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Barrier Lyapunov Function Based Controller Design for Euler-Lagrange Systems with Reduced Control Effort

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Abstract: In this paper, a novel nonlinear controller is designed for Euler- Lagrange (EL) systems while satisfying user-defined safety-constraints in position variables. The EL systems, which are highly popular for modeling a variety of non-linear mechanical plants like aircraft, spacecraft, quad-rotors, robotic manipulators, etc., critically demand safety-constraint satisfaction during trajectory tracking applications. Barrier Lyapunov Function (BLF) is widely used for systematically designing the controller to prevent safety-constraint violation during trajectory tracking. By proving the boundedness of BLF in a closed loop, safety-constraint satisfaction is analytically guaranteed. The fundamental principle of BLF is that if the states are reaching close to the boundary of a safe region, the high control input is applied to the system to push states far inside the boundary. In practical scenarios, such a high control effort requirement may lead to the critical problem of actuator saturation. This work alleviates the issue of high control effort requirements in the BLF framework. The proposed BLF-based controller is capable of guaranteeing safety-constraint satisfaction with less control effort, making the design practically viable. Simulation results validate significant improvement in terms of reduction in control input magnitude of the proposed algorithm in contrast to conventional BLF-based control algorithms.

Keywords: EL systems, BLF, safety-constraint, high control effort, actuator saturation.

1. INTRODUCTION

Control design for constrained nonlinear systems (Li et al., 2012; Wang et al., 2013; Cui et al., 2013), such as constrained robot, missile, quad-rotors, etc., has been an active area of research. In most of the real world systems, constraints are associated in various forms, such as saturation, physical stoppages, performance, safety parameters (Tee et al., 2011), (Chen et al., 2011). This work addresses the issue of safety constraint in the context of Euler Lagrange (EL) dynamics, which are used to model a wide class of nonlinear systems, such as robot manipulator, ship dynamics, quad-rotors, etc.

Lyapunov direct method is used to determine the stability of nonlinear systems, without knowing the explicit solution of the systems (Slotine et al., 1991). Besides the stability analysis, it is also used to design the stabilizing controllers via choosing a candidate Lyapunov function. For many practical systems, it is suitable to select the proper Lyapunov candidate from the physical intuition and systems knowledge to facilitate the controller design, as shown in different problems (Ortega et al., 2001). Several methods are developed in literature to handle constraint (Hu and Lin, 2001; Mayne et al., 2000; Bemporad, 1998; Gilbert and Kolmanovsky, 2002; Krstic and Bement, 2006). The works in (Ngo et al., 2005; Tee et al., 2009) have designed a novel method to tackle state constraints using Barrier Lyapunov Functions. Barrier Lyapunov candidate is such a function, which will approach to infinity, when it's argument approaches to some predefined safety constraint limit.

Although the BLF-based controller is a mathematically elegant way to systematically handle state constraints, the controller typically requires significantly high control efforts at those instances, when the state trajectory reaches close to the safety boundary. In fact, the fundamental principle of BLF based control is to push the states inward via high control input while the states are trying to transgress the barrier. Due to the high control input requirement, the BLF based control design looses its practical feasibility in many real world applications.

Unlike past literature, this work proposes a novel variant of BLF based controller, which overcomes the issue of high control input requirement. The work uses the framework of Euler-Lagrange systems to formulate the problem. However, with some modifications the idea can be applicable to various other class of nonlinear systems like strict-feedback systems etc. The key idea of the work is to strategically introduce an integral control action, which facilitates to transfer the demand of high magnitude from the actual control input to a virtual control input. The actual control action magnitude is reasonably low in the proposed scheme in contrast to conventional BLF based controllers.

This paper is organized as follows. Section II presents the problem statement and preliminaries, section III presents a new proposed BLF based control design. Simulation results

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and performance analysis are given in section IV. Section V concludes the paper with a summary of the contribution.

