Linjär Algebra

Pølse

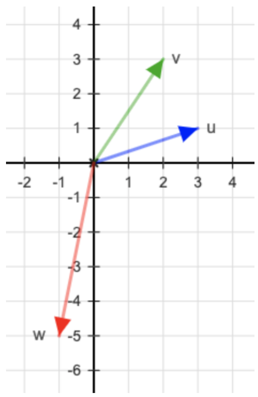
February 14, 2022

Contents

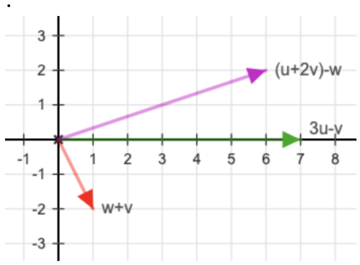
1 Geometriska vektorer

Avsnitt 1.1 och 1.2

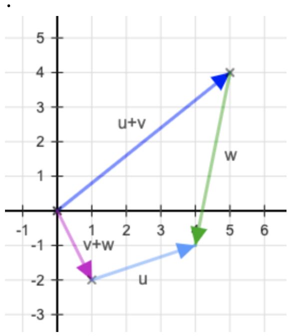
1.1



a) .



b) .



c) $u = \begin{pmatrix} 3 \\ 1 \end{pmatrix}, \quad v = \begin{pmatrix} 2 \\ 3 \end{pmatrix}, \quad w = \begin{pmatrix} -1 \\ -5 \end{pmatrix}$

$w = su + tv$

$\begin{pmatrix} -1 \\ -5 \end{pmatrix} = s \begin{pmatrix} 3 \\ 1 \end{pmatrix} + t \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

$\begin{cases} -1 = 3s + 2t \\ -5 = s + 3t \end{cases}$

$(3s + 2t) - 3(s + 3t) = (-1) - 3(-5)$
$3s + 2t - 3s - 9t = 14$
$-7t = 14$
$t = -2$
$s = (-5) - (3t) = (-5) - (-6) = 1$
$\begin{cases} s = 1 \\ t = -2 \end{cases}$
$w = u - 2v$

1.2

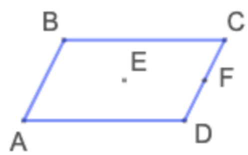
$v_t = \begin{pmatrix} 0 \\ -40 \end{pmatrix}$

a) $v_v = \begin{pmatrix} 10 \\ 0 \end{pmatrix}, \quad v_{total} = \begin{pmatrix} 10 \\ -40 \end{pmatrix}$
$||v_t|| = \sqrt{v_t \cdot v_t} = \sqrt{0^2 + (-40)^2} = \sqrt{1600} = 40km/h$
$||v_{total}|| = \sqrt{v_{total} \cdot v_{total}} = \sqrt{10^2 + (-40)^2} = \sqrt{100 + 1600} = \sqrt{1700} = 10\sqrt{17} \approx 41.23km/h$
$\cos(\theta) = \frac{v_t \cdot v_{total}}{||v_t|| * ||v_{total}||} = \frac{0*10+(-40)*(-40)}{40*10\sqrt{17}} = \frac{1600}{40*10\sqrt{17}} = \frac{4}{\sqrt{17}}$
$\theta = \cos^{-1}(\frac{4}{\sqrt{17}}) \approx 14.04°$

b) $||v_v|| = 10$
$v_v = \begin{pmatrix} \sqrt{50} \\ \sqrt{50} \end{pmatrix}, \quad v_{total} = \begin{pmatrix} \sqrt{50} \\ \sqrt{50} - 40 \end{pmatrix}$
$||v_t|| = \sqrt{v_t \cdot v_t} = \sqrt{0^2 + (-40)^2} = \sqrt{1600} = 40km/h$
$||v_{total}|| = \sqrt{v_{total} \cdot v_{total}} = \sqrt{\sqrt{50}^2 + (\sqrt{50} - 40)^2} = \sqrt{50 + (50 - 80\sqrt{50} + 1600)} = \sqrt{1700 - 80\sqrt{50}} = \sqrt{1700 - 400\sqrt{2}} = 10\sqrt{17 - 4\sqrt{2}} \approx 33.68km/h$
$\cos(\theta) = \frac{v_t \cdot v_{total}}{||v_t|| * ||v_{total}||} = \frac{0*\sqrt{50}+(-40)*(\sqrt{50}-40)}{400\sqrt{17-4\sqrt{2}}} = \frac{1600-40\sqrt{50}}{400\sqrt{17-4\sqrt{2}}} = \frac{1600-200\sqrt{2}}{400\sqrt{17-4\sqrt{2}}} = \frac{8-\sqrt{2}}{2\sqrt{17-4\sqrt{2}}}$
$\theta = \cos^{-1}(\frac{8-\sqrt{2}}{2\sqrt{17-4\sqrt{2}}}) \approx 12.12°$

