Swarm Update III

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1 Recap

The proposed control law to achieve heading consensus in the previous update was given by

$$\omega = \frac{A}{||\mathbf{p}||} \cdot \frac{\eta}{\pi} \tag{1}$$

where η is the angle betIen the agent's heading and the target heading $\hat{\theta}^{target}$

$$\hat{\theta}^{target} = \sum_i \frac{\mathbf{p}_i}{||\mathbf{p}_i||} \tag{2}$$

This control law observed $\dot{H} \leq 0 \ \forall \ t$ in a Monte Carlo. Based on this observation, I make the following claim:

Using Olfati's framework, namely the α-lattice potential ψ_α, I can then obtain a fixed-wing flock using Olfati's distributed potential field for α-lattice formation and my consensus term, which requires a complete communication architecture. This flight is also guaranteed to be collision free if the initial energy of the system does not exceed some k, exactly as in [**?**].

Note: My consensus term only includes heading control. The simplest way to achieving a collision-free flock using this is to assume all agents have the same speed. Thus $a_i = 0 \ \forall \ i$.

Let us recall the lyapunov stability equation:

$$H = V(\mathbf{q}) + \frac{1}{2}\sum_i ||\mathbf{v}_i||^2$$

Thus, if $a_i = 0$, then the problem reduces entirely to heading control. I note that in this model, $a_\perp = \omega \cdot ||\mathbf{p}||$ which is the centripetal force acting on the agent as it turns.

2 System Integration

I think this should work because of the following:

$$\begin{aligned}
\dot{H} &= \nabla V(\mathbf{q})\dot{\mathbf{q}} + \mathbf{v}^T\dot{\mathbf{v}} \\
\dot{H} &= \mathbf{v}^T\nabla V(\mathbf{q}) + \mathbf{v}^T(-\nabla V + f) \\
\dot{H} &= \mathbf{v}^T f
\end{aligned}$$

where f represents my consensus term. The consensus term can be understood in this manner: each agent tries to minimise the error of its own heading with respect to the average heading of the group. Thus, H must be reducing, as it is a positively weighted sum of the error in headings.

For actuator saturation, I note the following:

- the force applied on agents from (1) is bounded such that $a_\perp \in [0, A]$, where A is a positive constant
- the Olfati force (ϕ_α) is also bounded.

This allows us to set up the forces such that actuator saturation may be accounted for, even for fixed-wings. However, note that this means the **initial energy condition to ensure zero collisions will be much more restricted than it needs to be**. Let us recall the form of ϕ_α, Olfati's law.

$$\phi_\alpha(z) = \rho_h(\frac{z}{r_\alpha}) \cdot \phi(z - d_\alpha)$$

$$\phi(z) = \frac{1}{2}[(a+b)\sigma_1(z+c) + (a-b)]$$

with $0 < a \leq b, c = \frac{|a-b|}{\sqrt{4ab}}$ to maintain $\phi(0) = 0$, which allows us to define the potential associated with an α-lattice, ψ_α.

$$\psi_\alpha(z) = \int_{d_\alpha}^{z} \phi_\alpha(s)ds$$

In general, we can assume that $|\phi_\alpha|_{max}$ occurs at $z = 0$, to prevent collisions. Thus, the maximum force Olfati's law may apply to an agent is bounded by a constant based on system parameters. Let us denote this maximum force by B.

Thus, our simplistic model of a fixed-wing can apply a maximum turning force of $a_{||,max}, a_{\perp,max}$ in the directions parallel and perpendicular to the heading respectively. Setting A, B such that $a_{\perp,max} = A + B$ would completely ensure actuator saturation, **assuming $\mathbf{a}_{||,\mathbf{max}} \geq \mathbf{a}_{\perp,\mathbf{max}}$.**

