{a, b\} \cup \{c\} = \{a, b, c\}$.

Patches are only applicable to one repository state, and result in a new state. If they are compatible, we may string them together into a *patch sequence*. Denoting the previous example patch by P and the "do-nothing" patch by Id we have $\{a, b\}P\{a, b, c\}Id\{a, b, c\}$ – pulling $\{c\}$ followed by doing nothing. The repository state may be omitted from sequences.

Finally a notion of *commutation* of patches is defined. We say the patch sequence AB commute if there are patches A' and B' such that the following square commutes:

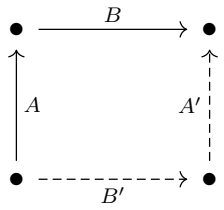

Figure 1.1: Commuting patches

and write $AB \leftrightarrow B'A'$. Note that the initial and final contexts (bottom left and top right, respectively) are the same, but the intermediary contexts need not be.

There are four axioms for patches and commutation:

1. Commutativity (lol) (3.1): $AB \leftrightarrow B'A' \iff B'A' \leftrightarrow AB$
2. Invertibility (3.2): for each A there is an A^{-1} s.t $AA^{-1} = A^{-1}A = Id$

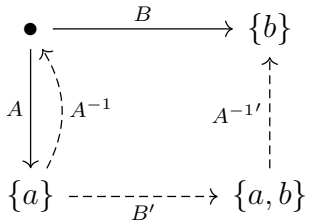

Figure 1.2: Merging A and B by commutation

3. Inv-cong (3.3): $AB \leftrightarrow B'A' \iff A^{-1}B' \leftrightarrow BA'^{-1}$. (we can start in the top left corner of Figure 1.1 if we want)

4. Circular (3.5/6): performing all pairwise commutations in a sequence gets us back to the beginning (or, a horrible equation)

These axioms allow us to define some useful operations on repositories. For example, given a span $\{a\} \xleftarrow{A} \bullet \xrightarrow{B} \{b\}$ we may want to incorporate the results of both patches to get $\{a, b\}$. We call this operation "merge" and proceed in three steps:

1. by invertibility, we can find a patch $\{a\}A^{-1}\bullet$

2. now that we have a sequence $A^{-1}B$, we commute it to get the sequence $B'A^{-1'}$

3. define merge(A, B) to be the sequence AB'.

This process is shown in Figure 1.2.

Another useful operation on repositories is "cherry picking". Cherry picking is the act of pulling some, but not all, patches from one repository into another. Consider the patch sequence $\{\}A\{a\}B\{a, b\}C\{a, b, c\}$ and a repository $\{a\}$. We want to incorporate the changes in C, but not the ones in B, but naively combining applying C does not work, since it is only applicable to the context $\{a, b\}$. The solution is to commute $BC \leftrightarrow C'B'$ (Figure 1.3) to obtain C' with the desired endpoints.

Problem: we cannot always commute patches, and Darcs does not have a great solution here.

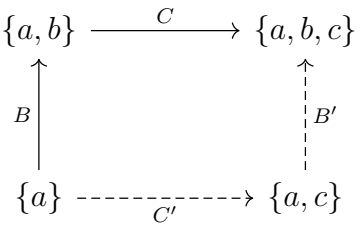

Figure 1.3: Commutation for cherry picking

1.1.2 A Categorical Theory of Patches

A Categorical Theory of Patches [9] defines a category of files and patches, such that a merge is a pushout. To ensure a merge is always possible they first construct the category $\mathcal{L}$ of files and patches, and then its conservative cocompletion $\mathcal{P}$.

$\mathcal{P}$ contains all finite colimits – and in particular all pushouts – so the merge of a span is always defined. The paper's chief achievement is the explicit construction of this category and these pushouts.

Interesting insights I'm not sure how to incorporate:

- the construction of $\mathcal{P}$ can be understood as the addition of *partially* ordered files to $\mathcal{L}$.
- "flattening" these partial orders leads to cyclic graphs. On editing text [4] objects, but maybe not correctly
- the poset structure of $\mathcal{L}$ and $\mathcal{P}$ is given explicitly by $\mathcal{G}$ and the nerve functor $N_{\_}$ (!!).

(maybe mention Pijul [2] (if so, figure out the relationship to [9])) (maybe some figures go here)

1.1.3 Homotopical Patch Theory

Homotopical Patch Theory [3] gives a formulation of patch theory in homotopy type theory. A patch theory is represented by a higher inductive type, and its interpretation by a function out of this type.