c) $v_v = \begin{pmatrix} 10 \\ 0 \end{pmatrix}, \quad v_{total} = \begin{pmatrix} 0 \\ x \end{pmatrix}, \quad v_t = \begin{pmatrix} -10 \\ x \end{pmatrix}$
$||v_t|| = 40$
$x = \sqrt{40^2 - (-10)^2} = \sqrt{1600 - 100} = \sqrt{1500} = 10\sqrt{15}$
$||v_{total}|| = 10\sqrt{15} \approx 38.73km/h$
$\cos(\theta) = \frac{v_t \cdot v_{total}}{||v_t|| * ||v_{total}||} = \frac{0*(-10)+(10\sqrt{15})^2}{40*10\sqrt{15}} = \frac{(10\sqrt{15})^2}{40*10\sqrt{15}} = \frac{10\sqrt{15}}{40} = \frac{\sqrt{15}}{4}$
$\theta = \cos^{-1}(\frac{\sqrt{15}}{4}) \approx 14.48°$

1.3



a) $E = \frac{1}{2}\vec{AC} = \frac{1}{2}(\vec{AB} + \vec{AD}) = \frac{1}{2}\vec{AB} + \frac{1}{2}\vec{AD}$

b) $\vec{AB} = \frac{1}{2}\vec{AC} - \frac{1}{2}\vec{BD}$
$\vec{AD} = \frac{1}{2}\vec{AC} + \frac{1}{2}\vec{BD}$

b) $\vec{AF} = \vec{AD} + \frac{1}{2}\vec{AB} = (\frac{1}{2}\vec{AC} + \frac{1}{2}\vec{BD}) + \frac{1}{2}(\frac{1}{2}\vec{AC} - \frac{1}{2}\vec{BD}) = \frac{1}{2}\vec{AC} + \frac{1}{2}\vec{BD} + \frac{1}{4}\vec{AC} - \frac{1}{4}\vec{BD} = \frac{3}{4}\vec{AC} + \frac{1}{4}\vec{BD}$

Avsnitt 1.3

1.4

$||u|| = 1, \quad ||v|| = 1, \quad \theta = \pi/3$

a) $u \cdot v = ||u|| * ||v|| * \cos(\theta) = 1 * 1 * \cos(\pi/3) = \cos(\pi/3) = \frac{1}{2}$

b) $(3u - 4v) \cdot (u + 5v) = 3u \cdot u + 3u \cdot 5v + (-4)v \cdot u + (-4)v \cdot 5v = 3(u \cdot u) + 15(u \cdot v) - 4(v \cdot u) - 20(v \cdot v) = 3 * 1 + 15 * 0.5 - 4 * 0.5 - 20 * 1 = 3 + 7.5 - 2 - 20 = \frac{6}{2} + \frac{15}{2} - \frac{4}{2} - \frac{40}{2} = \frac{-23}{2}$

c) $||3u{+}4v|| = \sqrt{(3u + 4v) \cdot (3u + 4v)} = \sqrt{3u \cdot 3u + 3u \cdot 4v + 4v \cdot 3u + 4v \cdot 4v} = \sqrt{9(u \cdot u) + 12(u \cdot v) + 12(v \cdot u) + 16(v \cdot v)} = \sqrt{9 * 1 + 12\frac{1}{2} + 12\frac{1}{2} + 16 * 1} = \sqrt{9 + 6 + 6 + 16} = \sqrt{37}$

d) $(u \cdot v)v = \frac{1}{2}v$

1.5

$||u|| = 2, \quad ||v|| = 3, \quad u \cdot v = -3$
$u \cdot v = ||u|| * ||v|| * \cos(\theta) = 2 * 3 * \cos(theta) = 6 * \cos(theta) = -3$
$\cos(\theta) = -\frac{3}{6} = \cos(\theta) = -\frac{1}{2}$
$\theta = \cos^{-1}(-\frac{1}{2}) = 120° = 2\pi/3$