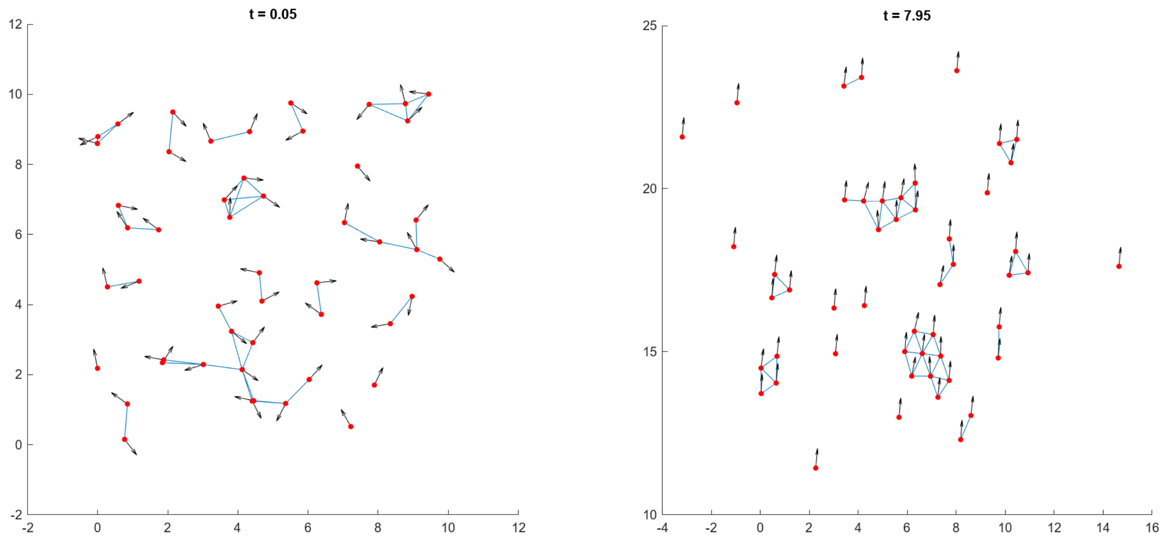
Caveat: Something I have not dealt with at all is the bounded velocity of the agents i.e, $v_{min} \leq v \leq v_{max}$. For now, it is assumed that the linear acceleration of the agent follows this scheme:

$$a_{||} = \begin{cases} a_{||} & \text{, if } v_{\min} < v < v_{\max}, \\ 0 & \text{, otherwise} \end{cases}$$

Relevant Simulations

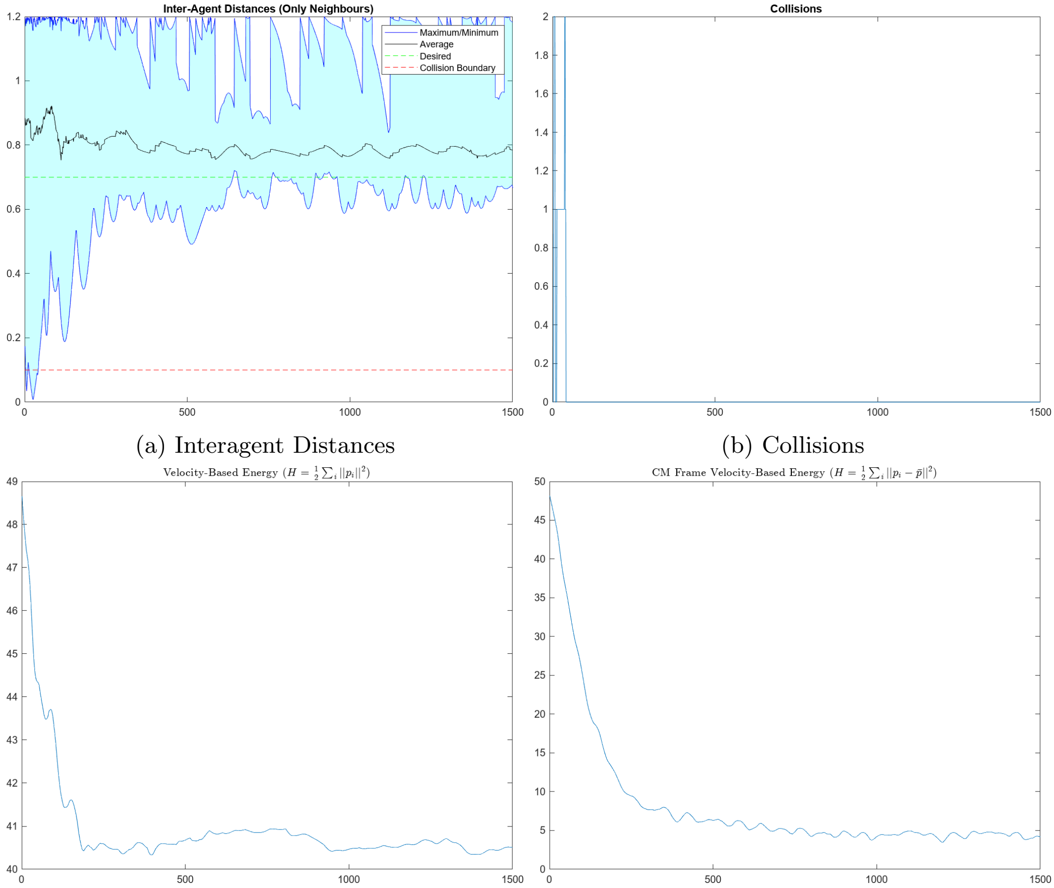
Here, I attempted to make observations about the energy profile of the flock over time under the action of the control law proposed above. I also observe other aspects of the simulation, namely collisions occurred and inter-agent distances. Note that two agents are considered to collide if they are within 0.1 units of each other.