2. PROBLEM STATEMENT AND PRELIMINARIES

Defines barrier Lyapunov candidates, mentions condition for state in R^k+1 to be bounded

Throughout this paper, $||.||$ denotes the Euclidean vector norm in $\mathbf{R}^n$, the R_+ denotes the set of non-negative real numbers, and norm $\mathscr{L}_\infty = max|x_i|$ for all i.

2.1 Model Description

Consider the standard Euler-Lagrange dynamics (Spong and Vidyasagar, 2008).

$$M(q)\ddot{q} + V_m(q,\dot{q})\dot{q} + F(\dot{q}) + G(q) = \tau \tag{1}$$

Where $q(t) \in \mathbf{R}^n$, $\dot{q}(t) \in \mathbf{R}^n$ and $\ddot{q}(t) \in \mathbf{R}^n$ represents generalized position, velocity and acceleration, respectively, $M(q) \in \mathbf{R}^{n\times n}$ represents a Inertia Matrix, $V_m(q,\dot{q}) \in \mathbf{R}^{n\times n}$ is a Coriolis and Centrifugal Matrix, $G(q) \in \mathbf{R}^n$ represents gravity, $F(\dot{q}) \in \mathbf{R}^n$ is frictional term, $\tau(t) \in \mathbf{R}^n$ denotes generalized torque, which is the control input. The EL dynamics (1) exhibits some special properties, which are subsequently described. These properties facilitate the control design and stability analysis, (Spong and Vidyasagar, 2008).

Property 1. $M(q) \in \mathbf{R}^{n\times n}$ is a symmetric, positive definite (PD) and bounded matrix. There exist two positive constant μ_1 and μ_2, such that

$$\mu_1 I \le M(q) \le \mu_2 I \tag{2}$$

Property 2. The Matrix $\dot{M}(q) - 2V_m(q,\dot{q})$ is a skew-symmetric, i.e.,

$$\xi^T(\dot{M}(q) - 2V_m(q,\dot{q}))\xi = 0, \quad \forall \xi \in \mathbf{R}^n \tag{3}$$

2.2 Control Objective

Assumes the agent is following a pre-defined trajectory, and is trying to minimise tracking error

The objective is to design a controller $\tau(t)$, such that $q(t)$ tracks a given desired trajectory $q_d(t) \in \mathbf{R}^n$, while the tracking error $z_1(t) \triangleq q(t) - q_d(t)$ should remain in a predefined safety constraint set i.e., $||z_1(t)|| < k_m \forall t \ge 0$, where k_m is a positive real constant.

The following assumption is considered for further analysis.

Assumption 1. The desired trajectory $q_d(t) \in C^3$ functions (i.e. it is continuous and thrice differentiable) and $q_d(t)$, $\dot{q}_d(t)$, $\ddot{q}_d(t)$, $\dddot{q}_d(t) \in \mathscr{L}_\infty$.

2.3 Preliminaries on BLF

To prevent the safety constraint violation of the states from a given dynamical system, a special Lyapunov function candidate called, Barrier Lyapunov Function (BLF) is introduced (Ngo et al., 2005; Tee et al., 2009) defined as follows.

Definition 1. (Tee et al., 2009) A Barrier Lyapunov Function is a energy like scalar function $V(z)$, defined with respect to the system $\dot{z} = g(z)$ on a open safety region Ω containing the origin, that is continuous, positive definite, has continuous first-order partial derivatives at every point of Ω, has the property $V(z) \to \infty$ as z approaches to the boundary of Ω, and satisfies $V(z(t)) \le k \; \forall t \ge 0$ along the solution of $\dot{z} = g(z)$ for $z(0) \in \Omega$ and some positive constant k.

The existence of a BLF for a system guarantees the stability of the equilibrium at the origin, and that Ω is a positively invariant region. A lemma from (Tee et al., 2009) is formulated for control design, which formalizes this notion for general forms of barrier functions.