Avsnitt 1.4

1.6

$||u|| = 2, \quad ||v|| = 3, \quad \theta = \pi/4$

a) $||u \times v|| = ||u|| * ||v|| * \sin(\pi/4) = 2 * 3 * \frac{\sqrt{2}}{2} = 3\sqrt{2}$

b) $Arean = ||u \times v|| = 3\sqrt{2}$

1.7

$||u|| = 1, \quad ||v|| = 1, \quad \theta = \pi/6$

a) $||u \times v|| = ||u|| * ||v|| * \sin(\theta) = 1 * 1 * \sin(\pi/6) = \sin(\pi/6) = \frac{1}{2}$

b) $||3u \times 4v|| = ||3u|| * ||4v|| * \sin(\theta) = 3 * 4 * \sin(\pi/6) = 12 \sin(\pi/6) = 12\frac{1}{2} = 6$

c) $Arean = ||u \times v|| = \frac{1}{2}$

d) $(3u - 4v) \times (u + 5v) = 3u \times u + 3u \times 5v + (-4)v \times u + (-4)v \times 5v = 3(u \times u) + 15(u \times v) - 4(v \times u) - 20(v \times v) = 3(u \times u) + 15(u \times v) + 4(u \times v) - 20(v \times v) = 3(u \times u) + 19(u \times v) - 20(v \times v) = 19(u \times v)$

1.8

(v,u,w)-vänsterorienterat
(-u,v,w)-vänsterorienterat
(v,u,-w)-högerorienterat
(-w,u,-v)-högerorienterat

Avsnitt 1.5

1.9

$P = (1, -4, -3), \quad Q = (-2 - 6, 1), \quad R = (5, 1, -1), \quad S = (2, -1, 3)$

a) $\vec{PQ} = \begin{pmatrix} (-2) - 1 \\ (-6) - (-4) \\ 1 - (-3) \end{pmatrix} = \begin{pmatrix} -3 \\ -2 \\ 4 \end{pmatrix}$

$\vec{SR} = \begin{pmatrix} 5 - 2 \\ 1 - (-1) \\ (-1) - 3 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \\ -4 \end{pmatrix}$

$\vec{QS} = \begin{pmatrix} 2 - (-2) \\ (-1) - (-6) \\ 3 - 1 \end{pmatrix} = \begin{pmatrix} 4 \\ 5 \\ 2 \end{pmatrix}$

b) $2\vec{PQ} + 3\vec{SR} = 2\begin{pmatrix} -3 \\ -2 \\ 4 \end{pmatrix} + 3\begin{pmatrix} 3 \\ 2 \\ -4 \end{pmatrix} = \begin{pmatrix} -6 \\ -4 \\ 8 \end{pmatrix} + \begin{pmatrix} 9 \\ 6 \\ -12 \end{pmatrix} = \begin{pmatrix} (-6) + 9 \\ (-4) + 6 \\ 8 - 12 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \\ -4 \end{pmatrix} = \vec{SR}$

$3\vec{RS} - \vec{QS} = 3\begin{pmatrix} -3 \\ -2 \\ 4 \end{pmatrix} - \begin{pmatrix} 4 \\ 5 \\ 2 \end{pmatrix} = \begin{pmatrix} -9 \\ -6 \\ 12 \end{pmatrix} - \begin{pmatrix} 4 \\ 5 \\ 2 \end{pmatrix} = \begin{pmatrix} (-9) - 4 \\ (-6) - 5 \\ 12 - 2 \end{pmatrix} = \begin{pmatrix} -13 \\ -11 \\ 10 \end{pmatrix}$

c) $\vec{PQ} = \vec{RS}$
$\vec{QS} = \vec{PR}$

1.10

$v = \begin{pmatrix} 5 \\ -7 \end{pmatrix}$

a) $||v|| = \sqrt{v \cdot v} = \sqrt{5^2 + (-7)^2} = \sqrt{25+49} = \sqrt{74}$

$v_{e1} = \frac{1}{\sqrt{74}} \begin{pmatrix} 5 \\ -7 \end{pmatrix}, \quad v_{e2} = \frac{1}{\sqrt{74}} \begin{pmatrix} -5 \\ 7 \end{pmatrix}, \quad$ Nej!

