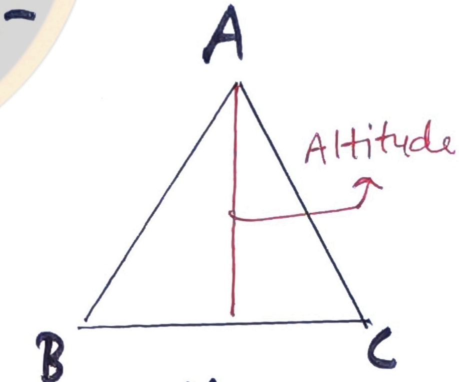
TRIANGLES

> 2 figures if congruent ; then they are similar ; but the converse is false

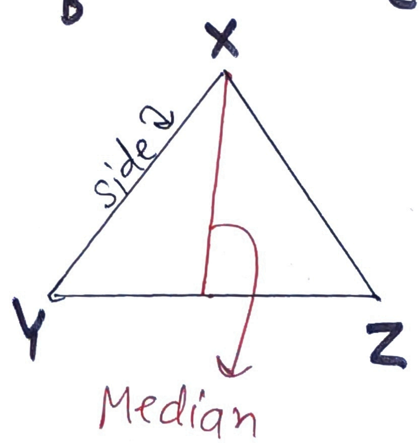
$$\text{Congruent} \underset{\times}{\overset{\checkmark}{\rightleftarrows}} \text{Similar}$$

> **In 2 similar triangles :-**

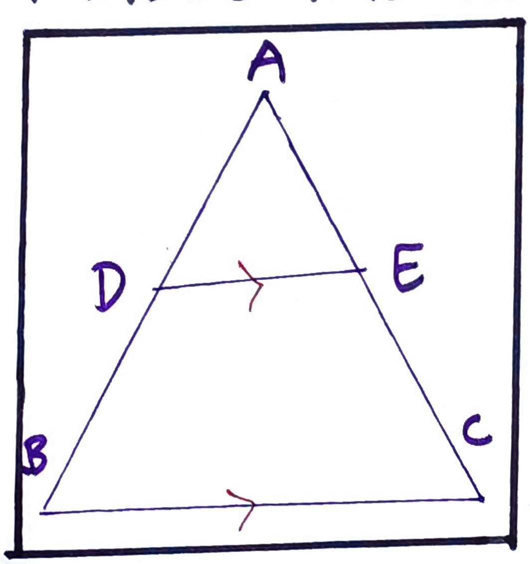
If $\Delta ABC \sim \Delta XYZ$



$$\frac{ar(\Delta ABC)}{ar(\Delta XYZ)} \rightarrow \begin{cases} (\text{side})^2 \text{ i.e. } \dfrac{AB^2}{XY^2} \\ (\text{altitude})^2 \\ (\text{median})^2 \\ (\text{angle-bisector})^2 \end{cases}$$



> **THALES THEOREM !**



If $DE \parallel BC$

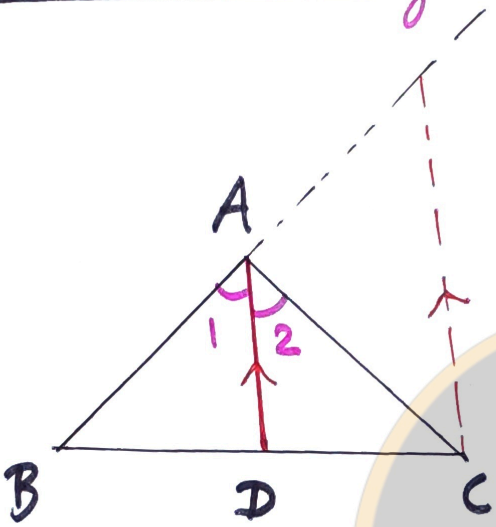
$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\frac{AD}{AB} = \frac{AE}{AC} \quad \Big| \quad \frac{DB}{AD} = \frac{EC}{AE} \quad \Big| \quad \frac{DB}{AB} = \frac{EC}{AC}$$