Lemma 1. For any positive constant k_{p_1}, k_{b_1}, let $\Phi := \{z_1 \in \mathbf{R} : -k_{p_1} < z_1 < k_{b_1}\} \subset \mathbf{R}$ and $\Psi := \mathbf{R}^k \times \Phi \subset \mathbf{R}^{k+1}$ be open sets.

Consider the system

$$\dot{\rho} = g(t,\rho) \tag{4}$$

Beta := vector in R^k

where $\rho := [\beta, z_1]^T \in \Psi$ is the state, and function $g : R_+ \times \Psi \subset \mathbf{R}^{k+1}$ is piecewise continuous in t and locally Lipschitz in z, uniformly in t. Suppose that there exist functions $V_2 : \mathbf{R}^k \to R_+$ and $V_1 : \Phi \to R_+$, continuously differentiable and positive definite in their respective domains, energy like scalar function, such that

$$V_1(z_1) \to \infty, \quad z_1 \to -k_{p_1} \quad or \quad z_1 \to k_{b_1} \tag{5}$$

V1 approaches infinity at the bounds -kp1, kb1, i.e, z1 stays bounded

and V_2 is a quadratic Lyapunov candidate, which is both positive definite and decrescent. Consider a combined Lyapunov function $V(\rho) := V_1(z_1) + V_2(\beta)$ and $z_1(0)$ belong to the set $z_1 \in (-k_{p_1}, k_{b_1})$. If the inequality holds:

$$\dot{V} = \frac{\partial V}{\partial \rho} g \le 0 \tag{6}$$

then $z_1(t)$ will always remain in the open set $(-k_{p_1}, k_{b_1})$ $\forall t \in [0,\infty)$.

Proof. For proof of lemma 1, see the reference (Tee et al., 2009).

Remark 1. In lemma 1, the state space is split out into z_1 and β, where z_1 is responsible for constrained state, which require Barrier Lyapunov function to prevent constraint voilation, and β are the free states, which are handled by quadratic Lyapunov function.

3. BLF BASED CONTROL DESIGN

To implement conventional as well as new proposed BLF based controller, standard Backstepping techniques are used Krstic et al. (1995).

To facilitate the design, a position tracking error is defined.

$$z_1 \triangleq q - q_d \tag{7}$$

Now taking the time derivative of (7)

$$\dot{z}_1 = \dot{q} - \dot{q}_d \pm \alpha_1 \tag{8}$$

where α_1 is a virtual stabilizing controller, which is based on the concept of standard backstepping technique Krstic et al. (1995) used in such type of controller design.

One alternate way to write the (8) is

$$\dot{z}_1 = z_2 - \dot{q}_d + \alpha_1 \tag{9}$$

where z_2 is an auxiliary error variable, defined as

$$z_2 = \dot{q} - \alpha_1 \tag{10}$$

Design the virtual controller α_1 as

$$\alpha_1 = \dot{q}_d - k_1 z_1 \tag{11}$$

Substituting (11) in (9) yields

$$\dot{z}_1 = z_2 - k_1 z_1 \tag{12}$$

where k_1 is positive constant and for stability analysis a special Lyapunov candidate called, Barrier Lyapunov Function is considered as in definition 1.

$$V_1 = \frac{1}{2} log \frac{k_m^2}{k_m^2 - z_1^T z_1} \tag{13}$$

where k_m is positive real constant, now taking the time derivative of (13) along the system trajectories becomes

$$\dot{V}_1 = \frac{z_1^T \dot{z}_1}{k_m^2 - z_1^T z_1} \tag{14}$$

Substituting (12) in (14) yields

$$\dot{V}_1 = \frac{z_1^T z_2}{k_m^2 - z_1^T z_1} - \frac{k_1 z_1^T z_1}{k_m^2 - z_1^T z_1} \tag{15}$$