b) $||v|| = \sqrt{v \cdot v} = \sqrt{5^2 + (-7)^2} = \sqrt{25+49} = \sqrt{74}$

$v_{e1} = \frac{1}{\sqrt{74}} \begin{pmatrix} 7 \\ 5 \end{pmatrix}, \quad v_{e2} = \frac{1}{\sqrt{74}} \begin{pmatrix} -7 \\ -5 \end{pmatrix}, \quad$ Nej!

c) $||w|| = 2, \quad ||v|| = \sqrt{v \cdot v} = \sqrt{5^2 + (-7)^2} = \sqrt{25+49} = \sqrt{74}$

$w = \frac{2}{\sqrt{74}} \begin{pmatrix} 5 \\ -7 \end{pmatrix}$

1.11

$v = \begin{pmatrix} 1 \\ 4 \\ -3 \end{pmatrix}$

a) $||v|| = \sqrt{v \cdot v} = \sqrt{1^2 + 4^2 + (-3)^2} = \sqrt{1+16+9} = \sqrt{26}$

$v_{e1} = \frac{1}{\sqrt{26}} \begin{pmatrix} 1 \\ 4 \\ -3 \end{pmatrix}, \quad v_{e2} = \frac{1}{\sqrt{26}} \begin{pmatrix} -1 \\ -4 \\ 3 \end{pmatrix}, \quad$ Nej!

b) $w = v \times \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 4*1-(-3)*0 \\ (-3)*0-1*1 \\ 1*0-4*0 \end{pmatrix} = \begin{pmatrix} 4-0 \\ 0-1 \\ 0-0 \end{pmatrix} = \begin{pmatrix} 4 \\ -1 \\ 0 \end{pmatrix}$

$||w|| = \sqrt{4^2 + (-1)^2} = \sqrt{16+1} = \sqrt{17}$

$w_{e1} = \frac{1}{\sqrt{17}} \begin{pmatrix} 4 \\ -1 \\ 0 \end{pmatrix}$

$w_{e2} = \frac{1}{\sqrt{17}} \begin{pmatrix} -4 \\ 1 \\ 0 \end{pmatrix} \quad$ Ja!

c) $||v|| = \sqrt{v \cdot v} = \sqrt{1^2 + 4^2 + (-3)^2} = \sqrt{1+16+9} = \sqrt{26}$

$w = \frac{7}{\sqrt{26}} \begin{pmatrix} 1 \\ 4 \\ -3 \end{pmatrix},$

1.12

$v = \begin{pmatrix} 1 \\ 4 \\ -3 \end{pmatrix}, \quad w = \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix}$

a) $v \cdot w = 1 * 2 + 4 * (-1) + (-3) * 0 = 2 - 4 + 0 = -2$

b) $v_L = \frac{w \cdot v}{w \cdot w} w = \frac{-2}{5} w = \frac{-2}{5} \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix}$

c) $v_S = 2v_L - v = 2\frac{w \cdot v}{w \cdot w} w - v = \frac{-4}{5} w - v = \frac{-4}{5} \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix} - \begin{pmatrix} 1 \\ 4 \\ -3 \end{pmatrix} = \begin{pmatrix} \frac{-8}{5} \\ \frac{4}{5} \\ 0 \end{pmatrix} - \begin{pmatrix} 1 \\ 4 \\ -3 \end{pmatrix} = \begin{pmatrix} \frac{-8}{5} - 1 \\ \frac{4}{5} - 4 \\ 0 + 3 \end{pmatrix} = \begin{pmatrix} \frac{-13}{5} \\ \frac{-16}{5} \\ 3 \end{pmatrix} = \begin{pmatrix} \frac{-13}{5} \\ \frac{-16}{5} \\ \frac{15}{5} \end{pmatrix} = \frac{1}{5} \begin{pmatrix} -13 \\ -16 \\ 15 \end{pmatrix}$

