University of Colorado - Boulder

ASEN 3113: Thermodynamics and Heat Transfer

October 3, 2022

Heat Conduction Lab

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Heat conduction is defined as the movement of heat between two systems in contact with each other across a temperature gradient. The objective for this lab is to analyze the steady state and transient behavior of heat conduction through three solid metal rods comprised of aluminum, brass, and stainless steel. The rods are assumed to be one-dimensional, allowing the use of the one-dimensional heat equation (I) in experimentation. The experiment for this lab employs the use of the NESLAB RTE-110 Water Chiller and a low-voltage power supply to facilitate heat transfer through the test rods. Each rod is equip with linearly-spaced thermocouples that take temperature readings for the duration of each test. Data from the thermocouples is then saved and imported into MATLAB simulations for analysis. This experiment aims to enforce the understanding of the fundamentals of heat conduction through constant mediums, and how the associated partial differential equations and boundary conditions affect both analytical and experimental models. To achieve this understanding, four models were generated. Model IA plots the steady-state distribution slopes against the thermocouple locations for each rod. Model IA generated large relative errors for the stainless steel and aluminum rods. To combat this, Model IB substitutes the analytical steady-state slope with the experimentally-determined slope in an effort to increase the accuracy of the model predictions. Model 1B was found to be far more accurate than model IA due to the large discrepancy between analytical and experimental distribution slopes. The brass test rods were exceptions to this, as they exhibited relative errors of 1.81% and 3.24% when supplied 26V and 29V, respectively. Model II aims to better Model 1B by performing a linear fit on the initial temperature distribution to extrapolate their initial state conditions. Model II provided a better estimate than Model 1B, as it employed a more accurate slope for the experimental data and accounted for the discrepancy in temperature. The final model, Model III, varies the thermal diffusivity value of each data set to best match the experimental and analytical results. The adjusted thermal diffusivities from Model III aided in calculating more accurate Fourier numbers, to better predict the propagation of heat transfer through the test rods, as well as the time to steady-state.

Nomenclature

α	Thermal diffusivity, m^2/s
$\dot{Q}$	Rate of heat, W
ρ	Density, kg/m^3
Fo	Fourier number
H_{an}	Analytical slope, K/m
H_{exp}	Experimental slope, K/m
k	Thermal conductivity, W/(m K)
T_0	Initial temperature of rod, °C
t_{ss}	Time to steady-state temperature, s

Contents

I. Introduction

Heat conduction in a metal rod is governed by several well-known equations, assuming one-dimensional heat transfer and constant heat flow (and no heat loss). In the simplest of models, the rod is assumed to have a constant temperature at one end (approximately 10 °C in this lab) and another, constant temperature at the other (determined by the power supply of the resistance band heater). Then, the heat is assumed to become linearly distributed along the length of the rod after the rod is allowed to heat for a long time. By placing several thermocouples, this steady-state solution can both be predicted and measured.

Using the following heat equation:

$$\frac{d^2u(x,t)}{d^2x} = \frac{1}{\alpha}\frac{du(x,t)}{dx}$$

and applying the following boundary conditions:

- The initial state of the rod is known $(u(x,0))$, and is independent of location.
- The temperature at any time t at $x = 0$ is known $(u(0,t))$ and is defined by T_0.
- The temperature at any time t at $x = L$ is known $(u(L,t))$ and is defined by $\frac{du}{dx}(L,t) = H$ where $H = \frac{\dot{Q}}{kA}$.

Once this is done, these equations and conditions can be applied to find both the transient and steady-state solutions, which can allow for a model of the temperature of the rod at any time and location along the rod. Many of these quantities are either unknown or must be approximated or experimentally determined, such as the slope H. In addition, some quantities such as the thermal diffusivity α and the initial temperature T_0 contain inherent error, either due to experimental error or reporting error (α may be different due to different material properties than those in the literature). In this lab, we seek to determine the effect of these variables on our prediction of $u(x,t)$, quantify them, and decide which are most (or least) important in order to develop the most accurate possible model.

II. Experimental Procedures

1) Set the water temperature to 10 °C.
2) Plug the thermocouple box to the computer, and plug the thermocouples from the heat conduction apparatus into the thermocouple box receptacles.
3) Run the VI to begin monitoring data.
4) Wait for the entire rod to be cooled to 10 °C.
5) Once the rod temperature has been verified, set the voltage and current to an acceptable value (15 V $\leq V \leq$ 30 V, $I \approxeq$ 400 mA).
6) Stop the VI, and begin running again to start collecting data.
7) Turn on the resistance heater power supply.
8) Monitor the rod temperature closely, watching for the time at which a steady state temperature has been reached.
9) Once a steady state condition has been reached, stop the VI and turn off the power supply, and save the data.
10) Repeat for as many samples as necessary.

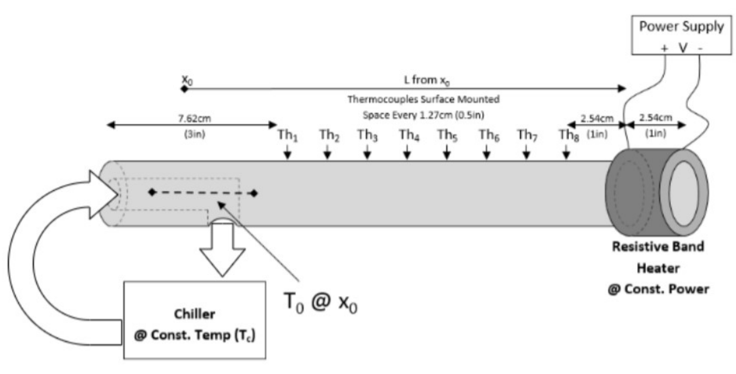

Fig. 1 Schematic of rod inside heating/cooling apparatus.

III. Results

The table containing the experimentally-determined steady-state slope and the analytically-determined steady-state slope for each material is shown below.

Material	Voltage (V)	Current (mA)	$T_0(C)$	$H_{exp}(K/m)$	$H_{an}(K/m)$	Percent Error
Aluminum	26	250	12.1045	51.36	98.67	47.9
Aluminum	28	269	12.5324	63.22	114.34	44.7
Brass	26	245	12.1024	107.34	109.32	1.81
Brass	29	273	12.2437	131.46	135.86	3.24
Steel	21	192	11.3509	261.79	491.19	46.7

Table 1 Experimental & Analytical Slope For Varying Materials.

