POLYNOMIALS

Quadratic

$k[x^2 - (\text{SUM})x + \text{PRODUCT}]$

Zeroes of polynomial

$\therefore$ Let them α & β

$$\alpha + \beta = \frac{-b}{a}$$

$$\alpha\beta = \frac{c}{a}$$

$ax^2 + bx + c$

e.g. $5x^2 + 2x + 8$

$k[x^2 - (\text{SUM})x + \text{PRODUCT}]$

$k[x^2 - (\alpha + \beta)x + (\alpha\beta)]$

Cubic

$k[x^3 - (\text{SUM})x^2 + (\text{PRODUCT}_2)x + P]$

Zeroes of cubic polynomial α, β, γ

$$\alpha + \beta + \gamma = \frac{-b}{a}$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$$

$$\alpha\beta\gamma = \frac{-d}{a}$$

$ax^3 + bx^2 + cx + d$

e.g. $7x^3 + 4x^2 + 3x + 49$

GRAPHS

Quadratic Polynomial

ax^2+bx+c

where ;

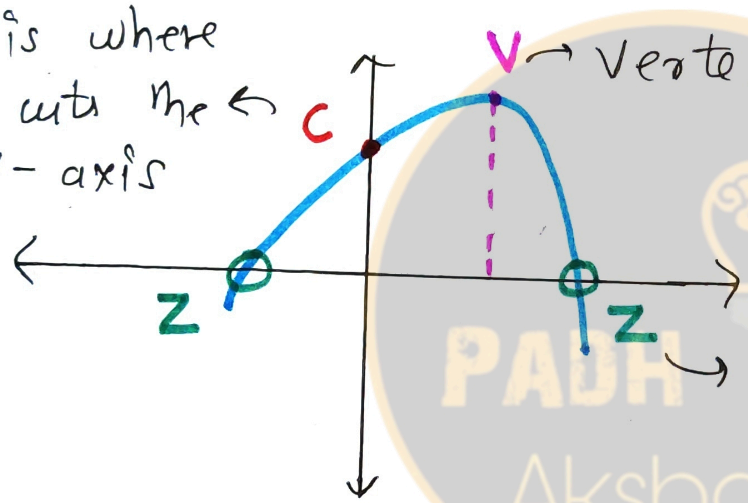
$D = b^2-4ac$

If $a>0$ i.e $a = +ve$

(smiley = parabola) 

If $a<0$ i.e $a = -ve$

'c' is where it cuts the Y-axis



Vertex → $\left(\frac{-b}{2a}, \frac{-D}{4a}\right)$

zeroes

where parabola cuts X-axis

Alert Notes

▷ If in a polynomial [i.e $ax^2+bx+c=0$]

→ $a>0$, $b>0$, $c>0$

→ $a<0$, $b<0$, $c<0$

→ Then ; Both the zeroes are of opposite signs

Chapter-2 Polynomial

Very Short Question

Q. 1. If α and β are the roots of $ax^2 - bx + c = 0$ $(a \neq 0)$, then calculate $\alpha + \beta$. [CBSE 2014]

Sol. Sum of the roots $(\alpha + \beta)$

$$= -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2} = -\left(-\frac{b}{a}\right) = \frac{b}{a}$$

Q. 4. Find a quadratic polynomial, the sum and product of whose zeroes are 0 and $-\sqrt{2}$, respectively. [CBSE 2015]

Sol. $\therefore$ Sum of zeroes $= 0$ and product of zeroes $= -\sqrt{2}$

$\therefore$ Required quadratic polynomial

$$= x^2 - (\text{Sum of zeroes}) + \text{Product of zeroes}$$

$$= x^2 - 0 \cdot x - \sqrt{2} = x^2 - \sqrt{2}$$

Q. 3. Find the quadratic polynomial whose zeroes are 3 and -4, respectively. [CBSE 2014]

Sol. We know that, if zeroes of a quadratic polynomial are given, then the quadratic polynomial is

$$x^2 - (\text{Sum of zeroes})\ x + \text{Product of zeroes}$$

Here, the zeroes of a quadratic polynomial are 3 and -4.