1.13

$v = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}, \quad w = \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix}$
$||v|| = \sqrt{1^2 + 3^2 + 2^2} = \sqrt{1 + 9 + 4} = \sqrt{14}$
$||w|| = \sqrt{3^2 + 2^2 + (-1)^2} = \sqrt{9 + 4 + 1} = \sqrt{14}$
$v \cdot w = 1 * 3 + 3 * 2 + 2 * (-1) = 3 + 6 - 2 = 7$
$\cos(\theta) = \frac{v \cdot w}{||v|| * ||w||} = \frac{7}{\sqrt{14} * \sqrt{14}} = \frac{7}{14} = \frac{1}{2}$
$\theta = \cos^{-1}(\frac{1}{2}) = 60°$

1.14

1.15

1.16

$v = \begin{pmatrix} 1 \\ 4 \\ -3 \end{pmatrix}, \quad w = \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix}$

a) $v \times w = \begin{pmatrix} 4 * 0 - (-3) * (-1) \\ (-3) * 2 - 0 * 1 \\ 1 * (-1) - 4 * 2 \end{pmatrix} = \begin{pmatrix} 0 - 3 \\ (-6) - 0 \\ (-1) - 8 \end{pmatrix} = \begin{pmatrix} -3 \\ -6 \\ -9 \end{pmatrix} = -3 \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$

b) $||v \times w|| = 3 * || \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} || = 3 * \sqrt{\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}} = 3\sqrt{1^2 + 2^2 + 3^2} = 3\sqrt{1 + 4 + 9} = 3\sqrt{14}$

c) $|| \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} || = \sqrt{14}$

$\frac{1}{\sqrt{14}}\begin{pmatrix}1\\2\\3\end{pmatrix}$

Avsnitt 1.6

1.17

$L1:\begin{cases}x=2+5t\\y=-3-t\\z=5+2t\end{cases}\quad L2:\begin{cases}x=-2-15t\\y=3+3t\\z=-5-6t\end{cases}\quad L3:\begin{cases}x=2-5t\\y=-3+t\\z=5+2t\end{cases}\quad L4:\begin{cases}x=-3+10t\\y=-2-2t\\z=3+4t\end{cases}$

$v_1=\begin{pmatrix}5\\-1\\2\end{pmatrix}\quad v_2=\begin{pmatrix}-15\\3\\-6\end{pmatrix}\quad v_3=\begin{pmatrix}-5\\1\\2\end{pmatrix}\quad v_4=\begin{pmatrix}10\\-2\\4\end{pmatrix}$

a) $v_1\times v_2=\begin{pmatrix}(-1)*(-6)-2*3\\2*(-15)-5*(-6)\\5*3-(-1)*(-15)\end{pmatrix}=\begin{pmatrix}6-6\\-30-(-30)\\15-15\end{pmatrix}=\begin{pmatrix}0\\0\\0\end{pmatrix}\rightarrow\theta=0$

$v_1\times v_3=\begin{pmatrix}(-1)*2-2*1\\2*(-5)-5*(2)\\5*1-(-1)*(-5)\end{pmatrix}=\begin{pmatrix}-2-2\\-10-10\\5-5\end{pmatrix}=\begin{pmatrix}-4\\-20\\0\end{pmatrix}\rightarrow\theta\neq 0$

$v_1\times v_4=\begin{pmatrix}(-1)*(4)-2*(-2)\\2*10-5*4\\5*(-2)-(-1)*10\end{pmatrix}=\begin{pmatrix}-4-(-4)\\20-20\\-10-(-10)\end{pmatrix}=\begin{pmatrix}0\\0\\0\end{pmatrix}\rightarrow\theta=0$

L1, L2 och L4 är parallella

b) L1 och L2: $\begin{cases}2+5t=-2\\-3-t=3\\5+2t=-5\end{cases}=\begin{cases}t=\frac{-4}{5}\\t=-6\\t=-5\end{cases}\rightarrow$ Ej samma

L1 och L4: $\begin{cases}2+5t=-3\\-3-t=-2\\5+2t=3\end{cases}=\begin{cases}t=-1\\t=-1\\t=-1\end{cases}\rightarrow$ Samma

