

Linjär Algebra

Pølse

February 14, 2022

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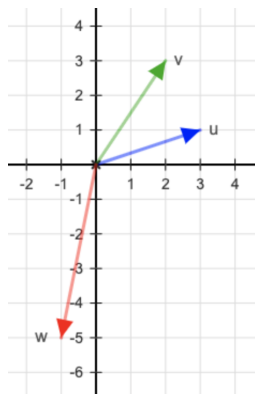
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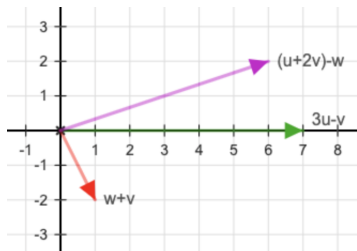
1 Geometrisk vektorer

Avsnitt 1.1 och 1.2

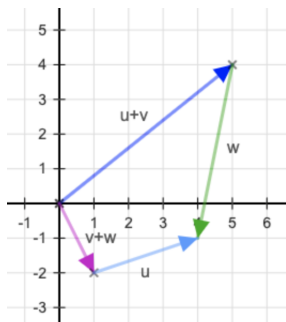
1.1



a) .



b) .



c) $u = \begin{pmatrix} 3 \\ 1 \end{pmatrix}, v = \begin{pmatrix} 2 \\ 3 \end{pmatrix}, w = \begin{pmatrix} -1 \\ -5 \end{pmatrix}$

$$w = su + tv$$

$$\begin{pmatrix} -1 \\ -5 \end{pmatrix} = s \begin{pmatrix} 3 \\ 1 \end{pmatrix} + t \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

$$\begin{cases} -1 = 3s + 2t \\ -5 = s + 3t \end{cases}$$

$$\begin{aligned}
(3s + 2t) - 3(s + 3t) &= (-1) - 3(-5) \\
3s + 2t - 3s - 9t &= 14 \\
-7t &= 14 \\
t &= -2 \\
s &= (-5) - (3t) = (-5) - (-6) = 1 \\
\begin{cases} s = 1 \\ t = -2 \end{cases} \\
w &= u - 2v
\end{aligned}$$

1.2

$$v_t = \begin{pmatrix} 0 \\ -40 \end{pmatrix}$$

$$\text{a) } v_v = \begin{pmatrix} 10 \\ 0 \end{pmatrix}, \quad v_{total} = \begin{pmatrix} 10 \\ -40 \end{pmatrix}$$

$$\|v_t\| = \sqrt{v_t \cdot v_t} = \sqrt{0^2 + (-40)^2} = \sqrt{1600} = 40 \text{ km/h}$$

$$\|v_{total}\| = \sqrt{v_{total} \cdot v_{total}} = \sqrt{10^2 + (-40)^2} = \sqrt{100 + 1600} = \sqrt{1700} = 10\sqrt{17} \approx 41.23 \text{ km/h}$$

$$\cos(\theta) = \frac{v_t \cdot v_{total}}{\|v_t\| \|v_{total}\|} = \frac{0 \cdot 10 + (-40) \cdot (-40)}{40 \cdot 10\sqrt{17}} = \frac{1600}{40 \cdot 10\sqrt{17}} = \frac{4}{\sqrt{17}}$$

$$\theta = \cos^{-1}\left(\frac{4}{\sqrt{17}}\right) \approx 14.04^\circ$$

$$\text{b) } \|v_v\| = 10$$

$$v_v = \begin{pmatrix} \sqrt{50} \\ \sqrt{50} \end{pmatrix}, \quad v_{total} = \begin{pmatrix} \sqrt{50} \\ \sqrt{50} - 40 \end{pmatrix}$$

$$\|v_t\| = \sqrt{v_t \cdot v_t} = \sqrt{0^2 + (-40)^2} = \sqrt{1600} = 40 \text{ km/h}$$

$$\|v_{total}\| = \sqrt{v_{total} \cdot v_{total}} = \sqrt{\sqrt{50}^2 + (\sqrt{50} - 40)^2} = \sqrt{50 + (50 - 80\sqrt{50} + 1600)} = \sqrt{1700 - 80\sqrt{50}} = \sqrt{1700 - 400\sqrt{2}} = 10\sqrt{17 - 4\sqrt{2}} \approx 33.68 \text{ km/h}$$

