Static Test Stand Report

Hayden Gebhardt, Maximillian Brown, Jacob Ecks, Ethan Hughes, Brad Millwater, Rishi Ayyagari
Ann & H.J. Smead Department of Aerospace Engineering Sciences

Using and manipulating experimental data to create models of physical systems and derive values is a common and necessary practice in the engineering world. With this lab, the goal is to use given data to learn about specific impulse, thrust, and the propulsion performance of a bottle rocket. In order to create the bottle rocket performance model, experimental data taken from various test flights was inputted into MATLAB to create a graph showing the thrust versus time of all the test flights. Then, to find the specific impulse (the parameter needed to maximize performance), the area under the curve was calculated with the use of a trapezoidal integration approach. By following this method, we were able to find peak thrust, the specific impulse, and the time of each test flight which will help in maximizing the rocket's performance for the physical test. By performing error analysis alongside every calculation, we ensured our results were reliable and as accurate as possible.

I. Nomenclature

I	=	impulse
I_{sp}	=	specific impulse
$F(t)$	=	rocket engine thrust as a function of time
m_{prop}	=	propellant mass
g_0	=	gravitational acceleration
$\bar{X}$	=	sample mean
SEM	=	standard error of the mean
N	=	number of samples
s	=	standard deviation of samples

II. Introduction and Theory

Specific impulse (I_{sp}) is an important factor to know when designing a rocket. Rocket engines work by expelling propellant mass out the nozzle, which changes the momentum, creating an upward force on the craft. The change in momentum caused by the rocket engine is called impulse, and normalizing with propellant mass yields specific impulse. Since specific impulse is a measurement of change in momentum per unit propellant mass, it acts as a measurement of a rocket engine's efficiency. To obtain specific impulse, the total thrust force must be integrated over time to obtain total impulse, then it must be normalized with propellant mass.

$$I = \int F(t)dt \tag{1}$$

$$I_{sp} = \frac{I}{m_{prop}g_0} \tag{2}$$

Given the nature of the experiment, the error of the specific impulse cannot simply be propagated analytically, due to the probability density functions being unknown. In order to determine uncertainty, the standard error of the mean is employed, which is mathematically defined as:

$$SEM = \frac{s}{\sqrt{N}} \tag{3}$$

where s is the standard deviation of the all the sample data, and N is the quantity of samples. Note that this process requires multiple samples, which are used to estimate the probability distributions. The SEM can be decreased, reducing uncertainty, by simply using more samples, since N is in the denominator.

The confidence interval is a method to determine the likelihood that the probability density function (PDF) mean is within a specified range of the sample mean, to a desired confidence level. Mathematically, it is defined as:

$$\bar{X} \pm z * SEM \tag{4}$$

where $\bar{X}$ is the mean of the samples, and z is a statistically-determined constant relating to the desired confidence level. By increasing z, the range around the sample mean in which the actual PDF mean may fall into widens, improving the confidence that it falls within that range.

If it is desired to compute the I_{sp} to a certain degree of accuracy with a given precision σ with a desired confidence, the number of samples required to reach that can be determined by simply working backwards with Eqs. 3 and 4. Rearranging yields the equation to determine the required quantity of samples:

$$N = (\frac{z * s}{\sigma})^2 \tag{5}$$

III. Materials and Methods

A. Experimental Process

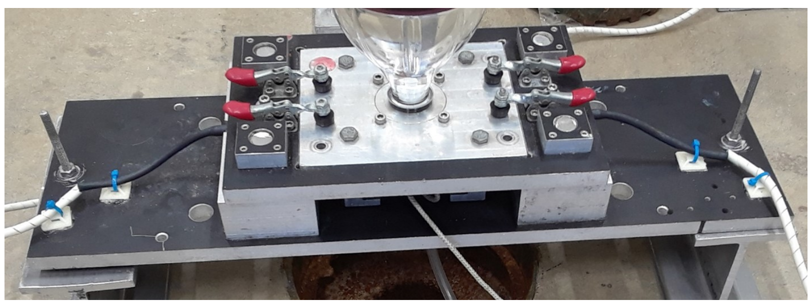

Figure 1 Static Test Stand Setup [4]