I have uploaded the videos of a few simulations here. Let's look at one of the simulations.



(a) Initial State (b) Final State

Figure 1: Initial & Final States



(a) Interagent Distances

(b) Collisions

(c) H

(d) H (CM Frame)

Figure 2: Some Stats

The simulations do not include the calculation of $V(q)$ in the energy profile as I was not able to obtain an accurate numerical value for $V(q)$. We can understand the energy profile of $V(q)$ from the plot of inter-agent distances. Thus the energy profiles only consider the kinetic energy terms. The following observations were made:

- The proposed control law does not completely get rid of collisions, as expected.
- The minimum inter-agent distance approaches the desired α-lattice parameter (d) as time goes on, whereas the maximum inter-agent distance does not change much. I suspect this is due to the absence of a damping term and oscillatory behaviour is prevalent.
- The velocity-based energy term in the CM Frame decreases appreciably and is almost monotonically decreasing. It does not go to 0 as there are disconnected clusters. In fact, Olfati's law is (energy) conservative, which is why we can see significant oscillatory behaviour in the agents in some of the simulations (link provided above). This oscillatory behaviour does die down over time however; although it might be due to error accumulation in the simulator, I am not certain about it.

3 Damping Term

I came up with a damping term, based on heuristics again. Here is the idea:

- If two agents are anti-parallel, both of them should slow down, in order to avoid collisions and attempt matching speeds (the lower bound on the speed is maintained).
- If two agents are aligned perpendicularly, then they should feel no need to speed up or slow down.
- If two agents are parallel, the faster one should slow down, and the slower should speed up in order to match velocities.

Mathematically,

$$f_d^i = \sum_{j \in \mathcal{N}_i} ((\mathbf{p}_j - \mathbf{p}_i) \cdot \frac{\mathbf{p}_i}{||\mathbf{p}_i||}) \cdot \frac{\mathbf{p}_j \cdot \mathbf{p}_i}{||\mathbf{p}_j|| \cdot ||\mathbf{p}_i||} \tag{3}$$

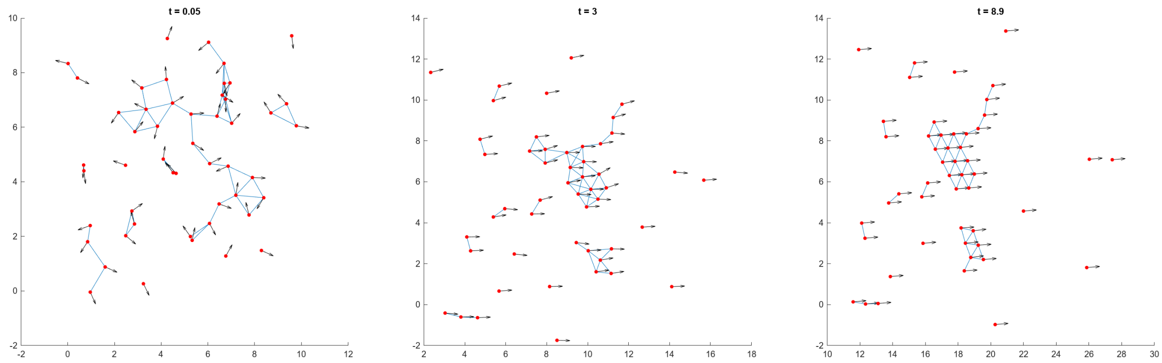
where $\mathcal{N}_i$ is the neighbour set of agent i. In simpler notation, the above evaluates to,

