Linjär Algebra

Pølse

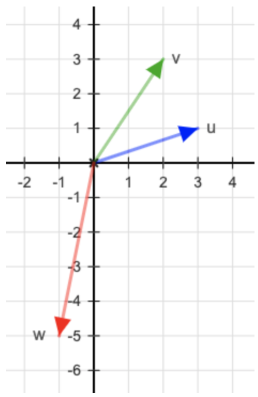
February 11, 2022

Contents

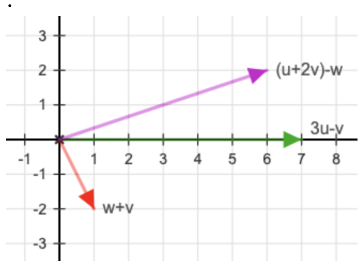
1 Geometriska vektorer

Avsnitt 1.1 och 1.2

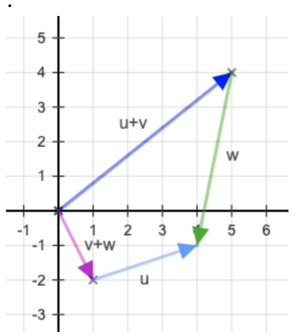
1.1



a) .



b) .



c) $u = \begin{pmatrix} 3 \\ 1 \end{pmatrix}, \quad v = \begin{pmatrix} 2 \\ 3 \end{pmatrix}, \quad w = \begin{pmatrix} -1 \\ -5 \end{pmatrix}$

$w = su + tv$

$\begin{pmatrix} -1 \\ -5 \end{pmatrix} = s \begin{pmatrix} 3 \\ 1 \end{pmatrix} + t \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

$\begin{cases} -1 = 3s + 2t \\ -5 = s + 3t \end{cases}$

$(3s+2t)-3(s+3t)=(-1)-3(-5)$
$3s+2t-3s-9t=14$
$-7t=14$
$t=-2$
$s=(-5)-(3t)=(-5)-(-6)=1$
$\begin{cases} s=1 \\ t=-2 \end{cases}$
$w=u-2v$

1.2

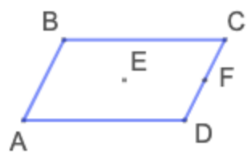
$v_t = \begin{pmatrix} 0 \\ -40 \end{pmatrix}$

a) $v_v = \begin{pmatrix} 10 \\ 0 \end{pmatrix}, \quad v_{total} = \begin{pmatrix} 10 \\ -40 \end{pmatrix}$
$||v_t|| = \sqrt{v_t \cdot v_t} = \sqrt{0^2+(-40)^2} = \sqrt{1600} = 40km/h$
$||v_{total}|| = \sqrt{v_{total} \cdot v_{total}} = \sqrt{10^2+(-40)^2} = \sqrt{100+1600} = \sqrt{1700} = 10\sqrt{17} \approx 41.23km/h$
$\cos(\theta) = \frac{v_t \cdot v_{total}}{||v_t|| * ||v_{total}||} = \frac{0*10+(-40)*(-40)}{40*10\sqrt{17}} = \frac{1600}{40*10\sqrt{17}} = \frac{4}{\sqrt{17}}$
$\theta = \cos^{-1}(\frac{4}{\sqrt{17}}) \approx 14.04°$

b) $||v_v|| = 10$
$v_v = \begin{pmatrix} \sqrt{50} \\ \sqrt{50} \end{pmatrix}, \quad v_{total} = \begin{pmatrix} \sqrt{50} \\ \sqrt{50}-40 \end{pmatrix}$
$||v_t|| = \sqrt{v_t \cdot v_t} = \sqrt{0^2+(-40)^2} = \sqrt{1600} = 40km/h$
$||v_{total}|| = \sqrt{v_{total} \cdot v_{total}} = \sqrt{\sqrt{50}^2+(\sqrt{50}-40)^2} = \sqrt{50+(50-80\sqrt{50}+1600)} = \sqrt{1700-80\sqrt{50}} = \sqrt{1700-400\sqrt{2}} = 10\sqrt{17-4\sqrt{2}} \approx 33.68km/h$
$\cos(\theta) = \frac{v_t \cdot v_{total}}{||v_t|| * ||v_{total}||} = \frac{0*\sqrt{50}+(-40)*(\sqrt{50}-40)}{400\sqrt{17-4\sqrt{2}}} = \frac{1600-40\sqrt{50}}{400\sqrt{17-4\sqrt{2}}} = \frac{1600-200\sqrt{2}}{400\sqrt{17-4\sqrt{2}}} = \frac{8-\sqrt{2}}{2\sqrt{17-4\sqrt{2}}}$
$\theta = \cos^{-1}(\frac{8-\sqrt{2}}{2\sqrt{17-4\sqrt{2}}}) \approx 12.12°$

