Expected Ara Rewards Analysis by Lester Kim

Network Model

Let us represent the Ara network of nodes as a complete graph [1] $G = (V, E)$ where V contains N vertices with each vertex representing a node and E containing $\frac{N(N-1)}{2}$ edges with each edge representing a communication channel between two nodes. Let C be some collection of content (in our case, a set of digital entertainment files) that all the nodes want to have, and let the subset $S \subseteq V$ contain all the nodes that have C (i.e. $S = \{v \in V : C \in v\}$).

Let time $t \in \mathbb{N}$. At $t = 0$, $|S| = 1$, so there is only one $v_0 \in V$ that has content C; thus, there is only one vertex that can deliver a copy of C to other nodes in the network. Assume that all other $N - 1$ nodes want C, and v_0 has enough bandwidth to deliver C to only one node. From $t = 0$ to $t = 1$, $|S|$ increases from 1 to 2. In general, at time t,

$$|S| = \begin{cases} 2^t & 0 \leq t < \log_2 N \\ N & t \geq \log_2 N. \end{cases} \tag{1}$$

Note that $|S| = N$ starting at $t = \lceil \log_2 N \rceil$.

$\forall s \in S, s$ will deliver C to some $v \in V \setminus S$ only if v pays s an amount p. Let M be the network's total budget for entertainment delivery. Dividing this evenly by N nodes gives $p = M/N$.

At $t = 0$, the sole $v_0 \in S$ receives p from some $v_1 \in V \setminus S$. Then, at $t = 1$, $v_0, v_1 \in S$ each receives p from some $v_2, v_3 \in V \setminus S$. At any $t < \lfloor \log_2 N \rfloor$, each of the $|S| = 2^t$ nodes in S receives p from 2^t nodes in $V \setminus S$. At $t = \lfloor \log_2 N \rfloor$, $|S| > \frac{N}{2}$, so there are more suppliers than demanders of C. When that occurs, $N - 2^t$ nodes from S are randomly selected to deliver C. At $t = \lceil \log_2 N \rceil$, $S = V$.

In this model, v_0 earns at least

$$\frac{M \lfloor \log_2 N \rfloor}{N}; \tag{2}$$

v_1 earns at least $\frac{M(\lfloor \log_2 N \rfloor - 1)}{N}$; v_k earns at least

$$\frac{M(\lfloor \log_2 N \rfloor - \lceil \log_2 (k+1) \rceil)}{N}. \tag{3}$$

The greatest k such that v_k earns at least $\frac{M}{N}$ is when

$$\lfloor \log_2 N \rfloor - \lceil \log_2 (k+1) \rceil \geq 1 \tag{4}$$

which implies

$$\log_2 (k+1) \leq \log_2 \frac{N}{2}. \tag{5}$$

Thus, the maximum value of k to earn at least $\frac{M}{N}$ is $k = \lfloor \frac{N}{2} \rfloor - 1$. On average, each earns

$$\frac{M - \frac{M}{N}}{2^{\lceil \log_2 N \rceil - 1}} = \frac{M(1 - \frac{1}{N})}{2^{\lceil \log_2 N \rceil - 1}}. \tag{6}$$

The numerator is $M - \frac{M}{N}$ to exclude v_0's entertainment budget since it had C at $t = 0$. The denominator is $2^{\lceil \log_2 N \rceil - 1}$ because at $t = \lceil \log_2 N \rceil - 1$, $|S| = 2^{\lceil \log_2 N \rceil - 1}$, and at that point, S consists of all the nodes that have the possibility of earning rewards throughout this process. This means that there are $N - 2^{\lceil \log_2 N \rceil - 1}$ nodes that will not be able to earn rewards.

Example

Approximately 80% of Americans have computers with Internet access [2]. Since there are 327M Americans living in the US [3], there are (0.8)(327M) = 261.6M Americans with devices connected to the Internet. Assuming each has one device, let $N = 261.6$M. The annual US entertainment consumption is \$734B [4] [5]. Let's assume most of this expenditure for future years will be digital, but let's only include the budget of the 80% of Americans who have Internet access such that the spending among them is (0.8)(734B) = \$587B. Let 10% of the spending be for covering distribution costs. Then, $M = (0.1)(\$587\text{B}) = \$58.7\text{B}$. Then, $p = M/N = \$58.7\text{B}/261.6\text{M} = \$224.39$. By (6), the average annual earnings is \$437.35 per node. By (2), the most v_0 can earn is \$6282.87. Thus, this example illustrates that the initial peers who share content will earn the most rewards.

References

[1] Wikipedia (2018, June 19), *Complete graph*, https://en.wikipedia.org/wiki/Complete_graph

[2] C. Ryan and J. M. Lewis, “Computer and Internet Use in the United States: 2015,” *American Community Survey Reports* U.S. Census Bureau, September 2017 https://www.census.gov/content/dam/Census/library/publications/2017/acs/acs-37.pdf

[3] World Population Review (2018, June 18) *United States Population 2018* http://worldpopulationreview.com/countries/united-states-population/

[4] SelectUSA (2018, August 22), *MEDIA AND ENTERTAINMENT SPOTLIGHT*, https://www.selectusa.gov/media-entertainment-industry-united-states

[5] Bureau of Labor Statistics (2017, August 29), *CONSUMER EXPENDITURES–2016* https://www.bls.gov/news.release/cesan.nr0.htm