To deal with the coupled term containing $z_1^T z_2$ in (15), the equation (10) is reshaped in the following form, which subsequently facilitate the control input design. another virtual term

$$M(q)\dot{z}_2 = M(q)\ddot{q} - M(q)\dot{\alpha}_1 \pm V_m z_2 \tag{16}$$

Substituting the $M(q)\ddot{q}$ from (1) in (16), the dynamics becomes

$$M(q)\dot{z}_2 = \tau - Gq - F\dot{q} - V_m\dot{q} - M(q)\dot{\alpha}_1 \pm V_m z_2 \tag{17}$$

Finally, the torque input is designed as

$$\tau = Gq + F\dot{q} + V_m\alpha_1 + M(q)\dot{\alpha}_1 - k_2 z_2 - \underbrace{\frac{z_1^T}{k_m^2 - z_1^T z_1}}_{\eta} \tag{18}$$

Substituting the (18) in (17), the error dynamics yields

$$M(q)\dot{z}_2 = -k_2 z_2 - V_m z_2 - \frac{z_1^T}{k_m^2 - z_1^T z_1} \tag{19}$$

A pictorial view of the conventional BLF based controller is shown in Fig. 1, while the following Theorem characterizes its stability and constraint satisfaction properties.

Theorem 1. For the model dynamics in (1), the control law (11), (18) under the assumption 1, ensure that the error dynamics $\zeta(t) = [z_1(t), z_2(t)]^T$ is asymptotically stable, while constrained set $||z_1(t)|| < k_m$ is not violated for all time, provided $\|z_1(0)\| < k_m$.

Proof. Consider the following Lyapunov candidate, which is a combination of two different type of Lyapunov candidate as

$$V_2 = \frac{1}{2} log \frac{k_m^2}{k_m^2 - z_1^T z_1} + \frac{1}{2} z_2^T M(q) z_2 \tag{20}$$

Here, the first component is for constrained state $z_1(t)$ same as in (13) and the second component of (20) is responsible for unconstrained state $z_2(t)$, same approach as in the lemma 1.

Taking the time derivative of (20) and substituting the error dynamics (12), (19) yields

$$\dot{V}_2 = \frac{z_1^T z_2}{k_m^2 - z_1^T z_1} - \frac{k_1 z_1^T z_1}{k_m^2 - z_1^T z_1} - z_2^T k_2 z_2 - z_2^T V_m z_2$$
$$- \frac{z_1^T z_2}{k_m^2 - z_1^T z_1} + \frac{1}{2} z_2^T \dot{M}(q) z_2 \tag{21}$$

After using the skew-symmetric property 2, (21) becomes

$$\dot{V}_2 = -\frac{k_1 z_1^T z_1}{k_m^2 - z_1^T z_1} - k_2 z_2^T z_2 \leq 0 \tag{22}$$

From (22), it can be claimed that $||z_1(t)|| < k_m$ holds for all $t \geq 0$, provided $\|z_1(0)\| < k_m$. And $\dot{V}_2$ is zero if and only if z_1, z_2 both are zero, then from both the above argument, it can be concluded that the error dynamics, $\zeta(t) = [z_1(t), z_2(t)]^T$ is asymptotically stable.

Remark 2. Although Theorem 1 establishes the guarantee of constraint satisfaction, the component $\eta(t)$ of the control input $\tau(t)$ can be unreasonably high at the specific instances, when $\|z_1(t)\|$ reaches near to the boundary k_m. This will lead to a high $\mathscr{L}_\infty$ norm of the torque, which may result in practical infeasibility of the controller.