Angle Bisector Theorem

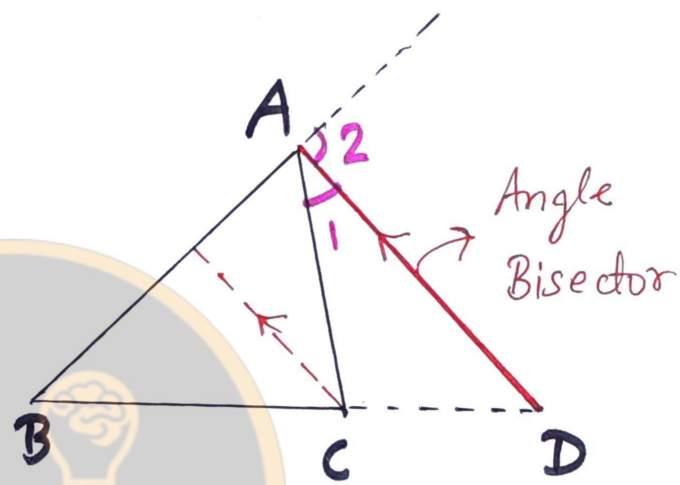
$\angle 1 = \angle 2$

Internal Angle



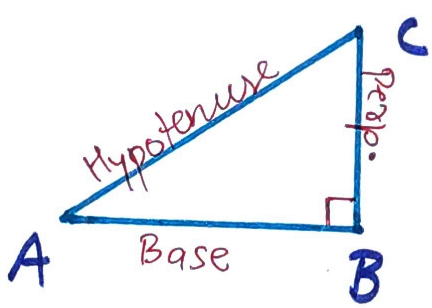
$$\boxed{\frac{AB}{AC} = \frac{BD}{DC}}$$

External Angle



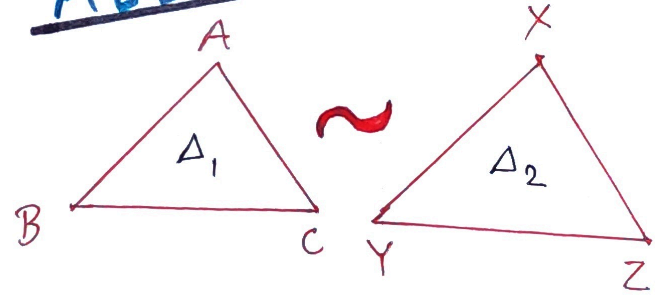
$$\boxed{\frac{AB}{AC} = \frac{BD}{CD}}$$

Pythagoras Theorem



$$AB^2 + BC^2 = AC^2$$

ALERT NOTE!



$$\triangle_1 \sim \triangle_2$$

$$\frac{AB}{XY} = \frac{BC}{YZ} = \frac{CA}{ZX} = \frac{Per.(\Delta_1)}{Per.(\Delta_2)}$$

Chapter-6 Triangle

Very Short Question

Q. 3. In ΔABC, $\angle B = 90°$ and $BD \perp AC$. If $DC = 7$ cm and $AD = 3$ cm, then find the length of BD. [CBSE 2014]

Sol. In ΔADB, $\angle 2 + \angle 4 = 90°$ ...(1)

In ΔCDB, $\angle 1 + \angle 3 = 90°$ ...(2)

In ΔABC, $\angle 1 + \angle 2 = 90°$ ...(3)

From eqs. (1), (2) and (3), we get

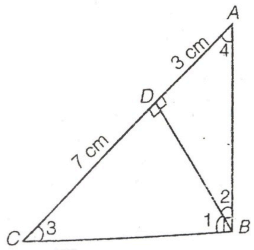
$$\angle 1 = \angle 4 \text{ and } \angle 3 = \angle 2$$