$\therefore$ Required quadratic polynomial

$$= x^2 - [3 + (-4)]\,x + 3 \times (-4)$$

$$= x^2 + x - 12$$

Short Question

Q. 2. If one zero of the polynomial $(a^2 + 9)\,x^2 + 13x + 6a$ is reciprocal of the other, then find the value of a. [CBSE 2015]

Sol. Let α and $\frac{1}{\alpha}$ be two zeroes of the given polynomial, which are reciprocal to each other.

On comparing the given polynomial with $Ax^2 + Bx + C$, we get

$$A = a^2 + 9;\ B = 13 \text{ and } C = 6a$$

Now, product of zeroes,

$$\alpha \times \frac{1}{\alpha} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

$\Rightarrow$ $1 = \dfrac{6a}{a^2 + 9}$

$\Rightarrow$ $a^2 + 9 = 6a$

$\Rightarrow$ $a^2 - 6a + 9 = 0$

$\Rightarrow$ $(a-3)^2 = 0$ $[\because (x-y)^2 = x^2 + y^2 - 2xy]$

$\therefore$ $a = 3$

Q. 5. Find the values of a and b, if they are the zeroes of polynomial $x^2 + ax + b$. [CBSE 2013]

Sol. Sum of zeroes $= -\dfrac{\text{Coeff. of } x}{\text{Coeff. of } x^2}$

$\Rightarrow$ $a + b = -a$ $\Rightarrow$ $2a + b = 0$

Product of zeroes $= \dfrac{\text{Constant term}}{\text{Coeff. of } x^2}$

$\Rightarrow$ $ab = b$ $\Rightarrow$ $a = 1$

Then, $b = -2$

Q. 7. Find all the zeroes of $f(x) = x^2 - 2x$ [CBSE 2013]

Sol. Here, $f(x) = x^2 - 2x = x\,(x-2)$

i.e., $f(x) = 0$

$\Rightarrow$ $x = 0$ or $x = 2$

Hence, zeroes are 0 and 2.

Q.10. Find the value of k, if -1 is a zero of the polynomial $p(x) = kx^2 - 4x + k$. [CBSE 2012]

Sol. Since, -1 is a zero of the polynomial

$$p(x) = kx^2 - 4x + k,$$

then $p(-1) = 0$

$\therefore$ $k(-1)^2 - 4(-1) + k = 0$

$\Rightarrow$ $k + 4 + k = 0$

$\Rightarrow$ $2k + 4 = 0$

$\Rightarrow$ $2k = -4$

Hence, $k = -2$

Q.14. Find a quadratic polynomial whose zeroes are $3+\sqrt{2}$ and $3-\sqrt{2}$. **[CBSE 2012]**

Sol. Sum of zeroes $= 3+\sqrt{2}+3-\sqrt{2} = 6$

and product of zeroes $= (3+\sqrt{2})(3-\sqrt{2})$

$$= 9 - 2 = 7$$

A quadratic polynomial, the sum and product of whose zeroes are 6 and 7 is given by $x^2 - 6x + 7$.

Q.17. Find the zeroes of the quadratic polynomial $x^2 - 2\sqrt{2}\,x$ and verify the relationship between the zeroes and the coefficients. **[CBSE 2015]**

Sol. Let $p(x) = x^2 - 2\sqrt{2}\,x = x(x - 2\sqrt{2})$,

$$p(x) = 0$$

$\Rightarrow \quad x(x - 2\sqrt{2}) = 0$

$\therefore$ zeroes are 0 and $2\sqrt{2}$

Sum of zeroes$= 2\sqrt{2} = -\dfrac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$

Product of zeroes $= 0 = \dfrac{\text{Constant term}}{\text{Coefficient of } x^2}$

Thus, relationship is verified.

