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NOTIFICATIONS

Introduction to Trigonometry

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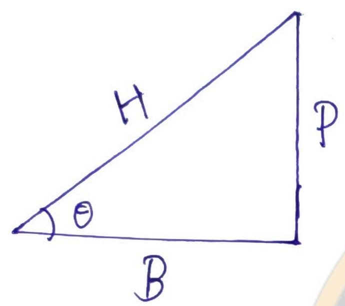


$$\sin\theta = \frac{P}{H} \qquad \cos\theta = \frac{B}{H} \qquad \tan\theta = \frac{P}{B}$$

> Complementary Angles

$$\sin(90^\circ - \theta) = \cos\theta$$

$$\tan(90^\circ - \theta) = \cot\theta$$

$$\sec(90^\circ - \theta) = \csc\theta$$

$$\frac{\text{Sin}}{\text{Cosec}} \qquad \frac{\text{Cos}}{\text{Sec}} \qquad \frac{\text{Tan}}{\text{Cot}}$$

> Trigonometric Identities

$$\sin^2\theta + \cos^2\theta = 1$$

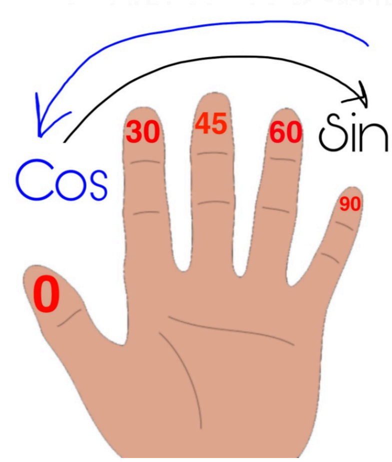
$$\sec^2\theta - \tan^2\theta = 1$$

$$\text{cosec}^2\theta - \cot^2\theta = 1$$

> Trigonometry Table

θ	$0°$	$30°$	$45°$	$60°$	$90°$
$\sin\theta$	0	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0
$\tan\theta$	0	$1/\sqrt{3}$	1	$\sqrt{3}$	∞
$\cot\theta$	∞	$\sqrt{3}$	1	$1/\sqrt{3}$	0
$\sec\theta$	1	$2/\sqrt{3}$	$\sqrt{2}$	2	∞
$\text{cosec}\,\theta$	∞	2	$\sqrt{2}$	$2/\sqrt{3}$	1

* The value of sine and cosine of any angle lies between – 1 to 1



Chapter-8 Introduction to Trigonometry

Very Short Question

Q. 2. If A and B are acute angles and $\operatorname{cosec} A = \sec B$, then find the value of $A+B$. **[CBSE 2015]**

Sol.
$$\operatorname{cosec} A = \sec B$$
$$\operatorname{cosec} A = \operatorname{cosec}(90° - B) \qquad [\because \operatorname{cosec}(90° - \theta) = \sec\theta]$$
$$A = 90° - B$$
$$\Rightarrow \quad A + B = 90°$$

Hence, the value of $A + B = 90°$

Q. 3. Find the value of $\dfrac{\sin 25°}{\cos 65°} + \dfrac{\tan 23°}{\cot 67°}$. **[CBSE 2015]**

Sol. Given, $\dfrac{\sin 25°}{\cos 65°} + \dfrac{\tan 23°}{\cot 67°} = \dfrac{\sin 25°}{\cos(90° - 25°)} + \dfrac{\tan 23°}{\cot(90° - 23°)} = \dfrac{\sin 25°}{\sin 25} + \dfrac{\tan 23°}{\tan 23°}$

$[\because \cos(90° - \theta) = \sin\theta$ and $=$ $\cot(90° - \theta) = \tan\theta]$

$= 1 + 1 = 2$

Q. 9. Find the value of $\cos 1° \cdot \cos 2° \cdot \cos 3° \ldots \cos 180°$. **[CBSE 2015, 17]**

Sol. Given, $\cos 1° \cdot \cos 2° \cdot \cos 3° \ldots \cos 180°$
$$= \cos 1° \cdot \cos 2° \ldots \cos 90° \ldots \cos 180°$$
$$= \cos 1° \cdot \cos 2° \ldots 0 \ldots \cos 180° = 0$$

Q.10. Evaluate: $\dfrac{1}{\sqrt{3}} \sec 60° - \operatorname{cosec} 60°$. **[CBSE 2015]**