The testing process begins by fixing the bottle rocket, loaded with a set amount of propellant, to the NI-9174 Chassis with NI-9234 Data Acquisition Module, shown in fig. 1. This is the apparatus that will secure the rocket, as well as record data from the two load cells. The bottle is then pressurized to 40±1 psi, the load cells are zeroed, then the release cord is pulled to start the test. The load cells will detect the mass flow of water and air out of the rocket's nozzle, giving a reading for the thrust force in lbf. After performing 16 tests with the same starting amount of propellant and water, the data is now ready to be used [4]. When first viewing the data, we noted some inconsistencies before and after the rocket is fired. Most inconsistencies before the test are likely due to the apparatus moving as the release cord is pulled. After the test, the reading across both load cells is not the same as before the test (i.e. zero), due to the bottle rocket losing propellant mass during testing. Though not as major, small air pressure differences or currents could also affect the data gathered.

B. Calculating Specific Impulse

Now that we have a large set of experimental data, we can begin calculating the specific impulse. The first step, because the thrust experiments do not start at the same times, is to find when the change between two data points reaches a certain amount. This point will be considered the beginning of the applied thrust. Similarly, the end points can be found. In this case, we need to find when the change between two data points is minimal. This can be done by iterating across every point in the data set, or by setting bounds that vary only slightly from the data after the thrust applied is zero. The next step is then to integrate the curve in order to find the total impulse over the individual thrust curves. This will have to be done by numerical integration methods such as a Riemann sum or trapezoidal integration approximations. The trapezoidal method is often more accurate than Riemann sums, so we will be using this method. Finally, we are only required to divide the total impulse value by m_{prop} and g_0 to obtain the I_{sp}. Each of these steps can be completed in a loop, iterating across all 16 given data sets.