$$f_d^i = \sum_{j \in \mathcal{N}_i} (||\mathbf{p}_j||cos\gamma - ||\mathbf{p}_i||)cos\gamma \tag{4}$$

where γ is the angle between agent i's and j's headings. This can be understood as the difference between agent j's and agent i's speeds along i's heading, scaled by a directional term, $cos\gamma$. It is also clear that this force term can also be made to fit the actuator saturation method above as it is upper bounded by $2v_{max}$.

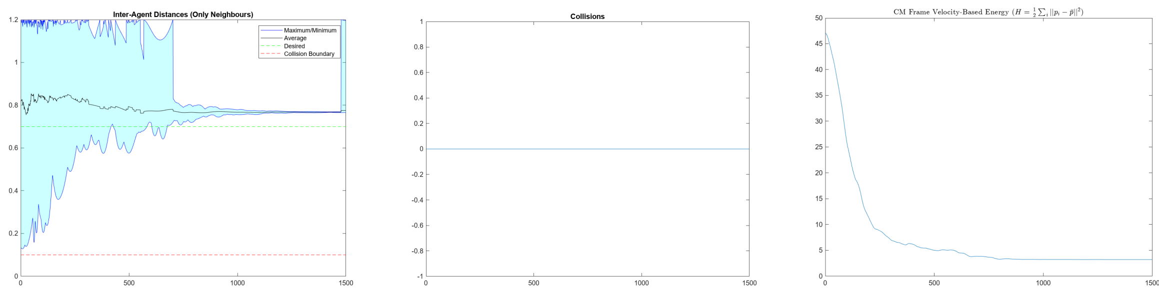
Relevant Simulations

This simulation has also been uploaded in the link above. Note that two agents are considered to collide if they are within 0.1 units of each other.



(a) Initial (b) Transient (c) Final

Figure 3: Damping Included



(a) Interagent Distances (b) Collisions (c) H (CM Frame)

Figure 4: Some Stats

We make the following observations:

- The energy term steadily declines implying the agents tend towards the same velocities. It does not fully go to zero because of disconnected clusters. The damping term gets rid of the oscillatory behaviour in the energy profile.
- This simulation was a clean run which had no collisions, despite the initial energy due to **ONLY** the velocity-based energy term being greater than c^*. This implies that the initial energy being lesser than c^* is a sufficient, but not necessary condition for 0 collisions. c^* is calculated based on simulation parameters in the appendix.
- The inter-agent distances also neatly converge to some value (that is not d? I'm not sure what's happening there). The large step at the end simply means that two previously unconnected clusters became connected then.

A Calculation of c^*

The value of c^* is calculated using the following formula:

$$\int_{d_\alpha}^{0} \phi_\alpha(z)dz = \int_{d_\alpha}^{0} \rho_h(\frac{z}{r_\alpha}) \cdot \phi(z - d_\alpha)dz < \int_{d_\alpha}^{0} \phi(z - d_\alpha)dz$$

$$\int_{d_\alpha}^{0} \phi(z - d_\alpha)dz = \int_{d_\alpha}^{0} \frac{1}{2}((a + b)\sigma_1(z - d_\alpha + c) + (a - b))dz$$

where σ_ϵ is given by

$$\sigma_\epsilon(z) = \frac{z}{\sqrt{1 + \epsilon||z||^2}}$$

Given $a = 50, b = 100, c = \frac{|a-b|}{\sqrt{4ab}} = \frac{1}{2\sqrt{2}}, d = 0.7, d_\alpha = ||d||_\sigma = 0.2207$, the above evaluates to the following

$$\int_{0.2207}^{0} \left(\frac{75(x + 0.132853)}{\sqrt{1 + (x + 0.132853)^2}} - 25 \right) dx = 1.62697 = c^*$$

B Simulator Validation

The simulator uses the forward euler method for forward propogation of the system. Thus error is bound to accumulate over time. More accurate simulation methods will be needed later on to scale this up.