Sol. Given, $\dfrac{1}{\sqrt{3}} \sec 60° - \operatorname{cosec} 60°$
$$= \frac{1}{\sqrt{3}} \times 2 - \frac{2}{\sqrt{3}} = 0$$

Q.12. If $\cot A = \dfrac{12}{5}$, then find the value of $(\sin A + \cos A) \operatorname{cosec} A$. **[CBSE 2014]**

Sol. Here, $(\sin A + \cos A) \times \operatorname{cosec} A$
$$= (\sin A + \cos A) \times \frac{1}{\sin A} = 1 + \frac{\cos A}{\sin A}$$
$$= 1 + \cot A \qquad \left[\because \cot A = \frac{\cos A}{\sin A}\right]$$
$$= 1 + \frac{12}{5} = \frac{17}{5}$$

Q.14. If $\tan \dfrac{5\theta}{2} = \sqrt{3}$ and θ is acute, then find the value of 2θ. **[CBSE 2014]**

Sol. Given, $\tan \dfrac{5\theta}{2} = \sqrt{3}$
$$\Rightarrow \quad \tan \frac{5\theta}{2} = \tan 60° \qquad [\because \tan 60° = \sqrt{3}]$$
$$\Rightarrow \quad \frac{5\theta}{2} = 60°$$
$$\Rightarrow \quad 5\theta = 120°$$
$$\Rightarrow \quad \theta = \frac{120°}{5} = 24°$$
$$\therefore \quad 2\theta = 2 \times 24° = 48°$$

Q.17. Find A if $\tan 2A = \cot(A - 24°)$ **[CBSE 2019]**

Sol. $\tan 2A = \cot(A - 24°)$
$$\Rightarrow \quad \cot(90° - 2A) = \cot(A - 24°)$$
$$\Rightarrow \quad 90° - 2A = A - 24°$$
$$\Rightarrow \quad 3A = 114°$$
$$\Rightarrow \quad A = 38°$$

Short Question

Q. 2. If $\sec A = \frac{2}{\sqrt{3}}$, find the value of $\frac{\tan A}{\cos A} + \frac{1+\sin A}{\tan A}$. **[CBSE 2015]**

Sol. Given that, $\sec A = \frac{2}{\sqrt{3}}$

$$\Rightarrow \quad \cos A = \frac{\sqrt{3}}{2}$$

$$\sin A = \sqrt{1-\cos^2 A} = \sqrt{1-\frac{3}{4}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$$

$$\therefore \quad \tan A = \frac{\sin A}{\cos A} = \frac{1}{\sqrt{3}}$$

Now, $\frac{\tan A}{\cos A} + \frac{1+\sin A}{\tan A}$

$$= \frac{\frac{1}{\sqrt{3}}}{\frac{\sqrt{3}}{2}} + \frac{1+\frac{1}{2}}{\frac{1}{\sqrt{3}}}$$

$$= \frac{2}{3} + \frac{3\sqrt{3}}{2} = \frac{2}{3} + \frac{3\sqrt{3}}{2} = \frac{4+9\sqrt{3}}{6}$$

Q.19. If $\tan 2A = \cot(A-18^\circ)$, where $2A$ is an angle, find the value of A. **[CBSE 2018]**

Sol. Give that,

$$\tan 2A = \cot(A-18^\circ)$$

We know that, $\tan\theta = \cot(90^\circ-\theta)$

$$\therefore \quad \cot(90^\circ-2A) = \cot(A-18^\circ)$$

$$\therefore \quad 90^\circ-2A = A-18^\circ$$

$$\therefore \quad 3A = 108^\circ$$

$$\therefore \quad A = 1-\frac{108^\circ}{3} = 36^\circ$$

Q. 4. If A, B and C are interior angles of a triangle ABC, then show that $\sin\left(\frac{B+C}{2}\right) = \cos\frac{A}{2}$. **[CBSE 2013]**