L2 och L4: $\begin{cases}-2-15t=-3\\3+3t=-2\\-5-6t=3\end{cases}=\begin{cases}t=\frac{1}{15}\\t=\frac{-5}{3}\\t=\frac{-4}{3}\end{cases}\rightarrow$ Ej samma

L1 och L4 är lika

1.18

$P_1=(1,2)\ \ P_2=(5,-1)$

a) $v=\begin{pmatrix}4\\-3\end{pmatrix}$

$Ax+By=3x+4y=3*1+4*2=3+8=11$

$3x+4y=11$

b) L: $\begin{pmatrix} x = 1 + 3t \\ y = 2 - 3t \end{pmatrix}$

1.19

1.20

L: $7x - 2y = 6$
$n = \begin{pmatrix} -7 \\ 2 \end{pmatrix} \quad v = \begin{pmatrix} 2 \\ 7 \end{pmatrix}$
$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ -3 \end{pmatrix} + t * \begin{pmatrix} 2 \\ 7 \end{pmatrix}$
L1: $\begin{cases} x = 2t \\ y = -3 + 7t \end{cases}$

1.21

$x + 3y + 4x + 7 = 0, \quad P = (2, -2, 2)$
$x + 3y + 4z = 2 - 6 + 8 = 4$
$x + 3y + 4z - 4 = 0$

1.22

$x + 3y + 4z + 7 = 0$
$n = \begin{pmatrix} 1 \\ 3 \\ 4 \end{pmatrix}$
$P_1 = (0, -1, -1), \quad P_2 = (-3, 0, -1), \quad P_3 = (-1, -2, 0)$
$P_1\vec{P}_2 = \begin{pmatrix} -3 \\ 1 \\ 0 \end{pmatrix}, \quad P_1\vec{P}_3 = \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix}$
$\begin{cases} x = -3s - t \\ y = -1 + s - t \\ z = -1 + t \end{cases}$

1.23

$\begin{cases} x = 2 + t \\ y = 2 + 2t \\ z = 3 + 3t \end{cases} \quad v = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \quad P_0 = (2, 2, 3)$
$s = P_0\vec{O} = \begin{pmatrix} -2 \\ -2 \\ -3 \end{pmatrix}$

$n = v \times s = \begin{pmatrix} 0 \\ -3 \\ 2 \end{pmatrix}$
$n \cdot v = 0 * 1 + (-3) * 2 + 2 * 3 = 0 - 6 + 6 = 0$
$n \cdot s = 0 * (-2) + (-3) * (-2) + 2 * (-3) = 0 + 6 - 6 = 0$
$-3y + 2z = 0$

1.24

$P_1 = (1, 2, 3) \quad P_2 = (4, 5, 6) \quad P_3 = (0, 1, 3)$
$P_1\vec{P}_2 = \begin{pmatrix} 3 \\ 3 \\ 3 \end{pmatrix} \quad P_1\vec{P}_3 = \begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix}$
$P_1\vec{P}_2 \times P_1\vec{P}_3 = \begin{pmatrix} 3 \\ -3 \\ 0 \end{pmatrix} \quad n = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$
$x - y + 1 = 0$

1.25

$P = (1, 3, 4) \quad Q = (2, -1, 5)$

a) $v = \begin{pmatrix} 1 \\ -4 \\ 1 \end{pmatrix}$

L: $\begin{cases} x = 1 + t \\ y = 3 - 4t \\ z = 4 + t \end{cases}$

b) $w = \vec{PO} = \begin{pmatrix} -1 \\ -3 \\ -4 \end{pmatrix}$

$n = v \times w = \begin{pmatrix} 19 \\ 3 \\ -7 \end{pmatrix}$
$n \cdot v = 19 * 1 + 3 * (-4) + (-7) * 1 = 19 - 12 - 7 = 0$
$n \cdot 2 = 19 * (-1) + 3 * (-3) + (-7) * (-4) = -19 - 9 + 28 = 0$
$19x + 3y - 7z + D = 0$
$19 * 1 + 3 * 3 - 7 * 4 = 19 + 9 - 28 = 0 \rightarrow D = 0$
$19x + 3y - 7z = 0$