Q.22. If α and β are the zeroes of the polynomial $f(x) = x^2 - 6x + k$, then find the value of k, such that $\alpha^2 + \beta^2 = 40$. **[CBSE 2015]**

Sol. Since, α and β are the zeroes of polynomial $f(x) = x^2 - 6x + k$.

So, $\quad \alpha + \beta = 6 \quad$ ...(1)

and $\quad \alpha\beta = k \quad$ ...(2)

It is given that, $\alpha^2 + \beta^2 = 40$

$\Rightarrow$

$(\alpha+\beta)^2 - 2\alpha\beta = 40 \quad [\because a^2 + b^2 = (a+b)^2 - 2ab]$

$\Rightarrow \quad (6)^2 - 2k = 40 \quad$ [from eqs. (1) and (2)]

$\Rightarrow \quad 36 - 2k = 40 \Rightarrow 2k = 36 - 40$

$\Rightarrow \quad 2k = -4 \Rightarrow k = -2$

2

Polynomials

Exercise 2.1 Multiple Choice Questions (MCQs)

Q. 1 If one of the zeroes of the quadratic polynomial $(k-1)x^2 + kx + 1$ is -3, then the value of k is

(a) $\frac{4}{3}$ (b) $\frac{-4}{3}$ (c) $\frac{2}{3}$ (d) $\frac{-2}{3}$

Thinking Process

If α is the one of the zeroes of the quadratic polynomial $f(x) = ax^2 + bx + c$. Then, $f(\alpha)$ must be equal to 0.

Sol. *(a)* Given that, one of the zeroes of the quadratic polynomial say $p(x) = (k-1)x^2 + kx + 1$ is -3, then $p(-3) = 0$

$$\Rightarrow \quad (k-1)(-3)^2 + k(-3) + 1 = 0$$
$$\Rightarrow \quad 9(k-1) - 3k + 1 = 0$$
$$\Rightarrow \quad 9k - 9 - 3k + 1 = 0$$
$$\Rightarrow \quad 6k - 8 = 0$$
$$\therefore \quad k = 4/3$$

Q. 2 A quadratic polynomial, whose zeroes are -3 and 4, is

(a) $x^2 - x + 12$ (b) $x^2 + x + 12$ (c) $\frac{x^2}{2} - \frac{x}{2} - 6$ (d) $2x^2 + 2x - 24$

Sol. *(c)* Let $ax^2 + bx + c$ be a required polynomial whose zeroes are -3 and 4.

Then, sum of zeroes $= -3 + 4 = 1$ $\quad \left[\because \text{sum of zeroes} = \frac{-b}{a}\right]$

$$\Rightarrow \quad \frac{-b}{a} = \frac{1}{1} \Rightarrow \frac{-b}{a} = -\frac{(-1)}{1} \quad \ldots \text{(i)}$$

and product of zeroes $= -3 \times 4 = -12$ $\quad \left[\because \text{product of zeroes} = \frac{c}{a}\right]$

$$\Rightarrow \quad \frac{c}{a} = \frac{-12}{1} \quad \ldots \text{(ii)}$$

From Eqs. (i) and (ii),

$$a = 1, b = -1 \text{ and } c = -12$$
$$= ax^2 + bx + c$$

$\therefore$ Required polynomial $= 1 \cdot x^2 - 1 \cdot x - 12$

$$= x^2 - x - 12$$

$$= \frac{x^2}{2} - \frac{x}{2} - 6$$

We know that, if we multiply/divide any polynomial by any constant, then the zeroes of polynomial do not change.

Alternate Method

Let the zeroes of a quadratic polynomial are $\alpha = -3$ and $\beta = 4$.