$$\cos(\theta) = \frac{v_t \cdot v_{total}}{\|v_t\| \|v_{total}\|} = \frac{0 \cdot \sqrt{50} + (-40) \cdot (\sqrt{50} - 40)}{40 \cdot 10\sqrt{17 - 4\sqrt{2}}} = \frac{1600 - 40\sqrt{50}}{400\sqrt{17 - 4\sqrt{2}}} = \frac{1600 - 200\sqrt{2}}{400\sqrt{17 - 4\sqrt{2}}} =$$

$$\frac{8 - \sqrt{2}}{2\sqrt{17 - 4\sqrt{2}}}$$

$$\theta = \cos^{-1}\left(\frac{8 - \sqrt{2}}{2\sqrt{17 - 4\sqrt{2}}}\right) \approx 12.12^\circ$$

$$\text{c) } v_v = \begin{pmatrix} 10 \\ 0 \end{pmatrix}, \quad v_{total} = \begin{pmatrix} 0 \\ x \end{pmatrix}, \quad v_t = \begin{pmatrix} -10 \\ x \end{pmatrix}$$

$$\|v_t\| = 40$$

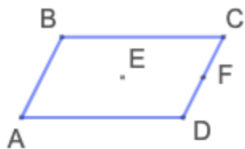
$$x = \sqrt{40^2 - (-10)^2} = \sqrt{1600 - 100} = \sqrt{1500} = 10\sqrt{15}$$

$$\|v_{total}\| = 10\sqrt{15} \approx 38.73 \text{ km/h}$$

$$\cos(\theta) = \frac{v_t \cdot v_{total}}{\|v_t\| \|v_{total}\|} = \frac{0 \cdot (-10) + (10\sqrt{15})^2}{40 \cdot 10\sqrt{15}} = \frac{(10\sqrt{15})^2}{40 \cdot 10\sqrt{15}} = \frac{10\sqrt{15}}{40} = \frac{\sqrt{15}}{4}$$

$$\theta = \cos^{-1}\left(\frac{\sqrt{15}}{4}\right) \approx 14.48^\circ$$

1.3



$$\text{a) } E = \frac{1}{2}\vec{AC} = \frac{1}{2}(\vec{AB} + \vec{AD}) = \frac{1}{2}\vec{AB} + \frac{1}{2}\vec{AD}$$

$$\begin{aligned} \text{b) } \vec{AB} &= \frac{1}{2}\vec{AC} - \frac{1}{2}\vec{BD} \\ \vec{AD} &= \frac{1}{2}\vec{AC} + \frac{1}{2}\vec{BD} \end{aligned}$$

$$\begin{aligned} \text{b) } \vec{AF} &= \vec{AD} + \frac{1}{2}\vec{AB} = \left(\frac{1}{2}\vec{AC} + \frac{1}{2}\vec{BD}\right) + \frac{1}{2}\left(\frac{1}{2}\vec{AC} - \frac{1}{2}\vec{BD}\right) = \frac{1}{2}\vec{AC} + \frac{1}{2}\vec{BD} + \\ &\quad \frac{1}{4}\vec{AC} - \frac{1}{4}\vec{BD} = \frac{3}{4}\vec{AC} + \frac{1}{4}\vec{BD} \end{aligned}$$

Avsnitt 1.3

1.4

$$||u|| = 1, \quad ||v|| = 1, \quad \theta = \pi/3$$

$$\text{a) } u \cdot v = ||u|| * ||v|| * \cos(\theta) = 1 * 1 * \cos(\pi/3) = \cos(\pi/3) = \frac{1}{2}$$

$$\begin{aligned} \text{b) } (3u - 4v) \cdot (u + 5v) &= 3u \cdot u + 3u \cdot 5v + (-4)v \cdot u + (-4)v \cdot 5v = 3(u \cdot \\ &\quad u) + 15(u \cdot v) - 4(v \cdot u) - 20(v \cdot v) = 3 * 1 + 15 * 0.5 - 4 * 0.5 - 20 * 1 = \\ &\quad 3 + 7.5 - 2 - 20 = \frac{6}{2} + \frac{15}{2} - \frac{4}{2} - \frac{40}{2} = \frac{-23}{2} \end{aligned}$$