The values obtained for the experimental and analytical steady state slope of each material are seen above in Table 1. The experimental slopes were obtained by plotting a best fit line to the data, while the analytical slopes were determined using the formula,

$$H_{an} = \frac{\dot{Q}}{kA} \tag{1}$$

where $\dot{Q}$ is the rate of heat into the system (calculated from P = IV), k is the thermal conductivity of the material, and A is the cross-sectional area of the rod. The heat transfer value is equivalent to the voltage multiplied by the amps value that the heat is supplied. For the aluminum and steel rods, this equation did not represent an accurate approximation for the slope as they were off by a factor of nearly two for both materials. However, for both brass rods, the equation provides a nearly perfect approximation. This result does not make physical sense since aluminum has higher thermal conductivity, electrical conductivity, and thermal diffusivity than brass and steel meaning its slope value should be the most accurate of the three materials tested. Additionally, the fact that brass is an alloy does not contribute to this phenomenon since steel is also an alloy and sees the opposite trend. The only possible reasons for this large of a discrepancy is that the aluminum rod was not properly insulated or there was some other source of error associated with the aluminum rod experiment. The fact that the slope values for the brass rods differ by a value of four is likely due to the accuracy of the analytical slope equation with the various assumptions it requires to be used.

The analytical solution is as follows:

$$u(x,t) = T_0 + Hx + \sum_{0}^{n} b_n sin(\lambda_n) e^{-\lambda^2 \alpha t}$$

where $b_n = (-1)^{n+1} \frac{8HL}{\pi^2(2n-1)^2}$ and $\lambda_n = \frac{(2n-1)\pi}{2L}$.
In this lab, n was chosen to vary from 1 to 10, as 10 terms reduced a lot of variability.

The plot showing the initial temperature of each rod at each thermocouple location is shown below.

For every rod except the steel rod, the assumption that the initial temperature was constant across the rod was correct to within approximately 1 degree Celsius. However, the steel rod temperature varies significantly across the length of the rod, meaning that this assumption cannot reasonably be made for this material at the time tested. Because steel has a very high thermal resistance, it was not sufficiently cooled from ambient temperatures. This could have been remedied by giving the steel rod more time to cool to its first, cooler steady state temperature. Even with steel not having a constant initial temperature value across the rod, the transient phase would not be greatly affected as the temperature only varies by roughly 5 degrees Celsius. In comparison to the final temperature of 45 degrees Celsius, this is not a significant difference and would not affect the transient phase greatly. This claim is corroborated by the fact that the temperature change over time for all rod materials are largest at the start of the heating process and therefore removes this source of error. Finally, the steady state value would not be affected by this initial difference in temperature because the final temperature achieved is a function of the heat supplied and the material properties. This small, additional, initial heat in the rod means the time it takes to reach its steady state temperature will be shorter, but that value for the steady state temperature would not change.

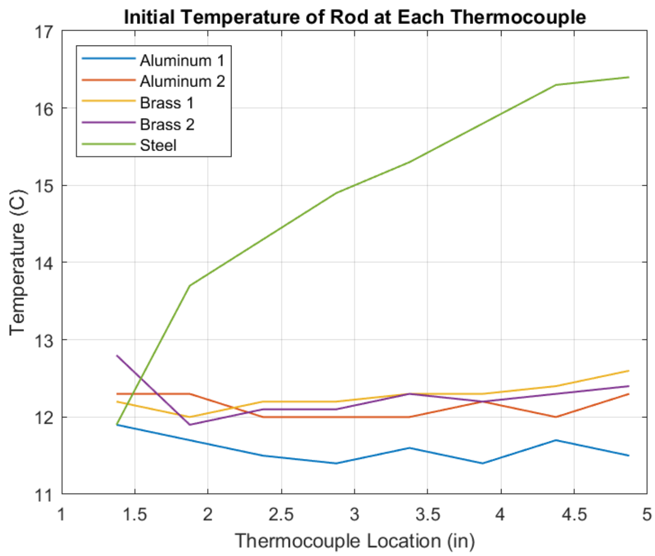

Fig. 2 Thermocouple temperatures at t = 0.

IV. Analysis and Discussion

A. Model I

Model Ia was calculated using the analytical slope, H_{an} (determined from Equation (1)), in the model for $u(x,t)$, whereas Model Ib used the experimental slope H_{exp} (determined from the graphs). The results of Model I are shown below.

1. Model IA.

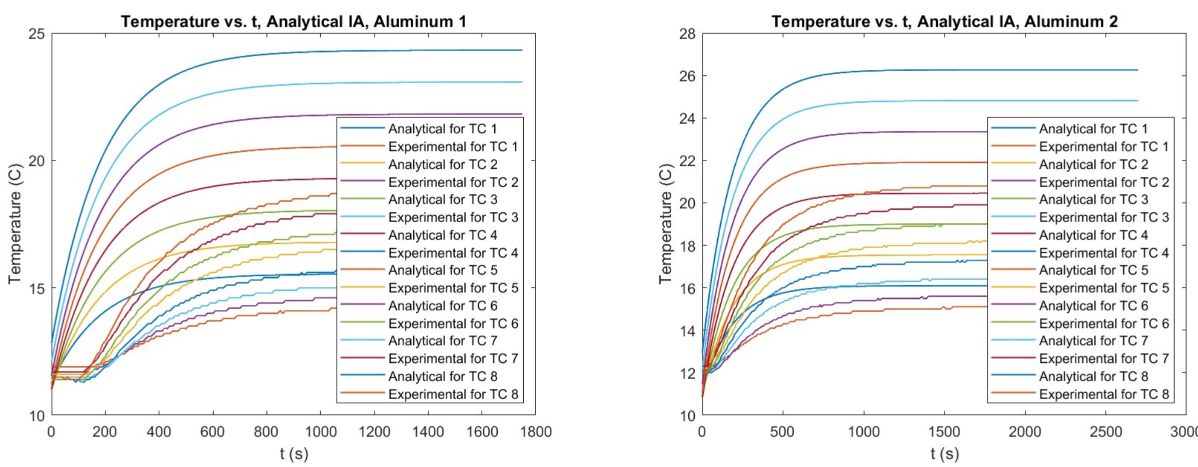

Fig. 3 Aluminum samples for Model IA.

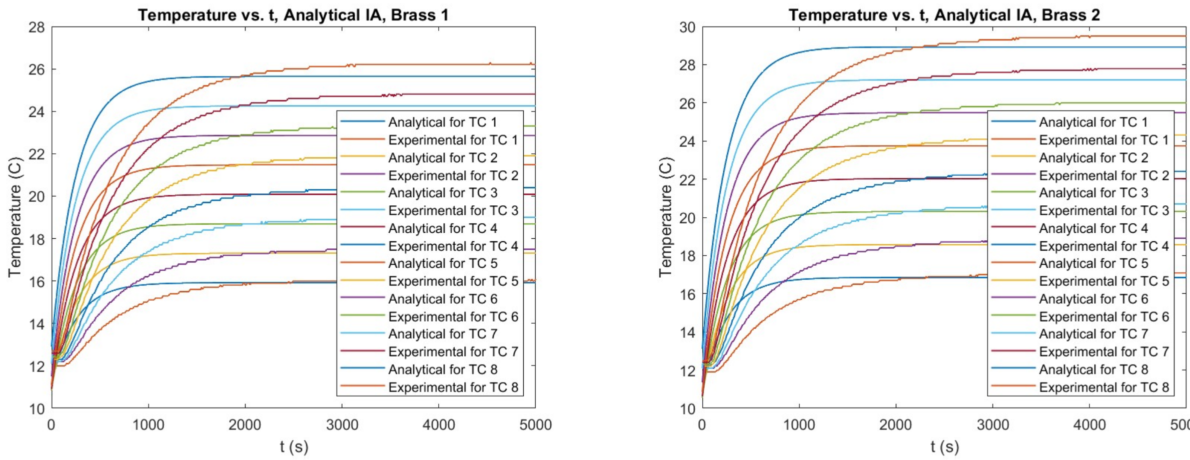

Fig. 4 Brass samples for Model IA.