Then, sum of zeroes $= \alpha + \beta = -3 + 4 = 1$

and product of zeroes $= \alpha\beta = (-3)(4) = -12$

$\therefore$ Required polynomial $= x^2 - (\text{sum of zeroes})x + (\text{product of zeroes})$

$$= x^2 - (1)x + (-12) = x^2 - x - 12$$

$$= \frac{x^2}{2} - \frac{x}{2} - 6$$

Q. 3 If the zeroes of the quadratic polynomial $x^2 + (a+1)x + b$ are 2 and -3, then

(a) $a = -7, b = -1$ (b) $a = 5, b = -1$

(c) $a = 2, b = -6$ (d) $a = 0, b = -6$

Sol. (d) Let $p(x) = x^2 + (a+1)x + b$

Given that, 2 and -3 are the zeroes of the quadratic polynomial $p(x)$.

$\therefore$ $p(2) = 0$ and $p(-3) = 0$

$\Rightarrow$ $2^2 + (a+1)(2) + b = 0$

$\Rightarrow$ $4 + 2a + 2 + b = 0$

$\Rightarrow$ $2a + b = -6$ ...(i)

and $(-3)^2 + (a+1)(-3) + b = 0$

$\Rightarrow$ $9 - 3a - 3 + b = 0$

$\Rightarrow$ $3a - b = 6$ ...(ii)

On adding Eqs. (i) and (ii), we get

$$5a = 0 \Rightarrow a = 0$$

Put the value of a in Eq. (i), we get

$$2 \times 0 + b = -6 \Rightarrow b = -6$$

So, the required values are $a = 0$ and $b = -6$.

Q. 4 The number of polynomials having zeroes as -2 and 5 is

(a) 1 (b) 2 (c) 3 (d) more than 3

Sol. (d) Let $p(x) = ax^2 + bx + c$ be the required polynomial whose zeroes are -2 and 5.

$\therefore$ Sum of zeroes $= \frac{-b}{a}$

$\Rightarrow$ $\frac{-b}{a} = -2 + 5 = \frac{3}{1} = \frac{-(-3)}{1}$ ...(i)

and product of zeroes $= \frac{c}{a}$

$\Rightarrow$ $\frac{c}{a} = -2 \times 5 = \frac{-10}{1}$ (ii)

From Eqs. (i) and (ii),

$$a = 1, b = -3 \text{ and } c = -10$$

$$\therefore \quad p(x) = ax^2 + bx + c = 1 \cdot x^2 - 3x - 10$$

$$= x^2 - 3x - 10$$

But we know that, if we multiply/divide any polynomial by any arbitrary constant. Then, the zeroes of polynomial never change.

$$\therefore \quad p(x) = kx^2 - 3kx - 10k \qquad \text{[where, } k \text{ is a real number]}$$

$$\Rightarrow \quad p(x) = \frac{x^2}{k} - \frac{3}{k}x - \frac{10}{k}, \qquad \text{[where, } k \text{ is a non-zero real number]}$$

Hence, the required number of polynomials are infinite *i.e.*, more than 3.

Q. 5 If one of the zeroes of the cubic polynomial $ax^3 + bx^2 + cx + d$ is zero, the product of then other two zeroes is

(a) $\frac{-c}{a}$ (b) $\frac{c}{a}$ (c) 0 (d) $\frac{-b}{a}$

Thinking Process

Firstly, we find the sum of product of two zeroes at a time and put the value of one of the zeroes i.e., zero, we get the required product of the other two zeroes.

Sol. *(b)* Let $p(x) = ax^3 + bx^2 + cx + d$

Given that, one of the zeroes of the cubic polynomial $p(x)$ is zero.

Let α, β and γ are the zeroes of cubic polynomial $p(x)$, where $a = 0$.