$$\begin{aligned} \text{c) } ||3u+4v|| &= \sqrt{(3u+4v) \cdot (3u+4v)} = \sqrt{3u \cdot 3u + 3u \cdot 4v + 4v \cdot 3u + 4v \cdot 4v} = \\ &\quad \sqrt{9(u \cdot u) + 12(u \cdot v) + 12(v \cdot u) + 16(v \cdot v)} = \sqrt{9 * 1 + 12\frac{1}{2} + 12\frac{1}{2} + 16 * 1} = \\ &\quad \sqrt{9 + 6 + 6 + 16} = \sqrt{37} \end{aligned}$$

$$\text{d) } (u \cdot v)v = \frac{1}{2}v$$

1.5

$$||u|| = 2, \quad ||v|| = 3, \quad u \cdot v = -3$$

$$u \cdot v = ||u|| * ||v|| * \cos(\theta) = 2 * 3 * \cos(\theta) = 6 * \cos(\theta) = -3$$

$$\cos(\theta) = -\frac{3}{6} = \cos(\theta) = -\frac{1}{2}$$

$$\theta = \cos^{-1}\left(-\frac{1}{2}\right) = 120^\circ = 2\pi/3$$

Avsnitt 1.4

1.6

$$||u|| = 2, \quad ||v|| = 3, \quad \theta = \pi/4$$

$$\text{a) } ||u \times v|| = ||u|| * ||v|| * \sin(\pi/4) = 2 * 3 * \frac{\sqrt{2}}{2} = 3\sqrt{2}$$

$$\text{b) } \text{Arean} = ||u \times v|| = 3\sqrt{2}$$

1.7

$$\|u\| = 1, \quad \|v\| = 1, \quad \theta = \pi/6$$

$$\text{a) } \|u \times v\| = \|u\| * \|v\| * \sin(\theta) = 1 * 1 * \sin(\pi/6) = \sin(\pi/6) = \frac{1}{2}$$

$$\text{b) } \|3u \times 4v\| = \|3u\| * \|4v\| * \sin(\theta) = 3 * 4 * \sin(\pi/6) = 12 \sin(\pi/6) = 12 \frac{1}{2} = 6$$

$$\text{c) } \text{Arean} = \|u \times v\| = \frac{1}{2}$$

$$\begin{aligned} \text{d) } (3u - 4v) \times (u + 5v) &= 3u \times u + 3u \times 5v + (-4)v \times u + (-4)v \times 5v = \\ &= 3(u \times u) + 15(u \times v) - 4(v \times u) - 20(v \times v) = 3(u \times u) + 15(u \times v) + 4(u \times v) - 20(v \times v) = \\ &= 3(u \times u) + 19(u \times v) - 20(v \times v) = 19(u \times v) \end{aligned}$$

1.8

(v,u,w)-vänsterorienterat

(-u,v,w)-vänsterorienterat

(v,u,-w)-högerorienterat

(-w,u,-v)-högerorienterat

Avsnitt 1.5

1.9

$$P = (1, -4, -3), \quad Q = (-2 - 6, 1), \quad R = (5, 1, -1), \quad S = (2, -1, 3)$$

$$\text{a) } \vec{PQ} = \begin{pmatrix} (-2) - 1 \\ (-6) - (-4) \\ 1 - (-3) \end{pmatrix} = \begin{pmatrix} -3 \\ -2 \\ 4 \end{pmatrix}$$

$$\vec{SR} = \begin{pmatrix} 5 - 2 \\ 1 - (-1) \\ (-1) - 3 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \\ -4 \end{pmatrix}$$

$$\vec{QS} = \begin{pmatrix} 2 - (-2) \\ (-1) - (-6) \\ 3 - 1 \end{pmatrix} = \begin{pmatrix} 4 \\ 5 \\ 2 \end{pmatrix}$$

$$\text{b) } 2\vec{PQ} + 3\vec{SR} = 2 \begin{pmatrix} -3 \\ -2 \\ 4 \end{pmatrix} + 3 \begin{pmatrix} 3 \\ 2 \\ -4 \end{pmatrix} = \begin{pmatrix} -6 \\ -4 \\ 8 \end{pmatrix} + \begin{pmatrix} 9 \\ 6 \\ -12 \end{pmatrix} = \begin{pmatrix} (-6) + 9 \\ (-4) + 6 \\ 8 - 12 \end{pmatrix} =$$

