Artificial intelligence
University of Rennes 1
ESIR
2020/21

Excercice1: Revision dot product Let $a = (a_1, \dots, a_n)$, and $b = (b_1, d \dots, b_n)$ be two n-dimensional vectors ($n \geq 1$). Their dot product is defined as

$$ab = \Sigma_{i=i}^{n} a_i b_i$$

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- Compute the dot product of the vectors $a(1, 3, -5)$ and $b(4, -2, -1)$
- What can we say about the dot product ab if a a and b are orthogonal? or if they are co-directional (their enclosed angle is 0)?
- What can we say about the value
$$\frac{ab}{|a||b|}$$
?

Excercice 2: Revision calculus Compute the derivatives of the following functions

- $f(x) = x^2$
- $f(x) = x^3$
- $f(x) = e^2$
- $f(x) = log(x)$
- Chain rule: let $f(x) = 6x + 3$ and $g(x) = -2x + 5$ and let $h(x) = f(g(x))$. Compute the derivative $\frac{dh(x)}{dx}$
- Let $f(x) = e^x$ and $g(x) = 4x$ and let $h(x) = f(g(x))$. Compute the derivative $\frac{dh(x)}{dx}$
- $f(x) = e^{x^2}$
- $f(x) = log(x^2)$
- $f(x) = \frac{1}{1+x^2}$

Excercice 3: Revision calculus (multiple variables)

- $f(x, y) = x^2 y^3$, Compute the following partial derivatives: $\frac{\partial f}{\partial x} =?$, $\frac{\partial f}{\partial y} =?$. Compute the gradient vector at $(3, 2)$, that is $\nabla f(3, 2)$. Also computer $\nabla f(0, 0)$ and $\nabla f(0, 1)$.
- $f(x, y) = (x + y)^2$ Compute the following partial derivatives: $\frac{\partial f}{\partial x} =?$, $\frac{\partial f}{\partial y} =?$.
- $f(x, y) = log(2x + y^2)$ Compute the following partial derivatives: $\frac{\partial f}{\partial x} =?$, $\frac{\partial f}{\partial y} =?$.

Excercice 4: Linear regression Assume that we have some data points $(x_1, y_1), \ldots, (x_n, y_n)$. We would like to use linear regression to fit a line to explain our data ($h(x) = \theta_1 x + \theta_0$). Use differential calculus to optimize the fitting of our models. Recall the cost function (mean squared errors): $J(\theta_0, \theta_1) = \frac{1}{n} \sum_{i=1}^{n} (h(x_i) - y_i)^2$.

- Computer the partial derivatives $\frac{\partial J}{\partial \theta_0}$, $\frac{\partial J}{\partial \theta_1}$
- Find the values of θ_0 and θ_1 for which the loss function is minimal.