lab1_final

January 26, 2025

Task 1: A classification example: fetal heart condition diagnosis

Step1

[2]:
```
import pandas as pd
from sklearn.model_selection import train_test_split

# Read the CSV file.
data = pd.read_csv("/Users/laqfire/code/Applied_ML/lab1/CTG.csv", skiprows=1)

# Select the relevant numerical columns.
selected_cols = ['LB', 'AC', 'FM', 'UC', 'DL', 'DS', 'DP', 'ASTV', 'MSTV',␣
 ↪'ALTV',
                 'MLTV', 'Width', 'Min', 'Max', 'Nmax', 'Nzeros', 'Mode',␣
 ↪'Mean',
                 'Median', 'Variance', 'Tendency', 'NSP']
data = data[selected_cols].dropna()

# Shuffle the dataset.
data_shuffled = data.sample(frac=1.0, random_state=0)

# Split into input part X and output part Y.
X = data_shuffled.drop('NSP', axis=1)

# Map the diagnosis code to a human-readable label.
def to_label(y):
    return [None, 'normal', 'suspect', 'pathologic'][(int(y))]

Y = data_shuffled['NSP'].apply(to_label)

# Partition the data into training and test sets.
Xtrain, Xtest, Ytrain, Ytest = train_test_split(X, Y, test_size=0.2,␣
 ↪random_state=0)
```

[3]:
```
X.head()
```

[3]:

	LB	AC	FM	UC	DL	DS	DP	ASTV	MSTV	ALTV	…	Width	\
658	130.0	1.0	0.0	3.0	0.0	0.0	0.0	24.0	1.2	12.0	…	35.0	

```
1734  134.0  9.0  1.0  8.0  5.0  0.0  0.0  59.0   1.2   0.0  …  109.0
1226  125.0  1.0  0.0  4.0  0.0  0.0  0.0  43.0   0.7  31.0  …   21.0
1808  143.0  0.0  0.0  1.0  0.0  0.0  0.0  69.0   0.3   6.0  …   27.0
825   152.0  0.0  0.0  4.0  0.0  0.0  0.0  62.0   0.4  59.0  …   25.0

        Min    Max  Nmax  Nzeros   Mode   Mean  Median  Variance  Tendency
658   120.0  155.0   1.0     0.0  134.0  133.0   135.0       1.0       0.0
1734   80.0  189.0   6.0     0.0  150.0  146.0   150.0      33.0       0.0
1226  120.0  141.0   0.0     0.0  131.0  130.0   132.0       1.0       0.0
1808  132.0  159.0   1.0     0.0  145.0  144.0   146.0       1.0       0.0
825   136.0  161.0   0.0     0.0  159.0  156.0   158.0       1.0       1.0

[5 rows x 21 columns]
```

Step2

[4]:
```python
from sklearn.dummy import DummyClassifier

clf = DummyClassifier(strategy='most_frequent')
from sklearn.model_selection import cross_val_score

import numpy as np
scores = cross_val_score(clf, Xtrain, Ytrain)
print("Scores:", scores)
print("Mean score:", np.mean(scores))
```

```
Scores: [0.78235294 0.78235294 0.77941176 0.77941176 0.77941176]
Mean score: 0.7805882352941176
```

Step3

[5]:
```python
from sklearn.tree import DecisionTreeClassifier
from sklearn.ensemble import RandomForestClassifier, GradientBoostingClassifier
from sklearn.linear_model import Perceptron, LogisticRegression
from sklearn.svm import LinearSVC
from sklearn.neural_network import MLPClassifier


# create dictionary of classifiers
classifiers = {
    'Decision Tree': DecisionTreeClassifier(),
    'Random Forest': RandomForestClassifier(),
    'Gradient Boosting': GradientBoostingClassifier(),
    'Perceptron': Perceptron(),
    'Logistic Regression': LogisticRegression(max_iter=2000),
    'Linear SVC': LinearSVC(),
    'Neural Network': MLPClassifier(random_state=0, max_iter=2000)
}
```

```python
# compute the cross-validation accuuracy
# 5-fold instead of 3-fold cross-validation
# Accuracy as evaluation metric
for name, clf in classifiers.items():
    score = cross_val_score(clf, Xtrain, Ytrain, cv=5, scoring='accuracy').
↪mean()
    print(f'{name}: {score}')
```

```
Decision Tree: 0.9211764705882353
Random Forest: 0.9388235294117647
Gradient Boosting: 0.95
Perceptron: 0.825294117647059
```

```
/Users/laqfire/code/sklearn-env/lib/python3.13/site-
packages/sklearn/linear_model/_logistic.py:465: ConvergenceWarning: lbfgs failed
to converge (status=1):
STOP: TOTAL NO. OF ITERATIONS REACHED LIMIT.

Increase the number of iterations (max_iter) or scale the data as shown in:
    https://scikit-learn.org/stable/modules/preprocessing.html
Please also refer to the documentation for alternative solver options:
    https://scikit-learn.org/stable/modules/linear_model.html#logistic-
regression
  n_iter_i = _check_optimize_result(
/Users/laqfire/code/sklearn-env/lib/python3.13/site-
packages/sklearn/linear_model/_logistic.py:465: ConvergenceWarning: lbfgs failed
to converge (status=1):
STOP: TOTAL NO. OF ITERATIONS REACHED LIMIT.

Increase the number of iterations (max_iter) or scale the data as shown in:
    https://scikit-learn.org/stable/modules/preprocessing.html
Please also refer to the documentation for alternative solver options:
    https://scikit-learn.org/stable/modules/linear_model.html#logistic-
regression
  n_iter_i = _check_optimize_result(
/Users/laqfire/code/sklearn-env/lib/python3.13/site-
packages/sklearn/linear_model/_logistic.py:465: ConvergenceWarning: lbfgs failed
to converge (status=1):
STOP: TOTAL NO. OF ITERATIONS REACHED LIMIT.

Increase the number of iterations (max_iter) or scale the data as shown in:
    https://scikit-learn.org/stable/modules/preprocessing.html
Please also refer to the documentation for alternative solver options:
    https://scikit-learn.org/stable/modules/linear_model.html#logistic-
regression
  n_iter_i = _check_optimize_result(
/Users/laqfire/code/sklearn-env/lib/python3.13/site-
```

```
packages/sklearn/linear_model/_logistic.py:465: ConvergenceWarning: lbfgs failed
to converge (status=1):
STOP: TOTAL NO. OF ITERATIONS REACHED LIMIT.

Increase the number of iterations (max_iter) or scale the data as shown in:
    https://scikit-learn.org/stable/modules/preprocessing.html
Please also refer to the documentation for alternative solver options:
    https://scikit-learn.org/stable/modules/linear_model.html#logistic-
regression
  n_iter_i = _check_optimize_result(
/Users/laqfire/code/sklearn-env/lib/python3.13/site-
packages/sklearn/linear_model/_logistic.py:465: ConvergenceWarning: lbfgs failed
to converge (status=1):
STOP: TOTAL NO. OF ITERATIONS REACHED LIMIT.

Increase the number of iterations (max_iter) or scale the data as shown in:
    https://scikit-learn.org/stable/modules/preprocessing.html
Please also refer to the documentation for alternative solver options:
    https://scikit-learn.org/stable/modules/linear_model.html#logistic-
regression
  n_iter_i = _check_optimize_result(
```

```
Logistic Regression: 0.8988235294117647
Linear SVC: 0.8952941176470588
Neural Network: 0.8847058823529412
```