$$\begin{pmatrix} 3 \\ 2 \\ -4 \end{pmatrix} = \vec{SR}$$

$$3\vec{RS} - \vec{QS} = 3 \begin{pmatrix} -3 \\ -2 \\ 4 \end{pmatrix} - \begin{pmatrix} 4 \\ 5 \\ 2 \end{pmatrix} = \begin{pmatrix} -9 \\ -6 \\ 12 \end{pmatrix} - \begin{pmatrix} 4 \\ 5 \\ 2 \end{pmatrix} = \begin{pmatrix} (-9) - 4 \\ (-6) - 5 \\ 12 - 2 \end{pmatrix} = \begin{pmatrix} -13 \\ -11 \\ 10 \end{pmatrix}$$

$$\begin{aligned} \text{c) } \vec{PQ} &= \vec{RS} \\ \vec{QS} &= \vec{PR} \end{aligned}$$

1.10

$$v = \begin{pmatrix} 5 \\ -7 \end{pmatrix}$$

$$\text{a) } \|v\| = \sqrt{v \cdot v} = \sqrt{5^2 + (-7)^2} = \sqrt{25 + 49} = \sqrt{74}$$

$$v_{e1} = \frac{1}{\sqrt{74}} \begin{pmatrix} 5 \\ -7 \end{pmatrix}, \quad v_{e2} = \frac{1}{\sqrt{74}} \begin{pmatrix} -5 \\ 7 \end{pmatrix}, \quad \text{Nej!}$$

$$\text{b) } \|v\| = \sqrt{v \cdot v} = \sqrt{5^2 + (-7)^2} = \sqrt{25 + 49} = \sqrt{74}$$

$$v_{e1} = \frac{1}{\sqrt{74}} \begin{pmatrix} 7 \\ 5 \end{pmatrix}, \quad v_{e2} = \frac{1}{\sqrt{74}} \begin{pmatrix} -7 \\ -5 \end{pmatrix}, \quad \text{Nej!}$$

$$\text{c) } \|w\| = 2, \quad \|v\| = \sqrt{v \cdot v} = \sqrt{5^2 + (-7)^2} = \sqrt{25 + 49} = \sqrt{74}$$

$$w = \frac{2}{\sqrt{74}} \begin{pmatrix} 5 \\ -7 \end{pmatrix}$$

1.11

$$v = \begin{pmatrix} 1 \\ 4 \\ -3 \end{pmatrix}$$

$$\text{a) } \|v\| = \sqrt{v \cdot v} = \sqrt{1^2 + 4^2 + (-3)^2} = \sqrt{1 + 16 + 9} = \sqrt{26}$$

$$v_{e1} = \frac{1}{\sqrt{26}} \begin{pmatrix} 1 \\ 4 \\ -3 \end{pmatrix}, \quad v_{e2} = \frac{1}{\sqrt{26}} \begin{pmatrix} -1 \\ -4 \\ 3 \end{pmatrix}, \quad \text{Nej!}$$

$$\text{b) } w = v \times \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 * 1 - (-3) * 0 \\ (-3) * 0 - 1 * 1 \\ 1 * 0 - 4 * 0 \end{pmatrix} = \begin{pmatrix} 4 - 0 \\ 0 - 1 \\ 0 - 0 \end{pmatrix} = \begin{pmatrix} 4 \\ -1 \\ 0 \end{pmatrix}$$

$$\|w\| = \sqrt{4^2 + (-1)^2} = \sqrt{16 + 1} = \sqrt{17}$$

$$w_{e1} = \frac{1}{\sqrt{17}} \begin{pmatrix} 4 \\ -1 \\ 0 \end{pmatrix}$$

$$w_{e2} = \frac{1}{\sqrt{17}} \begin{pmatrix} -4 \\ 1 \\ 0 \end{pmatrix} \quad \text{Ja!}$$

$$\text{c) } \|v\| = \sqrt{v \cdot v} = \sqrt{1^2 + 4^2 + (-3)^2} = \sqrt{1 + 16 + 9} = \sqrt{26}$$

