ENVT3362 Module 3 - Worked Solutions

Case Study 1

1. Using the concept of an energy balance (i.e. energy in = energy out), rearrange the equations to find the Earth's temperature, T:

$$\begin{aligned}
\text{Let } E_{\text{out}} &= \sigma 4\pi r^2 T^4 \\
\text{and } E_{\text{abs}} &= S\pi r^2(1-a) \\
\text{then } E_{\text{out}} &= E_{\text{abs}} \\
\sigma 4\pi r^2 T^4 &= S\pi r^2(1-a) \\
T^4 &= \frac{S\pi r^2(1-a)}{\sigma 4\pi r^2} \\
T^4 &= \frac{S\cancel{\pi r^2}(1-a)}{\sigma 4\cancel{\pi r^2}} \\
T &= \sqrt[4]{\frac{S(1-a)}{4\sigma}}
\end{aligned}$$

Case Study 2

1. Solve the following equation for T:

$$\begin{aligned}
\pi r^2 S(1-a) =& 4\pi r^2(x+yT) \\
\frac{\pi r^2 S(1-a)}{4\pi r^2} =& x+yT \\
yT =& \frac{\cancel{\pi r^2}S(1-a)}{4\cancel{\pi r^2}} - x \\
T =& \frac{\frac{S(1-a)}{4} - x}{y}
\end{aligned}$$

or

$$\begin{aligned}
\pi r^2 S(1-a) &= 4\pi r^2(x+yT) \\
\pi r^2 S(1-a) &= 4\pi r^2 x + 4\pi r^2 yT \\
\pi r^2 S(1-a) - 4\pi r^2 x &= 4\pi r^2 yT \\
T &= \frac{\pi r^2 S(1-a) - 4\pi r^2 x}{4\pi r^2 y}
\end{aligned}$$

Case Study 3

Energy-balance for the Earth-atmosphere system as a whole:

$$S_{av} + W = a_p S_{av} + \sigma T_u^4 + (1-\varepsilon)\sigma T_s^4 \tag{1}$$

Energy-balance for the upper layer of the atmosphere:

$$k_u S_{av} + \sigma T_l^4 + 0.5L = 2\sigma T_u^4 \tag{2}$$

Energy-balance for the lower layer of the atmosphere:

$$k_l S_{av} + \sigma T_u^4 + \varepsilon\sigma T_s^4 + 0.5L + H + W = 2\sigma T_l^4 \tag{3}$$

1. Rearrange Equation 1 for σT_u^4 in terms of T_s and parameters:

$$\begin{aligned} S_{av} + W &= a_p S_{av} + \sigma T_u^4 + (1-\varepsilon)\sigma T_s^4 \\ \sigma T_u^4 &= S_{av} + W - a_p S_{av} - (1-\varepsilon)\sigma T_s^4 \end{aligned}$$

2. Rearrange Equation 2 for σT_l^4 in terms of T_u and parameters:

$$\begin{aligned} k_u S_{av} + \sigma T_l^4 + 0.5L &= 2\sigma T_u^4 \\ \sigma T_l^4 &= 2\sigma T_u^4 - k_u S_{av} - 0.5L \end{aligned}$$

3. Substitute the results of Step 1 into the results of Step 2 to give an equation for σT_l^4 in terms of T_s and parameters.

$$\begin{aligned} \text{if } \sigma T_u^4 &= S_{av} + W - a_p S_{av} - (1-\varepsilon)\sigma T_s^4 \\ \text{and } \sigma T_l^4 &= 2\sigma T_u^4 - k_u S_{av} - 0.5L \\ \text{then } \sigma T_l^4 &= 2(S_{av} + W - a_p S_{av} - (1-\varepsilon)\sigma T_s^4) - k_u S_{av} - 0.5L \\ \sigma T_l^4 &= 2S_{av} + 2W - 2a_p S_{av} - 2(1-\varepsilon)\sigma T_s^4 - k_u S_{av} - 0.5L \end{aligned}$$