Step4 As we can see above, Gradient Boosting has the highest accuary.

[6]:
```
from sklearn.metrics import accuracy_score

gbc = GradientBoostingClassifier()
gbc.fit(Xtrain, Ytrain)
Yguess = gbc.predict(Xtest)
print(f"Accuracy: {accuracy_score(Ytest, Yguess)}")
```

```
Accuracy: 0.9295774647887324
```

Task 2: Decision trees for classification

[7]:
```
from sklearn.tree import DecisionTreeClassifier
from sklearn.metrics import accuracy_score
import numpy as np
clf = DecisionTreeClassifier(max_depth = 1)
clf.fit(Xtrain, Ytrain);
from sklearn.model_selection import cross_val_score

cross_val_score(clf, Xtrain, Ytrain, cv=5).mean()
```

[7]:
```
np.float64(0.8164705882352941)
```

[8]:
```
class DecisionTreeLeaf:

    def __init__(self, value):
            self.value = value

        # This method computes the prediction for this leaf node. This will just␣
↪return a constant value.\n",
    def predict(self, x):
            return self.value

        # Utility function to draw a tree visually using graphviz.\n",
    def draw_tree(self, graph, node_counter, names):
            node_id = str(node_counter)
            val_str = f'{self.value:.4g}' if isinstance(self.value, float) else␣
↪str(self.value)
            graph.node(node_id, val_str, style='filled')
            return node_counter+1, node_id

    def __eq__(self, other):
        if isinstance(other, DecisionTreeLeaf):
                return self.value == other.value
        else:
                return False

    def to_str(self, indent, lines, names):
            val_str = f'{self.value:.4g}' if isinstance(self.value, float) else␣
↪str(self.value)
            lines.append(' '*indent + val_str)
```

[9]:
```
class DecisionTreeBranch:

        def __init__(self, feature, threshold, low_subtree, high_subtree):
            self.feature = feature
            self.threshold = threshold
            self.low_subtree = low_subtree
            self.high_subtree = high_subtree

        # For a branch node, we compute the prediction by first considering the␣
↪feature, and then \n",
        # calling the upper or lower subtree, depending on whether the feature␣
↪is or isn't greater\n",
        # than the threshold.\n",
        def predict(self, x):
            if x[self.feature] <= self.threshold:
                return self.low_subtree.predict(x)
            else:
                return self.high_subtree.predict(x)
```

```
        # Utility function to draw a tree visually using graphviz.\n",
        def draw_tree(self, graph, node_counter, names):
            node_counter, low_id = self.low_subtree.draw_tree(graph,␣
↪node_counter, names)
            node_counter, high_id = self.high_subtree.draw_tree(graph,␣
↪node_counter, names)
            node_id = str(node_counter)
            fname = f'F{self.feature}' if names is None else names[self.feature]
            lbl = f'{fname} > {self.threshold:.4g}?'
            graph.node(node_id, lbl, shape='box', fillcolor='yellow',␣
↪style='filled, rounded')
            graph.edge(node_id, low_id, 'False')
            graph.edge(node_id, high_id, 'True')
            return node_counter+1, node_id

        def to_str(self, indent, lines, names):
            pad = ' '*indent
            fname = f'F{self.feature}' if names is None else names[self.feature]
            lines.append(pad + f'{fname} <= {self.threshold:.3g}?')
            lines.append(pad + '  True =>')
            self.low_subtree.to_str(indent+4, lines, names)
            lines.append(pad + '  False =>')
            self.high_subtree.to_str(indent+4, lines, names)
```