1.26

$P = (1, 2, 0)$ L: $\begin{cases} x = 2 + t \\ y = 2 + 2t \\ z = 3 + 3t \end{cases} \quad R = (2, 2, 3) \quad v = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$

$||\vec{RP}|| = || \begin{pmatrix} -1 \\ 0 \\ -3 \end{pmatrix} || = \sqrt{10}$

$||\vec{RP}_L|| = \frac{|\vec{RP} \cdot v|}{||v||} = \frac{|(-1)*1+0*2+(-3)*3|}{\sqrt{1^2+2^2+3^2}} = \frac{|-1-9|}{\sqrt{1+4+9}} = \frac{|-10|}{\sqrt{14}} = \frac{10}{\sqrt{14}}$

$d = ||\vec{PQ}|| = \sqrt{||\vec{RP}||^2 - ||\vec{RQ}||^2} = \sqrt{10 - \frac{100}{14}} = \sqrt{\frac{140}{14} - \frac{100}{14}} = \sqrt{\frac{40}{14}} = \sqrt{\frac{20}{7}} = \frac{2}{7}\sqrt{35}$

1.27

$P = (1, 0, 3), \quad x + 2y + 3x + 4 = 0, \quad n = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$

$d = \frac{|1+2*0+3*3+4|}{||n||} = \frac{1+0+9+4}{\sqrt{1^2+2^2+3^2}} = \frac{14}{\sqrt{14}} = \sqrt{14}$

1.28

1.29

$P = (1, 1, 1), \quad x+2y+3z+4 = 0, \quad n = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \quad P_0 = (-1, 0, -1), \quad \vec{P_0P} = \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$

$v_o = \frac{\vec{P_0P} \cdot n}{n \cdot n} n = \frac{10}{14} n = \frac{5}{7} n = \begin{pmatrix} \frac{5}{7} \\ \frac{10}{7} \\ \frac{15}{7} \end{pmatrix}$

$\vec{OP} - v_o = \begin{pmatrix} \frac{2}{7} \\ \frac{-3}{7} \\ \frac{-8}{7} \end{pmatrix}$

2 Matriser

Avsnitt 2.1

2.1

$A = \begin{pmatrix} 1 & -5 \\ 4 & 3 \end{pmatrix} \quad B = \begin{pmatrix} 2 & -2 \\ 4 & 1 \end{pmatrix} \quad C = \begin{pmatrix} 1 & 0 & -2 \\ -3 & 4 & 0 \\ 7 & 2 & 5 \end{pmatrix} \quad D = \begin{pmatrix} -4 & 5 & 6 \\ -2 & 3 & 1 \\ 0 & 9 & 3 \end{pmatrix}$

a) $A + B = \begin{pmatrix} 3 & -7 \\ 8 & 3 \end{pmatrix}$
B+C är ej definierad
$C + D = \begin{pmatrix} -3 & 5 & 4 \\ -5 & 7 & 1 \\ 7 & 11 & 8 \end{pmatrix}$

$$-3C = \begin{pmatrix} -3 & 0 & 6 \\ 9 & -12 & 0 \\ -21 & -6 & -15 \end{pmatrix}$$
$$2A + B = \begin{pmatrix} 4 & -12 \\ 12 & 7 \end{pmatrix}$$

b) $X = \frac{1}{2}(5B - 3A) = \frac{1}{2}\begin{pmatrix} 7 & 5 \\ 8 & -4 \end{pmatrix}$

2.2

$$C = \begin{pmatrix} 1 & 0 & -2 \\ -3 & 4 & 0 \\ 7 & 2 & 5 \end{pmatrix} \quad D = \begin{pmatrix} -4 & 5 & 6 \\ -2 & 3 & 1 \\ 0 & 9 & 3 \end{pmatrix} \quad u = \begin{pmatrix} 2 \\ 4 \\ 1 \end{pmatrix} \quad v = \begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix}$$

a) $Cu = \begin{pmatrix} 0 \\ 10 \\ 27 \end{pmatrix} \quad C(2u) = \begin{pmatrix} 0 \\ 20 \\ 54 \end{pmatrix}$