4. Rearrange Equation 3 for $\varepsilon\sigma T_s^4$ in terms T_l,T_u, and parameters.

$$\begin{aligned} k_l S_{av} + \sigma T_u^4 + \varepsilon\sigma T_s^4 + 0.5L + H + W &= 2\sigma T_l^4 \\ \varepsilon\sigma T_s^4 &= 2\sigma T_l^4 - k_l S_{av} - \sigma T_u^4 - 0.5L - H - W \end{aligned}$$

5. Put the results of Steps 1 and 3 into the results of Step 4 and simplify to give the equation for T_s.

$$
\begin{aligned}
\text{if } \varepsilon\sigma T_s^4 &= 2\sigma T_l^4 - k_l S_{av} - \sigma T_u^4 - 0.5L - H - W \\
\text{and } \sigma T_u^4 &= S_{av} + W - a_p S_{av} - (1-\varepsilon)\sigma T_s^4 \\
\text{and } \sigma T_l^4 &= 2S_{av} + 2W - 2a_p S_{av} - 2(1-\varepsilon)\sigma T_s^4 - k_u S_{av} - 0.5L \\
\text{then } \varepsilon\sigma T_s^4 &= 2(2S_{av} + 2W - 2a_p S_{av} - 2(1-\varepsilon)\sigma T_s^4 - k_u S_{av} - 0.5L) - k_l S_{av} - \\
&\quad (S_{av} + W - a_p S_{av} - (1-\varepsilon)\sigma T_s^4) - 0.5L - H - W \\
\varepsilon\sigma T_s^4 &= 4S_{av} + 4W - 4a_p S_{av} - 4(1-\varepsilon)\sigma T_s^4 - 2k_u S_{av} - L - k_l S_{av} - \\
&\quad S_{av} - W + a_p S_{av} + (1-\varepsilon)\sigma T_s^4 - 0.5L - H - W \\
\varepsilon\sigma T_s^4 &= 3S_{av} + 2W - 3a_p S_{av} - 3(1-\varepsilon)\sigma T_s^4 - 2k_u S_{av} - k_l S_{av} - 1.5L - H \\
\varepsilon\sigma T_s^4 + 3(1-\varepsilon)\sigma T_s^4 &= 3S_{av} + 2W - 3a_p S_{av} - 2k_u S_{av} - k_l S_{av} - 1.5L - H \\
\varepsilon\sigma T_s^4 + (3-3\varepsilon)\sigma T_s^4 &= 3S_{av} + 2W - 3a_p S_{av} - 2k_u S_{av} - k_l S_{av} - 1.5L - H \\
\varepsilon\sigma T_s^4 + 3\sigma T_s^4 - 3\varepsilon\sigma T_s^4 &= 3S_{av} + 2W - 3a_p S_{av} - 2k_u S_{av} - k_l S_{av} - 1.5L - H \\
T_s^4(\varepsilon\sigma + 3\sigma - 3\varepsilon\sigma) &= S_{av}(3 - 3a_p - 2k_u - k_l) - 1.5L - H + 2W \\
T_s^4 &= \frac{S_{av}(3 - 3a_p - 2k_u - k_l) - 1.5L - H + 2W}{\varepsilon\sigma + 3\sigma - 3\varepsilon\sigma} \\
T_s^4 &= \frac{S_{av}(3 - 3a_p - 2k_u - k_l) - 1.5L - H + 2W}{3\sigma - 2\varepsilon\sigma} \\
T_s^4 &= \frac{S_{av}(3 - 3a_p - 2k_u - k_l) - 1.5L - H + 2W}{(3-2\varepsilon)\sigma} \\
T_s &= \sqrt[4]{\frac{S_{av}(3 - 3a_p - 2k_u - k_l) - 1.5L - H + 2W}{(3-2\varepsilon)\sigma}}
\end{aligned}
$$

or

$$
T_s = \left(\frac{S_{av}(3 - 3a_p - 2k_u - k_l) - 1.5L - H + 2W}{(3-2\varepsilon)\sigma}\right)^{\frac{1}{4}}
$$