Swarm Aggregation Without Communication and Global Positioning

Dhruv Shah and Leena Vachhani

Abstract—**In this letter, we propose a novel decentralized controller for the aggregation of a swarm of unicycle agents in the absence of communication and exchange of global positions. The methodology implicitly facilitates tasks like obstacle avoidance and path-following, whereas the agents exercise local decision-making. The primary aim of this letter is to form stable aggregates of a swarm using only local sensing and no inter-agent communication. A theoretical analysis of the proposed algorithm is presented based on the theory of switched systems, establishing asymptotic stability of the formed aggregates. Simulation experiments are presented to show the stability of aggregates in the presence of large, extended obstacles and external disturbances. Furthermore, we demonstrate an implementation of the proposed controller on a real system using experiments on a swarm of differential drive microbots. The proposed technique without communication demonstrates similar performance when compared with existing swarm aggregation techniques using global and local communications.**

Index Terms—**Swarms, multi-robot systems, collision avoidance, sensor-based control, autonomous agents.**

I. Introduction

SWARM aggregation, or grouping all the robots in a region of the environment, is a useful building block for modeling collective behaviors [1]. An aggregated set of robots facilitates not only easy collection and interaction but also more complicated maneuvers like task sequencing and mission control [2] or decentralized flocking and coordination [3]–[5]. Inspired by the diverse mechanisms for self-organized aggregation in biological beings, there has been a move towards decentralized swarm robotic systems where the robots rely only on local communication and sensing capabilities and do not have access to global knowledge [1], [7]. The miniaturization of swarm robots, with the advent of robots like Alice [8], Kilobots [9], GRITSBots [10] etc., calls for algorithms with fewer sensing requirements and communication overheads.

An early effort in physics-based approaches, [11] developed continuous time motion dynamics for swarm aggregation and presented stability analysis ascertaining finite time cohesion. This method, however, assumes global knowledge of positions of all members. Moving away from global knowledge, authors in [5], [12]–[14] limit communication to a small neighborhood of each agent, while achieving the collective goal of aggregation. In particular, [5] models local interactions with Voronoi partitions, limiting each agent's interactions to its immediate neighbors (topological), while [14] limits local interactions within a fixed radius (metric). The performance of a tasked artificial swarm on the choice of communication model is analyzed in detail in [15]. Other approaches to swarm aggregation use formation control [13], [16], [17] as a means to explicitly model the structure of the aggregate. [13] enforces self-organization by moderating the topology of the agents; [16] uses pose information from neighbors to model formation control inspired by fish schools. The performance of these methods relies heavily on the communication between neighbors and accurate relative pose estimates. Therefore, we develop this work by asking the following questions: *Can we solve the aggregation problem for a fixed target with only on-board sensing and no inter-agent communication? Is there a guarantee that a system with no inter-agent communication is stable?*

Probabilistic approaches to aggregation [18]–[20] use finite state machines (FSMs) to model interactions and decisions (actions) that are stochastic in nature. BEECLUST [21], [22] is a popular bio-inspired aggregation algorithm which uses a probabilistic FSM and a primitive motion model ("turn" or "move forward") to mimic the collective decision making capabilities of bees. ODOCLUST [23] improvises on this by using an active odometry-based homing process to achieve tighter aggregates and faster convergence. [24] presents a similar approach to local communication inspired by pheromone trails used by ants. Such methods, however, have not yet provided convergence guarantees and do not account for motion dynamics beyond point robots.

With this work, we approach the swarm aggregation problem in an entirely different way – each agent independently moves towards a common target while the interaction with other agents is limited to avoiding inter-agent collision – and demonstrate provably stable aggregation without communication or global pose information. Simulation results show that our method achieves convergence on par with popular methods that use communication and complete pose information. Due to independent control-laws for each agent, the proposed technique is scalable. Further, the method is developed for agents with unicycle kinematics, hence easily implementable on mobile robots. In this work, we present:

1) A decentralized switching control-law for swarm aggregation of unicycle agents to a fixed target using only local sensing. The control-law avoids inter-agent communication and collision between agents and static obstacles without explicitly aiming for organizing.

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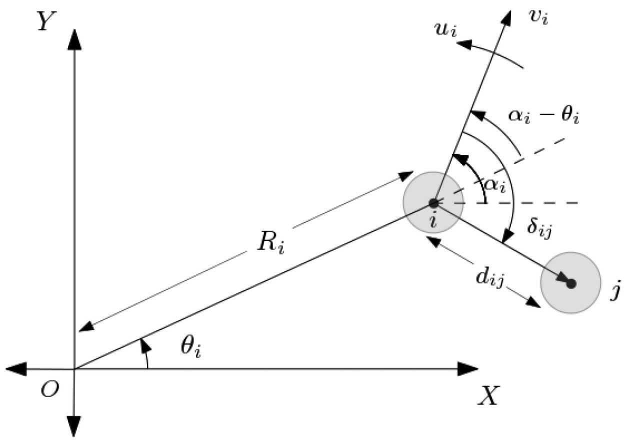


Fig. 1. Engagement geometry between an agent i and the aggregation target O. The position of the agent is uniquely determined by $\{R_i, \theta_i\}$. Interaction with another agent j is characterized by the distance between them (d_{ij}) and the bearing of agent j with respect to agent i (δ_{ij}).

2) Stability results are shown using the concept of multiple Lyapunov functions for switched systems ensuring aggregation of the swarm.
3) Experiments on simulated and real robots to demonstrate aggregation in the presence of multiple, large, extended obstacles and disturbances.

The remainder of the letter is organized as follows: in Section II we present preliminaries to build the proposed control-law, which include topics in stability for switched system, collision avoidance using artificial potential functions (APFs) and system description. Section III describes our proposed control-law with stability guarantees. Section IV presents results from experiments on simulated and real robots, demonstrating aggregation in the presence of obstacles and disturbances. Finally, we conclude in Section V.