$$w = \frac{7}{\sqrt{26}} \begin{pmatrix} 1 \\ 4 \\ -3 \end{pmatrix},$$

1.12

$$v = \begin{pmatrix} 1 \\ 4 \\ -3 \end{pmatrix}, \quad w = \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix}$$

$$\text{a) } v \cdot w = 1 * 2 + 4 * (-1) + (-3) * 0 = 2 - 4 + 0 = -2$$

$$\text{b) } v_L = \frac{w \cdot v}{w \cdot w} w = \frac{-2}{5} w = \frac{-2}{5} \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix}$$

$$\begin{aligned} \text{c) } v_S &= 2v_L - v = 2 \frac{w \cdot v}{w \cdot w} w - v = \frac{-4}{5} w - v = \frac{-4}{5} \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix} - \begin{pmatrix} 1 \\ 4 \\ -3 \end{pmatrix} = \begin{pmatrix} \frac{-8}{5} \\ \frac{4}{5} \\ 0 \end{pmatrix} - \\ &= \begin{pmatrix} 1 \\ 4 \\ -3 \end{pmatrix} = \begin{pmatrix} \frac{-8}{5} - 1 \\ \frac{4}{5} - 4 \\ 0 + 3 \end{pmatrix} = \begin{pmatrix} \frac{-13}{5} \\ \frac{-16}{5} \\ 3 \end{pmatrix} = \begin{pmatrix} \frac{-13}{5} \\ \frac{-16}{5} \\ \frac{15}{5} \end{pmatrix} = \frac{1}{5} \begin{pmatrix} -13 \\ -16 \\ 15 \end{pmatrix} \end{aligned}$$

1.13

$$\begin{aligned} v &= \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}, \quad w = \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix} \\ \|v\| &= \sqrt{1^2 + 3^2 + 2^2} = \sqrt{1 + 9 + 4} = \sqrt{14} \\ \|w\| &= \sqrt{3^2 + 2^2 + (-1)^2} = \sqrt{9 + 4 + 1} = \sqrt{14} \\ v \cdot w &= 1 * 3 + 3 * 2 + 2 * (-1) = 3 + 6 - 2 = 7 \\ \cos(\theta) &= \frac{v \cdot w}{\|v\| \|w\|} = \frac{7}{\sqrt{14} * \sqrt{14}} = \frac{7}{14} = \frac{1}{2} \\ \theta &= \cos^{-1}\left(\frac{1}{2}\right) = 60^\circ \end{aligned}$$

1.14

1.15

1.16

$$v = \begin{pmatrix} 1 \\ 4 \\ -3 \end{pmatrix}, \quad w = \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix}$$

$$\text{a) } v \times w = \begin{pmatrix} 4 * 0 - (-3) * (-1) \\ (-3) * 2 - 0 * 1 \\ 1 * (-1) - 4 * 2 \end{pmatrix} = \begin{pmatrix} 0 - 3 \\ (-6) - 0 \\ (-1) - 8 \end{pmatrix} = \begin{pmatrix} -3 \\ -6 \\ -9 \end{pmatrix} = -3 \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

$$\begin{aligned} \text{b) } \|v \times w\| &= 3 * \left\| \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \right\| = 3 * \sqrt{\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}} = 3\sqrt{1^2 + 2^2 + 3^2} = 3\sqrt{1 + 4 + 9} = \\ &= 3\sqrt{14} \end{aligned}$$

$$\text{c) } \left\| \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \right\| = \sqrt{14}$$

$$\frac{1}{\sqrt{14}} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

Avsnitt 1.6

1.17

$$L1: \begin{cases} x = 2 + 5t \\ y = -3 - t \\ z = 5 + 2t \end{cases} \quad L2: \begin{cases} x = -2 - 15t \\ y = 3 + 3t \\ z = -5 - 6t \end{cases} \quad L3: \begin{cases} x = 2 - 5t \\ y = -3 + t \\ z = 5 + 2t \end{cases} \quad L4: \begin{cases} x = -3 + 10t \\ y = -2 - 2t \\ z = 3 + 4t \end{cases}$$

$$v_1 = \begin{pmatrix} 5 \\ -1 \\ 2 \end{pmatrix} \quad v_2 = \begin{pmatrix} -15 \\ 3 \\ -6 \end{pmatrix} \quad v_3 = \begin{pmatrix} -5 \\ 1 \\ 2 \end{pmatrix} \quad v_4 = \begin{pmatrix} 10 \\ -2 \\ 4 \end{pmatrix}$$