It is good to note that the agents seem to orient themselves in random directions in different simulations; this is shown below. I am yet to perform a monte carlo to confirm this asymptotically.

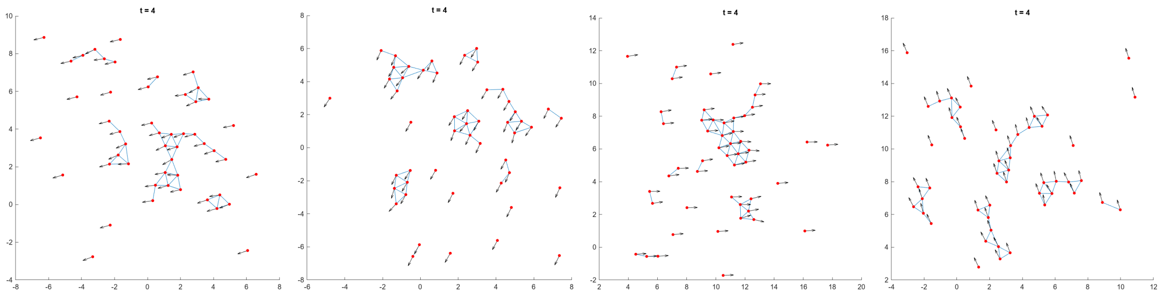
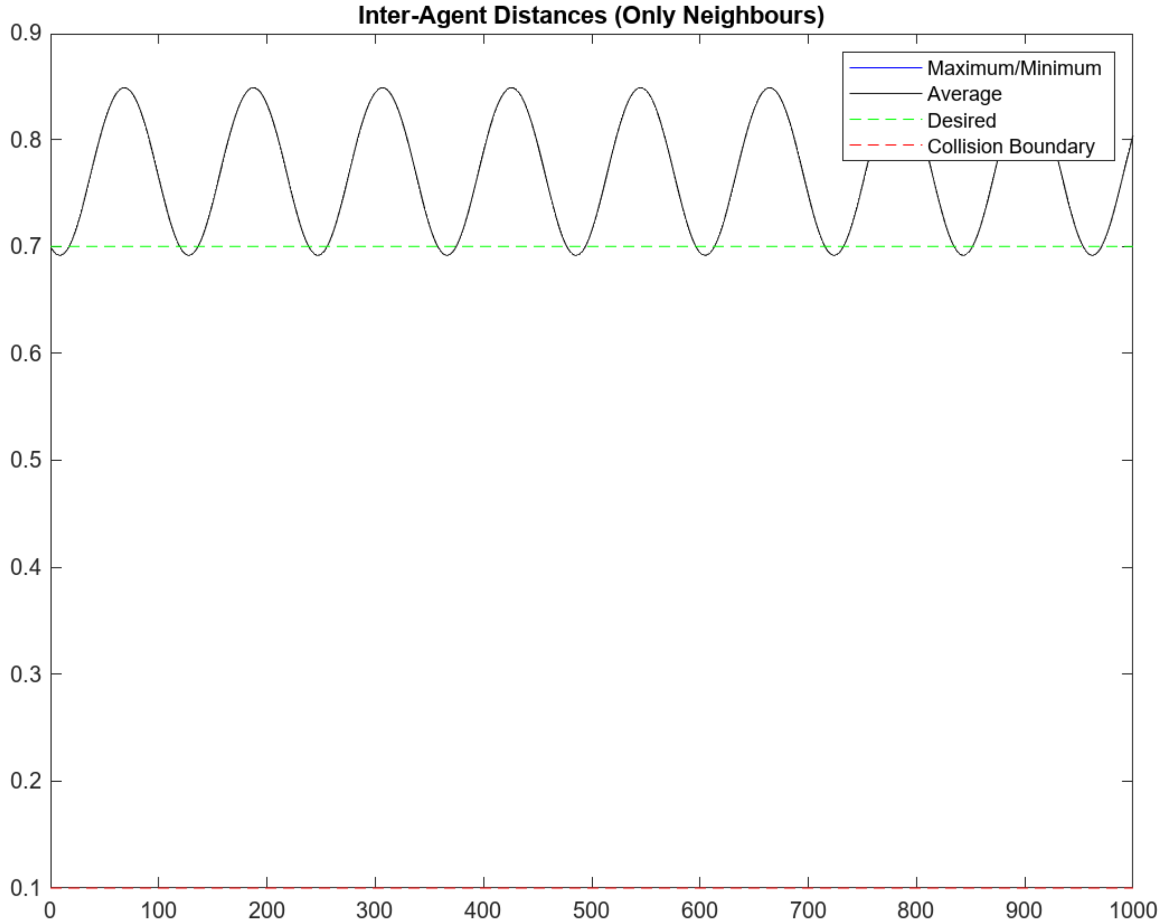
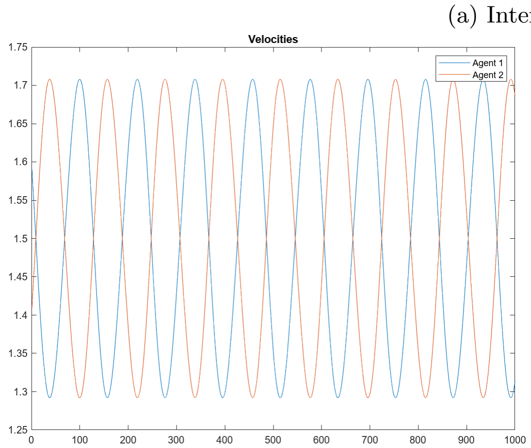