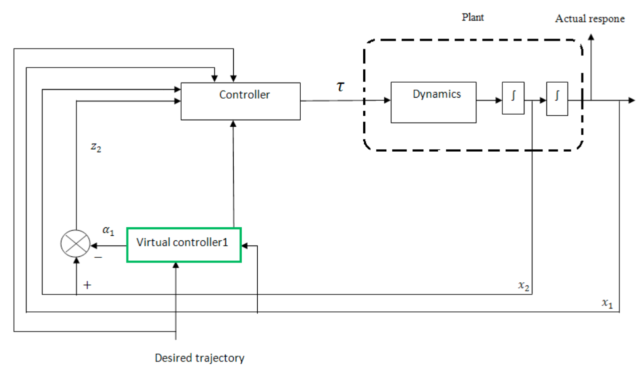

Fig. 1. conventional BLF based control design model

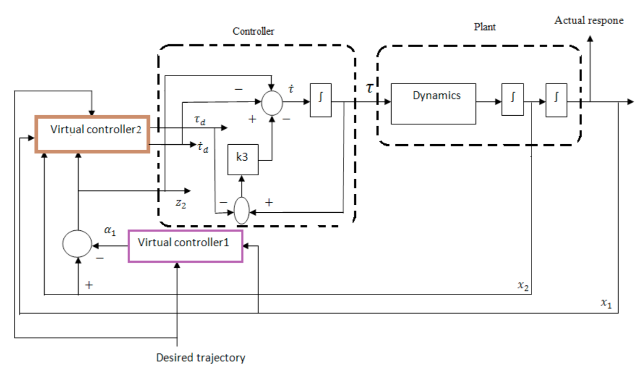

Fig. 2. A proposed new BLF based control design model

3.1 A proposed new BLF based control design

This section proposes a novel variant of BLF, where safety constraint on z_1 is maintained without demanding high control effort. The scheme introduce a strategically designed integrator in the controller, which plays the central role in reducing the $\mathscr{L}_\infty$ norm of the control input.

The error dynamics of z_2 in (17), can be modified as follows.

$$M(q)\dot{z}_2 = z_3 - Gq - F\dot{q} - V_m\dot{q} - M(q)\dot{\alpha}_1 \pm V_m z_2 + \tau_d \tag{23}$$

where another virtual controller τ_d and another corresponding auxiliary error variable z_3 are intentionally introduced, The error z_3 is defined as

$$z_3 = \tau - \tau_d \tag{24}$$

instead of the actual controller $\tau(t)$, the virtual controller $\tau_d(t)$ is designed by replicating the right-hand-side of (18).

$$\tau_d = Gq + F\dot{q} + V_m\alpha_1 + M(q)\dot{\alpha}_1 - k_2 z_2 - \underbrace{\frac{z_1^T}{K_m^2 - z_1^T z_1}}_{\eta} \tag{25}$$

Now taking the time derivative of (24) is

$$\dot{z}_3 = \dot{\tau} - \dot{\tau}_d \tag{26}$$

And selecting the $\dot{\tau}$ such that

$$\dot{\tau} = \dot{\tau}_d - k_3 z_3 - z_2 \tag{27}$$

In this paper the time derivative of τ_d, i.e., the derivative of (25) is done analytically. The following theorem establishes the stability and constraint satisfaction properties of the designed controller.

Theorem 2. For the model dynamics in (1), the control law evolves from (27) under the assumption 1, ensure that the error dynamics $\delta(t) = [z_1(t), z_2(t), z_3(t)]^T$ is asymptotically stable, while constrained set $||z_1|| < k_m$ is not violated for all time, provided $||z_1(0)|| < k_m$.

Proof. A suitable Lyapunov candidate is chosen for stability analysis such that

$$V_3 = V_1 + \frac{1}{2} z_2^T M(q) z_2 + \frac{1}{2} z_3^T z_3 \tag{28}$$

where V_1 is same as in (13), Now taking the time derivative of (28) along the modified system trajectories and substituting the (12) yields

$$\dot{V}_3 = \frac{z_1 z_2}{K_m^2 - z_1^T z} - \frac{k_1 z_1^T z_1}{K_m^2 - z_1^T z} + z_2^T M(q)\dot{z}_2 + \frac{1}{2} z_2^T \dot{M}(q) z_2 + z_3^T \dot{z}_3 \tag{29}$$

After putting (23), (26) and (27) into $\dot{V}_3$, using the property 2 yields

$$\dot{V}_3 = -\frac{k_1 z_1^T z_1}{K_m^2 - z_1^T z_1} - k_2 z_2^T z_2 - k_3 z_3^T z_3 \leq 0 \tag{30}$$

From (30), it can be claimed that $||z_1(t)|| < k_m$ holds for all $t \geq 0$, provided $||z_1(0)|| < k_m$. And V_3 is zero if and only if z_1, z_2 and z_3 all are zero, then from the above argument, it can be concluded that the error dynamics, $\delta(t) = [z_1(t), z_2(t), z_3(t)]^T$ is asymptotically stable.