Sol. As we know that,

$$\angle A + \angle B + \angle C = 180^\circ$$

$$\Rightarrow \quad \angle B + \angle C = 180^\circ - \angle A$$

$$\Rightarrow \quad \frac{\angle B + \angle C}{2} = \frac{180^\circ - \angle A}{2}$$

$$= \frac{180^\circ}{2} - \frac{\angle A}{2} = 90^\circ - \frac{\angle A}{2}$$

$$\text{L.H.S.} = \sin\left(\frac{B+C}{2}\right)$$

$$= \sin\left(90^\circ - \frac{\angle A}{2}\right)$$

$$= \cos\frac{A}{2} = \text{R.H.S.}$$

$\therefore$ L.H.S. = R.H.S. **Hence proved**

Q. 1. Evaluate :

$(\sec^2 37^\circ - \cot^2 53^\circ)\cdot\tan 21^\circ\cdot\tan 69^\circ - \sin 51^\circ\cdot\cos 39^\circ - \cos 51^\circ\cdot\sin 39^\circ$ **[CBSE 2015]**

Sol. Here, $(\sec^2 37^\circ - \cot^2 53^\circ)\cdot\tan 21^\circ\cdot\tan 69^\circ - \sin 51^\circ\cdot\cos 39^\circ - \cos 51^\circ\cdot\sin 39^\circ$

$$= \{\sec^2 37^\circ - \cot^2(90^\circ-37^\circ)\}\cdot\tan 21^\circ\cdot \tan(90^\circ-21^\circ) - \sin(90^\circ-39^\circ)\cos 39^\circ - \cos(90^\circ-39^\circ)\cdot\sin 39^\circ$$

$$= \{\sec^2 37^\circ - \tan^2 37^\circ\}\cdot\tan 21^\circ\cdot\cot 21^\circ - \cos 39^\circ\cdot\cos 39^\circ - \sin 39^\circ\cdot\sin 39^\circ$$

$$= 1\cdot\tan 21^\circ\frac{1}{\tan 21^\circ} - \cos^2 39^\circ - \sin^2 39^\circ$$

$$= 1-(\cos^2 39^\circ + \sin^2 39^\circ) = 1-1 = 0$$

Q. 18. Prove that $\tan^2\theta - \sin^2\theta = \tan^2\theta \times \frac{1}{\text{cosec}^2\theta}$. [CBSE 2014]

Sol. L.H.S. $= \tan^2\theta - \sin^2\theta$

$$= \frac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta \qquad \left[\because \tan\theta = \frac{\sin\theta}{\cos\theta}\right]$$

$$= \sin^2\theta\left(\frac{1-\cos^2\theta}{\cos^2\theta}\right)$$

$$= \frac{\sin^2\theta}{\cos^2\theta} \times \sin^2\theta \qquad [\because 1-\cos^2\theta = \sin^2\theta]$$

$$= \tan^2\theta \times \frac{1}{\text{cosec}^2\theta} \qquad \left[\because \sin^2\theta = \frac{1}{\text{cosec}^2\theta}\right]$$

= R.H.S. **Hence proved**

Q.20. If $\tan\theta + \sin\theta = m$ and $\tan\theta - \sin\theta = n$, show that : $m^2 - n^2 = 4\sqrt{mn}$. [CBSE 2013]

Sol. $\tan\theta + \sin\theta = m$ ...(1)

and $\tan\theta - \sin\theta = n$ ...(2)

Squaring and subtracting eqs.(1) and (2), we have

$$\tan^2\theta + \sin^2\theta + 2\tan\theta\sin\theta - \tan^2\theta - \sin^2\theta + 2\tan\theta\sin\theta = m^2 - n^2$$

$$\Rightarrow \quad 4\tan\theta\sin\theta = m^2 - n^2 \qquad ...(3)$$

Taking R.H.S.,

$$4\sqrt{mn} = 4\sqrt{(\tan\theta + \sin\theta)(\tan\theta - \sin\theta)}$$

$$= 4\sqrt{\tan^2\theta - \sin^2\theta}$$

$$= 4\sqrt{\frac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta}$$

$$= 4\sqrt{\frac{\sin^2\theta(1-\cos^2\theta)}{\cos^2\theta}}$$

$$= 4\sqrt{\tan^2\theta\sin^2\theta}$$

$$= 4\tan\theta\sin\theta \qquad ...(4)$$

From eqs.(3) and (4), we have $m^2 - n^2 = 4\sqrt{mn}$.