$$a) \quad v_1 \times v_2 = \begin{pmatrix} (-1) * (-6) - 2 * 3 \\ 2 * (-15) - 5 * (-6) \\ 5 * 3 - (-1) * (-15) \end{pmatrix} = \begin{pmatrix} 6 - 6 \\ -30 - (-30) \\ 15 - 15 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \rightarrow \theta = 0$$

$$v_1 \times v_3 = \begin{pmatrix} (-1) * 2 - 2 * 1 \\ 2 * (-5) - 5 * (2) \\ 5 * 1 - (-1) * (-5) \end{pmatrix} = \begin{pmatrix} -2 - 2 \\ -10 - 10 \\ 5 - 5 \end{pmatrix} = \begin{pmatrix} -4 \\ -20 \\ 0 \end{pmatrix} \rightarrow \theta \neq 0$$

$$v_1 \times v_4 = \begin{pmatrix} (-1) * (4) - 2 * (-2) \\ 2 * 10 - 5 * 4 \\ 5 * (-2) - (-1) * 10 \end{pmatrix} = \begin{pmatrix} -4 - (-4) \\ 20 - 20 \\ -10 - (-10) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \rightarrow \theta = 0$$

L1, L2 och L4 är parallella

$$b) \quad L1 \text{ och } L2: \begin{cases} 2 + 5t = -2 \\ -3 - t = 3 \\ 5 + 2t = -5 \end{cases} = \begin{cases} t = \frac{-4}{5} \\ t = -6 \\ t = -5 \end{cases} \rightarrow \text{Ej samma}$$

$$L1 \text{ och } L4: \begin{cases} 2 + 5t = -3 \\ -3 - t = -2 \\ 5 + 2t = 3 \end{cases} = \begin{cases} t = -1 \\ t = -1 \\ t = -1 \end{cases} \rightarrow \text{Samma}$$

$$L2 \text{ och } L4: \begin{cases} -2 - 15t = -3 \\ 3 + 3t = -2 \\ -5 - 6t = 3 \end{cases} = \begin{cases} t = \frac{1}{15} \\ t = \frac{-5}{3} \\ t = \frac{-4}{3} \end{cases} \rightarrow \text{Ej samma}$$

L1 och L4 är lika

1.18

$$P_1 = (1, 2) \quad P_2 = (5, -1)$$

$$a) \quad v = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$$

$$Ax + By = 3x + 4y = 3 * 1 + 4 * 2 = 3 + 8 = 11$$

$$3x + 4y = 11$$

$$\text{b) L: } \begin{pmatrix} x = 1 + 3t \\ y = 2 - 3t \end{pmatrix}$$

1.19

1.20

$$\begin{aligned} \text{L: } 7x - 2y &= 6 \\ n &= \begin{pmatrix} -7 \\ 2 \end{pmatrix} \quad v = \begin{pmatrix} 2 \\ 7 \end{pmatrix} \\ \begin{pmatrix} x \\ y \end{pmatrix} &= \begin{pmatrix} 0 \\ -3 \end{pmatrix} + t * \begin{pmatrix} 2 \\ 7 \end{pmatrix} \\ \text{L1: } \begin{cases} x = 2t \\ y = -3 + 7t \end{cases} \end{aligned}$$

1.21

$$\begin{aligned} x + 3y + 4z + 7 &= 0, \quad P = (2, -2, 2) \\ x + 3y + 4z &= 2 - 6 + 8 = 4 \\ x + 3y + 4z - 4 &= 0 \end{aligned}$$

1.22

$$\begin{aligned} x + 3y + 4z + 7 &= 0 \\ n &= \begin{pmatrix} 1 \\ 3 \\ 4 \end{pmatrix} \\ P_1 &= (0, -1, -1), \quad P_2 = (-3, 0, -1), \quad P_3 = (-1, -2, 0) \\ P_1 \vec{P}_2 &= \begin{pmatrix} -3 \\ 1 \\ 0 \end{pmatrix}, \quad P_1 \vec{P}_3 = \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix} \\ \begin{cases} x = -3s - t \\ y = -1 + s - t \\ z = -1 + t \end{cases} \end{aligned}$$