Remark 3. As depicted in Fig. 2, the proposed control architecture consists of two virtual controller blocks. The second virtual controller, i.e., the virtual torque $tau_d(t)$ is intentionally introduced to resolve the issue of high torque requirement in BLF framework. Unlike conventional BLF controller, where $\eta(t)$ is introduced in the actual torque $\tau(t)$ (see equation (18)), the proposed BLF- based controller incorporated a similar term in the vrtual torque $\tau_d(t)$ (see equation (25)), instead of the actual torque $\tau(t)$. Hence, the burden of high $\mathscr{L}_\infty$ norm requirement has been shifted from actual torque to the virtual torque in the proposed design. That is why, the $\mathscr{L}_\infty$ norm of the actual torque $\tau(t)$ has been reduced to a significantly low value, which is further validated in the simulation section subsequently.

4. SIMULATION RESULTS

The proposed new BLF based control design is simulated by considering the mathematical model of two link revolute joint robot manipulator (Spong and Vidyasagar, 2008). After that, a comparison study is carried out for the same model by considering both conventional as well as a newly proposed method.

$$M(q)\ddot{q} + V_m(q,\dot{q})\dot{q} + F(\dot{q}) = \tau \tag{31}$$

Respective matrices related to (31), described as following

$$M(q) = \begin{bmatrix} w_1 + 2w_3c_2 & w_2 + w_3c_2 \\ w_2 + w_3c_2 & w_2 \end{bmatrix} \tag{32}$$

$$V_m(q,\dot{q}) = \begin{bmatrix} -w_3s_2\dot{q}_2 & -w_3s_2(\dot{q}_1 + \dot{q}_2) \\ w_3s_2\dot{q}_1 & 0 \end{bmatrix} \tag{33}$$