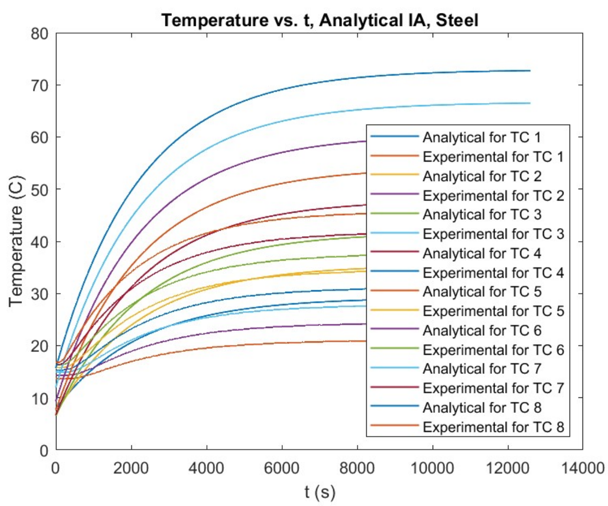

Fig. 5 Steel sample for Model IA.

2. Model IB.

As can be seen from the plots, Model IB was a lot more accurate, due to the sometimes large discrepancy between analytical and experimental slopes. (This discrepancy is discussed more in the preceding paragraph). However, for brass, in particular, the two models did not differ by much, due to the similarity in experimental and analytical slope.

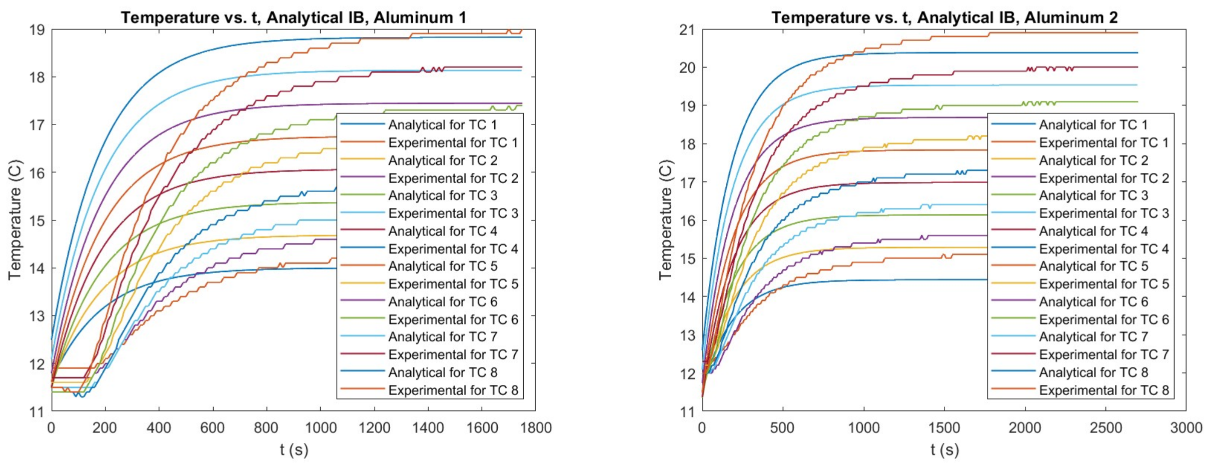

Fig. 6 Aluminum samples for Model IB.

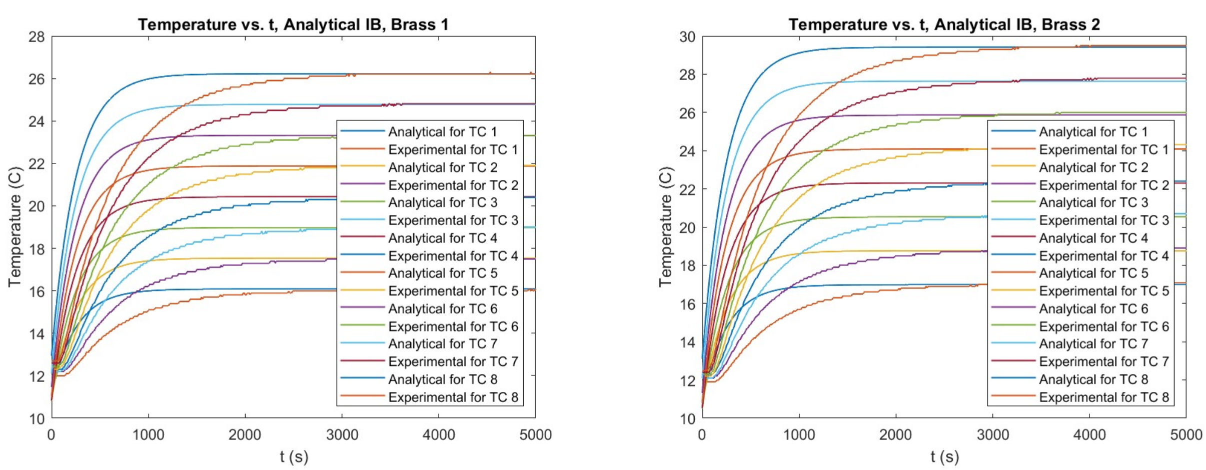

Fig. 7 Brass samples for Model IB.

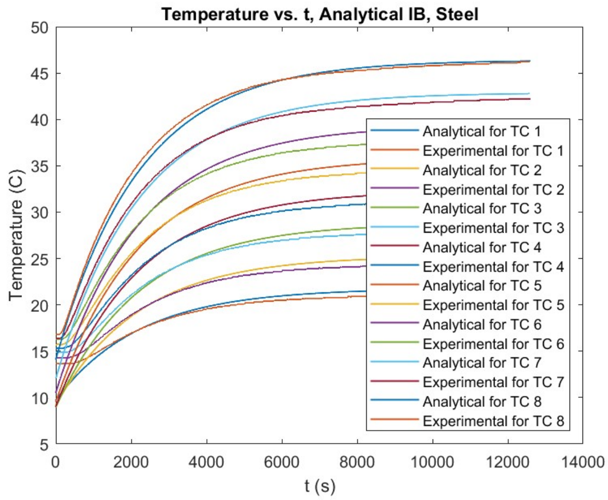

Fig. 8 Steel sample for Model IB.