[10]:
```
from graphviz import Digraph
from sklearn.base import BaseEstimator, ClassifierMixin
from abc import ABC, abstractmethod

class DecisionTree(ABC, BaseEstimator):

        def __init__(self, max_depth):
            super().__init__()
            self.max_depth = max_depth

        # As usual in scikit-learn, the training method is called *fit*. We␣
↪first process the dataset so that\n",
        # we're sure that it's represented as a NumPy matrix. Then we call the␣
↪recursive tree-building method\n",
        # called make_tree (see below).\n",
        def fit(self, X, Y):
            if isinstance(X, pd.DataFrame):
                self.names = X.columns
                X = X.to_numpy()
            elif isinstance(X, list):
                self.names = None
```

```
            X = np.array(X)
        else:
            self.names = None
        Y = np.array(Y)
        self.root = self.make_tree(X, Y, self.max_depth)

    def draw_tree(self):
        graph = Digraph()
        self.root.draw_tree(graph, 0, self.names)
        return graph

    # By scikit-learn convention, the method *predict* computes the classification or regression output\n",
    # for a set of instances.\n",
    # To implement it, we call a separate method that carries out the prediction for one instance.\n",
    def predict(self, X):
        if isinstance(X, pd.DataFrame):
            X = X.to_numpy()
        return [self.predict_one(x) for x in X]

    # Predicting the output for one instance.\n",
    def predict_one(self, x):
        return self.root.predict(x)

    # This is the recursive training \n",
    def make_tree(self, X, Y, max_depth):

        # We start by computing the default value that will be used if we'll return a leaf node.\n",
        # For classifiers, this will be the most common value in Y.\n",
        default_value = self.get_default_value(Y)

        # First the two base cases in the recursion: is the training set completely\n",
        # homogeneous, or have we reached the maximum depth? Then we need to return a leaf.\n",

        # If we have reached the maximum depth, return a leaf with the majority value.\n",
        if max_depth == 0:
            return DecisionTreeLeaf(default_value)

        # If all the instances in the remaining training set have the same output value,\n",
        # return a leaf with this value.\n",
```

```
            if self.is_homogeneous(Y):
                return DecisionTreeLeaf(default_value)

            # Select the \"most useful\" feature and split threshold. To rank␣
↪the \"usefulness\" of features,\n",
            # we use one of the classification or regression criteria.\n",
            # For each feature, we call best_split (defined in a subclass). We␣
↪then maximize over the features.\n",
            n_features = X.shape[1]

            _, best_feature, best_threshold = max(self.best_split(X, Y, feature)
                    for feature in range(n_features))

            if best_feature is None:
                return DecisionTreeLeaf(default_value)

            # Split the training set into subgroups, based on whether the␣
↪selected feature is greater than\n",
            # the threshold or not\n",
            X_low, X_high, Y_low, Y_high = self.split_by_feature(X, Y,␣
↪best_feature, best_threshold)

            # Build the subtrees using a recursive call. Each subtree is␣
↪associated\n",
            # with a value of the feature.\n",
            low_subtree = self.make_tree(X_low, Y_low, max_depth-1)
            high_subtree = self.make_tree(X_high, Y_high, max_depth-1)

            if low_subtree == high_subtree:
                return low_subtree

            # Return a decision tree branch containing the result.\n",
            return DecisionTreeBranch(best_feature, best_threshold,␣
↪low_subtree, high_subtree)

        # Utility method that splits the data into the \"upper\" and \"lower\"␣
↪part, based on a feature\n",
        # and a threshold.\n",
        def split_by_feature(self, X, Y, feature, threshold):
            low = X[:,feature] <= threshold
            high = ~low
            return X[low], X[high], Y[low], Y[high]

        def __str__(self):
            lines = []
            self.root.to_str(0, lines, self.names)
```

```
            return '\\n'.join(lines)

        # The following three methods need to be implemented by the␣
↪classification and regression subclasses.\n",

        @abstractmethod
        def get_default_value(self, Y):
            pass

        @abstractmethod
        def is_homogeneous(self, Y):
            pass

        @abstractmethod
        def best_split(self, X, Y, feature):
            pass
```

[11]:
```
from collections import Counter

class TreeClassifier(DecisionTree, ClassifierMixin):

    def __init__(self, max_depth=10, criterion='maj_sum'):
        super().__init__(max_depth)
        self.criterion = criterion

    def fit(self, X, Y):
        # For decision tree classifiers, there are some different ways to␣
↪measure
        # the homogeneity of subsets.
        if self.criterion == 'maj_sum':
            self.criterion_function = majority_sum_scorer
        elif self.criterion == 'info_gain':
            self.criterion_function = info_gain_scorer
        elif self.criterion == 'gini':
            self.criterion_function = gini_scorer
        else:
            raise Exception(f'Unknown criterion: {self.criterion}')
        super().fit(X, Y)
        self.classes_ = sorted(set(Y))

    # Select a default value that is going to be used if we decide to make a␣
↪leaf.
    # We will select the most common value.
    def get_default_value(self, Y):
        self.class_distribution = Counter(Y)
        return self.class_distribution.most_common(1)[0][0]
```

```
    # Checks whether a set of output values is homogeneous. In the␣
↪classification case,
    # this means that all output values are identical.
    # We assume that we called get_default_value just before, so that we can␣
↪access
    # the class_distribution attribute. If the class distribution contains just␣
↪one item,
    # this means that the set is homogeneous.
    def is_homogeneous(self, Y):
        return len(self.class_distribution) == 1

    # Finds the best splitting point for a given feature. We'll keep frequency␣
↪tables (Counters)
    # for the upper and lower parts, and then compute the impurity criterion␣
↪using these tables.
    # In the end, we return a triple consisting of
    # - the best score we found, according to the criterion we're using
    # - the id of the feature
    # - the threshold for the best split
    def best_split(self, X, Y, feature):

        # Create a list of input-output pairs, where we have sorted
        # in ascending order by the input feature we're considering.
        sorted_indices = np.argsort(X[:, feature])
        X_sorted = list(X[sorted_indices, feature])
        Y_sorted = list(Y[sorted_indices])

        n = len(Y)

        # The frequency tables corresponding to the parts *before and including*
        # and *after* the current element.
        low_distr = Counter()
        high_distr = Counter(Y)

        # Keep track of the best result we've seen so far.
        max_score = -np.inf
        max_i = None

        # Go through all the positions (excluding the last position).
        for i in range(0, n-1):

            # Input and output at the current position.
            x_i = X_sorted[i]
            y_i = Y_sorted[i]

            # Update the frequency tables.
            low_distr[y_i] += 1
```

```
            high_distr[y_i] -= 1

            # If the input is equal to the input at the next position, we will
            # not consider a split here.
            #x_next = XY[i+1][0]
            x_next = X_sorted[i+1]
            if x_i == x_next:
                continue

            # Compute the homogeneity criterion for a split at this position.
            score = self.criterion_function(i+1, low_distr, n-i-1, high_distr)

            # If this is the best split, remember it.
            if score > max_score:
                max_score = score
                max_i = i

        # If we didn't find any split (meaning that all inputs are identical), return
        # a dummy value.
        if max_i is None:
            return -np.inf, None, None

        # Otherwise, return the best split we found and its score.
        split_point = 0.5*(X_sorted[max_i] + X_sorted[max_i+1])
        return max_score, feature, split_point
```