Figure 5: Random Final Orientations

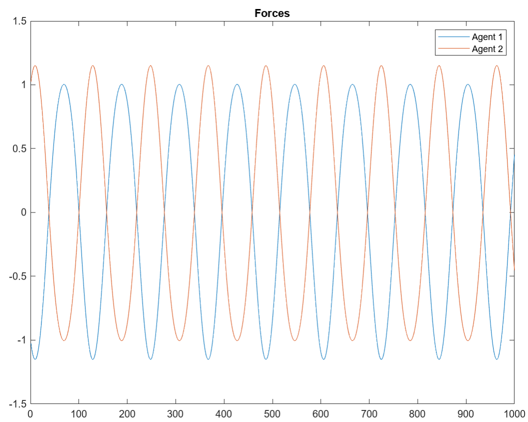
Simply using Olfati's law performs properly on a system with only two agents such that the total energy of the system is constant. This can be seen below. The system is initialised with two agents at $(x_1, y_1) = (0.3, 0), (x_2, y_2) = (1, 0)$ and with velocities $(\dot{x}_1, \dot{y}_1) = (1.6, 0), (\dot{x}_2, \dot{y}_2) = (1.4, 0)$.



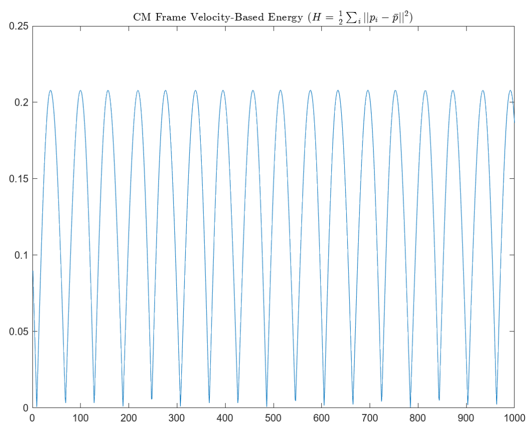
(a) Inter-Agent Distances



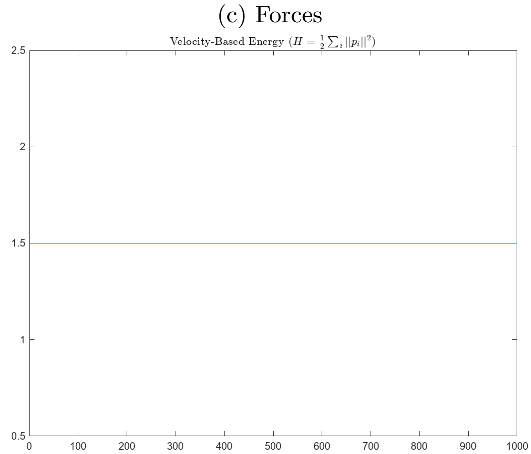
(b) Velocities



(c) Forces



(d)



(e)

Figure 6: Only Olfati acting on two aligned agents