II. Preliminaries and Problem Formulation

A. Modeling the Swarm

Consider a swarm of N agents in a two-dimensional Euclidean space. We model each agent $i = 1, 2, \ldots, N$ in the swarm with non-holonomic kinematics of a unicycle as follows:

$$\begin{aligned}\dot{R}_i &= v_i \cos(\alpha_i - \theta_i)\\ R_i\dot{\theta}_i &= v_i \sin(\alpha_i - \theta_i)\\ \dot{\alpha}_i &= u_i\end{aligned} \tag{1}$$

where (R_i, θ_i) denote agent i's position in polar coordinates, α_i is its yaw, and (u_i, v_i) are its yaw-rate and linear velocity, respectively. In this work, we consider $v_i \geq 0$. Figure 1 illustrates the engagement geometry between an agent i and the aggregation target O.

Accounting for practical limitations of sensing, the field-of-view (FoV) of an agent is modeled as a cone $\mathcal{C}_i$ with its axis aligned with the instantaneous velocity vector and defined by half-angle δ_c and sensing range d_c. An agent j is detected by agent i if and only if their separation $d_{ij} \leq d_c$ and its bearing $|\delta_{ij}| \leq \delta_c$. Figure 2 illustrates the interactions among 3 agents (i, j, k). For simplicity, we consider $\delta_c \leq \frac{\pi}{2}$.

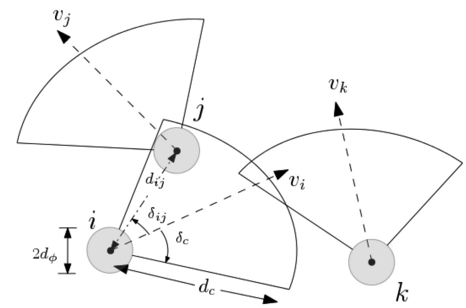


Fig. 2. Field-of-view of the agents can be seen in solid lines. In this example, robot $j \in \mathcal{C}_i$ and is detected by i, while i is not detected by j. Robot k is agnostic to the presence of both i, j and also remains undetected. The safe distance for an agent is given by d_ϕ.

B. Switched System With Two Subsystems and Its Stability Using Multiple Lyapunov Functions (MLF)

Consider a system constituted by two subsystems and a switching signal that toggles between the subsystems. As has been elucidated in several works on analyzing switched systems, the stability of independent subsystem is not adequate to ascertain stability of a switched system [25]–[29]. [28, Thm. 1] presents an MLF approach for establishing stability of a switched system by imposing constraints on the constituent subsystems and switching signal. The result for a system with two subsystems is reproduced for completeness.

For a switched system with two subsystems $\{s_1, s_2\}$ and initial subsystem s_1, let $\{t_q(k)\}$ be increasing sequence of switching times for $q \in \{s_1, s_2\}$ with k being a positive integer. In particular, $t_{s_1}(k)$ and $t_{s_2}(k)$ are the time instants when state x enters the subsystems s_1 and s_2 respectively for the k^{th} instance.

Let $\mathcal{S} := \{t_{s_1}(1), t_{s_2}(1), t_{s_1}(2), t_{s_2}(2), t_{s_1}(3)\ldots\}$ be any switching sequence associated with the system. The interval when s_1 is active is thus given by $\mathcal{I}(\mathcal{S}|s_1) = \bigcup_k \left[t_{s_1}(k),\ t_{s_2}(k)\right)$, whereas the interval when s_2 is active is given by $\mathcal{I}(\mathcal{S}|s_2) = \bigcup_k \left[t_{s_2}(k),\ t_{s_1}(k+1)\right)$. Further, let the dynamics of subsystem s_q be represented as $\dot{x} = f_q(x(t)),\ t \in \mathcal{I}(\mathcal{S}|q)$ and V_q be the corresponding candidate function using which it is shown that each subsystem q is Lyapunov stable. If for all switching sequences $\mathcal{S}$, the following conditions are satisfied,

1) $\dot{V}_{s_2}(x(t)) < 0\ \forall\, t \in \mathcal{I}(\mathcal{S}|s_2)$ and $V_{s_2}(t_{s_2}(k+1)) \leq V_{s_2}(t_{s_2}(k))$.
2) There exists a positive constant m, such that $|V_{s_1}(t)| \leq m|V_{s_2}(t_{s_2}(k))|$, for $t_{s_1}(k+1) \leq t < t_{s_2}(k+1)$ and any k.
3) x does not have a finite escape time in s_1 and is guaranteed to enter s_2.

then the system is *asymptotically* stable [30] and one of the following two conditions is satisfied

C1: the sequence $\{V_{s_2}(t_{s_2}(k))\} \to 0$ as $k \to \infty$.
C2: $\{t_{s_2(k)}\}$ is a finite sequence and the switched system stays in s_2 after the last switching.

C. Collision Avoidance Using Artificial Potential Fields (APFs)

The notion of using APFs to represent interactions between multi-agent systems and the environment is commonplace in both robotics and biological literature [31]–[33]. APFs have been used as a source of virtual forces between a robot and an obstacle, as a function of their relative position vectors, for collision avoidance [1] and as a means to model global path-planning [31]. In particular, an APF f can be expressed as $f_{\text{att}} + f_{\text{rep}}$, where *repulsive* interactions f_{rep} aimed at collision-avoidance interplay with the goal-directed *attractive* interactions f_{att} to provide a plan in the presence of obstacles.