[12]:
```
def majority_sum_scorer(n_low, low_distr, n_high, high_distr):
    maj_sum_low = low_distr.most_common(1)[0][1]
    maj_sum_high = high_distr.most_common(1)[0][1]
    return maj_sum_low + maj_sum_high

def entropy(distr):
    n = sum(distr.values())
    ps = [n_i/n for n_i in distr.values()]
    return -sum(p*np.log2(p) if p > 0 else 0 for p in ps)

def info_gain_scorer(n_low, low_distr, n_high, high_distr):
    return -(n_low*entropy(low_distr)+n_high*entropy(high_distr))/(n_low+n_high)

def gini_impurity(distr):
    n = sum(distr.values())
    ps = [n_i/n for n_i in distr.values()]
    return 1-sum(p**2 for p in ps)

def gini_scorer(n_low, low_distr, n_high, high_distr):
```

```
    return -(n_low*gini_impurity(low_distr)+n_high*gini_impurity(high_distr))/
↪(n_low+n_high)
```

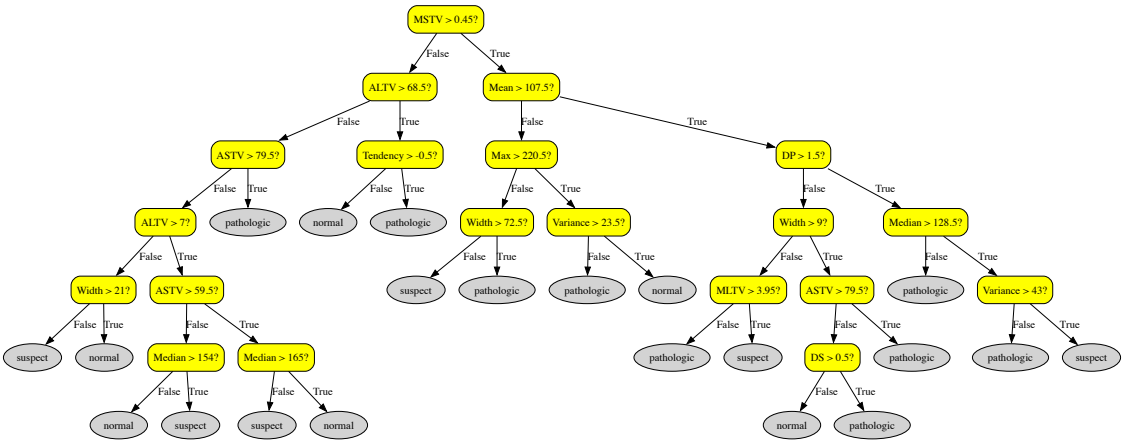
[13]:
```
# try different max_depth value
depths = range(1, 30)
scores = []

for depth in depths:
    clf = TreeClassifier(max_depth=depth)
    score = cross_val_score(clf, Xtrain, Ytrain, cv=5).mean()
    scores.append(score)

# find the best
best_depth = depths[np.argmax(scores)]

# evaluate the classifier
best_clf = TreeClassifier(max_depth=best_depth)
best_clf.fit(Xtrain, Ytrain)
test_score = best_clf.score(Xtest, Ytest)

print(f'Best depth: {best_depth}')
print(f'Test accuracy: {test_score}')
```

```
Best depth: 13
Test accuracy: 0.8732394366197183
```

draw a tree for illustration

[14]:
```
cls = TreeClassifier(max_depth=6)
cls.fit(Xtrain, Ytrain)
cls.draw_tree()
```

[14]:



0.1 Task 3: A regression example: predicting apartment prices

Step 1: Data Munging Read the `sberbank.csv` file, select specific columns, shuffle the data, seperate input matrix `X` and ouput vector `y`, and then do the test-training split.

The following code is quoted from the instructions.

[15]:
```
import pandas as pd
import numpy as np
from sklearn.model_selection import train_test_split

# Read the CSV file using Pandas.
alldata = pd.read_csv("sberbank.csv")

# Convert the timestamp string to an integer representing the year.
def get_year(timestamp):
    return int(timestamp[:4])
alldata['year'] = alldata.timestamp.apply(get_year)

# Select the 7 input columns and the output column.
selected_columns = ['price_doc', 'year', 'full_sq', 'life_sq', 'floor',␣
 ↪'num_room', 'kitch_sq', 'full_all']
alldata = alldata[selected_columns]
alldata = alldata.dropna()

# Shuffle.
alldata_shuffled = alldata.sample(frac=1.0, random_state=0)

# Separate the input and output columns.
X = alldata_shuffled.drop('price_doc', axis=1)
# For the output, we'll use the log of the sales price.
Y = alldata_shuffled['price_doc'].apply(np.log)

# Split into training and test sets.
Xtrain, Xtest, Ytrain, Ytest = train_test_split(X, Y, test_size=0.2,␣
 ↪random_state=0)
```

Step 2: Train and evaluate different regression models First, we perform cross validtion on the training set for all the regressors, and select the model with highest score to train on the whole training set and evaluate on the held-out test set.

The models used are shown in the code.

[16]:
```
from sklearn.dummy import DummyRegressor
from sklearn.linear_model import LinearRegression, Ridge, Lasso
from sklearn.ensemble import RandomForestRegressor, GradientBoostingRegressor
from sklearn.neural_network import MLPRegressor
from sklearn.tree import DecisionTreeRegressor
from sklearn.model_selection import cross_val_score
```

```
from sklearn.metrics import mean_squared_error

models = {
    "DummyRegressor": DummyRegressor(),
    "LinearRegression": LinearRegression(),
    "Lasso": Lasso(alpha=1.0, max_iter=1000, tol=0.0001, random_state=114514),
    "Ridge": Ridge(
            alpha=1.0, fit_intercept=True, solver='auto',
            max_iter=None, tol=0.001, random_state=42
        ),
    "RandomForestRegressor": RandomForestRegressor(
            n_estimators=100,
            max_depth=None, min_samples_split=2, min_samples_leaf=1,
            random_state=42, max_features= 1 / 3
        ),
    "GradientBoostingRegressor": GradientBoostingRegressor(
            n_estimators=100, learning_rate=0.1, max_depth=3,
            min_samples_split=2, min_samples_leaf=1, random_state=42
        ),
    "DecisionTreeRegressor": DecisionTreeRegressor(
            max_depth=None, min_samples_split=2,
            min_samples_leaf=1, random_state=42, ccp_alpha=0.01
        ),
    "MLPRegressor": MLPRegressor(
            random_state=198
        ),
}

#Preform cross validation for all models on the training set
#Print their scores
for name, regr_model in models.items():
    score = cross_val_score(regr_model, Xtrain, Ytrain,␣
 ↪scoring="neg_mean_squared_error").mean()
    print(f"Model: {name}; Score: {score}")
```

```
Model: DummyRegressor; Score: -0.38925247260237567
Model: LinearRegression; Score: -0.3013986588767176
Model: Lasso; Score: -0.3010470671748872
Model: Ridge; Score: -0.30139784232179745
Model: RandomForestRegressor; Score: -0.27988745119594965
Model: GradientBoostingRegressor; Score: -0.2645272118267597
Model: DecisionTreeRegressor; Score: -0.3115473038894633
Model: MLPRegressor; Score: -11.144804540767561
```

Clearly, `GradientBoostingRegressor` achieved highest score in cross validation.