c) $v_v = \begin{pmatrix} 10 \\ 0 \end{pmatrix}, \quad v_{total} = \begin{pmatrix} 0 \\ x \end{pmatrix}, \quad v_t = \begin{pmatrix} -10 \\ x \end{pmatrix}$
$||v_t|| = 40$
$x = \sqrt{40^2-(-10)^2} = \sqrt{1600-100} = \sqrt{1500} = 10\sqrt{15}$
$||v_{total}|| = 10\sqrt{15} \approx 38.73km/h$
$\cos(\theta) = \frac{v_t \cdot v_{total}}{||v_t|| * ||v_{total}||} = \frac{0*(-10)+(10\sqrt{15})^2}{40*10\sqrt{15}} = \frac{(10\sqrt{15})^2}{40*10\sqrt{15}} = \frac{10\sqrt{15}}{40} = \frac{\sqrt{15}}{4}$
$\theta = \cos^{-1}(\frac{\sqrt{15}}{4}) \approx 14.48°$

1.3



a) $E = \frac{1}{2}\vec{AC} = \frac{1}{2}(\vec{AB} + \vec{AD}) = \frac{1}{2}\vec{AB} + \frac{1}{2}\vec{AD}$

b) $\vec{AB} = \frac{1}{2}\vec{AC} - \frac{1}{2}\vec{BD}$
$\vec{AD} = \frac{1}{2}\vec{AC} + \frac{1}{2}\vec{BD}$

b) $\vec{AF} = \vec{AD} + \frac{1}{2}\vec{AB} = (\frac{1}{2}\vec{AC} + \frac{1}{2}\vec{BD}) + \frac{1}{2}(\frac{1}{2}\vec{AC} - \frac{1}{2}\vec{BD}) = \frac{1}{2}\vec{AC} + \frac{1}{2}\vec{BD} + \frac{1}{4}\vec{AC} - \frac{1}{4}\vec{BD} = \frac{3}{4}\vec{AC} + \frac{1}{4}\vec{BD}$

Avsnitt 1.3

1.4

$||u|| = 1, \quad ||v|| = 1, \quad \theta = \pi/3$

a) $u \cdot v = ||u|| * ||v|| * \cos(\theta) = 1 * 1 * \cos(\pi/3) = \cos(\pi/3) = \frac{1}{2}$

b) $(3u - 4v) \cdot (u + 5v) = 3u \cdot u + 3u \cdot 5v + (-4)v \cdot u + (-4)v \cdot 5v = 3(u \cdot u) + 15(u \cdot v) - 4(v \cdot u) - 20(v \cdot v) = 3 * 1 + 15 * 0.5 - 4 * 0.5 - 20 * 1 = 3 + 7.5 - 2 - 20 = \frac{6}{2} + \frac{15}{2} - \frac{4}{2} - \frac{40}{2} = \frac{-23}{2}$

c) $||3u{+}4v|| = \sqrt{(3u + 4v) \cdot (3u + 4v)} = \sqrt{3u \cdot 3u + 3u \cdot 4v + 4v \cdot 3u + 4v \cdot 4v} = \sqrt{9(u \cdot u) + 12(u \cdot v) + 12(v \cdot u) + 16(v \cdot v)} = \sqrt{9 * 1 + 12\frac{1}{2} + 12\frac{1}{2} + 16 * 1} = \sqrt{9 + 6 + 6 + 16} = \sqrt{37}$

d) $(u \cdot v)v = \frac{1}{2}v$

1.5

$||u|| = 2, \quad ||v|| = 3, \quad u \cdot v = -3$
$u \cdot v = ||u|| * ||v|| * \cos(\theta) = 2 * 3 * \cos(theta) = 6 * \cos(theta) = -3$
$\cos(\theta) = -\frac{3}{6} = \cos(\theta) = -\frac{1}{2}$
$\theta = \cos^{-1}(-\frac{1}{2}) = 120° = 2\pi/3$