Hence proved

Q.26. If $x = a\sin\theta + b\cos\theta$ and $y = a\cos\theta - b\sin\theta$ then prove that $x^2 + y^2 = a^2 + b^2$. [CBSE 2012, 13]

Sol. Given, $x = a\sin\theta + b\cos\theta$ ...(1)

and $y = a\cos\theta - b\sin\theta$ ...(2)

Squaring and adding eq.(1) and eq.(2), we have

$$x^2 + y^2 = a^2\sin^2\theta + b^2\cos^2\theta + 2ab\sin\theta\cos\theta + a^2\cos^2\theta + b^2\sin^2\theta - 2ab\sin\theta\cos\theta$$

$$= a^2(\sin^2\theta + \cos^2\theta) + b^2(\cos^2\theta + \sin^2\theta)$$

$$x^2 + y^2 = a^2 + b^2 \qquad [\because \cos^2\theta + \sin^2\theta = 1]$$

Hence proved

Q. 40. If $\tan(A+B) = 1$ and $\tan(A-B) = \frac{1}{\sqrt{3}}$, $0° < A+B < 90°$, $A > B$, then find the values of A and B. [CBSE 2019]

Sol. $\tan(A+B) = 1$ and $\tan(A-B) = \frac{1}{\sqrt{3}}$

$\therefore$ $\tan(A+B) = \tan 45°$ and $\tan(A-B) = \tan 30°$

$\therefore$ $A + B = 45°$ ...(i)

and $A - B = 30°$ ...(ii)

Adding (i) and (ii)

$2A = 75°$

$\therefore$ $A = 37.5°$

$\therefore$ $37.5 - B = 30°$ $\therefore$ $B = 7.5°$

8

Introduction *to* Trigonometry and its Applications

Exercise 8.1 Multiple Choice Questions (MCQs)

Q. 1 If $\cos A = \frac{4}{5}$, then the value of $\tan A$ is

(a) $\frac{3}{5}$ (b) $\frac{3}{4}$ (c) $\frac{4}{3}$ (d) $\frac{5}{3}$

Thinking Process

(i) First, we use the formula $\sin\theta = \sqrt{1-\cos^2\theta}$ *to get the value of* $\sin\theta$.

(ii) Second, we use the formula $\tan\theta = \frac{\sin\theta}{\cos\theta}$ *to get the value of* $\tan\theta$.

Sol. *(b)* Given, $\cos A = \frac{4}{5}$

$$\therefore \quad \sin A = \sqrt{1-\cos^2 A} \qquad \left[\begin{array}{l}\because \sin^2 A + \cos^2 A = 1\\ \therefore \sin A = \sqrt{1-\cos^2 A}\end{array}\right]$$

$$= \sqrt{1-\left(\frac{4}{5}\right)^2} = \sqrt{1-\frac{16}{25}} = \sqrt{\frac{9}{25}} = \frac{3}{5}$$

Now, $$\tan A = \frac{\sin A}{\cos A} = \frac{\frac{3}{5}}{\frac{4}{5}} = \frac{3}{4}$$

Hence, the required value of $\tan A$ is $3/4$.

Q. 2 If $\sin A = \frac{1}{2}$, then the value of $\cot A$ is

(a) $\sqrt{3}$ (b) $\frac{1}{\sqrt{3}}$ (c) $\frac{\sqrt{3}}{2}$ (d) 1

Thinking Process

(i) First, we use the formula $\cos\theta = \sqrt{1-\sin^2\theta}$ *to get the value of* $\cos\theta$.

(ii) Now, we use the trigonometric ratio $\cot\theta = \frac{\cos\theta}{\sin\theta}$ *to get the value of* $\cot\theta$.

Sol. ***(a)*** Given, $\sin A = \frac{1}{2}$

$$\therefore \quad \cos A = \sqrt{1-\sin^2 A} = \sqrt{1-\left(\frac{1}{2}\right)^2}$$

$$= \sqrt{1-\frac{1}{4}} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2} \quad [\because \sin^2 A + \cos^2 = 1 \Rightarrow \cos A = \sqrt{1-\sin^2 A}]$$

Now, $$\cot A = \frac{\cos A}{\sin A} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$$

Hence, the required value of $\cot A$ is $\sqrt{3}$.