We know that,

Sum of product of two zeroes at a time $= \frac{c}{a}$

$$\Rightarrow \quad \alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$$

$$\Rightarrow \quad 0 \times \beta + \beta\gamma + \gamma \times 0 = \frac{c}{a} \qquad [\because \alpha = 0, \text{ given}]$$

$$\Rightarrow \quad 0 + \beta\gamma + 0 = \frac{c}{a}$$

$$\Rightarrow \quad \beta\gamma = \frac{c}{a}$$

Hence, product of other two zeroes $= \frac{c}{a}$

Q. 6 If one of the zeroes of the cubic polynomial $x^3 + ax^2 + bx + c$ is -1, then the product of the other two zeroes is

(a) $b - a + 1$ (b) $b - a - 1$ (c) $a - b + 1$ (d) $a - b - 1$

Thinking Process

Firstly, we find the value of constant term 'c', by using $p(-1) = 0$. After that we find the product of all zeroes and put the value of one of the zeroes. Finally, we get the required result.

Sol. *(a)* Let $p(x) = x^3 + ax^2 + bx + c$

Let α, β and γ be the zeroes of the given cubic polynomial $p(x)$.

$$\therefore \quad \alpha = -1 \qquad \text{[given]}$$

$$\text{and} \quad p(-1) = 0$$

$$\Rightarrow \quad (-1)^3 + a(-1)^2 + b(-1) + c = 0$$
$$\Rightarrow \quad -1 + a - b + c = 0$$
$$\Rightarrow \quad c = 1 - a + b$$

We know that,

$$\text{Product of all zeroes} = (-1)^3 \cdot \frac{\text{Constant term}}{\text{Coefficient of } x^3} = -\frac{c}{1}$$

$$\alpha\beta\gamma = -c$$
$$\Rightarrow \quad (-1)\,\beta\gamma = -c \qquad [\because \alpha = -1$$
$$\Rightarrow \quad \beta\gamma = c$$
$$\Rightarrow \quad \beta\gamma = 1 - a + b \qquad \text{[from Eq. (i)]}$$

Hence, product of the other two roots is $1 - a + b$.

Alternate Method

Since, -1 is one of the zeroes of the cubic polynomial $f(x) = x^2 + ax^2 + bx + c$ *i.e.*, $(x + 1)$ is a factor of $f(x)$.

Now, using division algorithm,

$$\begin{array}{r|l} & x^2 + (a-1)x + (b-a+1) \\ \hline x+1 & x^3 + ax^2 + bx + c \\ & x^3 + x^2 \\ \hline & (a-1)x^2 + bx \\ & (a-1)x^2 + (a-1)\,x \\ \hline & (b-a+1)x + c \\ & (b-a+1)\,x\,(b-a+1) \\ \hline & (c-b+a-1) \end{array}$$

$$\therefore \quad x^3 + ax^2 + bx + c = (x+1) \times \{x^2 + (a-1)x + (b-a+1)\} + (c-b+a-1)$$
$$\Rightarrow \quad x^3 + ax^2 + bx + (b-a+1) = (x+1)\{x^2 + (a-1)x + (b-a+1)\}$$

Let α and β be the other two zeroes of the given polynomial, then

$$\text{Product of zeroes} = (-1)\,\alpha \cdot \beta = \frac{-\text{Constant term}}{\text{Coefficient of } x^3}$$

$$\Rightarrow \quad -\alpha \cdot \beta = \frac{-(b-a+1)}{1}$$

$$\Rightarrow \quad \alpha\beta = -a + b + 1$$

Hence, the required product of other two roots is $(-a + b + 1)$.

Q. 7 The zeroes of the quadratic polynomial $x^2 + 99x + 127$ are

(a) both positive (b) both negative

(c) one positive and one negative (d) both equal

Sol. (b) Let given quadratic polynomial be $p(x) = x^2 + 99x + 127$.