By representing repository state as points and patches as paths in a higher inductive type, the groupoid structure of the patch theory comes "for free". Paths come with composition, and by the groupoid laws this composition is associative, unital, and respects inverses. Additionally, functions (which are functors) respect this structure so any interpretation must also validate the groupoid laws.

Patch laws are represented by paths between paths (squares? disks? 2D-somethings). For example we may want the application of two independent patches to commute – this is done with a patch law.

While the HIT formulation gives a lot "for free", it also has some drawbacks. In particular, the requirement that all patches have inverses causes some problems. The workaround is to "type" patches with the history they are applicable to. This allows Angiuli et al. to define a merge operation in terms on only the "forward" patches, but leads to a fairly complex theory even for relatively simple settings.

An interesting feature of Angiuli et al.'s patch theories is that the type of repositories must be contractible. Since patches are represented by paths, any point can be retracted along them. As such, all repositories are – in a sense – "the same" and we need better notions of "sub-homotopical" [3] computations to reason about their differences.

1.1.4 Path Spaces of Higher Inductive Types

Path Spaces of Higher Inductive Types in Homotopy Type Theory [6] provides an induction principle for paths in coequalizers. This is extremely useful, since we want to define functions out of spans in HITs. (← rework this sentence)

Summarizing this will be very technical, and may become its own chapter if I successfully formalize the proof in cubical agda. Otherwise it goes here.

Chapter 2

Homotopy Type Theory

It's cool. [14]

The purpose of this section is to give the reader enough prerequisites to follow the ensuing development [pretentious af]. It follows, with numerous omissions, the development in Egbert Rijke's 2019 summer school [12]. [NOT REALLY, ANYMORE] For a more thorough treatment see [11] and for a complete textbook see The Book [14].

2.1 (Dependent) Type Theory

1. (dependent) types in computer science
2. type theories in math/foundations (the formal stuff)
3. Agda syntax?

Types are a familiar concept to the computer scientist. We are used to working with data, and this data often has a *data type* either explicitly or implicitly. For example, `42` is an `int`, `'c'` is a `char`, and `['a','b','c']` is a list of `char`s (henceforth denoted `[char]`). We call `int`, `char` and `[char]` *types* and `42`, `'c'`, `['a','b','c']` *terms* of those types. While this is a good basis for intuition, Type Theory (tm) is a bit different.

However, let us stick with the programming intuition to introduce a less familiar concept: *dependent* types. First, note that one of the types in the previous paragraph is a bit different than the others: ['a','b','c'] is a list *of* ***`char`****s*. Similarly we could have lists of `int`s, lists of `float`s or even lists of lists! Clearly "lists" comprises many different types, depending on the type of their elements. We could call `list` a family of types *parametrized* by types. Such a family is actually a whole collection of types – one for each other type we can make lists of. Dependent types extend this idea by allowing families to be parametrized by terms. Then we can create new and exciting types like `Vec 3` and `Vec 4` – the types of 3- and 4-dimensional vectors. Again `Vec` is actually a whole collection of types – one for each integer!

We now leave the familiar world of programming behind and venture in to the spooky (but exciting) world of foundational mathematics.

$$\frac{\Gamma \vdash a : A \qquad \Gamma \vdash f : A \to B}{\Gamma \vdash f(a) : B} \tag{2.1}$$

In this new and wondrous world, a type theory is a system of *inference rules* like 2.1 that can be used to make *derivations*.

This particular inference rule is the elimination rule for function types. It says that if a is a term of type A and f is a function from A to B, then $f(a)$ is a term of type B. Let us take it apart.

The part above the line is a list of hypotheses, and the part below is the conclusion.

Each piece of the rule is called a *judgement*. They consist of a context, some expression and a $\vdash$ separating the two. In this example our judgements are:

$$\Gamma \vdash a : A$$

"In any context Γ, a is a term of type A"

$$\Gamma \vdash f : A \to B$$

"In any context Γ, f is a function from A to B"

$$\Gamma \vdash f(a) : B$$

"In any context Γ, $f(a)$ is a term of type A"

In fact these are all the same kind of judgement: a particular term $(a, f, f(a))$ is of a particular type $(A, A \to B, B)$. There are three other kinds of judgements permitted in (Martin-Löf) type theory [NOTE: THESE ARE JUDGEMENTAL EQUALITIES, DISTINCT FROM IDENTITY TYPES. MAKE CLEAR AND SETTLE ON SYNTAX]:

$$\Gamma \vdash A\ Type$$

"A is a type."

$$\Gamma \vdash a \equiv b : A$$

"a and b are judgementally equal terms of type A."

$$\Gamma \vdash A \equiv B\ Type$$

"A and B are judgementally equal types."