Finally, train it on the full training set and evaluate on the held-out test set:

[17]:
```
regr_model = models["GradientBoostingRegressor"]
regr_model.fit(Xtrain, Ytrain)
mse = mean_squared_error(Ytest, regr_model.predict(Xtest))
print(f"Model: {name}; MSE: {mse}")
```

```
Model: MLPRegressor; MSE: 0.2713747983891566
```

0.1.1 Task 4: Decision trees for regression

1) Small Dataset Training We first train a tree regressor on a small dataset to see how it works. We will use the Russian dataset for training later.

[18]:
```
# Importing libraries
import numpy as np
import pandas as pd
from graphviz import Digraph
from sklearn.base import BaseEstimator, ClassifierMixin
from abc import ABC, abstractmethod
import matplotlib.pyplot as plt
from sklearn.base import RegressorMixin
```

[19]:
```
from sklearn.base import RegressorMixin

class TreeRegressor(DecisionTree, RegressorMixin):
    def __init__(self, max_depth=10, criterion='variance_reduction'):
        super().__init__(max_depth)
        self.criterion = criterion

    def fit(self, X, Y):

        super().fit(X, Y)
   # Select a default value that is going to be used if we decide to make a␣
 ↪leaf.
   # We will select the most common value.
    def get_default_value(self, Y):

        return np.mean(Y)

    # Checks whether a set of output values is homogeneous. In the␣
 ↪classification case,
    # this means that all output values are identical.
    # We assume that we called get_default_value just before, so that we can␣
 ↪access
    # the class_distribution attribute. If the class distribution contains just␣
 ↪one item,
    # this means that the set is homogeneous.
    def is_homogeneous(self, Y):
        variance_threshold = 1e-6
```

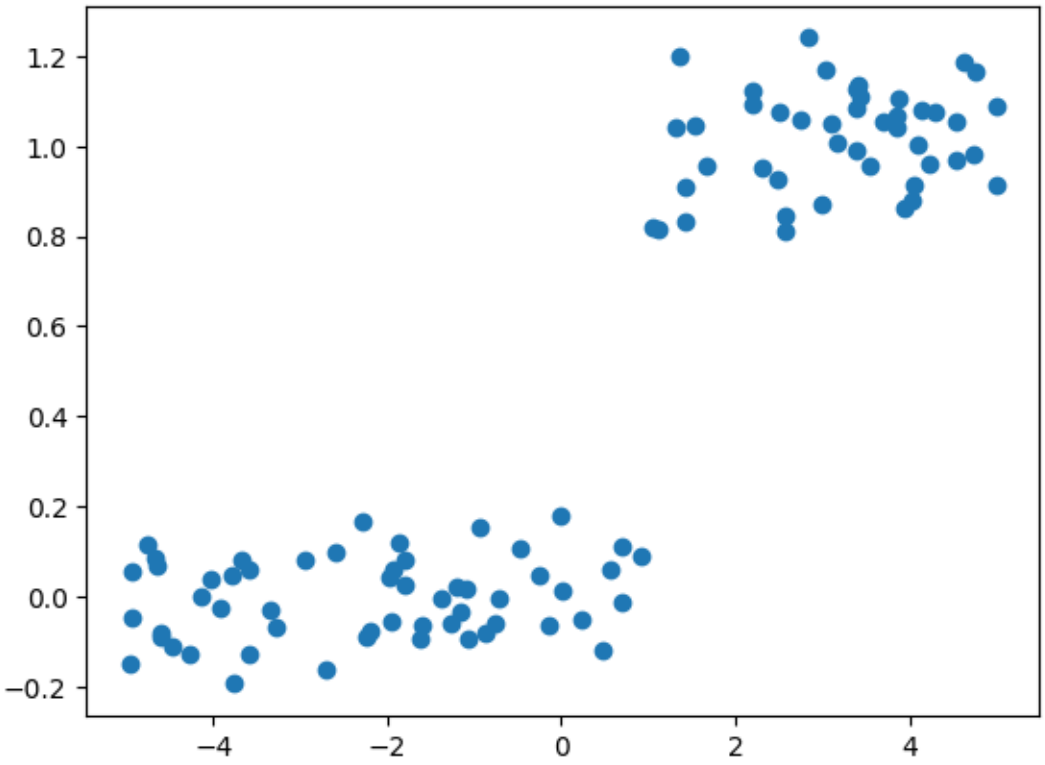
```
        return np.var(Y) < variance_threshold

    # Finds the best splitting point for a given feature. We'll keep frequency tables (Counters)
    # for the upper and lower parts, and then compute the impurity criterion using these tables.
    # In the end, we return a triple consisting of
    # - the best score we found, according to the criterion we're using
    # - the id of the feature
    # - the threshold for the best split
    def best_split(self, X, Y, feature, tot_var=None):

        # Create a list of input-output pairs, where we have sorted
        # in ascending order by the input feature we're considering.
        sorted_indices = np.argsort(X[:, feature])
        X_sorted = X[sorted_indices, feature]
        Y_sorted = Y[sorted_indices]
        tot_var = np.var(Y) * len(Y)

        n = len(Y)
        if n <= 1:
            return -np.inf, None, None
        total_variance = np.var(Y) * n

        # The frequency tables corresponding to the parts *before and including*
        # and *after* the current element.
        #low_distr = Counter()
        #high_distr = Counter(Y)

        # Keep track of the best result we've seen so far.
        max_score = -np.inf
        best_split_point = None

        # Initialize sum values for variance computation
        partial_low_squared_sum = 0
        partial_low_sum_squared = 0
        partial_high_squared_sum = np.sum(Y)
        partial_high_sum_squared = np.sum(Y ** 2)
     # Go through all the positions (excluding the last position).
                # Go through all the positions (excluding the last position).
        for i in range(0, n-1):

            # Input and output at the current position.
            x_i = X_sorted[i]
            y_i = Y_sorted[i]

            partial_low_squared_sum += y_i**2
```

```
            partial_low_sum_squared += y_i

            partial_high_squared_sum -= y_i**2
            partial_high_sum_squared -= y_i

            n_low = i + 1
            n_high = n - n_low
            low_var = partial_low_squared_sum / n_low -␣
↪partial_low_sum_squared**2 / (n_low**2)
            high_var = partial_high_squared_sum / n_high -␣
↪partial_high_sum_squared**2 / (n_high**2)

            # Update the frequency tables.
            #low_distr[y_i] += 1
            #high_distr[y_i] -= 1

            # If the input is equal to the input at the next position, we will
            # not consider a split here.
            #x_next = XY[i+1][0]
            x_next = X_sorted[i+1]
            if x_i == x_next:
                continue

            # Compute the homogeneity criterion for a split at this position.
            score = tot_var - (n_high * high_var + n_low * low_var) / n

            # If this is the best split, remember it.
            if score > max_score:
                max_score = score
                max_i = i

        # If we didn't find any split (meaning that all inputs are identical),␣
↪return
        # a dummy value.
        if max_i is None:
            return -np.inf, None, None

        # Otherwise, return the best split we found and its score.
        split_point = 0.5*(X_sorted[max_i] + X_sorted[max_i+1])
        return max_score, feature, split_point
```