On comparing $p(x)$ with $ax^2 + bx + c$, we get

$$a = 1,\ b = 99 \text{ and } c = 127$$

We know that,
$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$ [by quadratic formula]
$$=\frac{-99\pm\sqrt{(99)^2-4\times1\times127}}{2\times1}$$
$$=\frac{-99\pm\sqrt{9801-508}}{2}$$
$$=\frac{-99\pm\sqrt{9293}}{2}=\frac{-99\pm 96.4}{2}$$
$$=\frac{-99+96.4}{2},\frac{-99-96.4}{2}$$
$$=\frac{-2.6}{2},\frac{-195.4}{2}$$
$$=-1.3,-97.7$$

Hence, both zeroes of the given quadratic polynomial $p(x)$ are negative.

Alternate Method

We know that,

In quadratic polynomial, if $\left.\begin{matrix} a>0 \\ a<0 \end{matrix}\ \text{or}\ \begin{matrix} b>0,\ c>0 \\ b<0,\ c<0 \end{matrix}\right\}$, then both zeroes are negative.

In given polynomial, we see that
$$a=1>0,\ b=99>0 \text{ and } c=127>0$$
which satisfy the above condition.

So, both zeroes of the given quadratic polynomial are negative.

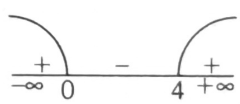
Q. 8 The zeroes of the quadratic polynomial x^2+kx+k where $k\neq 0$,

(a) cannot both be positive
(b) cannot both be negative
(c) are always unequal
(d) are always equal

Sol. *(a)* Let $p(x)=x^2+kx+k,\ k\neq 0$

On comparing $p(x)$ with ax^2+bx+c, we get
$$a=1,\ b=k \text{ and } c=k$$

Now,
$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$ [by quadratic formula]
$$=\frac{-k\pm\sqrt{k^2-4k}}{2\times1}$$
$$=\frac{-k\pm\sqrt{k(k-4)}}{2},\ k\neq 0$$



Here, we see that
$$k(k-4)>0$$
$$\Rightarrow\quad k\in(-\infty,0)\cup(4,\infty)$$

Now, we know that

In quadratic polynomial ax^2+bx+c

If $a>0, b>0, c>0$ or $a<0, b<0, c<0$,

then the polynomial has always all negative zeroes.

and if $a>0$, $c<0$ or $a<0, c>0$, then the polynomial has always zeroes of opposite sign.

Case I If $k \in (-\infty, 0)$ *i.e.*, $k < 0$

$\Rightarrow$ $a = 1 > 0, \quad b, c = k < 0$

So, both zeroes are of opposite sign.

Case II If $k \in (4, \infty)$ i.e., $k \geq 4$

$\Rightarrow$ $a = 1 > 0, \; b, c \geq 4$

So, both zeroes are negative.

Hence, in any case zeroes of the given quadratic polynomial cannot both be positive.

Q. 9 If the zeroes of the quadratic polynomial $ax^2 + bx + c$, where $c \neq 0$, are equal, then

(a) c and a have opposite signs
(b) c and b have opposite signs
(c) c and a have same signs
(d) c and b have the same signs

Sol. ***(c)*** The zeroes of the given quadratic polynomial $ax^2 + bx + c$, $c \neq 0$ are equal. If coefficient of x^2 and constant term have the same sign *i.e.*, c and *a* have the same sign. While *b i.e.*, coefficient of x can be positive/negative but not zero.

e.g., (i) $x^2 + 4x + 4 = 0$ (ii) $x^2 - 4x + 4 = 0$

$\Rightarrow$ $(x + 2)^2 = 0$ $\Rightarrow$ $(x - 2)^2 = 0$

$\Rightarrow$ $x = -2, -2$ $\Rightarrow$ $x = 2, 2$

Alternate Method

Given that, the zeroes of the quadratic polynomial $ax^2 + bx + c$, where $c \neq 0$, are equal *i.e.*, discriminant $(D) = 0$

$$\Rightarrow \quad b^2 - 4ac = 0$$
$$\Rightarrow \quad b^2 = 4ac$$
$$\Rightarrow \quad ac = \frac{b^2}{4}$$
$$\Rightarrow \quad ac > 0$$

which is only possible when *a* and *c* have the same signs.