1.23

$$\begin{aligned} \begin{cases} x = 2 + t \\ y = 2 + 2t \\ z = 3 + 3t \end{cases} \quad v &= \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \quad P_0 = (2, 2, 3) \\ s = P_0 \vec{O} &= \begin{pmatrix} -2 \\ -2 \\ -3 \end{pmatrix} \end{aligned}$$

$$n = v \times s = \begin{pmatrix} 0 \\ -3 \\ 2 \end{pmatrix}$$

$$n \cdot v = 0 * 1 + (-3) * 2 + 2 * 3 = 0 - 6 + 6 = 0$$

$$n \cdot s = 0 * (-2) + (-3) * (-2) + 2 * (-3) = 0 + 6 - 6 = 0$$

$$-3y + 2z = 0$$

1.24

$$P_1 = (1, 2, 3) \quad P_2 = (4, 5, 6) \quad P_3 = (0, 1, 3)$$

$$P_1 \vec{P}_2 = \begin{pmatrix} 3 \\ 3 \\ 3 \end{pmatrix} \quad P_1 \vec{P}_3 = \begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix}$$

$$P_1 \vec{P}_2 \times P_1 \vec{P}_3 = \begin{pmatrix} 3 \\ -3 \\ 0 \end{pmatrix} \quad n = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$$

$$x - y + 1 = 0$$

1.25

$$P = (1, 3, 4) \quad Q = (2, -1, 5)$$

$$\text{a) } v = \begin{pmatrix} 1 \\ -4 \\ 1 \end{pmatrix}$$

$$\text{L: } \begin{cases} x = 1 + t \\ y = 3 - 4t \\ z = 4 + t \end{cases}$$

$$\text{b) } w = \vec{PO} = \begin{pmatrix} -1 \\ -3 \\ -4 \end{pmatrix}$$

$$n = v \times w = \begin{pmatrix} 19 \\ 3 \\ -7 \end{pmatrix}$$

$$n \cdot v = 19 * 1 + 3 * (-4) + (-7) * 1 = 19 - 12 - 7 = 0$$

$$n \cdot 2 = 19 * (-1) + 3 * (-3) + (-7) * (-4) = -19 - 9 + 28 = 0$$

$$19x + 3y - 7z + D = 0$$

$$19 * 1 + 3 * 3 - 7 * 4 = 19 + 9 - 28 = 0 \rightarrow D = 0$$

$$19x + 3y - 7z = 0$$

1.26

$$P = (1, 2, 0) \quad \text{L: } \begin{cases} x = 2 + t \\ y = 2 + 2t \\ z = 3 + 3t \end{cases} \quad R = (2, 2, 3) \quad v = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

$$\begin{aligned}\|\vec{RP}\| &= \left\| \begin{pmatrix} -1 \\ 0 \\ -3 \end{pmatrix} \right\| = \sqrt{10} \\ \|\vec{RP}_L\| &= \frac{|\vec{RP} \cdot v|}{\|v\|} = \frac{|(-1)*1+0*2+(-3)*3|}{\sqrt{1^2+2^2+3^2}} = \frac{|-1-9|}{\sqrt{1+4+9}} = \frac{|-10|}{\sqrt{14}} = \frac{10}{\sqrt{14}} \\ d = \|\vec{PQ}\| &= \sqrt{\|\vec{RP}\|^2 - \|\vec{RQ}\|^2} = \sqrt{10 - \frac{100}{14}} = \sqrt{\frac{140}{14} - \frac{100}{14}} = \sqrt{\frac{40}{14}} = \\ &= \sqrt{\frac{20}{7}} = \frac{2}{7}\sqrt{35}\end{aligned}$$

1.27

$$\begin{aligned}P &= (1, 0, 3), \quad x + 2y + 3z + 4 = 0, \quad n = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \\ d &= \frac{|1+2*0+3*3+4|}{\|n\|} = \frac{1+0+9+4}{\sqrt{1^2+2^2+3^2}} = \frac{14}{\sqrt{14}} = \sqrt{14}\end{aligned}$$