The judgement form $\Gamma \vdash A\ Type$ lets us formally define lists and vectors. Lists are easy:

$$\frac{\Gamma \vdash A\ Type}{\Gamma \vdash [A]\ Type}$$

This rule says "if A is a type, then lists of A is a type". Using $\mathbb{N}$ for the type of natural numbers, vectors are very similar:

$$\frac{\Gamma \vdash n : \mathbb{N}}{\Gamma \vdash Vec(n)\ Type}$$

Finally, we consider two important families of dependent types: Σ-types (sometimes called "dependent pairs") and Π-types ("dependent functions"). These are, respectively, pairs / functions where we let the second projection / codomain depend on the value of the first projection / domain. [THIS IS IMPRECISE AND USES WORDS WE HAVE NOT INTRODUCED].

$$\frac{\Gamma \vdash A\ Type \qquad x : A \vdash B(x)\ Type}{\Gamma \vdash \Sigma_A B\ Type} \qquad \frac{\Gamma \vdash x : A \qquad \Gamma \vdash y : B(x)}{\Gamma \vdash (x, y) : \Sigma_A B}$$

This pair of rules (called an introduction/formation rule and an elimination(?) rule) tells us that

1. if A is a type, and B is a type family over A [INTRODUCE PHRASING?], then we can make the type $\Sigma_A B$ of dependent pairs
2. if we have a term x of type A and a term y of $B(x)$ we can create a term (x, y) of type $\Sigma_A B$

The analogous rules for dependent functions are:

$$\frac{\Gamma \vdash A\ Type \qquad x : A \vdash B(x)\ Type}{\Gamma \vdash \Pi_A B\ Type} \qquad \frac{x : A \vdash b(x) : B(x)}{\Gamma \vdash \lambda x.b(x) : \Pi_A B}$$

When introducing a new type, we do so with a collection of rules. [NOT SURE ABOUT THIS BIT, MAYBE EXPLAIN WITH Π AND/OR Σ?]

$$\frac{\Gamma \vdash A\ Type \qquad \Gamma \vdash B\ Type}{\Gamma \vdash A \to B\ Type} \tag{2.2}$$

$$\frac{\Gamma \vdash f : A \to B}{\Gamma, a : A \vdash f(a) : B} \tag{2.3}$$

$$\frac{\Gamma \vdash B\ Type \qquad \Gamma, a : A \vdash f(a) : B}{\Gamma \vdash \lambda x.f(x) : A \to B} \tag{2.4}$$

$$\frac{\Gamma \vdash f : A \to B}{\Gamma \vdash \lambda x.f(x) \equiv f : A \to B} \tag{2.5}$$

$$\frac{\Gamma \vdash B\ Type \qquad \Gamma, a : A \vdash f(a) : B}{\Gamma, a : A \vdash (\lambda y.f(y))(a) \equiv f(a) : B} \tag{2.6}$$

Theses are (respectively, I will find a better way to label them) an introduction rule, an elimination rule, lambda abstraction (what do we call this one in general?) and two computation rules known as β- and η-reduction.

2.2 Types and Propositions (and spaces?)

1. types represent propositions (and spaces)
2. implication and simple and/or $(\to, \times, +)$
3. quantifiers and dependent types (fibers) (Σ, Π)

In this section we consider an important interpretation of type theory: the Howard-Curry Isomorphism (which isn't an isomorphism, but we're not going into those details).

Under this "isomorphism" types are identified with logical propositions, and terms with proofs of those propositions. This means we can consider a proposition "true" (or at least "proved") if we can construct a term of the corresponding type.

Two very simple types are the empty type $\bot$ which has not terms, and the unit type $\top$ which has one term denoted by $\mathbf{1}$. [MAYBE INTRODUCE THE TYPES FIRST]

Under the "types as propositions" interpretation, $\bot$ represents *false*. The type has no terms so there are no proofs of "false", just like we would expect from a sound system. (Of course this alone does not prove our type theory sound.) Similarly, $\top$ represents *true*. It always has a proof: $\mathbf{1}$.