B. Model II

For Model II, the initial temperature of each rod was considered, as well as the extrapolated value of $T(x = 0, t = 0)$. Model II used a new model for the temperature of the thermocouple by taking into account the initial steady state slope of the thermocouple. This new model was derived using the transient heat solution slides provided on Canvas, and the new derivation can be found in the Appendix of this document.

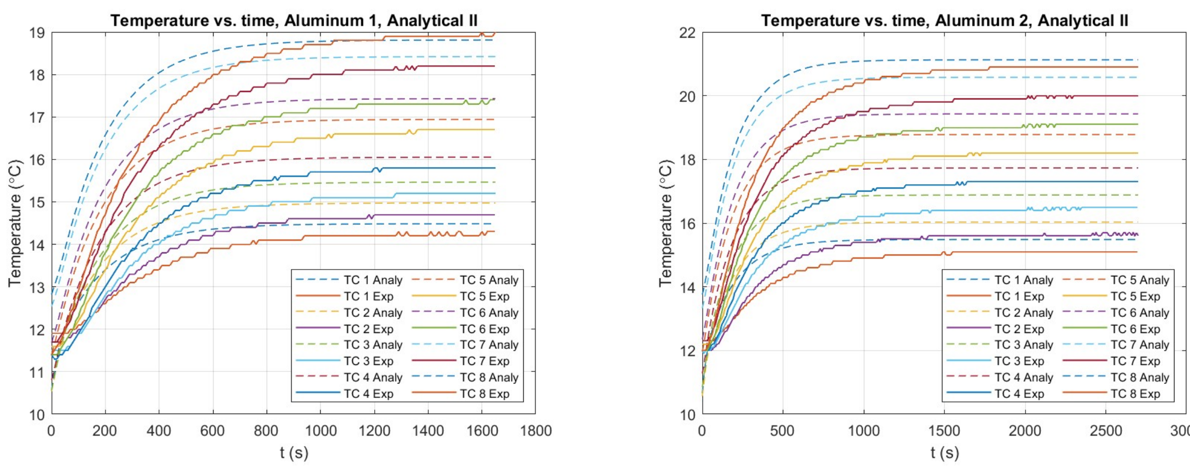

Fig. 9 Aluminum samples for Model II.

The corresponding table of slopes is also shown.

These slopes were used in the alternative derivation of the temperature function $u(x, t)$ that did not include the assumption that the beginning slopes were zero. This model was slightly better than the first two (except in the case of steel, for which our starting temperature data is misleading), especially since it used a more accurate slope for the experimental data and accounted for the discrepancy in temperature.

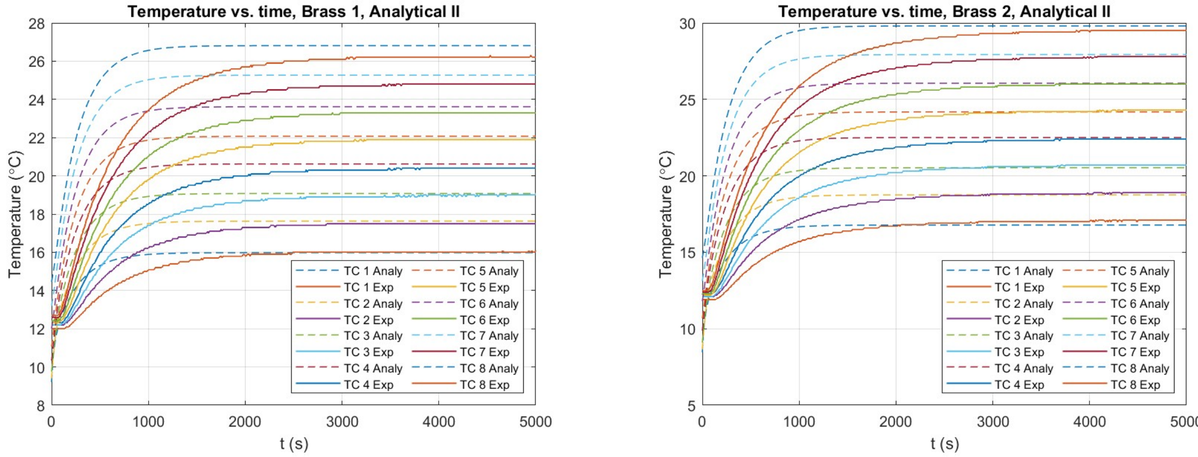

Fig. 10 Brass samples for Model II.

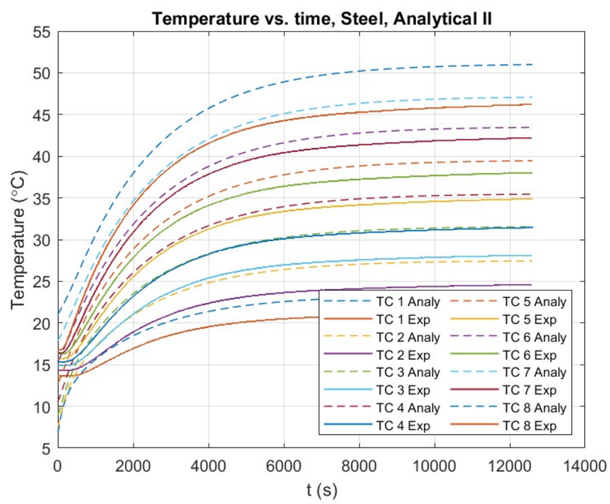

Fig. 11 Steel sample for Model II.

Material	Initial State Slope, M_{exp} (°C/m)	Extrapolated T_0 (°C)
Aluminum 1	-3.37	11.89
Aluminum 2	-0.375	12.13
Brass 1	7.03	11.78
Brass 2	5.81	11.76
Steel	34.6	12.69

Table 2 Initial State Slopes & Extrapolated Starting Temperatures.

C. Model III

For Model III, the differing slopes were used as well as adjusting the thermal diffusivity α from its nominal value to one that caused the model to best fit the experimental data.

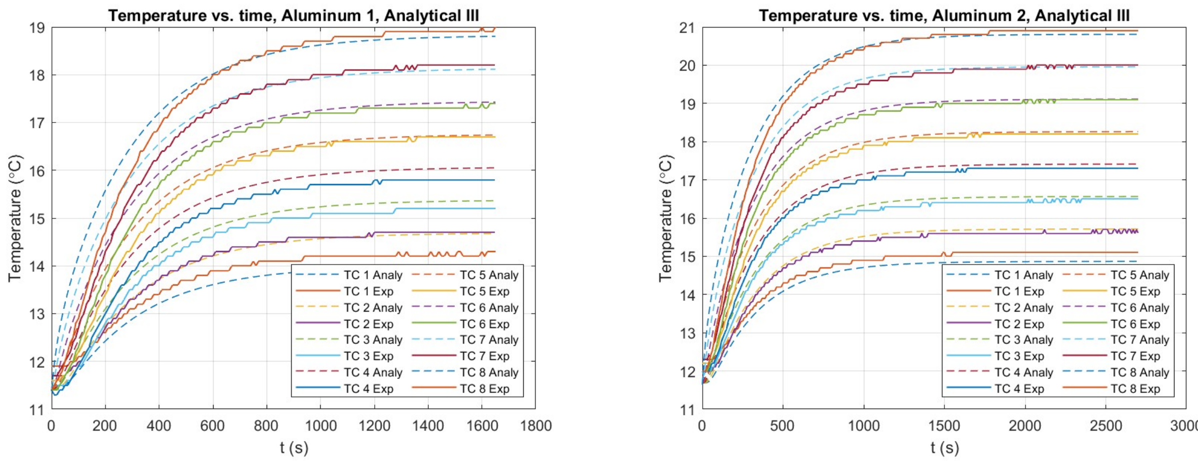

Fig. 12 Aluminum samples for Model III.