Sanity check

[20]:
```
def make_some_data(n):
    x = np.random.uniform(-5, 5, size=n)
    Y = (x > 1) + 0.1*np.random.normal(size=n)
    X = x.reshape(n, 1) # X needs to be a 2-dimensional matrix
```

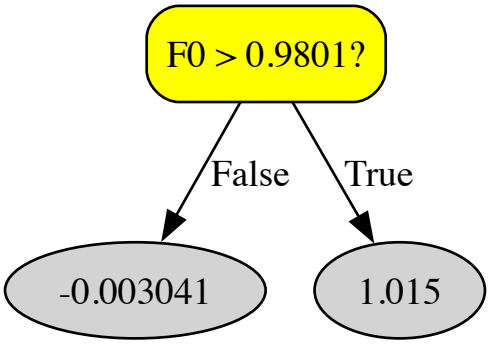
```
    return X, Y
X, Y = make_some_data(100)
plt.scatter(X, Y)
plt.show()
```



It generated a small number of training examples for a simple regression task with one input variable. The output figure is similar to the one in the assignment.

[21]:
```
clf = TreeRegressor(max_depth=1)
clf.fit(X, Y)
clf.draw_tree()
```

[21]:



After sanity check and small dataset testing(with max_depth=1), we found out that it runs well and can perform well on the small dataset. Now we can proceed to the Russian housing prices prediction.

For the data-generating function, the decision tree would be like a binary segmentation based on whether "x $>$ 1". We hope the decision tree with a depth of 1 would be sufficient to describe this data. The tree would have a single split point at x $=$ 1, creating two leaf nodes, one representing the average value of the data where x 1 and the other where x $>$ 1, just like the figure above.

A tree depth of 1 was selected based on the simplicity of the data-generating function. The resulting tree successfully splits the data at the optimal threshold (e.g., x $=$ 1.027). The leaf nodes represent the average values of the two partitions, which matches the expected structure of the function. The result makes sense as it aligns with the binary nature of the data.

It would cause overfitting if the data is too complex and the tree is too deep. And we are going to simulate and show the data in the following sections.

2) Russian Housing Prices Prediction In this part we will use the Sberbank dataset to predict the apartment prices in Russia. We will use the decision tree regression algorithm to predict the prices. Before we start, we need to preprocess the data.

[22]:
```
# Data preprocessing
import numpy as np
import pandas as pd
# Read the CSV file using Pandas.
alldata = pd.read_csv("sberbank.csv")

# Convert the timestamp string to an integer representing the year.
def get_year(timestamp):
    return int(timestamp[:4])
alldata['year'] = alldata.timestamp.apply(get_year)

# Select the 7 input columns and the output column.
selected_columns = ['price_doc', 'year', 'full_sq', 'life_sq', 'floor',␣
 ↪'num_room', 'kitch_sq', 'full_all']
alldata = alldata[selected_columns]
alldata = alldata.dropna()

# Shuffle.
alldata_shuffled = alldata.sample(frac=1.0, random_state=0)

# Separate the input and output columns.
X = alldata_shuffled.drop('price_doc', axis=1)
# For the output, we'll use the log of the sales price.
Y = alldata_shuffled['price_doc'].apply(np.log)
```

```
# Split into training and test sets.
Xtrain, Xtest, Ytrain, Ytest = train_test_split(X, Y, test_size=0.2,␣
 ↪random_state=0)

from sklearn.dummy import DummyRegressor
from sklearn.model_selection import cross_validate
m1 = DummyRegressor()
cross_validate(m1, Xtrain, Ytrain, scoring='neg_mean_squared_error')
```

[22]:
```
{'fit_time': array([0.00043797, 0.00031185, 0.00034714, 0.00030017,
0.00034809]),
 'score_time': array([0.00018525, 0.00014114, 0.00013685, 0.00013208,
0.00013089]),
 'test_score': array([-0.39897319, -0.37113485, -0.38083108, -0.39057156,
-0.40475168])}
```