Let us make some more elaborate propositions. For example given the types (and hence propositions) A and B what would it mean to prove $A \wedge B$? Well if both A and B are true, we should be able to give a proof of A *and* and proof of B. But since proofs are terms of the corresponding type, this is the same as having terms $a : A$ and $b : B$. To keep track of both, lets form the ordered pair (a, b). This is precisely an element of the product type $A \times B$! Hence this product type represents the proposition $A \wedge B$, since its terms correspond exactly to proofs of A and B.

As a sanity check, consider the truth table of $A \wedge B$ (2.1a) alongside the terms of $A \times B$ (2.1b) using $\top$ and $\bot$ to represent true and false. $A \wedge B$ is true when both A and B are true, and similarly $A \times B$ is inhabited exactly when both A and B are inhabited.

A	B	$A \wedge B$
false	false	false
false	true	false
true	false	false
true	true	true

(a) logic

A	B	$A \times B$
$\bot$	$\bot$	()
$\bot$	$\top$	()
$\top$	$\bot$	()
$\top$	$\top$	$(\mathbf{1}, \mathbf{1})$

(b) types "()" meaning there are no terms of this type

As another example, what does it mean to prove an implication $A \to B$? One reasonable answer is that given a proof of A, I can produce a proof of B. In terms of types, that means a way to produce a term of type B given a term of type A, which is exactly a function from A to B! Finally, note that logical "or" is represented by the sum type (disjoint union) $A + B$.

[NOTE: this results in a *constructive* logic (good)]

We have the basic building blocks of propositional logic, but what about first-order logic with $\exists$ and $\forall$? This is where our dependent types come in handy.

First, let us note that a predicate on a variable is a lot like a dependent type. If simple types can be interpreted as propositions, and a predicate on some variable is a proposition that *depends* on a variable, then it stands to reason that a predicate can be represented with a dependent type. As such, we may view a term of the type $B(x)$ as a proof that B holds for the term x. [WHATEVER THAT MEANS, LOL]

Extending this thinking to quantifiers and considering what it means to provide a proof, a proof of $\exists x.P(x)$ should consist of some $x : A$ [CHEATING IN THE A] and a proof that P is true of x. Such a pair is a term of a type we have seen before: the dependent pair $\Sigma_A P$. Note that this term actually contains *more* data than just asserting $\exists x.P(x)$ – it gives us an x.

Similarly, a proof of $\forall x.P(x)$ can be seen as an assertion that whatever $x : A$ you give me, I can show that produce a proof (term) of $P(x)$. We use $\Pi_A P$ to represent this quantification. Note that both of these constructions quantify *over* some base type A, not over "every x in the universe," whatever that means. [REVISIT]

2.3 Identity Types

1. what about things that are equal?
2. J-rule (intuition: reflexive closure? groupoid structure?)
3. paths in space

Given this notion of propositions as types, one of the things we may want to propose (and prove) is the equality of two terms. That is, given two terms of some type, how do we show that they are equal? Note that this is different from the *judgemental* equality discussed in section 2.1. [HOW EXACTLY?]

Since propositions are types and "x is equal to y" is a proposition, there should be a corresponding type. Also, the truth of this proposition depends on x and y (clearly "2 is equal to 2" should be different from "2 is equal to 3") so the type should depend on x and y as well. But how should this type be constructed? What are the terms of such a type?

The solution, proposed by Per Martin-Löf [8], is an inductive family of dependent types called the *identity type*. For each type A and pair of terms $x, y : A$ we construct the identity type $x =_A y$ (the subscript may be dropped when the type of x and y is clear). It has the following formation and introduction rules [12]:

$$\frac{\Gamma \vdash a : A}{\Gamma, x : A \vdash a =_A x \; Type} \qquad \frac{\Gamma \vdash a : A}{\Gamma \vdash refl_a : a =_A a}$$

and an induction principle given by:

$$\frac{\Gamma \vdash a : A \qquad \Gamma, x : A, p : a =_A x \vdash P(x,p) Type}{\Gamma \vdash J_a : P(a, refl_a) \to \Pi_{x:A} \Pi_{p:a=_A x} P(x,p)}$$

This is astonishingly simple! The identity type has one constructor: $refl\_$, and in order to use its terms it is enough to know how to use $refl$.