$$F(\dot{q}) = \begin{bmatrix} f_1 & 0 \\ 0 & f_2 \end{bmatrix} \tag{34}$$

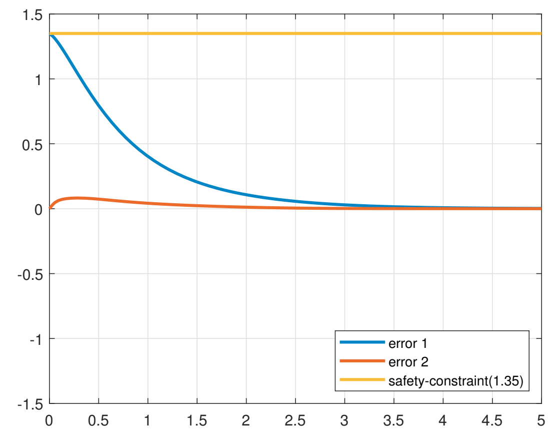

Fig. 3. tracking-error $z_1(t)$

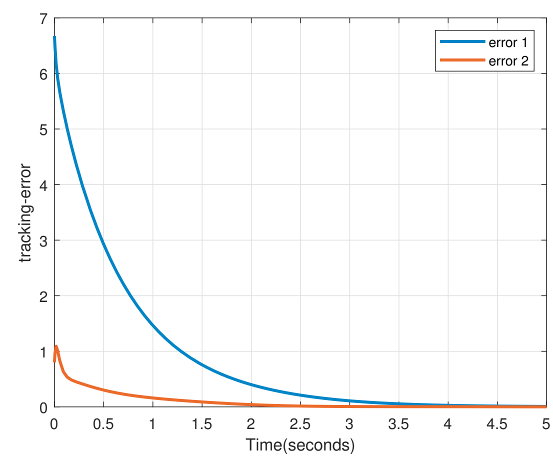

Fig. 4. tracking-error $z_2(t)$

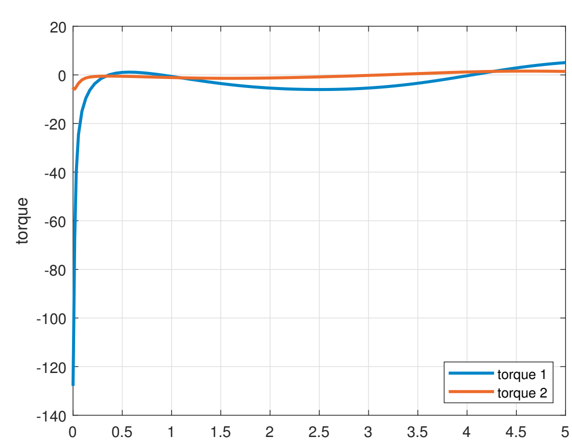

Fig. 5. conventional BLF based controller $\tau(t)$

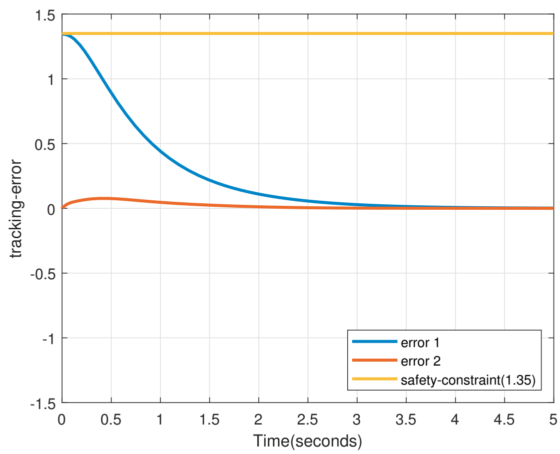

Fig. 6. tracking-error $z_1(t)$

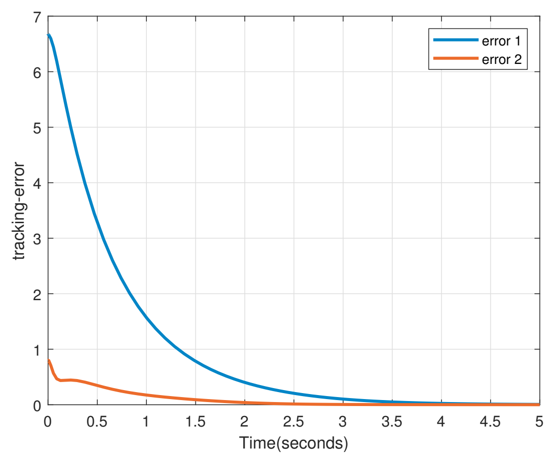

Fig. 7. tracking-error $z_2(t)$

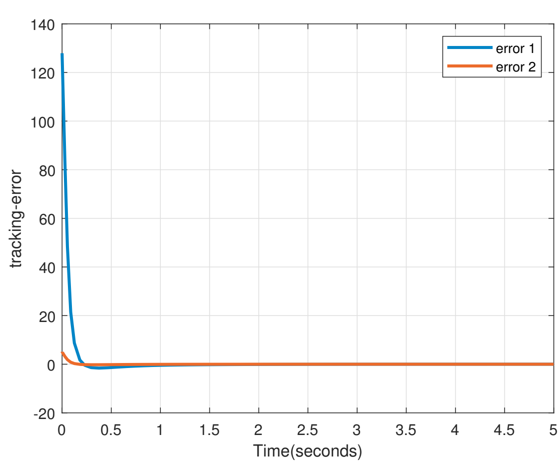

Fig. 8. tracking-error $z_3(t)$

System parameter are given by $w_1 = 3.473\ kg.m^2$, $w_2 = 0.196\ kg.m^2$, $w_3 = 0.242\ kg.m^2$, $f_1 = 5.3\ Nm.sec$, $f_2 = 1.1\ Nm.sec$. Further $c_2 = cos(q_2)$ and $s_2 = sin(q_2)$. The gains are chosen such that $k_1 = 5I_2$, $k_2 = 5I_2$ and $k_3 = 5I_2$. Where I_2, denotes the identity matrix. Desired trajectories are $x_d(t) = [sint; cost]^T$. Fig.3-Fig.5 is for conventional BLF based control design for

