

ENVT3362 Module 3 - Worked Solutions

Case Study 1

1. Using the concept of an energy balance (i.e. energy in = energy out), rearrange the equations to find the Earth's temperature, T :

$$\text{Let } E_{\text{out}} = \sigma 4\pi r^2 T^4$$

$$\text{and } E_{\text{abs}} = S\pi r^2(1 - a)$$

$$\text{then } E_{\text{out}} = E_{\text{abs}}$$

$$\sigma 4\pi r^2 T^4 = S\pi r^2(1 - a)$$

$$T^4 = \frac{S\pi r^2(1 - a)}{\sigma 4\pi r^2}$$

$$T^4 = \frac{S\cancel{\pi r^2}(1 - a)}{\sigma 4\cancel{\pi r^2}}$$

$$T = \sqrt[4]{\frac{S(1 - a)}{4\sigma}}$$

Case Study 2

1. Solve the following equation for T :

$$\pi r^2 S(1 - a) = 4\pi r^2(x + yT)$$

$$\pi r^2 S(1 - a) = 4\pi r^2 x + 4\pi r^2 yT$$

$$\pi r^2 S(1 - a) - 4\pi r^2 x = 4\pi r^2 yT$$

$$T = \frac{\pi r^2 S(1 - a) - 4\pi r^2 x}{4\pi r^2 y}$$

Case Study 3

Energy-balance for the Earth-atmosphere system as a whole:

$$S_{av} + W = a_p S_{av} + \sigma T_u^4 + (1 - \varepsilon)\sigma T_s^4 \quad (1)$$

Energy-balance for the upper layer of the atmosphere:

$$k_u S_{av} + \sigma T_l^4 + 0.5L = 2\sigma T_u^4 \quad (2)$$

Energy-balance for the lower layer of the atmosphere:

$$k_l S_{av} + \sigma T_u^4 + \varepsilon\sigma T_s^4 + 0.5L + H + W = 2\sigma T_l^4 \quad (3)$$

1. Rearrange Equation 1 for σT_u^4 in terms of T_s and parameters:

$$S_{av} + W = a_p S_{av} + \sigma T_u^4 + (1 - \varepsilon)\sigma T_s^4$$

$$\sigma T_u^4 = S_{av} + W - a_p S_{av} - (1 - \varepsilon)\sigma T_s^4$$

2. Rearrange Equation 2 for σT_l^4 in terms of T_u and parameters:

$$\begin{aligned} k_u S_{av} + \sigma T_l^4 + 0.5L &= 2\sigma T_u^4 \\ \sigma T_l^4 &= 2\sigma T_u^4 - k_u S_{av} - 0.5L \end{aligned}$$

3. Substitute the results of Step 1 into the results of Step 2 to give an equation for σT_l^4 in terms of T_s and parameters.

$$\begin{aligned} \text{if } \sigma T_u^4 &= S_{av} + W - a_p S_{av} - (1 - \varepsilon)\sigma T_s^4 \\ \text{and } \sigma T_l^4 &= 2\sigma T_u^4 - k_u S_{av} - 0.5L \\ \text{then } \sigma T_l^4 &= 2(S_{av} + W - a_p S_{av} - (1 - \varepsilon)\sigma T_s^4) - k_u S_{av} - 0.5L \\ \sigma T_l^4 &= 2S_{av} + 2W - 2a_p S_{av} - 2(1 - \varepsilon)\sigma T_s^4 - k_u S_{av} - 0.5L \end{aligned}$$

4. Rearrange Equation 3 for $\varepsilon\sigma T_s^4$ in terms T_l, T_u , and parameters.

$$\begin{aligned} k_l S_{av} + \sigma T_u^4 + \varepsilon\sigma T_s^4 + 0.5L + H + W &= 2\sigma T_l^4 \\ \varepsilon\sigma T_s^4 &= 2\sigma T_l^4 - k_l S_{av} - \sigma T_u^4 - 0.5L - H - W \end{aligned}$$

5. Put the results of Steps 1 and 3 into the results of Step 4 and simplify to give the equation for T_s .

$$\begin{aligned} \text{if } \varepsilon\sigma T_s^4 &= 2\sigma T_l^4 - k_l S_{av} - \sigma T_u^4 - 0.5L - H - W \\ \text{and } \sigma T_u^4 &= S_{av} + W - a_p S_{av} - (1 - \varepsilon)\sigma T_s^4 \\ \text{and } \sigma T_l^4 &= 2S_{av} + 2W - 2a_p S_{av} - 2(1 - \varepsilon)\sigma T_s^4 - k_u S_{av} - 0.5L \\ \text{then } \varepsilon\sigma T_s^4 &= 2(2S_{av} + 2W - 2a_p S_{av} - 2(1 - \varepsilon)\sigma T_s^4 - k_u S_{av} - 0.5L) - k_l S_{av} - \\ &\quad (S_{av} + W - a_p S_{av} - (1 - \varepsilon)\sigma T_s^4) - 0.5L - H - W \\ \varepsilon\sigma T_s^4 &= 4S_{av} + 4W - 4a_p S_{av} - 4(1 - \varepsilon)\sigma T_s^4 - 2k_u S_{av} - L - k_l S_{av} - \\ &\quad S_{av} - W + a_p S_{av} + (1 - \varepsilon)\sigma T_s^4 - 0.5L - H - W \\ \varepsilon\sigma T_s^4 &= 3S_{av} + 2W - 3a_p S_{av} - 3(1 - \varepsilon)\sigma T_s^4 - 2k_u S_{av} - k_l S_{av} - 1.5L - H \\ \varepsilon\sigma T_s^4 + 3(1 - \varepsilon)\sigma T_s^4 &= 3S_{av} + 2W - 3a_p S_{av} - 2k_u S_{av} - k_l S_{av} - 1.5L - H \\ \varepsilon\sigma T_s^4 + (3 - 3\varepsilon)\sigma T_s^4 &= 3S_{av} + 2W - 3a_p S_{av} - 2k_u S_{av} - k_l S_{av} - 1.5L - H \\ \varepsilon\sigma T_s^4 + 3\sigma T_s^4 - 3\varepsilon\sigma T_s^4 &= 3S_{av} + 2W - 3a_p S_{av} - 2k_u S_{av} - k_l S_{av} - 1.5L - H \\ T_s^4(\varepsilon\sigma + 3\sigma - 3\varepsilon\sigma) &= S_{av}(3 - 3a_p - 2k_u - k_l) - 1.5L - H + 2W \\ T_s^4 &= \frac{S_{av}(3 - 3a_p - 2k_u - k_l) - 1.5L - H + 2W}{\varepsilon\sigma + 3\sigma - 3\varepsilon\sigma} \\ T_s^4 &= \frac{S_{av}(3 - 3a_p - 2k_u - k_l) - 1.5L - H + 2W}{3\sigma - 2\varepsilon\sigma} \\ T_s^4 &= \frac{S_{av}(3 - 3a_p - 2k_u - k_l) - 1.5L - H + 2W}{(3 - 2\varepsilon)\sigma} \\ T_s &= \sqrt[4]{\frac{S_{av}(3 - 3a_p - 2k_u - k_l) - 1.5L - H + 2W}{(3 - 2\varepsilon)\sigma}} \\ \text{or} \\ T_s &= \left(\frac{S_{av}(3 - 3a_p - 2k_u - k_l) - 1.5L - H + 2W}{(3 - 2\varepsilon)\sigma} \right)^{\frac{1}{4}} \end{aligned}$$