Q. 3 The value of the expression $\text{cosec}\,(75° + \theta) - \sec\,(15° - \theta) - \tan\,(55° + \theta) + \cot\,(35° - \theta)$ is

(a) -1 (b) 0 (c) 1 (d) $\frac{3}{2}$

Thinking Process

We see that, the given trigonometric angle of the ratio are the reciprocal in the sense of sign. Then, use the following formulae

(i) $\text{cosec}\,(90° - \theta) = \sec\theta$ *(ii)* $\cot\,(90° - \theta) = \tan\theta$

Sol. ***(b)*** Given, expression $= \text{cosec}\,(75° + \theta) - \sec\,(15° - \theta) - \tan\,(55° + \theta) + \cot\,(35° - \theta)$

$= \text{cosec}\,[90° - (15° - \theta)] - \sec\,(15° - \theta) - \tan\,(55° + \theta) + \cot\,\{90° - (55° + \theta)\}$

$= \sec\,(15° - \theta) - \sec\,(15° - \theta) - \tan\,(55° + \theta) + \tan\,(55° + \theta)$

$[\because \text{cosec}\,(90° - \theta) = \sec\theta$ and $\cot\,(90° - \theta) = \tan\theta]$

$= 0$

Hence, the required value of the given expression is 0.

Q. 4 If $\sin\theta = \frac{a}{b}$, then $\cos\theta$ is equal to

(a) $\frac{b}{\sqrt{b^2 - a^2}}$ (b) $\frac{b}{a}$ (c) $\frac{\sqrt{b^2 - a^2}}{b}$ (d) $\frac{a}{\sqrt{b^2 - a^2}}$

Sol. ***(c)*** Given, $\sin\theta = \frac{a}{b}$ $\quad [\because \sin^2\theta + \cos^2\theta = 1 \Rightarrow \cos\theta = \sqrt{1-\sin^2\theta}]$

$$\therefore \quad \cos\theta = \sqrt{1-\sin^2\theta}$$

$$= \sqrt{1-\left(\frac{a}{b}\right)^2} = \sqrt{1-\frac{a^2}{b^2}} = \frac{\sqrt{b^2 - a^2}}{b}$$

Q. 5 If $\cos(\alpha + \beta) = 0$, then $\sin(\alpha - \beta)$ can be reduced to

(a) $\cos\beta$ (b) $\cos 2\beta$ (c) $\sin\alpha$ (d) $\sin 2\alpha$

Sol. *(b)* Given, $\cos(\alpha + \beta) = 0 = \cos 90°$ $[\because \cos 90° = 0]$

$\Rightarrow \alpha + \beta = 90°$

$\Rightarrow \alpha = 90° - \beta$ ...(i)

Now, $\sin(\alpha - \beta) = \sin(90° - \beta - \beta)$ [put the value from Eq. (i)]

$= \sin(90° - 2\beta)$

$= \cos 2\beta$ $[\because \sin(90° - \theta) = \cos\theta]$

Hence, $\sin(\alpha - \beta)$ can be reduced to $\cos 2\beta$.

Q. 6 The value of $(\tan 1° \tan 2° \tan 3° \ldots \tan 89°)$ is

(a) 0 (b) 1 (c) 2 (d) $\frac{1}{2}$

Thinking Process

Use the transformation $\tan(90° - \theta) = \cot\theta$ *from greater than trigonometric angle* $\tan 45°$ *after that we use the trigonometric ratio,* $\cot\theta = \frac{1}{\tan\theta}$.

Sol. *(b)* $\tan 1° \cdot \tan 2° \cdot \tan 3° \ldots \tan 89°$

$= \tan 1° \cdot \tan 2° \cdot \tan 3° \ldots \tan 44° \cdot \tan 45° \cdot \tan 46° \ldots \tan 87° \cdot \tan 88° \cdot \tan 89°$

$= \tan 1° \cdot \tan 2° \cdot \tan 3° \ldots \tan 44° \cdot (1) \cdot \tan(90° - 44°) \ldots \tan(90° - 3°) \cdot \tan(90° - 2°) \cdot \tan(90° - 1°)$ $(\because \tan 45° = 1)$

$= \tan 1° \cdot \tan 2° \cdot \tan 3° \ldots \tan 44° \cdot (1) \cdot \cot 44° \ldots \cot 3° \cdot \cot 2° \cdot \cot 1°$

$[\because \tan(90° - \theta) = \cot\theta]$

$= \tan 1° . \tan 2° . \tan 3° \ldots \tan 44° (1) . \frac{1}{\tan 44°} \ldots \frac{1}{\tan 30°} . \frac{1}{\tan 2°} . \frac{1}{\tan 1°}$ $\left[\because \cot\theta = \frac{1}{\tan\theta}\right]$

$= 1$

Q. 7 If $\cos 9\alpha = \sin\alpha$ and $9\alpha < 90°$, then the value of $\tan 5\alpha$ is

(a) $\frac{1}{\sqrt{3}}$ (b) $\sqrt{3}$ (c) 1 (d) 0

Sol. *(c)* Given, $\cos 9\alpha = \sin\alpha$ and $9\alpha < 90°$ *i.e.*, acute angle.