D. Sensing

For the task of *homing*, or reporting to a fixed home position, it has been shown that the knowledge of position or direction with respect to home position is not necessary. [34], [35] show that coarse sensor measurements, like the signum of the gradient of bearing θ_i for each agent i, are sufficient for visual homing, without explicitly estimating the bearing θ_i or position (R_i, θ_i). In this work, we assume that every agent i can measure $\text{sgn}(\dot{\theta}_i)$: 1 for $\dot{\theta}_i \geq 0$ and -1 otherwise. In addition, for every agent $j \in \mathcal{C}_i$, agent i has access to the signum of agent j's bearing with respect to itself, or $\text{sgn}(\delta_{ij})$, and can identify the closest agent. It is worth noting that both the measurements are available from monocular camera images and depth estimation from images can identify the closest agent. Unlike existing methods [11]–[14], we do not require the poses of neighboring agents. Following [36], we represent an obstacle as a set of *virtual* agents. For each agent and obstacle pair, a virtual agent is placed on the boundary of the obstacle at the closest distance to the agent. An agent is capable of identifying such a virtual agent for any obstacle that lies in its FoV.

E. Problem Formulation

Our objective is to design swarm aggregation system when there is no inter-agent communication. Therefore, the problem is to design a decentralized controller for each member such that the swarm aggregates to a *known* target. In practice, the environment may have obstacles and hence, the objective also includes designing a controller that prevents collisions with obstacles as well as other agents. In particular, the only interaction between members of the swarm is inter-agent collision avoidance (CA). The proposed control-law is designed in the following setting:

- Environment is populated with obstacles at unknown locations. However, the target is reachable, i.e., there exists a path to the target from the initial locations of each agent in the swarm.
- Each agent is a unicycle and avoids collision with obstacles with the safety distance d_ϕ.
- There is no communication between the agents.
- Each agent i is equipped with sensor(s) that can measure $\text{sgn}(\dot{\theta}_i)$ and $\text{sgn}(\delta_{ij})$ for the closest agent $j \in \mathcal{C}_i$.
- The target is described by a circular ring with radius R_{th} centered at O. An agent knows if it is within the target or outside the target.

III. Proposed Control-Law for Swarm Aggregation

We propose a switching controller for each agent i with two subsystems based on the obstacle in vicinity (within FoV of the local sensor): 1) *free* subsystem, wherein there are no agents (or obstacles) in its FoV, and 2) *engaged* subsystem, wherein there exist at least one agent (or obstacle) in its FoV. In this section, we design the controller for each subsystem and establish asymptotic stability of the switched system thus formed.

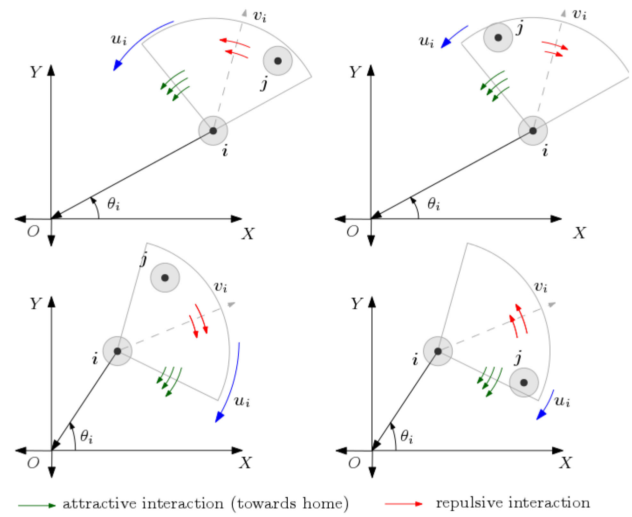


Fig. 3. Demonstrating the steering control input u_i in the engaged subsystem. The two factors, attraction towards target O ($\beta_i \,\text{sgn}(\theta_i)$, shown in green) and repulsion from agent j ($k_i \,\text{sgn}(\delta_{ij})$, shown in red), represent inter-agent interaction, resulting in net control as per (6) (shown in blue).

A. Free Subsystem

In the free subsystem, an agent has no obstacle in its FoV and the objective is to move towards the target centred at O. The steering control-law for an agent i in this subsystem, moving with linear velocity bounded by $\bar{v}_i = \max v_i$, is borrowed from [35]. It is reproduced as Lemma 1 for completeness.

Lemma 1: For an agent i in the free subsystem, the agent asymptotically converges to the neighborhood of O when it is controlled by (2)