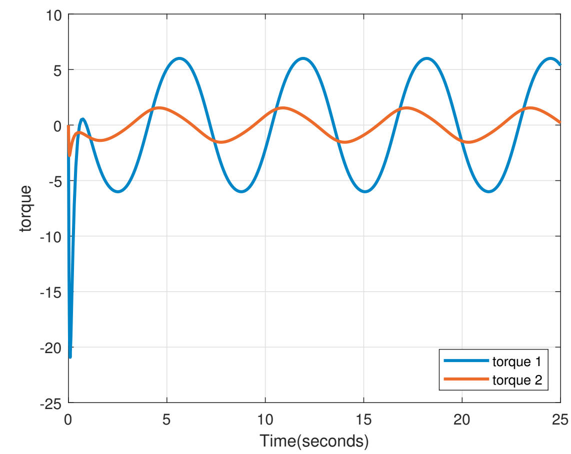

Fig. 9. A proposed new BLF based controller $\tau(t)$

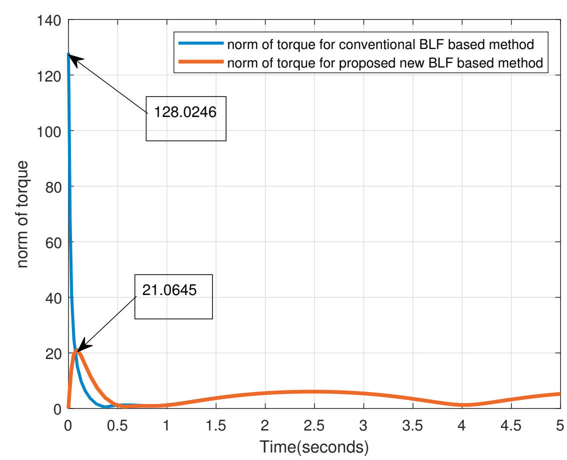

Fig. 10. A comparison for norm of torque $\tau(t)$ for both conventional as well as proposed new BLF based method

two link rigid robotic manipulators, where Fig.3, $z_1(t)$ shows tracking error for the safety constraint set $||z_1(t)|| < 1.35$. Fig.4, $z_2(t)$ represents tracking error for unconstrained states. Fig.5 shows a huge torque is required when constrained state $z_1(t)$ is near to the boundary of the safety constraint $k_m = 1.35$. Fig.6-Fig.9 is responsible for the proposed new BLF based control design, which uses one more extra layer of Integrator dynamics. Fig.6-Fig.8 shows the tracking error and Fig.9 shows the torque, which is not so much large in magnitude as compare to the conventional BLF based controller design, when the safety constrained is near to the boundary. Finally a comparison study is carried out for norm of torque $\tau(t)$ for both conventional as well as a proposed new BLF based control design in Fig.10., which indicates that a significant improvement is justified in terms of torque requirement.

5. CONCLUSION

This work proposes a novel variant of BLF based controller design for constraint Euler-Lagrange systems, which reduces the control torque requirement significantly. An additional integral action is intentionally introduced which transfers the burden of the high magnitude requirement from the actual control to a virtual control. A Lyapunov analysis is performed to establish the stability and constraint satisfaction properties of the proposed controller. Simulation is done by considering the two link revolute joint rigid robot manipulator example, which validate the efficacy of new proposed BLF based control design. Extending the work in the presence of parametric uncertainty using an adaptive controller is a worthy scope of future research.

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