$\sin(90° - 9\alpha) = \sin\alpha$ $[\because \cos A = \sin(90° - A)]$

$\Rightarrow 90° - 9\alpha = \alpha$

$\Rightarrow 10\alpha = 90°$

$\Rightarrow \alpha = 9°$

$\therefore \tan 5\alpha = \tan(5 \times 9°) = \tan 45° = 1$ $[\because \tan 45° = 1]$

Q. 8 If ΔABC is right angled at C, then the value of $\cos(A+B)$ is

(a) 0 (b) 1 (c) $\frac{1}{2}$ (d) $\frac{\sqrt{3}}{2}$

Sol. *(a)* We know that, in ΔABC, sum of three angles $= 180°$

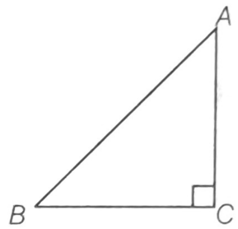
i.e., $\angle A + \angle B + \angle C = 180°$

But right angled at C i.e., $\angle C = 90°$ [given]

$\angle A + \angle B + 90° = 180°$

$\Rightarrow \quad A + B = 90° \quad [\because \angle A = A]$

$\therefore \quad \cos(A+B) = \cos 90° = 0$



Q. 9 If $\sin A + \sin^2 A = 1$, then the value of $(\cos^2 A + \cos^4 A)$ is

(a) 1 (b) $\frac{1}{2}$ (c) 2 (d) 3

Sol. *(a)* Given, $\sin A + \sin^2 A = 1$

$\Rightarrow \quad \sin A = 1 - \sin^2 A = \cos^2 A \quad [\because \sin^2\theta + \cos^2\theta = 1]$

On squaring both sides, we get

$$\sin^2 A = \cos^4 A$$

$\Rightarrow \quad 1 - \cos^2 A = \cos^4 A$

$\Rightarrow \quad \cos^2 A + \cos^4 A = 1$

Q. 10 If $\sin\alpha = \frac{1}{2}$ and $\cos\beta = \frac{1}{2}$, then the value of $(\alpha + \beta)$ is

(a) 0° (b) 30° (c) 60° (d) 90°

Sol. *(d)* Given, $\sin\alpha = \frac{1}{2} = \sin 30° \quad \left[\because \sin 30° = \frac{1}{2}\right]$

$\Rightarrow \quad \alpha = 30°$

and $\cos\beta = \frac{1}{2} = \cos 60° \quad \left[\because \cos 60° = \frac{1}{2}\right]$

$\Rightarrow \quad \beta = 60°$

$\therefore \quad \alpha + \beta = 30° + 60° = 90°$

Q. 11 The value of the expression

$$\left(\frac{\sin^2 22° + \sin^2 68°}{\cos^2 22° + \cos^2 68°} + \sin^2 63° + \cos 63° \sin 27°\right) \text{ is}$$

(a) 3 (b) 2 (c) 1 (d) 0

Sol. *(b)* Given expression, $\dfrac{\sin^2 22° + \sin^2 68°}{\cos^2 22° + \cos^2 68°} + \sin^2 63° + \cos 63° \sin 27°$

$$= \frac{\sin^2 22° + \sin^2(90° - 22°)}{\cos^2(90° - 68°) + \cos^2 68°} + \sin^2 63° + \cos 63° \sin(90° - 63°)$$

$$= \frac{\sin^2 22° + \cos^2 22°}{\sin^2 68° + \cos^2 68°} + \sin^2 63° + \cos 63° \cdot \cos 63° \quad \left[\begin{array}{l}\because \sin(90° - \theta) = \cos\theta \\ \text{and } \cos(90° - \theta) = \sin\theta\end{array}\right]$$

$$= \frac{1}{1} + (\sin^2 63° + \cos^2 63°) \quad [\because \sin^2\theta + \cos^2\theta = 1]$$

$$= 1 + 1 = 2$$

Q. 12 If $4\tan\theta = 3$, then $\left(\dfrac{4\sin\theta - \cos\theta}{4\sin\theta + \cos\theta}\right)$ is equal to

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{3}{4}$

Sol. ***(c)*** Given, $4\tan\theta = 3$

$\Rightarrow$ $\tan\theta = \frac{3}{4}$ ...(i)