$$u_i = \kappa_i \,\text{sgn}(\dot{\theta}_i) \tag{2}$$

κ_i is a positive free parameter such that

$$\kappa_i > \frac{\bar{v}_i}{R_{\text{th}}}. \tag{3}$$

Proof: Considering the Lyapunov function

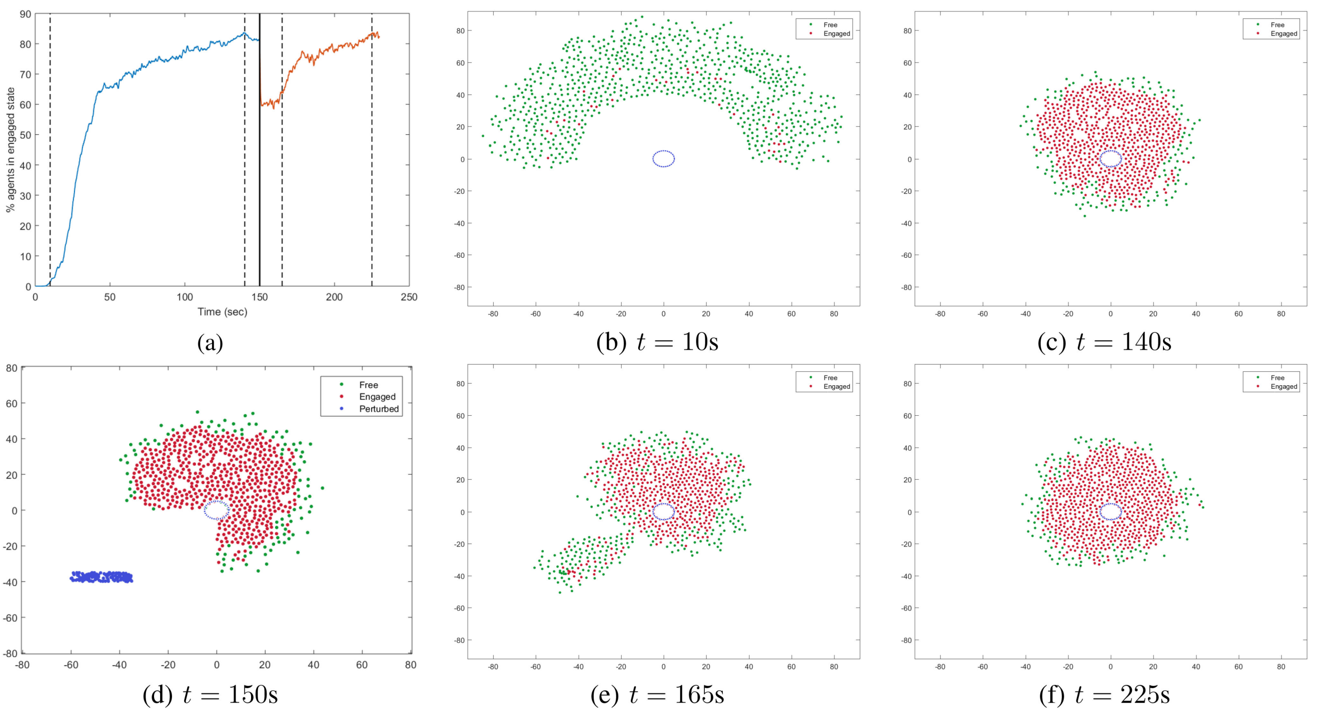
$$V_f^i = 1 + \cos(\alpha_i - \theta_i) \tag{4}$$

for a system corresponding to the agent i, the stability of this system described by (1) is shown in [35, Thm. 1]. ■

A particular choice of the linear velocity control in the free subsystem is presented in Remark 1.

B. Engaged Subsystem

In this subsystem, an agent i has at least one other agent in its FoV $\mathcal{C}_i$. Towards the idea of simpler sensing, we design the control-law that is only affected by interactions with the *closest* agent in its sensing range. In particular, the interacting agent with agent i is $j = \arg\min_{\rho \in \mathcal{C}_i} d_{i\rho}$. This idea is inspired by prior work in CA, where only the repulsive interactions due to the closest agent are considered for ensuring no collision [38].



(a) (b) $t = 10$s (c) $t = 140$s

(d) $t = 150$s (e) $t = 165$s (f) $t = 225$s

Fig. 4. Subsystem composition of a swarm undergoing aggregation: solid line shows perturbation instant in (a). Snapshots (color-coded by subsystem) are shown in (b)–(f) at times indicated by the black lines in (a). Agents were spawned in the top half of the arena ≈ 80 m from target.

This is a worst-case consideration and finite agent velocities ensure that CA with respect to the closest agent is sufficient. Therefore, this subsystem has one measured state, δ_{ij} for which the dynamics is given by

$$\dot{\delta_{ij}} = -u_i + \frac{\hat{v}_{ji}}{d_{ij}}, \tag{5}$$

where $\hat{v}_{ji}$ is the projection of relative velocity of agent j with respect to i perpendicular to the line joining them. We present the control-law in this subsystem using the following lemma:

Lemma 2: Let the control-law for each agent i be given by

$$\begin{aligned} u_i &= \beta_i \operatorname{sgn}(\dot{\theta}_i) - k_i \operatorname{sgn}(\delta_{ij}) \\ v_i &= \max\{v_i(t_e^i) - (t - t_e^i)\lambda_i, 0\} \end{aligned} \tag{6}$$

where t_e^i is the time at which the agent i enters in engaged subsystem, $v_i(t_e^i)$ is the velocity of agent i at t_e^i and β_i, k_i, λ_i are positive free parameters. Let $\bar{v}$ be $\max_i\{\bar{v}_i\}$ over all agents. This control-law ensures CA and steers the agent out of the engaged subsystem when

$$\beta_i > \frac{\bar{v}_i}{R_{\text{th}}} \tag{7}$$

$$k_i > \frac{\bar{v}_i}{R_{\text{th}}} + 2\frac{\bar{v}}{d_\phi} \tag{8}$$

$$\lambda_i > \frac{\bar{v}^2}{d_c - d_\phi} \tag{9}$$

Proof: Let the candidate Lyapunov-like function in the engaged subsystem V_e^i for each agent i be the sum of attractive and repulsive potentials corresponding to the tasks of aggregation and CA, respectively. Thus,

$$V_e^i = V_{\text{att}}^i + \gamma_i V_{\text{rep}}^i \tag{10}$$

with $\gamma_i > 0$ and

$$\begin{aligned} V_{\text{att}}^i &= 1 + \cos(\alpha_i - \theta_i) \\ V_{\text{rep}}^i &= \cos(\delta_{ij}) - \cos(\delta_c). \end{aligned} \tag{11}$$