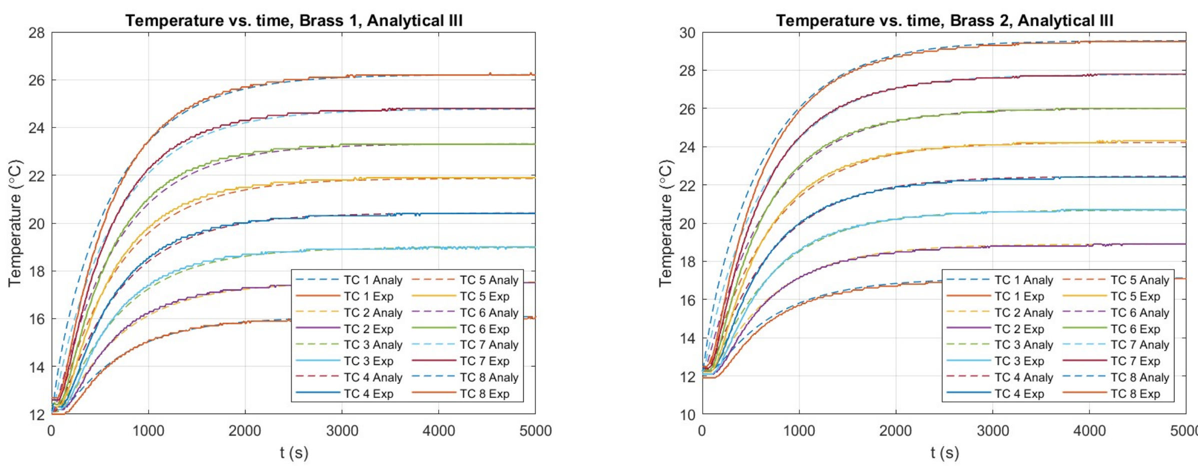

Fig. 13 Brass samples for Model III.

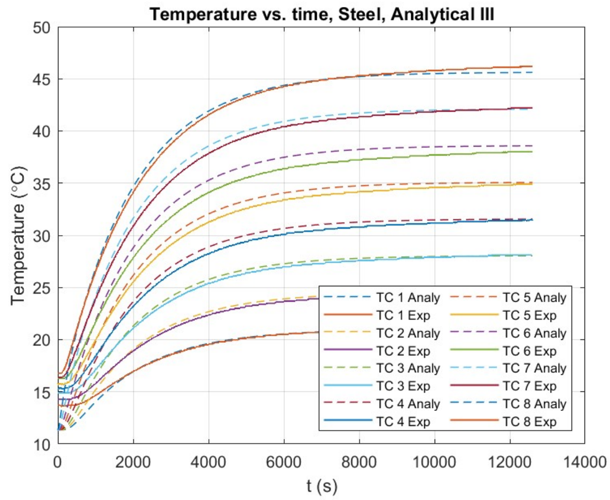

Fig. 14 Steel sample for Model III.

The adjusted α was found largely by trial and error. If the predictions were on average overestimates and the slope too large, then α was too large; underestimates, or a flatter slope during the transient phase, and it was too small. By adjusting the values and noting the adherence to the experimental curve, a reasonable estimate could be made in most cases. However, this is not true of the steel curve, given that the initial temperatures at each thermocouple were not distributed evenly. Because the experimental data displayed some concavity during the initial heating, which was not true of the analytical solutions, the initial data points did not conform as nicely as the transient solution data points. Therefore, more effort was made to match the long-term solution data. The first and last thermocouples, and the nearest to them, were monitored the most closely, and the greatest effort was made to match those; in general, if this happened, the middle thermocouples matched up nicely enough as well.

The thermal diffusivities from the online data sets and adjusted thermal diffusivities are tabulated below.

Material	Nominal α (m^2/s)	Adjusted α (m^2/s)
Aluminum 1	$4.82 * 10^{-5}$	$3.133 * 10^{-5}$
Aluminum 2	$4.82 * 10^{-5}$	$2.892 * 10^{-5}$
Brass 1	$3.59 * 10^{-5}$	$1.4 * 10^{-5}$
Brass 2	$3.59 * 10^{-5}$	$1.382 * 10^{-5}$
Steel	$4.05 * 10^{-6}$	$4.86 * 10^{-6}$

Table 3 Nominal & Adjusted Thermal Diffusivity For Varying Materials.

The adjusted diffusivities are in the same order of magnitude as their nominal counterparts, which implies that the discrepancy is normal and can be explained. In the case of aluminum and brass, whose adjusted diffusivities are lower, this could be from manufacturing inconsistencies, experimental error, or heat loss due to radiation (which occurred, but was not modeled). In the case of steel, which was larger, this is likely due to the initial temperature of the sample not being consistent (higher than it should have been).

Model III is the most accurate of all the models, as it also accounts for the differences in the transient solution as well as the beginning and end conditions. Overall, as will be discussed in the analysis section, Model III exhibited the least error and predicted the temperature $u(x, t)$ the most accurately.

A larger Fourier Number corresponds to a faster propagation of heat transfer through the test rod, and is a ratio of the thermal diffusivity, multiplied by the time to reach steady state, divided by the length of the rod squared. The

Fourier Numbers determined below were found using an approximation of the steady state time based on the Model III plots, and they varied based on the original and adjusted thermal diffusivity. Model III times were chosen, because the model followed the trend of the data better as time increased. As a result, all times to steady state are an approximation, based on the experimental data and Model III plots.

Material	t_{ss}	F_0	t_{ss} for 1 term approx with F_0 = 0.2
Aluminum 1	1400	3.030	177.69
Aluminum 2	2000	4.329	192.50
Brass 1	4000	6.448	397.62
Brass 2	4500	7.254	402.78
Steel	8000	1.454	1145.50

As listed above the Fourier numbers for brass tend to be the largest, while steel is the smallest followed by the "Aluminum 2" then "Aluminum 1" samples. Samples Brass 1 and 2 have the largest Fourier number; this indicates that the brass had a faster propagation of heat through the rod. This result makes sense, when looking at our adjusted thermal diffusivity values in table 3 it is clear that this change may have also had an impact on the large Fourier number. One other reason that brass has a larger Fourier number is due to the fact that the experiments when recording the data for aluminum were not conducted properly. We would expect aluminum to have the best propagation of heat, because it has the highest thermal conductivity out of all of the materials. but because of the error in the original experiment with the aluminum rod not being correctly insulated, the time to steady state may have not been the most accurate when recording the temperature at each thermocouple.