$\therefore$ $\dfrac{4\sin\theta - \cos\theta}{4\sin\theta + \cos\theta} = \dfrac{4\dfrac{\sin\theta}{\cos\theta} - 1}{4\dfrac{\sin\theta}{\cos\theta} + 1}$

[divide by $\cos\theta$ in both numerator and denominator]

$= \dfrac{4\tan\theta - 1}{4\tan\theta + 1}$ $\left[\because \tan\theta = \dfrac{\sin\theta}{\cos\theta}\right]$

$= \dfrac{4\left(\frac{3}{4}\right) - 1}{4\left(\frac{3}{4}\right) + 1} = \dfrac{3-1}{3+1} = \dfrac{2}{4} = \dfrac{1}{2}$ [put the value from Eq. (i)]

Q. 13 If $\sin\theta - \cos\theta = 0$, then the value of $(\sin^4\theta + \cos^4\theta)$ is

(a) 1 (b) $\frac{3}{4}$ (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

Thinking Process

Firstly, from $\sin\theta - \cos\theta = 0$ get the value of θ. After that put the value of θ in the given expression to get the desired result.

Sol. ***(c)*** Given, $\sin\theta - \cos\theta = 0$

$\Rightarrow$ $\sin\theta = \cos\theta \Rightarrow \dfrac{\sin\theta}{\cos\theta} = 1$

$\Rightarrow$ $\tan\theta = 1$ $\left[\because \tan\theta = \dfrac{\sin\theta}{\cos\theta} \text{ and } \tan 45^\circ = 1\right]$

$\Rightarrow$ $\tan\theta = \tan 45^\circ$

$\therefore$ $\theta = 45^\circ$

Now, $\sin^4\theta + \cos^4\theta = \sin^4 45^\circ + \cos^4 45^\circ$

$= \left(\dfrac{1}{\sqrt{2}}\right)^4 + \left(\dfrac{1}{\sqrt{2}}\right)^4$ $\left[\because \sin 45^\circ = \cos 45^\circ = \dfrac{1}{\sqrt{2}}\right]$

$= \dfrac{1}{4} + \dfrac{1}{4} = \dfrac{2}{4} = \dfrac{1}{2}$

Q. 14 $\sin(45^\circ + \theta) - \cos(45^\circ - \theta)$ is equal to

(a) $2\cos\theta$ (b) 0 (c) $2\sin\theta$ (d) 1

Sol. ***(b)*** $\sin(45^\circ + \theta) - \cos(45^\circ - \theta) = \cos[90^\circ - (45^\circ + \theta)] - \cos(45^\circ - \theta)$ $[\because \cos(90^\circ - \theta) = \sin\theta]$

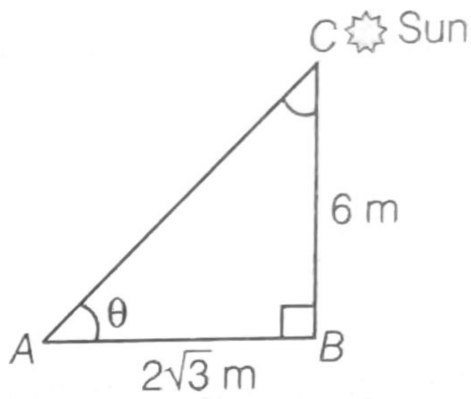
$= \cos(45^\circ - \theta) - \cos(45^\circ - \theta)$

$= 0$

Q. 15 If a pole 6 m high casts a shadow $2\sqrt{3}$ m long on the ground, then the Sun's elevation is

(a) $60°$ (b) $45°$ (c) $30°$ (d) $90°$

Sol. *(a)* Let $BC = 6$ m be the height of the pole and $AB = 2\sqrt{3}$ m be the length of the shadow on the ground. let the Sun's makes an angle θ on the ground.



Now, in ΔBAC,
$$\tan\theta = \frac{BC}{AB}$$
$$\Rightarrow \quad \tan\theta = \frac{6}{2\sqrt{3}} = \frac{3}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$
$$\Rightarrow \quad \tan\theta = \frac{3\sqrt{3}}{3} = \sqrt{3} = \tan 60° \qquad [\because \tan 60° = \sqrt{3}]$$
$$\therefore \quad \theta = 60°$$

Hence, the Sun's elevation is $60°$.