It is worth noting that $V_{\text{rep}}^i = 0$, i.e., V_{att}^i is the same as V_f^i when the agent i leaves the engaged subsystem. The function V_e^i is smooth and satisfies $V_e^i = 0$ when $\alpha_i - \theta_i = (2k+1)\pi$ for any integer k and $|\delta_{ij}| = \delta_c$. We have $V_{\text{att}}^i > 0$ for $\alpha_i - \theta_i \neq (2k+1)\pi$ and $V_{\text{rep}}^i > 0$ for $|\delta_{ij}| < \delta_c \leq \pi/2$, since the cosine function is decreasing on $[0, \pi/2]$. Since each component function is positive definite when the agent is in the engaged subsystem, V_e^i is positive definite. The derivative along the trajectories of the subsystem is given by

$$\dot{V_e^i} = -\sin(\alpha_i - \theta_i)(\dot{\alpha_i} - \dot{\theta}_i) - \gamma_i \sin(\delta_{ij})\dot{\delta_{ij}}. \tag{12}$$

Using $\operatorname{sgn}(\sin(\alpha_i - \theta_i)) = \operatorname{sgn}(\dot{\theta}_i)$, (5) and (6), we have

$$\begin{aligned} \dot{V_e^i} = \gamma_i &\left(\beta_i \sin(\delta_{ij}) \operatorname{sgn}(\dot{\theta}_i) - k_i|\sin(\delta_{ij})| - \sin(\delta_{ij})\frac{\hat{v}_{ji}}{d_{ij}}\right) \\ &- |\sin(\alpha_i - \theta_i)| \left(\beta_i - k_i \operatorname{sgn}(\dot{\theta}_i)\operatorname{sgn}(\delta_{ij}) - |\dot{\theta}_i|\right) \end{aligned} \tag{13}$$

To ensure that the agents never collide, we need $d_{ij} > d_\phi$. Defining $\tilde{v}_{ji}$ as the projection of the relative velocity of agent j with respect to i along the line joining their positions, we have $\tilde{v}_{ji} \leq v_i + v_j$. Further, let t_0 be the instant when agent j enters $\mathcal{C}_i$, and t^* be the instant when the two agents are closest to each other. Using the equations of motion, we get

$$\begin{aligned} d_{ij}(t^*) &\geq d_c - \int_{t_0}^{t^*} (v_i + v_j)dt \\ &\geq d_c - \frac{\bar{v}_i{}^2 + \bar{v}_j^2}{2\lambda_i} \geq d_c - \frac{\bar{v}^2}{\lambda_i} \end{aligned} \tag{14}$$

We can ensure $d_{ij}(t^*) \geq d_\phi$ by satisfying condition (9), thereby ensuring CA. We now divide the analysis into two cases based on the value of $\operatorname{sgn}(\dot{\theta}_i)\operatorname{sgn}(\delta_{ij})$.

- *Case 1:* $\operatorname{sgn}(\dot{\theta}_i)\operatorname{sgn}(\delta_{ij}) = -1$. For this case, (13) renders

$$\begin{aligned} \dot{V}_e^i &= -\gamma_i\,|\sin(\delta_{ij})|\left(\beta_i + k_i + \operatorname{sgn}(\delta_{ij})\frac{\hat{v}_{ji}}{d_{ij}}\right) \\ &\quad - |\sin(\alpha_i - \theta_i)|(\beta_i + k_i - |\dot{\theta}_i|) \end{aligned} \tag{15}$$

  With $\beta_i + k_i > \max(|\dot{\theta}_i|, |\frac{\hat{v}_{ji}}{d_{ij}}|)$, we ensure $\dot{V}_e^i < 0$. Since $\max(|\dot{\theta}_i|) = \bar{v}_i/R_{th}$, $\max(\hat{v}_{ji}) = 2\bar{v}$ and $\min(d_{ij}) = d_\phi$, we have

$$\beta_i + k_i > \max\left(\frac{\bar{v}_i}{R_{th}}, \frac{2\bar{v}}{d_\phi}\right) \tag{16}$$

- *Case 2:* $\operatorname{sgn}(\dot{\theta}_i)\operatorname{sgn}(\delta_{ij}) = 1$. For this case, (13) renders

$$\begin{aligned} \dot{V}_e^i &= |\sin(\alpha_i - \theta_i)|(k_i + |\dot{\theta}| - \beta_i) \\ &\quad - \gamma_i|\sin(\delta_{ij})|(k_i - \beta_i) - \gamma_i \sin(\delta_{ij})\frac{\hat{v}_{ji}}{d_{ij}} \end{aligned} \tag{17}$$

  For $\dot{V}_e^i < 0$, we use $|\sin(\alpha_i - \theta_i)| \leq 1$ and $\sin(\delta_{ij}) \geq -1$ in (17). Hence,

$$(k_i + |\dot{\theta}_i| - \beta_i) - \gamma_i|\sin(\delta_{ij})|(k_i - \beta_i) + \gamma_i\frac{\hat{v}_{ji}}{d_{ij}} < 0 \tag{18}$$

  Choosing $\beta_i > \max(|\dot{\theta}_i|)$, we get

$$k_i > \frac{\gamma_i}{(\gamma_i - 1)}\left(\beta_i + \frac{\hat{v}_{ji}}{d_{ij}}\right), \qquad \text{with } \gamma_i > 1 \tag{19}$$

  We have $\hat{v}_{ji} \leq 2\bar{v}$, $d_{ij} > d_\phi$. Therefore, (8) and an appropriate γ_i guarantees $\dot{V}_e^i < 0$.

Now, following cases may occur when $\dot{V}_e^i < 0$:

a) $\dot{V}_{\text{att}}^i < 0$ and $\dot{V}_{\text{rep}}^i < 0$: The agent aligns itself with the target direction, hence promoting aggregation, and also steers away from obstacle, avoiding collision.

b) $\dot{V}_{\text{att}}^i > 0$ and $\dot{V}_{\text{rep}}^i < 0$: The agent steers away from obstacle, hence avoiding collision, but may move away from target.

c) $\dot{V}_{\text{att}}^i < 0$ and $\dot{V}_{\text{rep}}^i > 0$: The agent aligns itself with the target direction, but may steer towards the obstacle.