IV. Results: Descriptive Statistics

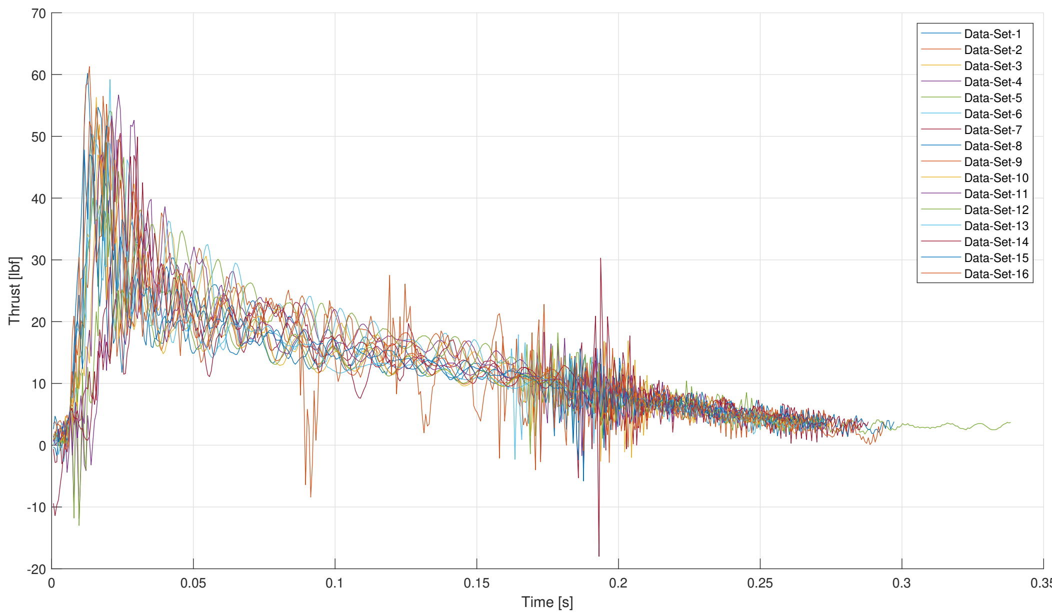

Figure 2 Adjusted Thrust Curves

Trial Number	I_{sp} [s]	Peak Thrust [lbf]	Time of Test [s]
1	1.67	60.2	0.272
2	1.87	61.3	0.256
3	1.78	56.3	0.268
4	1.81	56.7	0.264
5	1.98	54.1	0.338
6	1.82	59.2	0.253
7	1.57	49.9	0.285
8	1.65	50.4	0.297
9	1.65	55.2	0.276
10	1.54	47.0	0.274
11	1.63	53.3	0.273
12	1.54	49.0	0.271
13	1.55	48.8	0.273
14	1.66	50.5	0.288
15	1.64	51.7	0.273
16	1.75	52.4	0.293

Table 1 Calculated I_{sp} and given thrust data

The I_{sp} was calculated using given data from [3], while the peak thrust and test times are from the raw data in table 1. From these values, we obtained the mean and standard deviation for the I_{sp}, the peak thrust, and the time of each test. The mean values are 1.7 seconds, 53.5 lbf, and 0.278 seconds respectively. Also, the standard deviations are as follows: 0.129 seconds, 4.32 lbf, and 0.02 seconds. For all of these calculations, m_{prop} was assumed to be 1000 grams, or 0.0685 slugs.

V. Results: Standard Error of the Mean Analysis

The standard error of the mean (SEM) is a quantitative representation of the difference between the mean of the sample and the mean of the actual population. This is closely related to the standard deviation; the formula for SEM even includes the standard deviation, but while the standard deviation measures the inconsistency between individual values and the mean, SEM measures the discrepancy between a sample mean and the whole population's mean [1]. SEM helps us find out the reliability of the average when using large, highly variable sets of data. It stands to reason that having more data helps us make better approximations, as the mean and SEM are more accurate. By plotting SEM of the specific impulse versus the number of tests, N, we can see a clear trend.

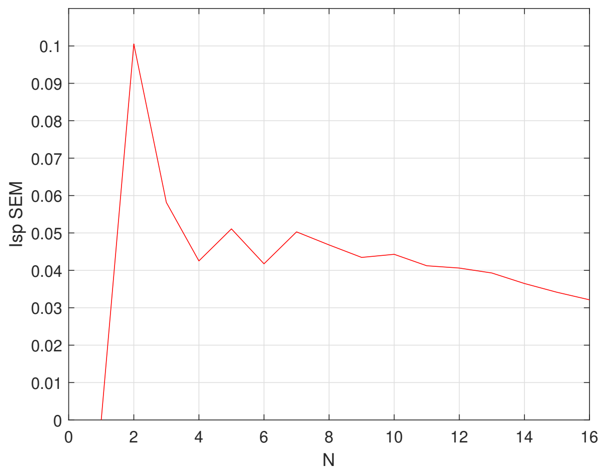

Figure 3 SEM vs number of trials

From fig. 3 we can gather that with a larger sample set the SEM decreases. To get more reliable approximations for SEM, we need to get as large of an N value as possible. Calculating the I_{sp} SEM with the whole data set gives us 0.0321 seconds. We know this is the most reliable estimate for I_{sp} SEM as it uses the most data and thus has the highest value for N. This in turn makes our vallues for I_{sp} more reliable as they have the smallest possible error over the whole sample population.