Now, in right angle ΔADB and ΔBDC,

$\angle 2 = \angle 3$, $\angle 4 = \angle 1$ and $\angle D = 90°$

$\therefore$ $\Delta ADB \sim \Delta BDC$

[by AAA similarity criterion]



$\Rightarrow$ $\frac{BD}{CD} = \frac{AD}{BD}$

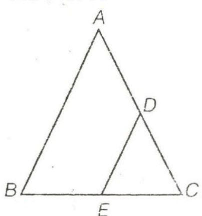
$\Rightarrow$ $BD^2 = CD \times AD$

$= 7 \times 3 = 21$

$\Rightarrow$ $BD = \sqrt{21}$ cm

Hence, the length of BD is $\sqrt{21}$ cm.

Q. 7. In the figure of ΔABC, $DE \parallel AB$. If $AD = 2x$, $DC = x + 3$, $BE = 2x - 1$ and $CE = x$, then find the value of x. [CBSE 2015]



Sol. Given, $AD = 2x$, $DC = x + 3$, $BE = 2x - 1$ and $CE = x$

In ΔABC, $DE \parallel AB$

$\frac{CD}{CA} = \frac{CE}{CB}$ [by Thales's Theorem]

$\frac{CD}{CD + AD} = \frac{CE}{CE + BE}$

$\Rightarrow$ $\frac{x+3}{x+3+2x} = \frac{x}{x+2x-1}$

$\Rightarrow$ $\frac{x+3}{3x+3} = \frac{x}{3x-1}$

$\Rightarrow$ $(x+3)(3x-1) = x(3x+3)$

$\Rightarrow$ $3x^2 - x + 9x - 3 = 3x^2 + 3x$

$\Rightarrow$ $8x - 3 = 3x$

$\Rightarrow$ $8x - 3x = 3$

$\Rightarrow$ $5x = 3$ $\quad x = \frac{3}{5}$

Hence the value of x is $\frac{3}{5}$.

Q. 4. If $\Delta ABC \sim \Delta PQR$, $AB = 6.5$ cm, $PQ = 10.4$ cm and perimeter of $\Delta ABC = 60$ cm, then find the perimeter of ΔPQR. [CBSE 2015]

Sol. Since, $\Delta ABC \sim \Delta PQR$

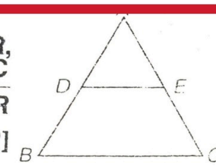
$\therefore$ $\frac{\text{Perimeter of } \Delta PQR}{\text{Perimeter of } \Delta ABC} = \frac{PQ}{AB}$

$\Rightarrow$ $\frac{\text{Perimeter of } \Delta PQR}{60} = \frac{10.4}{6.5}$

$\Rightarrow$ Perimeter of $\Delta PQR = 60 \times \frac{10.4}{6.5} = 96$ cm

Hence, the perimeter of ΔPQR is 96 cm.

Q. 8. Given $\Delta ABC \sim \Delta PQR$, if $\frac{AB}{PQ} = \frac{1}{3}$, then find $\frac{ar\Delta ABC}{ar\Delta PQR}$. [CBSE 2019]



Sol. $\frac{ar\Delta ABC}{ar\Delta PQR} = \frac{AB^2}{PQ^2}$ (Ratio of area of similar triangle is equal to square of their proportional sides)

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$$

Q. 7. ABC is an isosceles triangle with $AC = BC$. If $AB^2 = 2AC^2$, prove that ABC is a right angle triangle. [CBSE 2014]

Sol. In ΔABC, $AC = BC$ and $AB^2 = 2AC^2$ given

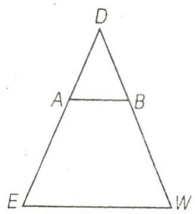
$\Rightarrow$ $AB^2 = AC^2 + AC^2$

$\Rightarrow$ $AB^2 = AC^2 + BC^2$ $\quad [\because AC = BC]$

Hence, by the converse of Pythagoras Theorem, ΔABC is a right triangle, right-angled at C.