Cases (a), (b) ensure that the agent exits engaged subsystem, since $\dot{V}_{\text{rep}}^i < 0$. In case (c), (14) ensures CA while the control input $u_i \neq 0$ makes the agent j out of FoV of agent i. Therefore, state of the agent i either corresponds to cases (a)/(b) or switches to free subsystem.

Motivation for the choice of control-law in this subsystem is illustrated using Figure 3.

Remark 1: The linear velocity control proposed in (6) is only representative and the proof holds for any control input v_i that ensures $d_{ij} > d_\phi$ (14). Since the agent decelerates in the engaged subsystem, an accelerating linear velocity control for the free state allows the agent to regain maximum velocity. A particular choice of velocity profile satisfying $\max(v_i) = \bar{v}_i$ is given by

$$v_i = v_p\left(1 - \frac{\bar{v}_i - v_i(t_f^i)}{\bar{v}_i}e^{-(t - t_f^i)}\right) \tag{20}$$

Remark 2: The control law given by Lemma 2 depends on $\bar{v} = \max_i\{\bar{v}_i\}$, the maximum linear velocity attainable by an agent in the swarm. This is a constant quantity known a priori and hence, agents are not required to exchange or update this information after deployment.

C. Switched System Stability

In this section, we establish stability of the proposed switching-based controller using the MLF approach described

TABLE I
PARAMETER VALUES USED FOR EXPERIMENTS (SI UNITS)

Parameter	$\bar{v}_i$	κ_i	β_i	k_i	λ_i	d_ϕ	d_c	δ_c
Simulation	2	0.6	0.6	8.6	2.1	0.5	2.5	85°
Testbed [39]	0.1	0.45	0.45	3	0.14	0.1	0.2	85°

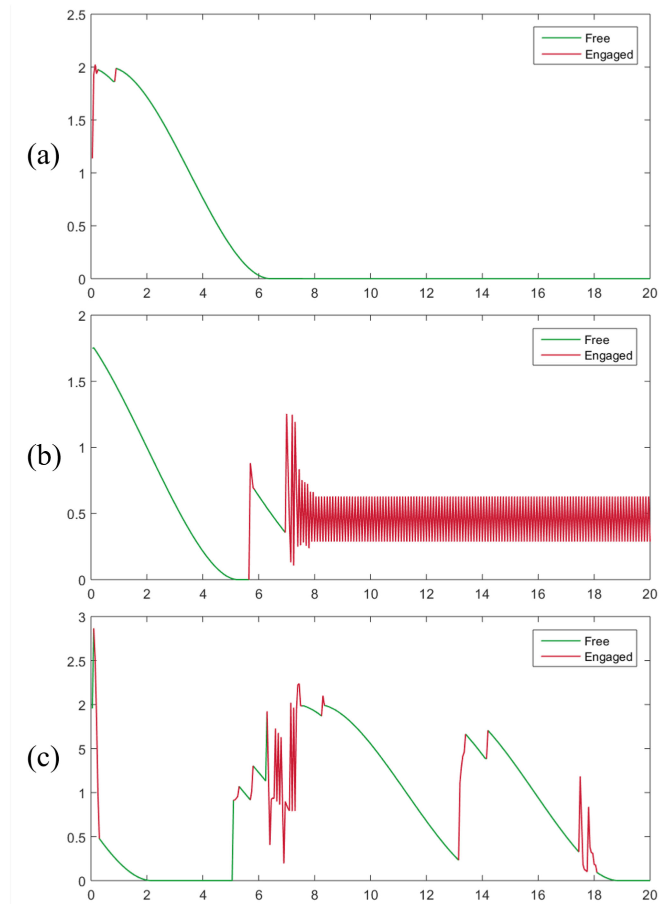
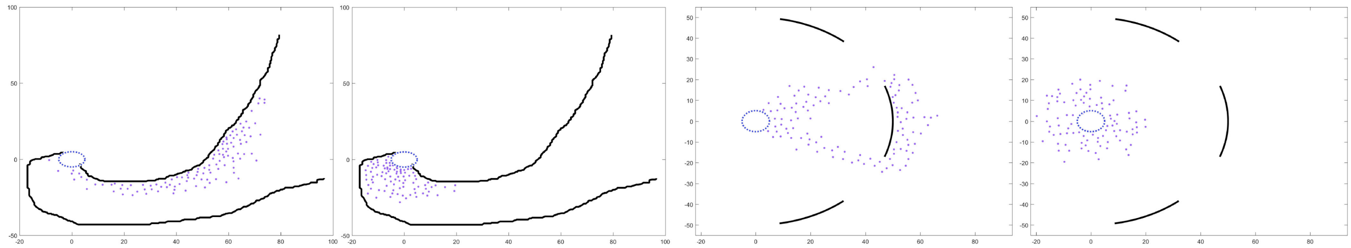


Fig. 5. Individual Lyapunov functions of three agents in a swarm of 50 agents undergoing aggregation (color-coded by subsystem).



(a) (b)

Fig. 6. Swarm aggregation in the presence of static obstacles: (a) snapshots of the swarm in the presence of path markers at 65 s and 190 s, respectively; (b) aggregation in the presence of multiple, large, extended obstacles at 40 s and 115 s.

in Section II-B. For an agent i, we define s_1 as its free subsystem f and s_2 as its engaged subsystem e, i.e., $V_1 \triangleq V_f^i$ and $V_2 \triangleq V_e^i$. The switching signal uses the condition $\delta_{ij} < \delta_c$ to switch to engaged state and vice-versa.