VI. Results: Confidence Analysis

	95%	97.5%	99%
Confidence Interval [s]	[1.632, 1.758]	[1.623, 1.767]	[1.613, 1.778]

Table 2 Confidence in calculated I_{sp} for all 16 trials

By working backwards through our calculations, we were able to compute the necessary amount of tests needed to obtain an I_{sp} value to a desired degree of accuracy. If we wished to find an I_{sp} value within 0.1 seconds of the true mean, 7 tests would need to be completed for 95% confidence, 9 for 97.5% confidence, and 11 for 99% confidence. For a desired accuracy of ± 0.01 seconds, we would need to conduct one hundred times as many tests: 634 for 95% confidence, 828 for 97.5% confidence, and 1100 for 99% confidence. These results make sense; namely, that as higher accuracy is desired, more tests must be conducted.

VII. Conclusion

The goal of this lab was to experimentally calculate the specific impulse of a bottle rocket, and to determine the amount of trials needed to reach a certain confidence interval using the standard error of the mean. The experiment was conducted using a test stand to which the rocket stays fixed throughout the launch, and the thrust was measured with two load cells connected to the stand. The data from 16 launch trials was compiled to create Fig. 2, and analyzed to provide the specific impulse, peak thrust, and duration for each trial, as shown in Table 1. From this data, the standard error of the mean analysis gave the confidence intervals and trials required to obtain 0.1 s and 0.01 s of uncertainty. These results concluded that our experiment could guarantee 99% confidence with 0.1 s of uncertainty, but that the experiment would need many more trials to approach 0.01 s. Some contributing factors to this uncertainty were the movement of the stand while the release cord was pulled, slight shifting of the load cells during the launch, and small air pressure differences in between trials. While more trials would have provided greater confidence, the labor involved with cleaning the data and compiling it so that the launch times aligned placed a limitation on the analysis. For future experiments, we learned that an automated method of cleaning each trial's data would greatly benefit the experimental analysis and provide the group with more time to run trials and achieve a smaller confidence interval.

References

1) Tuovila, A., "Standard error of the mean vs. standard deviation: The difference," Investopedia Available: https://www.investopedia.com/ask/answers/042415/what-difference-between-standard-error-means-and-standard-deviation.asp.
2) Hall, N., "Specific impulse," National Aeronautics and Space Administration Available: https://www.grc.nasa.gov/www/k-12/airplane/specimp.html.
3) Gravel, W., "Static Test Stand Data - 16 trials," Mar. 2022.
4) Instructional Team, ASEN 2004, Boulder, CO: 2022.

Appendix

Appendix A.1: `main()`

```
%% ASEN 2004 Rocket Lab
2 % Static Test Stand Report
3 % Hayden Gebhardt, Max Brown
clear; clc; close all;
% Mass of the propellant
propMass = 0.0685; %[slug]

%% Reading in Static Test Stand Data files
%Save the folder of files in the current directory
path_directory='Static Test Stand Data/Fixed Mass';
original_files=dir(path_directory);
plotnum = 1;
figure('WindowState','maximized');
% Initialize
%data = zeros(14000,3,18);
% Iterate through each data set
for k=3:length(original_files)
    % load file
    filename=[path_directory '/' original_files(k).name];
    data(:,:,k) = readmatrix(filename,'Range',9);
    % create time vector based on sample rate
    time =  1:length(data);
    time = time' / 1652;

    peaks(k,1) = max(data(:,3,k));

    idx = find(data(:,3,k) > 5);
    starts(k,1) = idx(1)-10;

    idx2 = data(1:6608,3,k) < 3.7 & data(1:6608,3,k) > 0;
    idx3 = find(idx2 == 0);
    ends(k,1) = idx3(end);

    % plot each trial individually

    hold on;
```

```
    subplot(4,4,plotnum)
    plot(time(1:end),data(1:end,3,k),'DisplayName','Thrust')
    xline(time(starts(k)),'r','DisplayName','Start')
    xline(time(ends(k)),'b','DisplayName','Stop')
    title(original_files(k).name)
    xlabel('Time [s]')
    ylabel('Thrust [lbf]')
    %legend('show')
    grid on;
    hold off;
    plotnum = plotnum + 1;
end


% clean out first two entries which are blank
starts(1) = [];
starts(1) = [];
peaks(1) = [];
peaks(1) = [];
ends(1) = [];
ends(1) = [];
% don't need the individual readings, just the sum
data(:,:,1:2) = [];
data(:,1:2,:) = [];


timeStep = time(2)-time(1);

sizeData = size(data);

% plotting every trial dataset on one plot
figure('WindowState','maximized');
hold on;
for i = 1:sizeData(3)

    % bounds
    idx1 = starts(i);
    idx2 = ends(i);

    % trimmed data
    cleanData{:,1,i} = data(idx1:idx2,1,i);
    % use trapezoidal integration to approximate area un the curve
    impulse(i,1) = trapz(timeStep,cleanData{:,1,i});
    % divide by g0 and mass of propellant to get Isp
    Isp(i,1) = impulse(i) / (32.2 * propMass);

    timeLength(i,1) = time(idx2)-time(idx1);

    % plot each set
    legendEntry = strcat('Data-Set-',num2str(i));
    plot(time(1:length(cleanData{:,1,i})),cleanData{:,1,i},'DisplayName',legendEntry)
end

% Calc Means and Std dev
meanIsp = mean(Isp);
stdIsp = std(Isp);
meanPeakThrust = mean(peaks);
stdPeakThrust = std(peaks);
meanTimeLength = mean(timeLength);
stdTimeLength = std(timeLength);

xlabel('Time [s]')
ylabel('Thrust [lbf]')
legend('show')
grid on;
hold off;
saveas(gcf,'adjustedData','epsc')