Factors that affect the time to steady-state (t_{ss}) include: the initial temperature of the rod, the thermal diffusivity of the material, the length of the rod, properties inherent to the material, and the material composition. If we hold F_0 constant, we can affect t_{ss} in the following ways: If we shorten the rod, t_{ss} will decrease. If we lengthen the rod, t_{ss} will increase. If we increase α, t_{ss} will decrease. If we decrease α, t_{ss} will increase.

Model II is likely very accurate for the first portion of the data, given that the initial temperatures are exactly aligned. However, Model III is likely accurate for the middle and ending portions of the data since it was adjusted to conform to these sections. This is more or less the case; Model II is certainly more accurate for the beginning section of the data than for the steady-state solution, while Model III is more accurate for the steady-state solution than for the beginning heating portion. In both cases, the error seen in the steady-state falls within the given temperature precision of +/- 2 deg C, with the exception of stainless steel. The methods used in Model III reduce the steady-state error greatly, but it is reasonable to assume that this uncertainty in our data contributes to the error seen in both models.

Given that Model III had the least error overall, especially when considering the transient phase, one could extrapolate that it would be the most accurate to predict when the transient phase would start, i.e. when we could consider the solution to be "steady-state." It also is the only model that was tailored to fit the curve exactly, rather than a result at the beginning or end (Models IA & IB sought to find the exact solution at the end, while Model II sought to model the temperature at the beginning of heating. Only Model III focused on intermediate time). Therefore, Model III would be the most useful model for a researcher desiring to predict the duration of the transient phase.

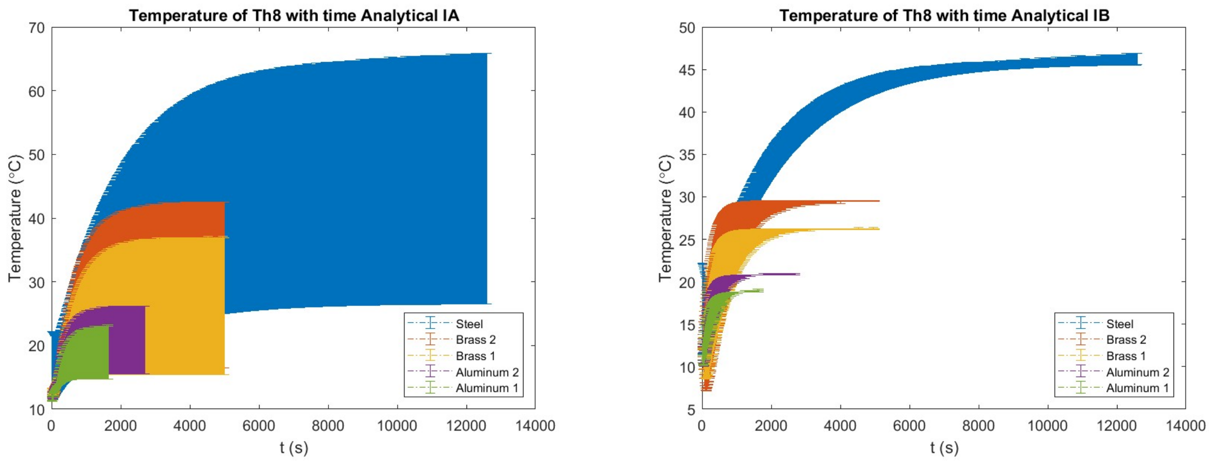

Fig. 15 Error for Model I.

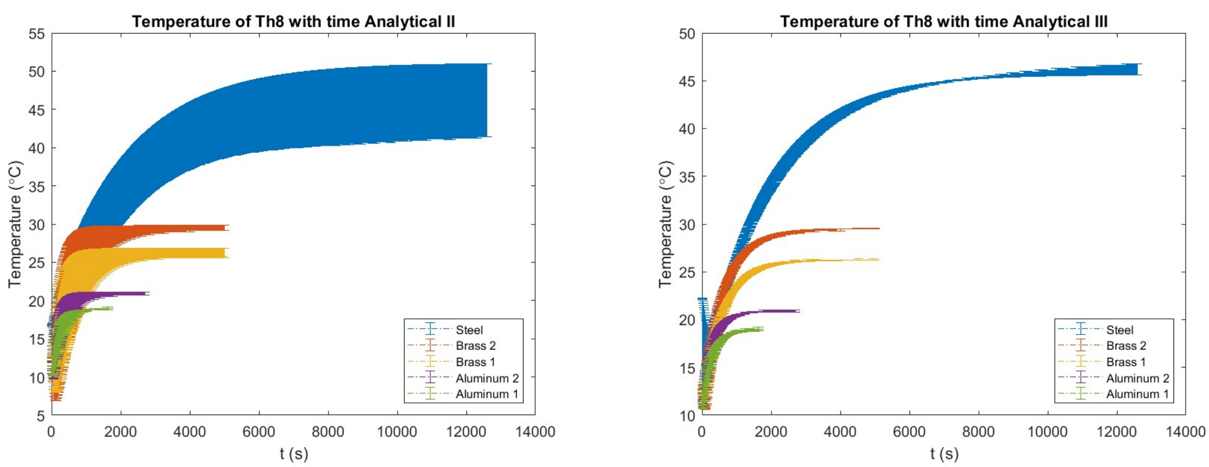

Fig. 16 Error for Models II & III.

V. Acknowledgements

We would like to acknowledge the TA Kate for informing us about the error in the experimental procedure when the instructional team recorded the data. We would also like to acknowledge Professor Li for providing the documents and Professor Schwartz for providing the Experimental Data.