Theorem 1: For a swarm of agents where every agent i independently follows the proposed switched-state control-law, each member is asymptotically stable.

Proof: For brevity, we write $V(x(t))$ as $V(t)$. Verifying the conditions for asymptotic stability of a switched system stated in Section II-B, we have the following

1) From Lemma 2, we have $\dot{V}_e^i(t) < 0$ for $t \in \mathcal{I}(\mathcal{S}|e)$. For consecutive switching instances $(t_e^i(k)$ and $t_e^i(k+1))$ into e subsystem, we have the difference Δ_k given by

$$\begin{aligned}\Delta_k &= V_e^i(t_e^i(k+1)) - V_e^i(t_e^i(k))\\ &= \left(V_e^i(t_e^i(k+1)) - V_e^i(t_f^i(k+1))\right)\\ &\quad + \left(V_e^i(t_f^i(k+1)) - V_e^i(t_e^i(k))\right)\\ &\leq \left(V_e^i(t_e^i(k+1)) - V_e^i(t_f^i(k+1))\right)\\ &= \left(V_f^i(t_e^i(k+1)) - V_f^i(t_f^i(k+1))\right) < 0\end{aligned} \tag{21}$$

Here, we use (i) $V_e^i(t_f^i(k+1)) - V_e^i(t_e^i(k)) < 0$ since V_e^i is the Lyapunov function in engaged subsystem and $t_f^i(k+1) > t_e^i(k)$, and (ii) $V_f^i(t_f^i(k)) = V_e^i(t_f^i(k))\ \forall k$, since $\delta_{ij} = \delta_c$ at $t_f^i(k)$ (exit time of e subsystem). Thus, we have $V_e^i(t_e^i(k+1)) < V_e^i(t_e^i(k))$ for all k.

2) After the k^{th} switching instant into f, the state stays in f subsystem for $t \in \left(t_f^i(k), t_e^i(k)\right)$ and we have

$$V_f^i(t) < V_f^i(t_f^i(k)) \leq V_e^i(t_f^i(k)) < V_e^i(t_e^i(k-1))$$

3) When the state is in free subsystem, each member (agent) i in the swarm is approaching the target. Therefore, the agents will come in the vicinity of each other. Hence, x_i does not have a finite escape time in subsystem f and is guaranteed to enter subsystem e.

Invoking [28, Thm. 1] guarantees asymptotic stability of agent i for all switching sequences. ■

This result shows that every agent in the swarm is asymptotically stable and falls under one of two categories: 1) agents that reach the target, and 2) exploratory agents, that are on the lookout for a path to the target since agents in category 1 have made the target inaccessible. In this regard, the system observes a state minimizing the average distance to target, with category 2 agents continuously exploring their surroundings. A perturbation to the system may alter the agent category from 1 to 2 or vice-versa and allow other exploratory agents to reach the target. In general, such a perturbation can cause the system to settle in a new state that minimizes average distance to target.

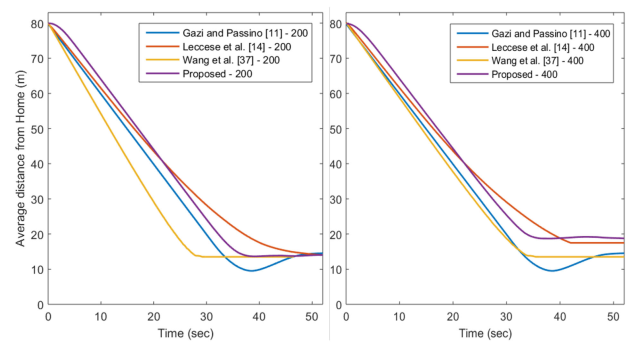
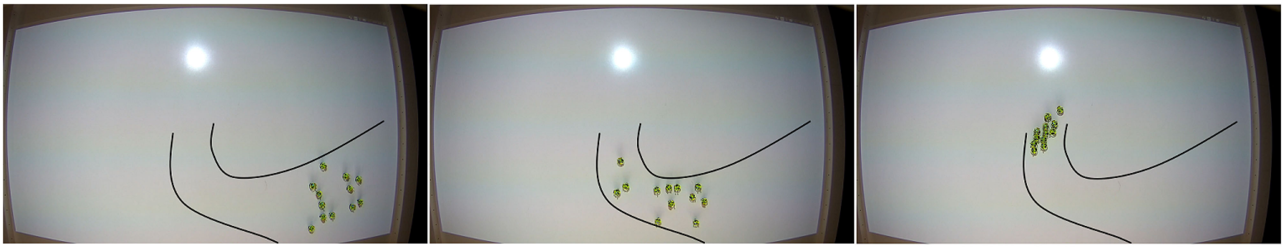


Fig. 7. Comparison of proposed method with [11], [14], [37] for the aggregation task with (left) 200 agents, and (right) 400 agents.

IV. Experimental Validation

A. Stability Analysis

To evaluate the stability of aggregation, we randomly spawn a swarm of 800 agents at 80 m from the target $O(0,0)$, on the upper half of the plane, with identical parameters for all agents (see Table I) and $R_{\text{th}} = 5$. We observe that the agents spread out and aggregate around O, avoiding inter-agent collisions. To test the stability of the swarm under disturbances, $\approx 20\%$ agents were removed and re-spawned away from O at $t = 150$ s. The swarm automatically reorganizes to fill the voids, while the perturbed agents move towards O to form a new aggregate. We choose the percentage of agents in the engaged subsystem as a metric to quantify convergence of the system. Figure 4 shows snapshots of the swarm and variation of this metric with time as the agents are spawned and perturbed. At $t = 140$ s (before perturbation), notice that 85% of the agents have converged to the engaged subsystem (red) and stay there forever. As described in Section II-B, one among the two conditions are satisfied for an agent in the engaged subsystem. Category 1 agents that reached O within the distance R_{th} are aware and hence, stop completely ($v_i = u_i = 0$), satisfying the condition C1. The remaining (exploratory) agents stay in engaged subsystem with ($v_i = 0, u_i \neq 0$). Therefore, condition C2 is satisfied for stability of these agents (shown with red color in Fig. 4(c) and (f)).



