Update II

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Recap: we aim to replace the consensus term in the Olfati's flocking law that accommodates for turning in fixed-wings i.e, a head-on collision setting should result in maximum turning instead of just deceleration.

Disclaimer: all simulations thus far have been done in a setting where agents do not move; this was done to test the consensus algorithm under a time-invariant topology. This ensures connectivity of the flock. In a real setting, connectivity cannot be guaranteed as agents may become isolated. It is seen below that in this setting, certain local minima exist. I conjecture that such local minima are only stable due to the time-invariant topology. In a setting where agents move in space, leading to a time-varying adjacency matrix, such local minima probably will not be stable.

The idea remains simple, each agent tries to minimise the difference in heading between itself and its neighbours. The first consensus term I attempted was highly nonlinear.

$$\omega_j = -\frac{A}{||p_j||} \cdot tanh(B \sum_k a_{jk}(q) sin(\frac{\eta_{jk}}{2})) \tag{1}$$

where $a_{jk}(q)$ and η_{jk} represent the weights of the adjacency matrix and the angle between the headings of agents j, k respectively. I then linearised the term, noting that the angle between the headings of any two agents is always bounded within $(-\pi, \pi]$.

$$\omega_i = \frac{A}{||p_i||} \cdot \frac{\sum_j a_{ij}(q)\eta_{ij}}{\sum_j a_{ij}(q)} \tag{2}$$

which corresponds to the following matrix equation

$$\omega = -D^{-1}\hat{L}\theta \tag{3}$$

where θ_i is measured with respect to the x-axis. However, this failed to account for the fact that the state-space of θ was circular i.e, it wrapped around, unlike the unbounded state-spaces of Cartesian coordinates x, y. This caused the control law to get stuck in many local minima as can be seen below in figure 1.

In an attempt to fix this, I made it so that each agent instead attempts to converge to a "target" heading, the weighted average of its neighbours.

$$\bar{\theta}_i^{target} = \sum_j a_{ij}(q) \frac{p_j}{||p_j||} \tag{4}$$

$$\omega_i = \frac{\eta}{\pi} \tag{5}$$

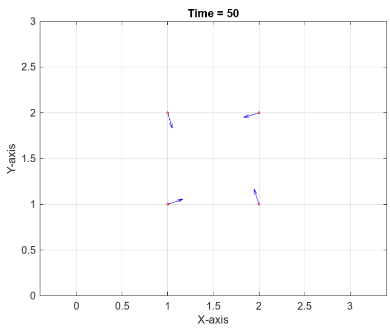

Figure 1: Stable local minima

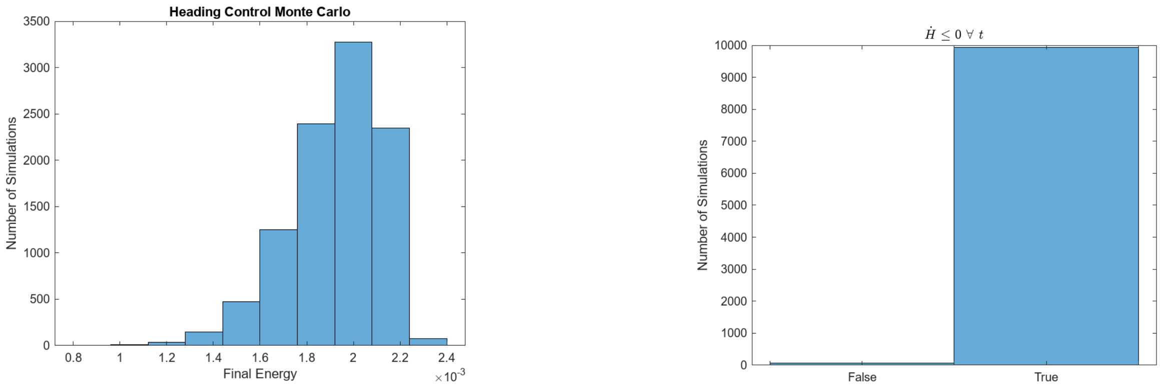

Figure 2: Monte Carlo results under target heading with complete connectivity

where η is the angle between agent i's heading and $\bar{\theta}_i^{target}$.

This formulation had greater success. In fact, under the condition of complete connectivity i.e, where each agent can sense the heading of all other agents and assumes $\theta^{target} = \sum_i \frac{p_i}{||p_i||}$, it was even observed that $\dot{H} \leq 0 \; \forall \; t$ for all 10000 monte carlo simulations as shown in figure 2. However, this is obviously undesirable whilst trying to develop a distributed algorithm.

I also attempted testing this control law in a distributed manner i.e, the agents can only sense neighbouring agents in the grid. This led to the observation of certain stable local minima not shown here, but were much fewer in number than those observed using (2).

Further Approach:

- Based on the results of figure 2, a proof for $\dot{H} \leq 0$ might be attempted for complete connectivity, using which one can easily show that no collisions will take place in Olfati's framework assuming the initial energy of the system is lesser than some E_0.
- The idea of each agent trying to achieve a target heading under complete connectivity seems rather promising, based on figure 2. To achieve this in a distributed manner, a proof of local convergence (to nearby agents) can be used to draw conclusions about global convergence, provided connectivity of the flock. This might lead to some idea about "safe" initial conditions.
- As mentioned in the disclaimer, I conjecture that these local minima are stable only because the agents do not move. Once again, assuming connectivity of the graph, one might be able to draw conclusions about global heading convergence using time-variant analysis.