Step 1. Implementing the regression model

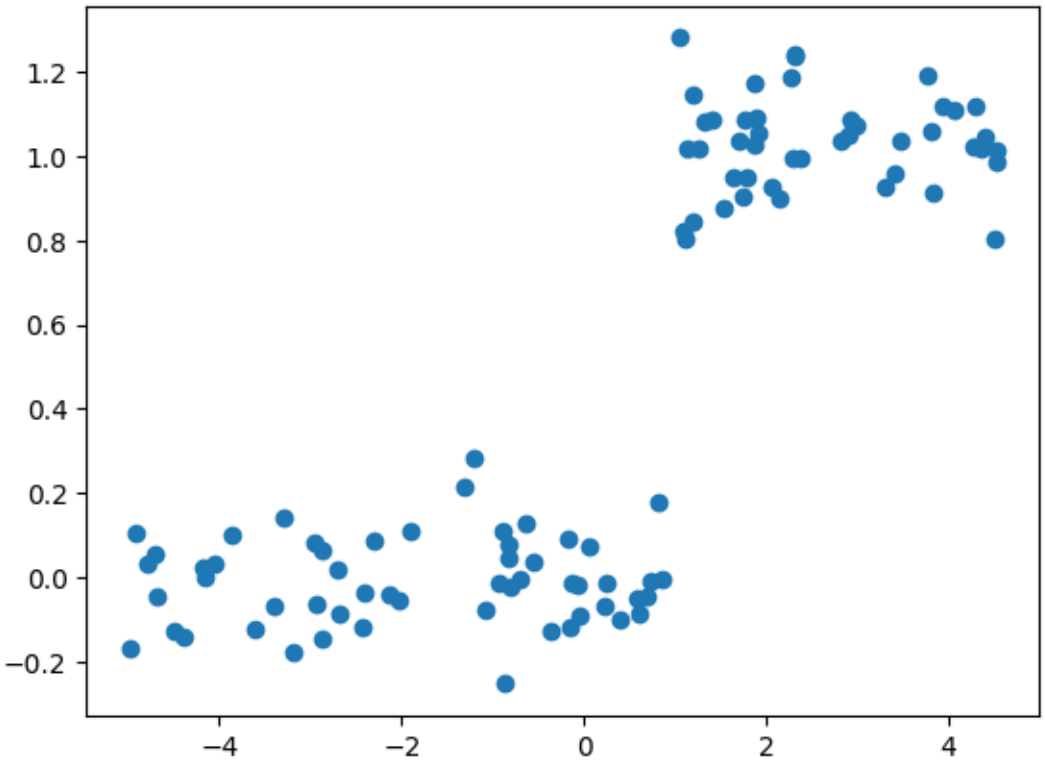
[23]:
```
class TreeRegressor(DecisionTree, RegressorMixin):

    def __init__(self, max_depth=10):
        super().__init__(max_depth)

    def fit(self, X, Y):
        super().fit(X, Y)

    # Select a default value that is going to be used if we decide to make a␣
 ↪leaf.
    # We will select the most common value.
    def get_default_value(self, Y):
        return np.mean(Y)

    # Checks whether a set of output values is homogeneous. In the␣
 ↪classification case,
    # this means that all output values are identical.
    # We assume that we called get_default_value just before, so that we can␣
 ↪access
    # the class_distribution attribute. If the class distribution contains just␣
 ↪one item,
    # this means that the set is homogeneous.
    def is_homogeneous(self, Y):
        variance_threshold = 0.01
        return np.var(Y) < variance_threshold

    # Finds the best splitting point for a given feature. We'll keep frequency␣
 ↪tables (Counters)
```

```
    # for the upper and lower parts, and then compute the impurity criterion␣
↪using these tables.
    # In the end, we return a triple consisting of
    # - the best score we found, according to the criterion we're using
    # - the id of the feature
    # - the threshold for the best split
    def best_split(self, X, Y, feature):

        # Create a list of input-output pairs, where we have sorted
        # in ascending order by the input feature we're considering.
        sorted_indices = np.argsort(X[:, feature])
        X_sorted = list(X[sorted_indices, feature])
        Y_sorted = list(Y[sorted_indices])

        n_row = len(Y)

        # The frequency tables corresponding to the parts *before and including*
        # and *after* the current element.
        low_part = []
        high_part = Y_sorted.copy()

        # Keep track of the best result we've seen so far.
        max_score = -np.inf
        max_i = None

        # Go through all the positions (excluding the last position).
        for i in range(0, n_row-1):

            # Input and output at the current position.
            x_i = X_sorted[i]
            y_i = Y_sorted[i]

            # Update the frequency tables.
            low_part.append(y_i)
            high_part.pop(0)

            # If the input is equal to the input at the next position, we will
            # not consider a split here.
            #x_next = XY[i+1][0]
            x_next = X_sorted[i+1]
            if x_i == x_next:
                continue

            # Compute the homogeneity criterion for a split at this position.
            s = low_part + high_part
            n = n_row
```

```
            score = np.var(s) - ((i + 1) * np.var(low_part) + (n_row - i - 1) *␣
↪np.var(high_part)) / n
            #score = variance_reduction(i+1, low_part, n_row-i-1, high_part)

            # If this is the best split, remember it.
            if score > max_score:
                max_score = score
                max_i = i

        # If we didn't find any split (meaning that all inputs are identical),␣
↪return
        # a dummy value.
        if max_i is None:
            return -np.inf, None, None

        # Otherwise, return the best split we found and its score.
        split_point = 0.5*(X_sorted[max_i] + X_sorted[max_i+1])
        return max_score, feature, split_point
```

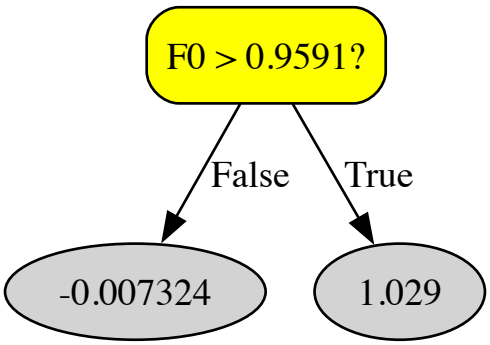
Step 2. Sanity check

[24]:
```
def make_some_data(n):
    x = np.random.uniform(-5, 5, size=n)
    Y = (x > 1) + 0.1*np.random.normal(size=n)
    X = x.reshape(n, 1) # X needs to be a 2-dimensional matrix
    return X, Y
X, Y = make_some_data(100)
plt.scatter(X, Y)
plt.show()
```



[25]:
```
# Small dataset training
clf = TreeRegressor(max_depth=1)
clf.fit(X, Y)
clf.draw_tree()
```