(a) $t = 0s$ (b) $t = 22s$ (c) $t = 91s$

Fig. 8. Swarm aggregation using 12 physical agents on the Robotarium testbed [39] in the presence of path markers.

When a few agents are perturbed, a void is created (see Fig. 4(d)), which enables some of the engaged agents to enter free state (green) and re-organize, while the disturbed agents (perturbed agents as well as ones that enter into free state) attempt to aggregate at the target. At $t = 225$ s, 84% of the agents are engaged while agents in the outer ring of the swarm continue exploring a path to O. This demonstrates the stability of the aggregate to external disturbances.

In order to interpret the implications of Theorem 1, we conduct an experiment with 50 agents spawned 20 m from the target (no perturbation). Figure 5 shows individual Lyapunov functions (4) and (11) of three agents in this aggregating swarm: 5(a) shows a category 1 agent that reaches the target at $t = 6.3$ s; 5(b) shows a category 2 exploratory agent that is in the engaged state from $t = 6.9$ s; 5(c) shows a category 2 agent switching continuously between the free and engaged states till it found a path to target: this agent reaches the target at $t = 18.8$ s and becomes a category 1 agent. In terms of the result stated in Section II-A, (a) and (c) demonstrate instances where the Lyapunov function eventually goes to zero and satisfy condition C1; (b) demonstrates an instance with finite switching where the agent stays in the engaged subsystem after last switching, satisfying condition C2.

In the experiments presented, there is no path to target after a few category 1 agents have reached R_{th}, and hence some exploratory agents never stop. To elucidate the convergence properties of Theorem 1 when a path to the target exists, we present an experiment of a swarm of 500 agents aggregating in the same setup. However, instead of the agents *stopping* when they reach the circle at R_{th}, we allow the agents to enter the circle and reach O. Since this allows a path to O for all agents, every agent eventually becomes a category 1 agent and reaches the target. The accompanying supplementary video provides a summary of the experiments reported in this section.

B. Handling Obstacles

By projecting obstacles as a set of virtual agents, the proposed control-law inherently guarantees collision-avoidance from static obstacles. This also gives a method for autonomous path planning in the presence of static walls/obstacles that guide an agent to the target, referred to as path markers. Figure 6(a) shows snapshots from an experiment where path markers (shown in black) were used to guide the agents to the target; Figure 6(b) shows snapshots from an experiment where the arena was littered with multiple, large, extended obstacles between the spawning region and target. Results show that proposed control-law is able to facilitate swarm aggregation in such scenarios while avoiding collisions with obstacles as well as other agents. Note that the agents have no information about the position or geometry of obstacles and only rely on the measurements described in Section II-D.

C. Comparative Study

For the task of aggregation in the absence of external obstacles, we compare our method against three popular techniques: [11] uses communication and global pose information of every agent, whereas [14] relies on local one-hop interactions, in terms of relative poses of neighboring agents. [37] uses a minimally invasive secondary controller on top of a nominal controller, which is only activated when a collision is imminent, to ensure collision-free behavior using barrier certificates. For this comparison, we use the authors' implementation of centralized barrier certificates on top of a straight-line collision-agnostic nominal controller. This centralized formulation assumes full pose information of every agent in the swarm and the safety controller is computed across all agents simultaneously. The parameters used ensure consistency in peak velocities and safety distance. The metric chosen is the average distance of the agents to the target. Simulation results with 200 and 400 agents (see Fig. 7) show that despite lack of complete position information and inter-agent communication, the proposed method is on par with methods that utilize such information. In particular, for the aggregation of 200 agents, all the methods converge at the same distance. Notice that [37] uses a centralized controller which is activated only near the safety distance, facilitating faster and more efficient aggregation.

D. Experiments on the Robotarium Testbed

To validate our controller on a real robotic platform, we use a swarm of GRITSBots [10] – inexpensive differential drive microbots – via Georgia Institute of Technology's Robotarium project [39]. While the GRITSBots are equipped with centralized communication capabilities, we limit our usage to provide each agent with only the necessary measurements (see Section II-D). To demonstrate, we setup 12 agents in an environment with extended obstacles as path markers, and allow them to aggregate using the proposed controller. Figure 8(a) shows the initial configuration of the swarm approaching the target from the bottom-right. Figure 8(b) shows the agents avoiding collision with the markers, and hence, moving towards the target along the path. Finally, Figure 8(c) shows the final aggregated configuration of the swarm.

V. Conclusion

In this letter, we present a decentralized controller for swarm aggregation. Compared to previous work, our approach uses limited sensor measurements and does not depend on global positioning or inter-agent communication overheads. By establishing theoretical guarantees on the asymptotic stability of the aggregation process, we showed that the resulting aggregates are robust to disturbances, positions of path markers and obstacles in the environment. Without explicitly planning for grouping of swarm members, limited local interactions among the agents in the form of collision avoidance is shown to be sufficient for swarm aggregation. Compared to the techniques developed for global and local communications, the on-par performance of the proposed technique without communication demonstrates that inter-agent communication is not a necessary factor in swarm-aggregation. Moreover, a study on the behavior of swarm of heterogeneous agents having varying communication capabilities (including deaf agents) is now possible. Future work will be aimed at analysis of closed-loop dynamics of the swarm from a global perspective and aggregation to moving targets.

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