```

```
%% Finding confidence intervals

% Each data set

% [Ci95, Ci975, Ci99, SEM, N, N_s]
[stats{1,1}, stats{1,2}, stats{1,3}, stats{1,4}, stats{1,5}, stats{1,6}] = getCi(Isp);


for i = 1:16
    SEMtest(i) = std(Isp(1:i))/sqrt(i);
end

figure();
plot(1:16,SEMtest, '-r')
xlabel('N')
ylabel('Isp SEM')
axis([0 16 0 0.11])
grid on;
saveas(gcf,'semvn','epsc')
```

Appendix A.2: `getCi()`

```
%% ASEN 2004 Rocket Lab
% Static Test Stand Report
% Hayden Gebhardt, Max Brown
function [Ci95, Ci975, Ci99, SEM, N, N_s] = getCi(data)
    %finding std and mean of a sample:
    Xbar = mean(data);
    s = std(data);

    %calculating SEM
    N = length(data);
    SEM = s/sqrt(N);

    %z's
    z95 = 1.96;
    z975 = 2.24;
    z99 = 2.58;

    %Confidence interval
    Ci95 = [Xbar - z95*SEM, Xbar + z95*SEM];
    Ci975 = [Xbar - z975*SEM, Xbar + z975*SEM];
    Ci99 = [Xbar - z99*SEM, Xbar + z99*SEM];

    %\\ Need to calculate I_sp for the data set (or theoretically?) so that
    %we can then work backwards and find the N to get within 0.1 and 0.01
    %of I_sp for each value of z, a total of 6 values of N with varying
    %confidence.

    Ci01 = Xbar*0.1;
    Ci001 = Xbar*0.01;

    SEM9501 = (Xbar-Ci01)/z95;
    SEM95001 = (Xbar-Ci001)/z95;
    SEM97501 = (Xbar-Ci01)/z975;
    SEM975001 = (Xbar-Ci001)/z975;
    SEM9901 = (Xbar-Ci01)/z99;
    SEM99001 = (Xbar-Ci001)/z99;

    N9501 = (z95*s/0.1)^2;
    N95001 = (z95*s/0.01)^2;
    N97501 = (z975*s/0.1)^2;
    N975001 = (z975*s/0.01)^2;
    N9901 = (z99*s/0.1)^2;
    N99001 = (z99*s/0.01)^2;
```

```
    N_s = [N9501;N95001;N97501;N975001;N9901;N99001];
end
```