[25]:



Step 3. Predicting apartment prices using decision tree regression

[26]:
```
Rus = TreeRegressor(max_depth=5)
Rus.fit(Xtrain, Ytrain)
```

```
Rus.score(Xtest, Ytest)
```

[26]: 0.27799386048394825

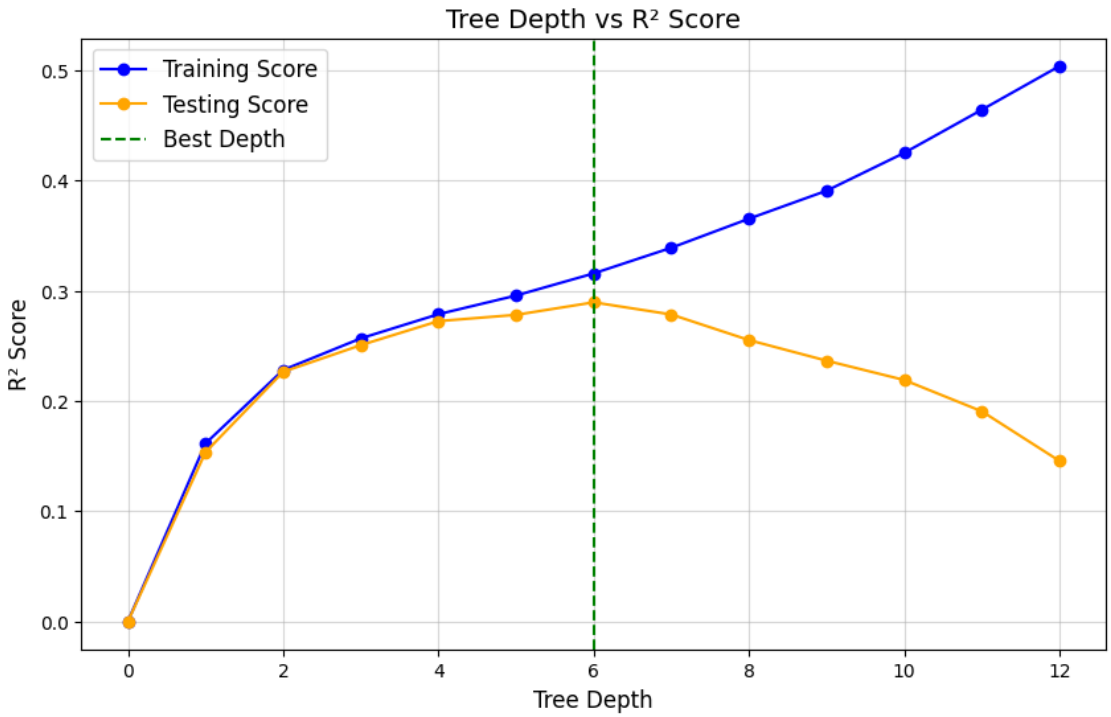
For this prediction, as we studied before, we knew there would be a difference in performance if we choose different depths. Thus, we chose 5 as the value of max_depth to test. The evaluation score is 0.28. This score indicates the proportion of variance explained by the model on the test set, which reflects the predictive performance of the decision tree.

Step 4. Underfitting and overfitting

[27]:
```
from sklearn.metrics import r2_score

def test_tree_regressor_depth(X_train, X_test, Y_train, Y_test, max_depth=13):

    depths = np.arange(0, max_depth)
    train_scores = []
    test_scores = []

    print("Depth | Training Score | Testing Score")
    print("-------------------------------------")

    for depth in depths:
        rus = TreeRegressor(max_depth=depth)

        rus.fit(X_train.values, Y_train.values)

        Y_train_pred = rus.predict(X_train.values)
        Y_test_pred = rus.predict(X_test.values)

        train_score = r2_score(Y_train.values, Y_train_pred)
        test_score = r2_score(Y_test.values, Y_test_pred)

        train_scores.append(train_score)
        test_scores.append(test_score)

        print(f"{depth:<5} | {train_score:<14.4f} | {test_score:<14.4f}")

    plt.figure(figsize=(10, 6))
    plt.plot(depths, train_scores, label='Training Score', marker='o',␣
↪color='blue')
    plt.plot(depths, test_scores, label='Testing Score', marker='o',␣
↪color='orange')
    plt.axvline(x=np.argmax(test_scores), color='green', linestyle='--',␣
↪label='Best Depth')
    plt.title('Tree Depth vs R² Score', fontsize=14)
    plt.xlabel('Tree Depth', fontsize=12)
    plt.ylabel('R² Score', fontsize=12)
```

```
    plt.legend(fontsize=12)
    plt.grid(alpha=0.5)
    plt.show()

    best_depth = np.argmax(test_scores)
    print("\nBest cooperation:")
    print(f"Best depth: {best_depth}")
    print(f"Max test score: {max(test_scores):.4f}")

test_tree_regressor_depth(Xtrain, Xtest, Ytrain, Ytest, max_depth=13)
```

```
Depth | Training Score | Testing Score
-------------------------------------
0     | 0.0000         | -0.0001
1     | 0.1614         | 0.1534
2     | 0.2280         | 0.2263
3     | 0.2566         | 0.2505
4     | 0.2786         | 0.2723
5     | 0.2955         | 0.2780
6     | 0.3156         | 0.2893
7     | 0.3389         | 0.2783
8     | 0.3652         | 0.2551
9     | 0.3906         | 0.2364
10    | 0.4249         | 0.2189
11    | 0.4641         | 0.1904
12    | 0.5037         | 0.1454
```



```
Best cooperation:
Best depth: 6
Max test score: 0.2893
```

In the final underfitting and overfitting analysis, the blue curve represents the training score and the orange curve represents the testing score. The green line represents the best depth that achieves the highest testing score. For the blue curve, the score rises as the depth increases, indicating that the model is overfitting the training data. For the orange curve, the score increases as the depth increases, however, when reaching a pole, the score starts to decrease, indicating that the model is starting to overfit the test data. And among the range of the assignment, the best depth is 6 after comparing to each other. the Max test score is 0.2893, which is the best score we can get. The chart shows the total depths we tried and the corresponding scores also with the figure above.