Normalized Trajectories

Author: Michel Barbeau, Carleton University

Version: February 23, 2019

This is a companion Live Script to the paper "Tuning the Demodulation Frequency Based on a Normalized Trajectory Model for Slow Speed Underwater Acoustic Communications".

Example normalization

In the Cartesian plane, let point p be the coordinate pair $(-3535.5, 3535.5)$, which corresponds to a distance of 5 km. Let the velocity vector $\overrightarrow{V}$ be $(1, 0)$, which corresponds to a velocity of 3.6 km/s. The required clockwise rotation for normalization is

$$-\theta = -\arctan(3535.5/-3535.5) = 45 \text{ degrees.}$$

The corresponding rotation matrix is

$$R(\theta) = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} 0.71 & 0.71 \\ -0.71 & 0.71 \end{bmatrix}.$$

In MATLAB:

```
clear;
p= [-3535.5 3535.5]; % point
V = [1 0];
R=[0.71 0.71; -0.71 0.71]; % rotation matrix
present(R*p'); % rotation of point
```

```
th = 2×1
10^3 ×
  -0.0000
   5.0204
```

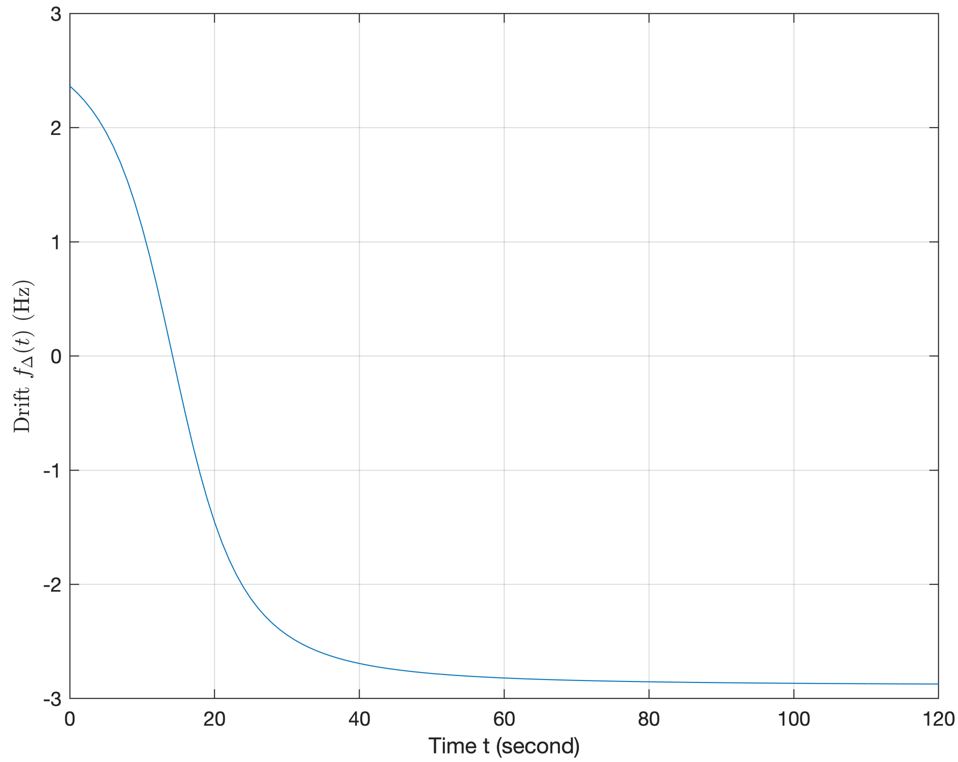
```
present(R*V'); % rotation of velocity vector
```

```
th = 2×1
   0.7100
  -0.7100
```

Plot of Doppler shift:

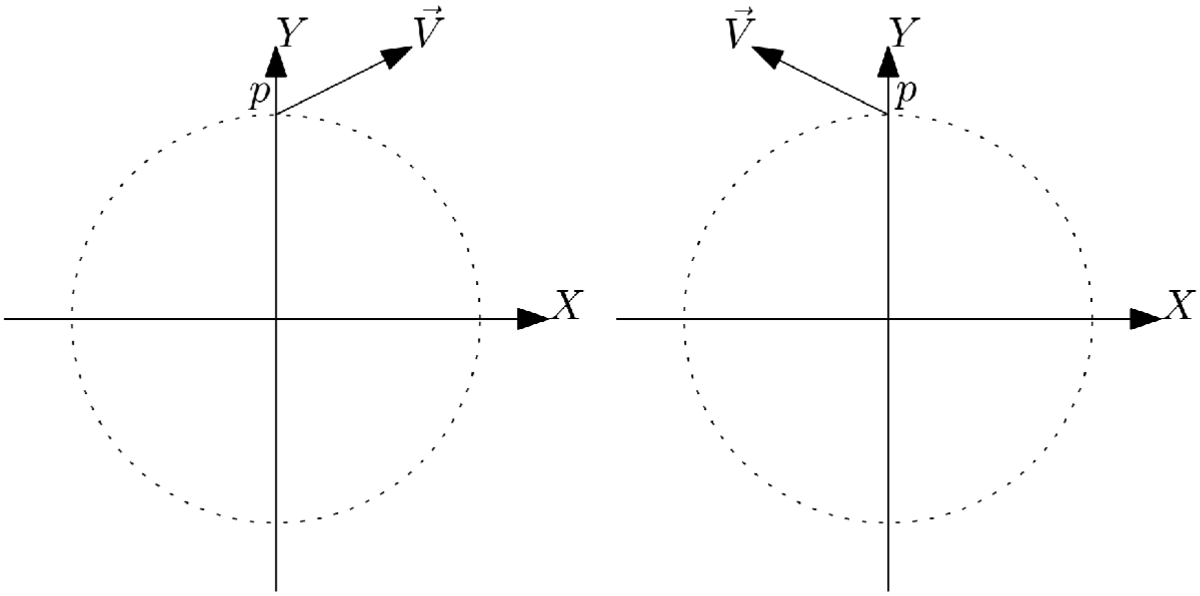
```
% time points
t=0:120; % seconds
% initial position
p = [0 50]; % meters
% velocity vector
V = (R*[0.5 -2.83]')'; % m/s
% nominal frequency
f0 = 1500; % Hz
% sound speed
c = 1500; % m/s
```

```
fs = dopplervstime(p, V, t, f0, c);
figure;
plot(t,fs);
xlabel('Time t (second)');
ylabel('Drift $$f_{\Delta } (t)$$ (Hz)','interpreter','latex');
grid on;
```

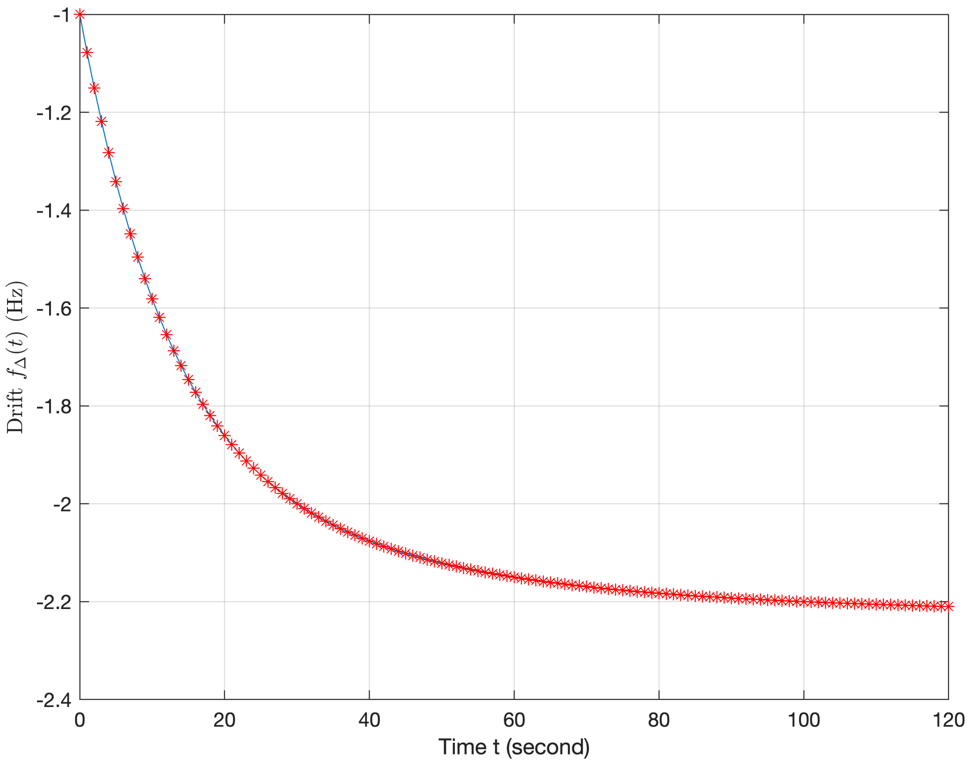


Example symmetry

Two different trajectories, after 90 degrees rotation for the case on the left, both cases have the same initial position p and symmetrical velocity vectors $\vec{V}$:



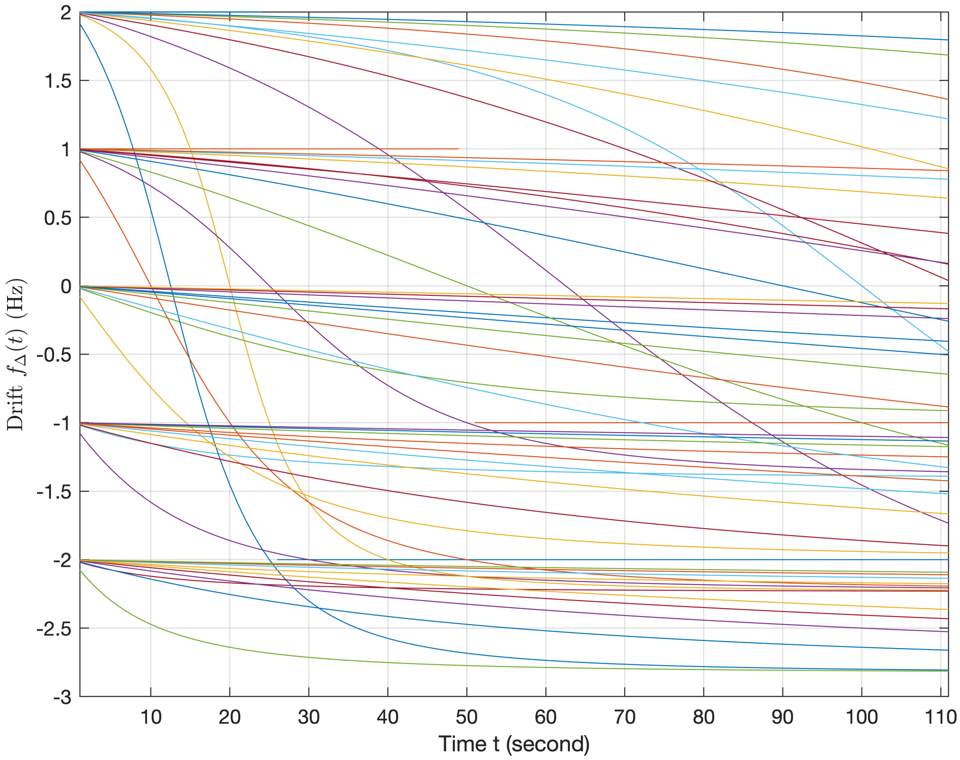

```
% time points
t=0:120; % seconds
%
%%% left case
%
% rotation matrix, for normalization
R=[0 1; -1 0]; % clockwise 90 degrees
% normalized initial position
p = (R*[-50 0]')'; % meters
% normalized velocity vector
V = (R*[-1 2]')'; % m/s
% nominal frequency
f0 = 1500; % Hz
% sound speed
c = 1500; % m/s
fs  = dopplervstime(p, V, t, f0, c);
figure;
plot(t,fs);
xlabel('Time t (second)');
ylabel('Drift $$f_{\Delta } (t)$$ (Hz)','interpreter','latex');
grid on;
hold on;
% normalized initial position
p = [0 50]; % meters
% normalizd velocity vector
V = [-2 1]; % m/s
% nominal frequency
f0 = 1500; % Hz
% sound speed
c = 1500; % m/s
fs  = dopplervstime(p, V, t, f0, c);
plot(t,fs,'r*');
```



Plot of an enumration of normal trajectories

```
clear;
figure;
% nominal frequency
f0 = 1500; % Hz
t=1:111; % seconds
% sound speed
c = 1500; % m/s
for p2=50:200:850 % initial position loop
    % initial position
    p = [0 p2]; % meters
    for V1=0:1:2 % 1st component of velocity vector loop
            for V2=-2:1:2 % 2nd component of velocity vector loop
                % when velocity vector and y-axis are co-linear,
                % skip if velocity is positive or vehicle does not cross
                % x-axis
                if ~(V1==0 && V2>=0) && ~(V1==0 && -p2/V2>111)
                    % velocity vector
                    V = [V1 V2]; % m/s
                    fs  = dopplervstime(p, V, t, f0, c);
                    plot(t,fs);
                    hold on;
                end
            end
    end
```

```
end
xlabel('Time t (second)');
ylabel('Drift $$f_{\Delta } (t)$$ (Hz)','interpreter','latex');
grid on;
xlim([1 111]);
```



MATLAB implementation of Corollary 2

```
function fdeltas = dopplervstime(p, V, t, f0, c)
% p = relative initial vehicle positions
% v = relative velocity (m/s)
% t = time points
% f0 = nominal frequency (in Hz)
% c = sound speed (in m/s)
% outputs
    fdeltas = []; % frequency shit vs time
    for i=1:length(t)
        q=t(i)*V+p;
        fdelta = -sign(q(1)*V(1)+q(2)*V(2))*...
            abs(dot(V,q))/sqrt(q(1)^2+q(2)^2)*f0/c;
        fdeltas = [ fdeltas fdelta ];
    end
end
```