Short Question

Q. 6. In ΔDEW, $AB \| EW$. If $AD = 4$ cm, $DE = 12$cm and $DW = 24$ cm, then find the value of DB. [CBSE 2015]



Sol. Given, $AD = 4$ cm, $DE = 12$ cm, $DW = 24$ cm and $AB \| EW$

In ΔDEW, $AB \| EW$

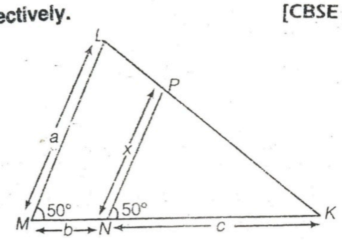
$$\frac{DA}{DE} = \frac{DB}{DW} \Rightarrow \frac{4}{12} = \frac{DB}{24}$$

$$\Rightarrow DB = \frac{4 \times 24}{12} = 8$$

(Using Basic Proportionality Theorem)

Hence, the value of DB is 8 cm.

Q.14. In the given figure, $\angle M = \angle N = 50°$. Express x in terms of a, b and c, where a, b and c are the lengths of LM, MN and NK respectively. [CBSE 2012]



Sol. As $\angle K = \angle K$

and $\angle LMK = \angle PNK = 50°$

$\Delta PNK \sim \Delta LMK$

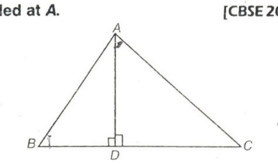
[by using AAA similarity criterion,]

$$\Rightarrow \frac{NK}{MK} = \frac{PN}{LM}$$

$$\Rightarrow PN = \frac{NK \times LM}{MK} = \frac{NK \times LM}{MN + NK}$$

$$\Rightarrow x = \frac{ac}{b+c}$$

Q.22. In ΔABC, $AD \perp BC$, such that $AD^2 = BD \times CD$. Prove that ΔABC is right angled at A. [CBSE 2015]



Sol. Given that,

$AD^2 = BD \times CD$

$\Rightarrow AD \times AD = BD \times CD$

$\Rightarrow \frac{AD}{CD} = \frac{BD}{AD}$

$\angle ADC = \angle ADB = 90°$ [given]

$\therefore \Delta ADC \sim \Delta BDA$

[by SAS similarity criterion]

$\Rightarrow \angle BAD = \angle ACD$ ...(1)

$\angle DAC = \angle DBA$ ...(2)

In ΔABC, by angle sum property, we get

$\angle BAD + \angle ACD + \angle DAC + \angle DBA = 180°$

$\angle BAD + \angle BAD + \angle DAC + \angle DAC = 180°$

[using eqs.(1) and (2)]

$2\angle BAD + 2\angle DAC = 180°$

$\angle BAD + \angle DAC = 90°$

$\angle BAC = 90°$

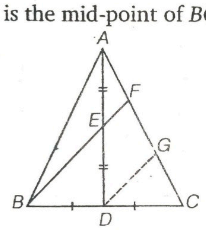
or $\angle A = 90°$

Hence, ΔABC is right angled at A.

Q.26. In a ΔABC, D is the mid-point of BC and E is the mid-point of AD. If BE produced meets AC in F, then prove that $AF = \frac{1}{3}AC$. [CBSE 2013]

Sol. Through D, draw $DG \| BF$ intersecting AC at G.

In ΔCBF, D is the mid-point of BC and $DG \| BF$



$\Rightarrow G$ is the mid-point of CF.

i.e., $CG = CF$ ...(1)

Again, in ΔADG, E is the mid-point of AD and EF is parallel to DG.