Q. 10 If one of the zeroes of a quadratic polynomial of the form $x^2 + ax + b$ is the negative of the other, then it

(a) has no linear term and the constant term is negative
(b) has no linear term and the constant term is positive
(c) can have a linear term but the constant term is negative
(d) can have a linear term but the constant term is positive

Sol. ***(a)*** Let $p(x) = x^2 + ax + b.$

Now, product of zeroes $= \dfrac{\text{Constant term}}{\text{Coefficient of } x^2}$

Let α and β be the zeroes of $p(x)$.

$\therefore$ Product of zeroes $(\alpha \cdot \beta) = \dfrac{b}{1}$

$$\Rightarrow \quad \alpha\beta = b \quad \text{...(i)}$$

Given that, one of the zeroes of a quadratic polynomial $p(x)$ is negative of the other.

$$\therefore \quad \alpha\beta < 0$$

So, $b < 0$ [from Eq. (i)]

Hence, b should be negative

Put $a = 0$, then, $p(x) = x^2 + b = 0$

$\Rightarrow$ $x^2 = -b$

$\Rightarrow$ $x = \pm\sqrt{-b}$ $[\because b < 0]$

Hence, if one of the zeroes of quadratic polynomial $p(x)$ is the negative of the other, then it has no linear term *i.e.*, $a = 0$ and the constant term is negative *i.e.*, $b < 0$.

Alternate Method

Let $f(x) = x^2 + ax + b$

and by given condition the zeroes are α and $-\alpha$.

$\therefore$ Sum of the zeroes $= \alpha - \alpha = a$

$\Rightarrow$ $a = 0$

$\therefore$ $f(x) = x^2 + b$, which cannot be linear.

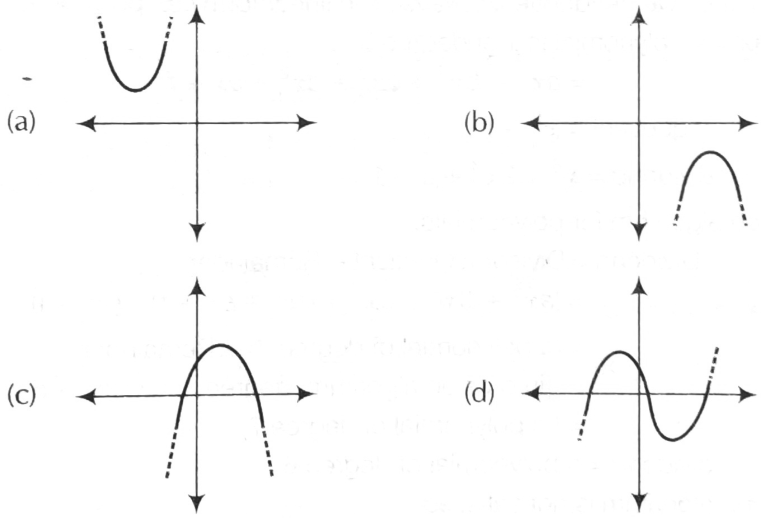
and product of zeroes $= \alpha \cdot (-\alpha) = b$

$\Rightarrow$ $-\alpha^2 = b$

which is possible when, $b < 0$.

Hence, it has no linear term and the constant term is negative.

Q. 11 Which of the following is not the graph of a quadratic polynomial?



Sol. *(d)* For any quadratic polynomial $ax^2 + bx + c, a \neq 0$, the graph of the Corresponding equation $y = ax^2 + bx + c$ has one of the two shapes either open upwards like $\cup$ or open downwards like $\cap$ depending on whether $a > 0$ or $a < 0$. These curves are called parabolas. So, option (d) cannot be possible.

Also, the curve of a quadratic polynomial crosses the X-axis on at most two points but in option (d) the curve crosses the X-axis on the three points, so it does not represent the quadratic polynomial.