Avsnitt 1.4

1.6

$||u|| = 2, \quad ||v|| = 3, \quad \theta = \pi/4$

a) $||u \times v|| = ||u|| * ||v|| * \sin(\pi/4) = 2 * 3 * \frac{\sqrt{2}}{2} = 3\sqrt{2}$

b) $Arean = ||u \times v|| = 3\sqrt{2}$

1.7

$||u|| = 1, \quad ||v|| = 1, \quad \theta = \pi/6$

a) $||u \times v|| = ||u|| * ||v|| * \sin(\theta) = 1 * 1 * \sin(\pi/6) = \sin(\pi/6) = \frac{1}{2}$

b) $||3u \times 4v|| = ||3u|| * ||4v|| * \sin(\theta) = 3 * 4 * \sin(\pi/6) = 12 \sin(\pi/6) = 12\frac{1}{2} = 6$

c) $Arean = ||u \times v|| = \frac{1}{2}$

d) $(3u - 4v) \times (u + 5v) = 3u \times u + 3u \times 5v + (-4)v \times u + (-4)v \times 5v = 3(u \times u) + 15(u \times v) - 4(v \times u) - 20(v \times v) = 3(u \times u) + 15(u \times v) + 4(u \times v) - 20(v \times v) = 3(u \times u) + 19(u \times v) - 20(v \times v) = 19(u \times v)$

1.8

(v,u,w)-vänsterorienterat
(-u,v,w)-vänsterorienterat
(v,u,-w)-högerorienterat
(-w,u,-v)-högerorienterat

Avsnitt 1.5

1.9

$P = (1, -4, -3), \quad Q = (-2 - 6, 1), \quad R = (5, 1, -1), \quad S = (2, -1, 3)$

a) $\vec{PQ} = \begin{pmatrix} (-2) - 1 \\ (-6) - (-4) \\ 1 - (-3) \end{pmatrix} = \begin{pmatrix} -3 \\ -2 \\ 4 \end{pmatrix}$

$\vec{SR} = \begin{pmatrix} 5 - 2 \\ 1 - (-1) \\ (-1) - 3 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \\ -4 \end{pmatrix}$

$\vec{QS} = \begin{pmatrix} 2 - (-2) \\ (-1) - (-6) \\ 3 - 1 \end{pmatrix} = \begin{pmatrix} 4 \\ 5 \\ 2 \end{pmatrix}$

b) $2\vec{PQ} + 3\vec{SR} = 2 \begin{pmatrix} -3 \\ -2 \\ 4 \end{pmatrix} + 3 \begin{pmatrix} 3 \\ 2 \\ -4 \end{pmatrix} = \begin{pmatrix} -6 \\ -4 \\ 8 \end{pmatrix} + \begin{pmatrix} 9 \\ 6 \\ -12 \end{pmatrix} = \begin{pmatrix} (-6) + 9 \\ (-4) + 6 \\ 8 - 12 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \\ -4 \end{pmatrix} = \vec{SR}$

$3\vec{RS} - \vec{QS} = 3 \begin{pmatrix} -3 \\ -2 \\ 4 \end{pmatrix} - \begin{pmatrix} 4 \\ 5 \\ 2 \end{pmatrix} = \begin{pmatrix} -9 \\ -6 \\ 12 \end{pmatrix} - \begin{pmatrix} 4 \\ 5 \\ 2 \end{pmatrix} = \begin{pmatrix} (-9) - 4 \\ (-6) - 5 \\ 12 - 2 \end{pmatrix} = \begin{pmatrix} -13 \\ -11 \\ 10 \end{pmatrix}$

c) $\vec{PQ} = \vec{RS}$
$\vec{QS} = \vec{PR}$

1.10

$v = \begin{pmatrix} 5 \\ -7 \end{pmatrix}$

a) $||v|| = \sqrt{v \cdot v} = \sqrt{5^2 + (-7)^2} = \sqrt{25+49} = \sqrt{74}$
   $v_{e1} = \frac{1}{\sqrt{74}} \begin{pmatrix} 5 \\ -7 \end{pmatrix}, \quad v_{e2} = \frac{1}{\sqrt{74}} \begin{pmatrix} -5 \\ 7 \end{pmatrix}, \quad \text{Nej!}$

b) $||v|| = \sqrt{v \cdot v} = \sqrt{5^2 + (-7)^2} = \sqrt{25+49} = \sqrt{74}$
   $v_{e1} = \frac{1}{\sqrt{74}} \begin{pmatrix} 7 \\ 5 \end{pmatrix}, \quad v_{e2} = \frac{1}{\sqrt{74}} \begin{pmatrix} -7 \\ -5 \end{pmatrix}, \quad \text{Nej!}$

c) $||w|| = 2, \quad ||v|| = \sqrt{v \cdot v} = \sqrt{5^2 + (-7)^2} = \sqrt{25+49} = \sqrt{74}$
   $w = \frac{2}{\sqrt{74}} \begin{pmatrix} 5 \\ -7 \end{pmatrix}$