$\Rightarrow$ F is the mid-point of AG.

i.e., $GF = FA$ ...(2)

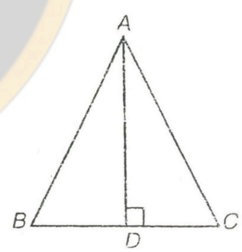
From eqs. (1) and (2), we get

$CG = GF = FA \Rightarrow AC = CG + GF + FA$

$\Rightarrow AC = FA + FA + FA \Rightarrow AC = 3FA$

$\Rightarrow \frac{1}{3}AC = FA$ or $AF = \frac{1}{3}AC$

Q.18. In the figure, ABC is a triangle in which $AD \perp BC$. Show that $AC^2 = AB^2 + BC^2 - 2BC \cdot BD$. [CBSE 2015]



Sol. Given ABC is a triangle and $AD \perp BC$.

To prove

$$AC^2 = AB^2 + BC^2 - 2BC \cdot BD$$

Proof In ΔABC,

Now, in ΔADC, $\angle D = 90°$

$$\therefore AC^2 = AD^2 + DC^2$$

(using Pythagoras theorem)

$$AC^2 = AB^2 - BD^2 + (BC - BD)^2$$

$[\because$ In ΔADB, $AB^2 = AD^2 + BD^2]$

$$AC^2 = AB^2 - BD^2 + BC^2 + BD^2 - 2BC \cdot BD$$

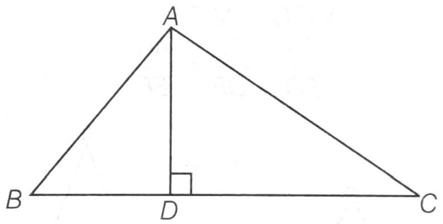
$$AC^2 = AB^2 + BC^2 - 2BC \cdot BD$$

6

Triangles

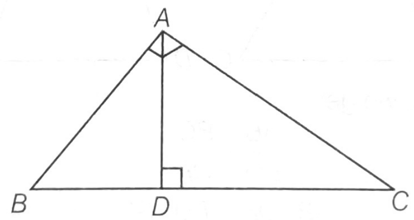
Exercise 6.1 Multiple Choice Questions (MCQs)

Q. 1 In figure, if $\angle BAC = 90^\circ$ and $AD \perp BC$. Then,



(a) $BD \cdot CD = BC^2$ (b) $AB \cdot AC = BC^2$ (c) $BD \cdot CD = AD^2$ (d) $AB \cdot AC = AD^2$

Sol. *(c)* In ΔADB and ΔADC,



$$\angle D = \angle D = 90^\circ$$

$$\angle DBA = \angle DAC \quad \text{[each equal to } 90^\circ - < C]$$

$$\therefore \quad \Delta ADB \sim \Delta ADC \quad \text{[by AAA similarity criterion]}$$

$$\therefore \quad \frac{BD}{AD} = \frac{AD}{CD}$$

$$\Rightarrow \quad BD \cdot CD = AD^2$$

Q. 2 If the lengths of the diagonals of rhombus are 16 cm and 12 cm. Then, the length of the sides of the rhombus is

(a) 9 cm (b) 10 cm (c) 8 cm (d) 20 cm

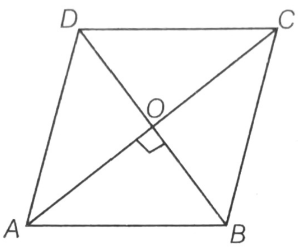
Sol. *(b)* We know that, the diagonals of a rhombus are perpendicular bisector of each other.

Given, $AC = 16\,\text{cm}$ and $BD = 12\,\text{cm}$ [let]

