Linjär Algebra

Pølse

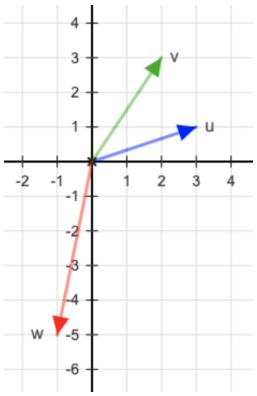
February 9, 2022

Contents

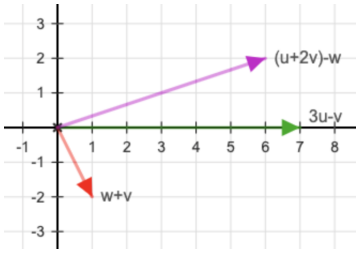
1 Geometriska vektorer

Avsnitt 1.1 och 1.2

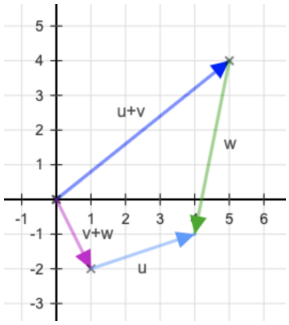
1.1



a) .



b) .



c) $u = \begin{pmatrix} 3 \\ 1 \end{pmatrix}, v = \begin{pmatrix} 2 \\ 3 \end{pmatrix}, w = \begin{pmatrix} -1 \\ -5 \end{pmatrix}$

$w = su + tv$

$\begin{pmatrix} -1 \\ -5 \end{pmatrix} = s \begin{pmatrix} 3 \\ 1 \end{pmatrix} + t \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

$\begin{cases} -1 = 3s + 2t \\ -5 = s + 3t \end{cases}$
$(3s + 2t) - 3(s + 3t) = (-1) - 3(-5)$
$3s + 2t - 3s - 9t = 14$
$-7t = 14$
$t = -2$
$s = (-5) - (3t) = (-5) - (-6) = 1$
$\begin{cases} s = 1 \\ t = -2 \end{cases}$
$w = u - 2v$

1.2

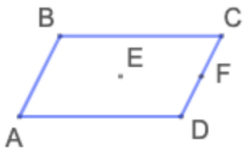
$v_{\text{tärna}} = \begin{pmatrix} 0 \\ -40 \end{pmatrix}$

a) $v_{\text{vind}} = \begin{pmatrix} 10 \\ 0 \end{pmatrix}, v_{\text{total}} = \begin{pmatrix} 10 \\ -40 \end{pmatrix}$
$||v_{\text{tärna}}|| = \sqrt{v_{\text{tärna}} \cdot v_{\text{tärna}}} = \sqrt{0^2 + (-40)^2} = \sqrt{1600} = 40km/h$
$||v_{\text{total}}|| = \sqrt{v_{\text{total}} \cdot v_{\text{total}}} = \sqrt{10^2 + (-40)^2} = \sqrt{100 + 1600} = \sqrt{1700} = 10\sqrt{17} \approx 41.23km/h$
$\cos(\theta) = \frac{v_{\text{tärna}} \cdot v_{\text{total}}}{||v_{\text{tärna}}|| * ||v_{\text{total}}||} = \frac{0*10+(-40)*(-40)}{40*10\sqrt{17}} = \frac{1600}{40*10\sqrt{17}} = \frac{4}{\sqrt{17}}$
$\theta = \cos^{-1}(\frac{4}{\sqrt{17}}) \approx 14.04°$