1.11

$v = \begin{pmatrix} 1 \\ 4 \\ -3 \end{pmatrix}$

a) $||v|| = \sqrt{v \cdot v} = \sqrt{1^2 + 4^2 + (-3)^2} = \sqrt{1+16+9} = \sqrt{26}$
   $v_{e1} = \frac{1}{\sqrt{26}} \begin{pmatrix} 1 \\ 4 \\ -3 \end{pmatrix}, \quad v_{e2} = \frac{1}{\sqrt{26}} \begin{pmatrix} -1 \\ -4 \\ 3 \end{pmatrix}, \quad \text{Nej!}$

b) $w = v \times \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 4*1-(-3)*0 \\ (-3)*0-1*1 \\ 1*0-4*0 \end{pmatrix} = \begin{pmatrix} 4-0 \\ 0-1 \\ 0-0 \end{pmatrix} = \begin{pmatrix} 4 \\ -1 \\ 0 \end{pmatrix}$
   $||w|| = \sqrt{4^2 + (-1)^2} = \sqrt{16+1} = \sqrt{17}$
   $w_{e1} = \frac{1}{\sqrt{17}} \begin{pmatrix} 4 \\ -1 \\ 0 \end{pmatrix}$
   $w_{e2} = \frac{1}{\sqrt{17}} \begin{pmatrix} -4 \\ 1 \\ 0 \end{pmatrix} \quad \text{Ja!}$

c) $||v|| = \sqrt{v \cdot v} = \sqrt{1^2 + 4^2 + (-3)^2} = \sqrt{1+16+9} = \sqrt{26}$
   $w = \frac{7}{\sqrt{26}} \begin{pmatrix} 1 \\ 4 \\ -3 \end{pmatrix},$

1.12

$v = \begin{pmatrix} 1 \\ 4 \\ -3 \end{pmatrix}, \quad w = \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix}$

a) $v \cdot w = 1 * 2 + 4 * (-1) + (-3) * 0 = 2 - 4 + 0 = -2$

b) $v_L = \frac{w \cdot v}{w \cdot w} w = \frac{-2}{5} w = \frac{-2}{5} \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix}$

c) $v_S = 2v_L - v = 2\frac{w \cdot v}{w \cdot w} w - v = \frac{-4}{5} w - v = \frac{-4}{5} \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix} - \begin{pmatrix} 1 \\ 4 \\ -3 \end{pmatrix} = \begin{pmatrix} \frac{-8}{5} \\ \frac{4}{5} \\ 0 \end{pmatrix} - \begin{pmatrix} 1 \\ 4 \\ -3 \end{pmatrix} = \begin{pmatrix} \frac{-8}{5} - 1 \\ \frac{4}{5} - 4 \\ 0 + 3 \end{pmatrix} = \begin{pmatrix} \frac{-13}{5} \\ \frac{-16}{5} \\ 3 \end{pmatrix} = \begin{pmatrix} \frac{-13}{5} \\ \frac{-16}{5} \\ \frac{15}{5} \end{pmatrix} = \frac{1}{5} \begin{pmatrix} -13 \\ -16 \\ 15 \end{pmatrix}$

1.13

$v = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}, \quad w = \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix}$
$||v|| = \sqrt{1^2 + 3^2 + 2^2} = \sqrt{1 + 9 + 4} = \sqrt{14}$
$||w|| = \sqrt{3^2 + 2^2 + (-1)^2} = \sqrt{9 + 4 + 1} = \sqrt{14}$
$v \cdot w = 1 * 3 + 3 * 2 + 2 * (-1) = 3 + 6 - 2 = 7$
$\cos(\theta) = \frac{v \cdot w}{||v|| * ||w||} = \frac{7}{\sqrt{14} * \sqrt{14}} = \frac{7}{14} = \frac{1}{2}$
$\theta = \cos^{-1}(\frac{1}{2}) = 60°$

2 Matriser

3 Geometriska linjära avbildningar

4 Rummet R^n

5 Linjära ekvationssytem

6 Determinant

7 Baser

8 Egenvärden och vektorer

9 Grafer och grannmatriser