b) $D(u + v) = Du + Dv = \begin{pmatrix} 47 \\ 23 \\ 69 \end{pmatrix}$

c) $(C + D)v = Cv + Dv = \begin{pmatrix} 25 \\ 32 \\ 27 \end{pmatrix}$

2.3

$$A = \begin{pmatrix} 1 & -5 \\ 4 & 3 \end{pmatrix} \quad B = \begin{pmatrix} 2 & -2 \\ 4 & 1 \end{pmatrix} \quad C = \begin{pmatrix} 1 & 0 & -2 \\ -3 & 4 & 0 \\ 7 & 2 & 5 \end{pmatrix} \quad D = \begin{pmatrix} -4 & 5 & 6 \\ -2 & 3 & 1 \\ 0 & 9 & 3 \end{pmatrix}$$

a) $AB = \begin{pmatrix} -18 & -7 \\ 20 & -5 \end{pmatrix} \quad BA = \begin{pmatrix} -6 & -16 \\ 8 & -17 \end{pmatrix} \quad A^2 = \begin{pmatrix} -19 & -20 \\ 16 & -11 \end{pmatrix}$ BC är ej definierad $CD = \begin{pmatrix} -4 & -13 & 0 \\ 4 & -3 & -14 \\ -32 & 86 & 59 \end{pmatrix} \quad DC = \begin{pmatrix} 23 & 32 & 38 \\ -4 & 14 & 9 \\ -6 & 42 & 15 \end{pmatrix}$

b) $A(A + B) = A^2 + AB = \begin{pmatrix} -37 & -27 \\ 36 & -16 \end{pmatrix}$

c) $A(BA) = (AB)A = \begin{pmatrix} -46 & 69 \\ 0 & -115 \end{pmatrix}$

2.4

$$A = \begin{pmatrix} 1 & -5 \\ 4 & 3 \end{pmatrix} \quad B = \begin{pmatrix} 2 & -2 \\ 4 & 1 \end{pmatrix} \quad C = \begin{pmatrix} 1 & 0 & -2 \\ -3 & 4 & 0 \\ 7 & 2 & 5 \end{pmatrix} \quad D = \begin{pmatrix} -4 & 5 & 6 \\ -2 & 3 & 1 \\ 0 & 9 & 3 \end{pmatrix}$$

a) $A^T = \begin{pmatrix} 1 & 4 \\ -5 & 3 \end{pmatrix}$ $B^T = \begin{pmatrix} 2 & 4 \\ -2 & 1 \end{pmatrix}$ $C^T = \begin{pmatrix} 1 & -3 & 7 \\ 0 & 4 & 2 \\ -2 & 0 & 5 \end{pmatrix}$ $D^T = \begin{pmatrix} -4 & -2 & 0 \\ 5 & 3 & 9 \\ 6 & 1 & 3 \end{pmatrix}$

b) Ingen

c) $(AB)^T = \begin{pmatrix} -18 & 20 \\ -7 & -5 \end{pmatrix}$ $A^T B^T = \begin{pmatrix} -6 & 8 \\ -16 & -17 \end{pmatrix} = (BA)^T$ $B^T A^T =$
$(AB)^T = \begin{pmatrix} -18 & 20 \\ -7 & -5 \end{pmatrix}$

Avsnitt 2.2

2.5

$A = \begin{pmatrix} a1 & a2 \end{pmatrix} = \begin{pmatrix} 1 & 4 \\ 3 & -5 \end{pmatrix}$ $B = \begin{pmatrix} 2 & -2 \\ 4 & 1 \end{pmatrix}$ $C = \begin{pmatrix} c1 & c2 & c3 \end{pmatrix} = \begin{pmatrix} -4 & 5 & 6 \\ -2 & 3 & 1 \\ 0 & 9 & 3 \end{pmatrix}$

a) $det(A) = -17$
$det(B) = 10$
$det(C) = 0 - (-30) + (-18) = 12$
$det(C^T) = det(\begin{pmatrix} -4 & -2 & 0 \\ 5 & 3 & 9 \\ 6 & 1 & 3 \end{pmatrix}) = 0 - (78) + 0 = -78$

b)

c)

d)

e)

f)

2.6

a)

2.7

a)

3 Geometriska linjära avbildningar

4 Rummet R^n

5 Linjära ekvationssytem

6 Determinant

7 Baser

8 Egenvärden och vektorer

9 Grafer och grannmatriser