b) $||v_{\text{vind}}|| = 10$
$v_{\text{vind}} = \begin{pmatrix} \sqrt{50} \\ \sqrt{50} \end{pmatrix}, v_{\text{total}} = \begin{pmatrix} \sqrt{50} \\ \sqrt{50} - 40 \end{pmatrix}$
$||v_{\text{tärna}}|| = \sqrt{v_{\text{tärna}} \cdot v_{\text{tärna}}} = \sqrt{0^2 + (-40)^2} = \sqrt{1600} = 40km/h$
$||v_{\text{total}}|| = \sqrt{v_{\text{total}} \cdot v_{\text{total}}} = \sqrt{\sqrt{50}^2 + (\sqrt{50} - 40)^2} = \sqrt{50 + (50 - 80\sqrt{50} + 1600)} = \sqrt{1700 - 80\sqrt{50}} = \sqrt{1700 - 400\sqrt{2}} = 10\sqrt{17 - 4\sqrt{2}} \approx 33.68km/h$
$\cos(\theta) = \frac{v_{\text{tärna}} \cdot v_{\text{total}}}{||v_{\text{tärna}}|| * ||v_{\text{total}}||} = \frac{0*\sqrt{50}+(-40)*(\sqrt{50}-40)}{400\sqrt{17-4\sqrt{2}}} = \frac{1600-40\sqrt{50}}{400\sqrt{17-4\sqrt{2}}} = \frac{1600-200\sqrt{2}}{400\sqrt{17-4\sqrt{2}}} = \frac{8-\sqrt{2}}{2\sqrt{17-4\sqrt{2}}}$
$\theta = \cos^{-1}(\frac{8-\sqrt{2}}{2\sqrt{17-4\sqrt{2}}}) \approx 12.12°$

c) $v_{\text{vind}} = \begin{pmatrix} 10 \\ 0 \end{pmatrix}, v_{\text{total}} = \begin{pmatrix} 0 \\ x \end{pmatrix}, v_{\text{tärna}} = \begin{pmatrix} -10 \\ x \end{pmatrix}$
$||v_{\text{tärna}}|| = 40$
$x = \sqrt{40^2 - (-10)^2} = \sqrt{1600 - 100} = \sqrt{1500} = 10\sqrt{15}$
$||v_{\text{total}}|| = 10\sqrt{15} \approx 38.73km/h$
$\cos(\theta) = \frac{v_{\text{tärna}} \cdot v_{\text{total}}}{||v_{\text{tärna}}|| * ||v_{\text{total}}||} = \frac{0*(-10)+(10\sqrt{15})^2}{40*10\sqrt{15}} = \frac{(10\sqrt{15})^2}{40*10\sqrt{15}} = \frac{10\sqrt{15}}{40} = \frac{\sqrt{15}}{4}$
$\theta = \cos^{-1}(\frac{\sqrt{15}}{4}) \approx 14.48°$

1.3



a) $E = \frac{1}{2}\vec{AC} = \frac{1}{2}(\vec{AB} + \vec{AD}) = \frac{1}{2}\vec{AB} + \frac{1}{2}\vec{AD}$

b) $\vec{AB} = \frac{1}{2}\vec{AC} - \frac{1}{2}\vec{BD}$
$\vec{AD} = \frac{1}{2}\vec{AC} + \frac{1}{2}\vec{BD}$

b) $\vec{AF} = \vec{AD} + \frac{1}{2}\vec{AB} = (\frac{1}{2}\vec{AC} + \frac{1}{2}\vec{BD}) + \frac{1}{2}(\frac{1}{2}\vec{AC} - \frac{1}{2}\vec{BD}) = \frac{1}{2}\vec{AC} + \frac{1}{2}\vec{BD} + \frac{1}{4}\vec{AC} - \frac{1}{4}\vec{BD} = \frac{3}{4}\vec{AC} + \frac{1}{4}\vec{BD}$

Avsnitt 1.3

1.4

$||u|| = 1, ||v|| = 1, \theta = \pi/3$

a) $u \cdot v = ||u|| * ||v|| * \cos(\theta) = 1 * 1 * \cos(\pi/3) = \cos(\pi/3) = \frac{1}{2}$

b) $(3u - 4v) \cdot (u + 5v) = 3u \cdot u + 3u \cdot 5v + (-4)v \cdot u + (-4)v \cdot 5v = 3(u \cdot u) + 15(u \cdot v) - 4(v \cdot u) - 20(v \cdot v) = 3 * 1 + 15 * 0.5 - 4 * 0.5 - 20 * 1 = 3 + 7.5 - 2 - 20 = \frac{6}{2} + \frac{15}{2} - \frac{4}{2} - \frac{40}{2} = \frac{-23}{2}$

c) $||3u + 4v||$

2 Matriser

3 Geometriska linjära avbildningar

4 Rummet R^n

5 Linjära ekvationssytem

6 Determinant

7 Baser

8 Egenvärden och vektorer

9 Grafer och grannmatriser