Appendix

A. Derivations

$$b_n = \frac{2}{L}\int_0^L g(x)\sin(\lambda_n x)\,dx \qquad \begin{matrix} u = g(x) & dv = \sin(\lambda_n x)\,dx \\ du = g'(x)\,dx & v = -\frac{1}{\lambda_n}\cos(\lambda_n x) \end{matrix}$$

$$= \frac{2}{L}\left[-\frac{1}{\lambda_n} g(x)\cos(\lambda_n x)\Big|_0^L + \int_0^L \frac{g'(x)}{\lambda_n}\cos(\lambda_n x)\,dx\right]$$

Note: $g'(x) = M - H$

$\therefore$ no longer dependent on x

$$= \frac{2}{L}\left[-\frac{g(L)}{\lambda_n}\cos(\lambda_n L) + \frac{1}{\lambda_n}\int_0^L (M-H)\cos(\lambda_n x)\,dx\right]$$

Note: $g(x) = (M-H)x$

$\therefore g(0) = 0$

$$= \frac{2}{L}\left[-\frac{g(L)}{\lambda_n}\cos(\lambda_n L) + \frac{(M-H)}{\lambda_n^2}\left(\sin(\lambda_n x)\Big|_0^L\right)\right]$$

$$= \frac{2}{L}\left[-\frac{g(L)}{\lambda_n}\cos(\lambda_n L) + \frac{(M-H)}{\lambda_n^2}\sin(\lambda_n L)\right]$$

$$= -\frac{2g(L)}{\lambda_n \cdot L}\cos(\lambda_n L) + \frac{2(M-H)}{\lambda_n^2 L}\sin(\lambda_n L) \qquad \lambda_n = \frac{(2n-1)\pi}{2L}$$

$$= -\frac{2g(L)}{\frac{(2n-1)\pi}{2\cancel{L}}\cdot\cancel{L}}\cos\left(\overset{0}{\cancel{\frac{(2n-1)\pi}{2}}}\right) + \frac{2(M-H)}{\left(\frac{(2n-1)\pi}{2L}\right)^2 L}\sin\left(\overset{(-1)^n}{\cancel{\frac{(2n-1)\pi}{2}}}\right)$$