1.28

1.29

$$\begin{aligned}P &= (1, 1, 1), \quad x + 2y + 3z + 4 = 0, \quad n = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \quad P_0 = (-1, 0, -1), \quad P_0 \vec{P} = \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} \\ v_o &= \frac{P_0 \vec{P} \cdot n}{n \cdot n} n = \frac{10}{14} n = \frac{5}{7} n = \begin{pmatrix} \frac{5}{7} \\ \frac{10}{7} \\ \frac{15}{7} \end{pmatrix} \\ \vec{OP} - v_o &= \begin{pmatrix} \frac{2}{7} \\ \frac{3}{7} \\ \frac{8}{7} \end{pmatrix}\end{aligned}$$

2 Matriser

Avsnitt 2.1

2.1

$$A = \begin{pmatrix} 1 & -5 \\ 4 & 3 \end{pmatrix} \quad B = \begin{pmatrix} 2 & -2 \\ 4 & 1 \end{pmatrix} \quad C = \begin{pmatrix} 1 & 0 & -2 \\ -3 & 4 & 0 \\ 7 & 2 & 5 \end{pmatrix} \quad D = \begin{pmatrix} -4 & 5 & 6 \\ -2 & 3 & 1 \\ 0 & 9 & 3 \end{pmatrix}$$

$$\begin{aligned}\text{a) } A + B &= \begin{pmatrix} 3 & -7 \\ 8 & 3 \end{pmatrix} \\ B + C &\text{ är ej definierad} \\ C + D &= \begin{pmatrix} -3 & 5 & 4 \\ -5 & 7 & 1 \\ 7 & 11 & 8 \end{pmatrix}\end{aligned}$$

$$-3C = \begin{pmatrix} -3 & 0 & 6 \\ 9 & -12 & 0 \\ -21 & -6 & -15 \end{pmatrix}$$

$$2A + B = \begin{pmatrix} 4 & -12 \\ 12 & 7 \end{pmatrix}$$

$$\text{b) } X = \frac{1}{2}(5B - 3A) = \frac{1}{2} \begin{pmatrix} 7 & 5 \\ 8 & -4 \end{pmatrix}$$

2.2

$$C = \begin{pmatrix} 1 & 0 & -2 \\ -3 & 4 & 0 \\ 7 & 2 & 5 \end{pmatrix} \quad D = \begin{pmatrix} -4 & 5 & 6 \\ -2 & 3 & 1 \\ 0 & 9 & 3 \end{pmatrix} \quad u = \begin{pmatrix} 2 \\ 4 \\ 1 \end{pmatrix} \quad v = \begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix}$$

$$\text{a) } Cu = \begin{pmatrix} 0 \\ 10 \\ 27 \end{pmatrix} \quad C(2u) = \begin{pmatrix} 0 \\ 20 \\ 54 \end{pmatrix}$$

$$\text{b) } D(u + v) = Du + Dv = \begin{pmatrix} 47 \\ 23 \\ 69 \end{pmatrix}$$

$$\text{c) } (C + D)v = Cv + Dv = \begin{pmatrix} 25 \\ 32 \\ 27 \end{pmatrix}$$

2.3

$$A = \begin{pmatrix} 1 & -5 \\ 4 & 3 \end{pmatrix} \quad B = \begin{pmatrix} 2 & -2 \\ 4 & 1 \end{pmatrix} \quad C = \begin{pmatrix} 1 & 0 & -2 \\ -3 & 4 & 0 \\ 7 & 2 & 5 \end{pmatrix} \quad D = \begin{pmatrix} -4 & 5 & 6 \\ -2 & 3 & 1 \\ 0 & 9 & 3 \end{pmatrix}$$

a)

2.4

$$A = \begin{pmatrix} 1 & -5 \\ 4 & 3 \end{pmatrix} \quad B = \begin{pmatrix} 2 & -2 \\ 4 & 1 \end{pmatrix} \quad C = \begin{pmatrix} 1 & 0 & -2 \\ -3 & 4 & 0 \\ 7 & 2 & 5 \end{pmatrix} \quad D = \begin{pmatrix} -4 & 5 & 6 \\ -2 & 3 & 1 \\ 0 & 9 & 3 \end{pmatrix}$$

a)

2.5

a)

2.6

a)

2.7

a)

3 Geometrisk linjära avbildningar

4 Rummet R^n

5 Linjära ekvationssystem

6 Determinant

7 Baser

8 Egenvärden och vektorer

9 Grafer och grannmatriser