[MAYBE SOMETHING ABOUT THE GROUPOID STRUCTURE AND UIP]

One way to make sense of identity types is through the homotopy theory. With this interpretation a term of $x =_A y$ is like a path in A from x to y. In fact the collection of all such paths is itself a space (and thus a type): the path space. Additionally there may be paths between paths, paths between paths between paths and so on. These higher paths are the eponymous "homotopies" and provide a rich field of study on their own. [REVISIT]

2.4 Higher Inductive Types

1. inductive types: base case(s) and point generator(s)
2. example(s)
3. HIGHER inductive types: terms and identities
4. ie. points and paths between points (and paths between paths (and paths between paths between paths))
5. example(s)
6. elimination rules? they need to go somewhere, but this might not be it

2.4.1 Inductive Types

One way to construct more elaborate types is by induction. An inductive type is defined by a number of constructors, which can be either constant terms or functions. Let us return to the type of lists. It can be constructed from the empty list and the function `cons` which takes an element and affixes it to the start of a list. Using [] for the empty list and :: for the (infix) cons function we have a pair of introduction rules:

$$\frac{\Gamma \vdash A\ Type}{\Gamma \vdash [] : [A]} \qquad \frac{\Gamma \vdash a : A \qquad \Gamma \vdash as : [A]}{\Gamma \vdash (a :: as) : [A]}$$

From these we can construct arbitrarily long lists by starting with the empty list and affixing new terms of A to obtain [], $(a :: [])$, $(a' :: (a :: []))$ etc.

In order to use this type, we also need an elimination rule (or recursion principle in the non-dependent case). The recursion principle tells us how to use terms of the type by defining functions out of it and will be familiar to anyone who has written a recursive function on lists.

$$\frac{\Gamma \vdash b_0 : B \qquad \Gamma \vdash b_{cons} : A \times [A] \to B}{\Gamma \vdash rec_{[A]}(b_0, b_{cons}) : [A] \to B}$$

In words, this rule states that you can construct a function from $[A]$ to B if you have a term b_0 and a function that takes a pair of an A and a list to produce a B. As one might expect, the resulting function maps [] to b_0 and $(a :: as)$ to $b_{cons}(a, as)$. (By a computation rule that we also need to specify).

[MAYBE GIVE A MORE GENERAL TREATMENT LIKE IN [12] (4.1)]

2.4.2 Higher Inductive Types

This doesn't really make sense without the interpretation of types as spaces, huh..

1. motivation: "natural" (sic) extension, synthetic topology
2. example: the circle
3. cubical and non-cubical elimination

When constructing ever more complicated types, it would be nice to have some control over which terms are identified. [Examples? Just quotients, maybe?]

One way to do this is *Higher Inductive Types*. Like inductive types, HITs are constructed from generators, but while the generators of an inductive type may only generate terms, the generators of a HIT may also generate paths.

[ELIMINATION RULES. "VARY CONTINUOUSLY"]

[SYNTHETIC TOPOLOGY TIME]

The prototypical example of a HIT is the circle S^1, because it is very simple comprising only a single point and one path. Its introduction and formation rules are:

$$\frac{}{\Gamma \vdash S^1\ Type} \qquad \frac{}{\Gamma \vdash base : S^1} \qquad \frac{}{\Gamma \vdash loop : base =_{S^1} base} \tag{2.7}$$

2.5 Agda

Introduce the Agda syntax so we can use it later (maybe)

2.6 Cubical?

Why not take "= is a path" seriously?

Chapter 3

Version Control Systems

They're not always cool. [10]

Version control systems are ubiquitous in software development, where they help facilitate cooperation and documentation of the development process. Their basic use is to record (*commit*) changes to a codebase (*repository*). Systems may also include ways for the codebase to diverge (*branch*) into different versions, and ways to reunite (*merge*) these versions.

The purpose of this section is to introduce the terminology, requirements and hopes for models of version control systems,

What do we need?

1. terms
    - repository
    - patch
    - merge
    - branch?
2. requirements
    - repo - accurately represents contents

- patch - applicable in a context, groupoid structure
- merge - "pushout property"/reconcile, symmetric (for distributed systems), (do we need associativity as well?)

3. hopes/goals

    - repos - modular/composable, somehow polymorphic
    - patches - *semantic* in some sense
    - merge - easily definable [sic.], considers semantics of patches

Chapter 4

Conclusion

We did some things and they worked out — or maybe they didn't.

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Appendix A

This is an appendix, if need be