$$= (-1)^n \frac{2(M-H)}{\frac{(2n-1)^2\pi^2}{4L^{\cancel{2}}}\cdot\cancel{L}}$$

$$= \boxed{(-1)^n \frac{8L(M-H)}{(2n-1)^2\pi^2}}$$

Fig. 17 Derivation of the transient heat solution used in Model II

B. MATLAB

```matlab
%% ASEN 3113
% Exp Prelab 2
% Hayden Gebhardt, Max Feinland
% 10/17/22

% Reading in data
alum_1 = readmatrix('Aluminum_26V_250mA');
```

```
alum_1 = alum_1(11:end, :);
alum_1(:, 1) = alum_1(:,1) - alum_1(1,1);
alum_2 = readmatrix('Aluminum_28V_269mA');
alum_2(:,2) = [];
brass_1 = readmatrix('Brass_26V_245mA');
brass_1(:,2) = [];
brass_2 = readmatrix('Brass_29V_273mA');
brass_2(:,2) = [];
steel = readmatrix('Steel_21V_192mA');
steel(:,2) = [];

% Declaring variables
R = 1/39.37; % m
L = 5.875/39.37; % m

loc_alum = ([1.875 2.375 2.875 3.375 3.875 4.375 4.875 5.375]')/39.37;
loc_brass = ([1.375 1.875 2.375 2.875 3.375 3.875 4.375 4.875]')/39.37;
k = [130, 130, 115, 115, 16.2];
A = pi*R^2;
alpha = [4.82e-5; 3.59e-5; 4.05e-6]; % m^2/s

%% Calling Functions
% Initial Plot: temperature w/ thermocouples at t = 0
plott0(alum_1, 'Aluminum 1', loc_alum)
plott0(alum_2, 'Aluminum 2', loc_alum)
plott0(brass_1, 'Brass 1', loc_brass)
plott0(brass_2, 'Brass 2', loc_brass)
plott0(steel, 'Steel', loc_brass)

% calling function to find analytical & experimental slope, plus T(x0)
[Han, Hexp, T0] = findH(alum_1, 26, .25, k(1), A, loc_alum);
[Han2, Hexp2, T02] = findH(alum_2, 28, .269, k(2), A, loc_alum);
[Han3, Hexp3, T03] = findH(brass_1, 26, .245, k(3), A, loc_brass);
[Han4, Hexp4, T04] = findH(brass_2, 29, .273, k(4), A, loc_brass);
[Han5, Hexp5, T05] = findH(steel, 21, .192, k(5), A, loc_brass);

% finding slope of initial state temperature distribution
[Hz, T0z] = findHz(alum_1, loc_alum, 'Aluminum 1');
[Hz2, T0z2] = findHz(alum_2, loc_alum, 'Aluminum 2');
[Hz3, T0z3] = findHz(brass_1, loc_brass, 'Brass 1');
[Hz4, T0z4] = findHz(brass_2, loc_brass, 'Brass 2');
[Hz5, T0z5] = findHz(steel, loc_brass, 'Steel');


% Models IA & IB
al1a = findu(alum_1, alpha(1), Han, T0, loc_alum, 'Analytical IA', 'Aluminum 1', 10);
al1b = findu(alum_1, alpha(1), Hexp, T0, loc_alum, 'Analytical IB', 'Aluminum 1', 10);
al21a = findu(alum_2, alpha(1), Han2, T02, loc_alum, 'Analytical IA', 'Aluminum 2', 10);
al21b = findu(alum_2, alpha(1), Hexp2, T02, loc_alum, 'Analytical IB', 'Aluminum 2', 10);
brass11a = findu(brass_1, alpha(2), Han3, T03, loc_brass, 'Analytical IA', 'Brass 1', 10);
brass11b = findu(brass_1, alpha(2), Hexp3, T03, loc_brass, 'Analytical IB', 'Brass 1', 10);
brass21a = findu(brass_2, alpha(2), Han4, T04, loc_brass, 'Analytical IA', 'Brass 2', 10);
brass21b = findu(brass_2, alpha(2), Hexp4, T04, loc_brass, 'Analytical IB', 'Brass 2', 10);
steel1a = findu(steel, alpha(3), Han5, T05, loc_brass, 'Analytical IA', 'Steel', 10);
steel1b = findu(steel, alpha(3), Hexp5, T05, loc_brass, 'Analytical IB', 'Steel', 10);
% %
% Model II
al12 = findumod2(alum_1, alpha(1), Hexp, Hz, loc_alum, 'Analytical II', 'Aluminum 1', 10);
al22 = findumod2(alum_2, alpha(1), Hexp2, Hz2, loc_alum, 'Analytical II', 'Aluminum 2', 10);
```

```
brass12 = findumod2(brass_1, alpha(2), Hexp3, Hz3, loc_brass, 'Analytical II', 'Brass 1', 10);
brass22 = findumod2(brass_2, alpha(2), Hexp4, Hz4, loc_brass, 'Analytical II', 'Brass 2', 10);
steel2 = findumod2(steel, alpha(3), Hexp5, Hz5, loc_brass, 'Analytical II', 'Steel', 10);

% Model III
al13 = findu(alum_1, alpha(1)*.65, Hexp, T0, loc_alum, 'Analytical III', 'Aluminum 1', 10);
al23 = findu(alum_2, alpha(1)*.6, Hexp2, T02, loc_alum, 'Analytical III', 'Aluminum 2', 10);
brass13 = findu(brass_1, alpha(2)*.39, Hexp3, T03, loc_brass, 'Analytical III', 'Brass 1', 10);
brass23 = findu(brass_2, alpha(2)*.385, Hexp4, T04, loc_brass, 'Analytical III', 'Brass 2', 10);
steel3 = findu(steel, alpha(3)*1.2, Hexp5, T05, loc_brass, 'Analytical III', 'Steel', 10);

% Error
figure()
err(steel, steel1a, 'Steel')
hold on
err(brass_2, brass21a, 'Brass 2')
err(brass_1, brass11a, 'Brass 1')
err(alum_2, al21a, 'Aluminum 2')
err(alum_1, al11a, 'Aluminum 1')
title(['Temperature of Th8 with time ', 'Analytical IA'])
hold off

figure()
err(steel, steel1b, 'Steel')
hold on
err(brass_2, brass21b, 'Brass 2')
err(brass_1, brass11b, 'Brass 1')
err(alum_2, al21b, 'Aluminum 2')
err(alum_1, al11b, 'Aluminum 1')
title(['Temperature of Th8 with time ', 'Analytical IB'])
hold off

figure()
err(steel, steel2, 'Steel')
hold on
err(brass_2, brass22, 'Brass 2')
err(brass_1, brass12, 'Brass 1')
err(alum_2, al22, 'Aluminum 2')
err(alum_1, al12, 'Aluminum 1')
title(['Temperature of Th8 with time ', 'Analytical II'])
hold off

figure()
err(steel, steel3, 'Steel')
hold on
err(brass_2, brass23, 'Brass 2')
err(brass_1, brass13, 'Brass 1')
err(alum_2, al23, 'Aluminum 2')
err(alum_1, al13, 'Aluminum 1')
title(['Temperature of Th8 with time ', 'Analytical III'])
hold off
%% Defining Functions
% finds analytical & experimental H as well as extrapolated T0
function [Han, Hexp, T0] = findH(data, v, i, k, A, loc)
temp = data(end, 2:end);
X = [ones(length(loc), 1) loc];
b = X\temp';
% Y = [0; loc]*b(2) + b(1);
Hexp = b(2);
```

```
Han = v*i/(k*A);
T0 = b(1);
end

% plots initial state temperature distribution
function [] = plott0(data, name, loc)
plot(loc, data(1, 2:9), 'LineWidth', 1, 'DisplayName', num2str(name))
xlabel('Thermocouple Location (in)'), ylabel('Temperature (C)')
title('Initial Temperature of Rod at Each Thermocouple')
hold on
grid on
legend('Location', 'northwest')
end

% finds slope of initial state temp distribution
function [H_z, T0_z] = findHz(data, loc, name)
temp = data(1, 2:end);
X = [ones(length(loc), 1) loc];
b = X\temp';
Y = [0; loc]*b(2) + b(1);
H_z = b(2);
T0_z = b(1);

plot(loc, data(1, 2:9), 'LineWidth', 1, 'DisplayName', [num2str(name), ' Exp'])
xlabel('Thermocouple Location (m)'), ylabel('Temperature ({\circ}C)', 'Interpreter', 'tex')
title('Initial Temperature of Rod at Each Thermocouple')
hold on
plot([0; loc], Y, 'k--', 'LineWidth', 1, 'DisplayName', [num2str(name), ' Linear Fit'])
grid on
legend('Location', 'northwest', 'FontSize', 7)
end

% finds u(x,t) with n sums
function [u] = findu(data, alpha, H, T0, loc, soltype, varname, n)
t = data(:,1);
u = zeros(length(t),1);

L = 5.875/39.37; % m

for j = 1:length(loc)
    for i = 1:length(t)
        for k = 1:n
            bn(k) = (-1)^k*8*H*L/((pi^2)*((2*k - 1)^2));
            lambda(k) = (2*k - 1)*pi/(2*L);
            summer(k) = bn(k)*sin(lambda(k)*loc(j))*exp(-(lambda(k))^2*alpha*t(i));
        end
        u(i,j) = T0 + H*loc(j) + sum(summer(1:k));
    end
end

figure()
for j = 1:length(loc)
    plot(t, u(:, j, end), '--', 'LineWidth', .75, 'DisplayName', ['TC ', num2str(j), ' Analy'])
    hold on
    plot(t, data(:, j+1), '-', 'LineWidth', 1, 'DisplayName', ['TC ', num2str(j), ' Exp'])
end
grid on
title(['Temperature vs. time, ', varname, ', ' soltype])
xlabel('t (s)'), ylabel('Temperature ({\circ}C)','Interpreter','tex')
```

```
% yline(u_limit, 'r--', 'LineWidth', 1, 'DisplayName', 'Limiting Value')
legend('Location', 'southeast', 'FontSize', 6)
lgd = legend;
lgd.NumColumns = 2;
end

% finds u(x,t) with n sums, inputting actual T0 values rather than
% calculated
function [u] = findumod2(data, alpha, H, M, loc, soltype, varname, n)
t = data(:,1);
u = zeros(length(t),1);

L = 5.875/39.37; % m

for j = 1:length(loc)
    for i = 1:length(t)
        for k = 1:n
            % bn(k) = (-1)^(k+1)*4*(H-M)*L^2/((pi^2)*(2*k - 1)^2);
            bn(k) = -8*H*L/((pi^2)*((2*k - 1)^2));
            lambda(k) = (2*k - 1)*pi/(2*L);
            summer(k) = bn(k)*sin(lambda(k)*loc(j))*exp(-(lambda(k))^2*alpha*t(i));
        end
        u(i,j) = data(1,j+1) + H*loc(j) + sum(summer(1:k));
    end
end

figure()
for j = 1:length(loc)
    plot(t, u(:, j, end), '--', 'LineWidth', .75, 'DisplayName', ['TC ', num2str(j), ' Analy'])
    hold on
    plot(t, data(:, j+1), '-', 'LineWidth', 1, 'DisplayName', ['TC ', num2str(j), ' Exp'])
end
grid on
title(['Temperature vs. time, ', varname, ', ' soltype])
xlabel('t (s)'), ylabel('Temperature ({\circ}C)','Interpreter','tex')
% yline(u_limit, 'r--', 'LineWidth', 1, 'DisplayName', 'Limiting Value')
legend('Location', 'southeast', 'FontSize', 6)
lgd = legend;
lgd.NumColumns = 2;
end

% plots errorbars
function [] = err(data, u, varname)
t = data(:,1);
err = u(:,end) - data(:,end);
% for j = 1:80
%     for i = 1:err(end)/2
%     t(2*i) = [];
%     data(2*i) = [];
%     err(2*i) = [];
%     end
% end
errorbar(t, data(:,end), err, '-.', 'LineWidth', 0.1, 'DisplayName', varname)
hold on
xlabel('t (s)'), ylabel('Temperature ({\circ}C)','Interpreter','tex')
legend('Location', 'southeast', 'FontSize', 8)
end
```

C. References

1. MATLAB 2022, The MathWorks, Inc., Natick, Massachusetts, United States.
2. CU Boulder College of Engineering ASEN 3113 - Transient Heat Solution
3. ASM Engineered Materials Reference Book, Second Edition, Michael Bauccio, Ed. ASM International, Materials Park, OH, 1994.