$\therefore$ $AO = 8\,\text{cm}$, $BO = 6\,\text{cm}$

and $\angle AOB = 90^\circ$

In right angled ΔAOB,



$$AB^2 = AO^2 + OB^2 \quad \text{[by Pythagoras theorem]}$$

$\Rightarrow$ $$AB^2 = 8^2 + 6^2 = 64 + 36 = 100$$

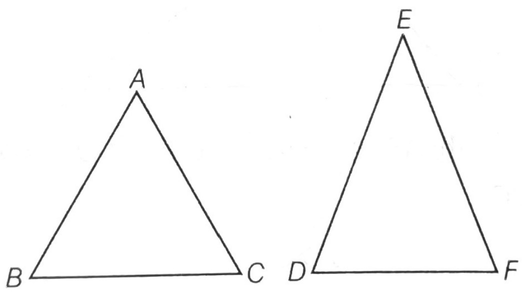
$\therefore$ $$AB = 10\,\text{cm}$$

Q. 3 If $\Delta ABC \sim \Delta EDF$ and ΔABC is not similar to ΔDEF, then which of the following is not true?

(a) $BC \cdot EF = AC \cdot FD$
(b) $AB \cdot EF = AC \cdot DE$
(c) $BC \cdot DE = AB \cdot EF$
(d) $BC \cdot DE = AB \cdot FD$

Sol. *(c)* Given, $$\Delta ABC \sim \Delta EDF$$

$\therefore$ $$\frac{AB}{ED} = \frac{BC}{DF} = \frac{AC}{EF}$$



Taking first two terms, we get

$$\frac{AB}{ED} = \frac{BC}{DF}$$

$\Rightarrow$ $$AB \cdot DF = ED \cdot BC$$

or $$BC \cdot DE = AB \cdot DF$$

So, option (d) is true.

Taking last two terms, we get

$$\frac{BC}{DF} = \frac{AC}{EF}$$

$\Rightarrow$ $$BC \cdot EF = AC \cdot DF$$

So, option (a) is also true.

Taking first and last terms, we get

$$\frac{AB}{ED} = \frac{AC}{EF}$$

$\Rightarrow$ $$AB \cdot EF = ED \cdot AC$$

Hence, option (b) is true.

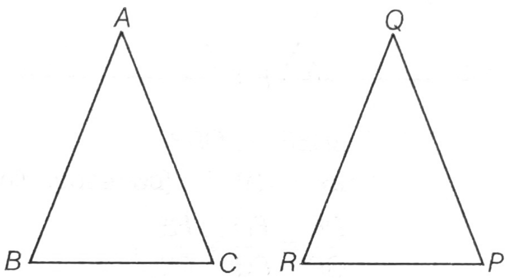
Q. 4 If in two ΔABC and ΔPQR, $\frac{AB}{QR} = \frac{BC}{PR} = \frac{CA}{PQ}$, then

(a) $\Delta PQR \sim \Delta CAB$ (b) $\Delta PQR \sim \Delta ABC$
(c) $\Delta CBA \sim \Delta PQR$ (d) $\Delta BCA \sim \Delta PQR$

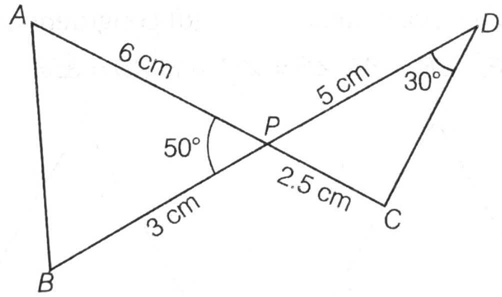
Sol. ***(a)*** Given, in two ΔABC and ΔPQR, $\frac{AB}{QR} = \frac{BC}{PR} = \frac{CA}{PQ}$

which shows that sides of one triangle are proportional to the side of the other triangle, then their corresponding angles are also equal, so by SSS similarity, triangles are similar.

i.e., $$\Delta CAB \sim \Delta PQR$$



Q. 5 In figure, two line segments AC and BD intersect each other at the point P such that $PA = 6$ cm, $PB = 3$ cm, $PC = 2.5$ cm, $PD = 5$ cm, $\angle APB = 50°$ and $\angle CDP = 30°$. Then, $\angle PBA$ is equal to



(a) 50° (b) 30° (c) 60° (d) 100°

Sol. ***(d)*** In ΔAPB and ΔCPD, $\angle APB = \angle CPD = 50°$ [vertically opposite angles]

$$\frac{AP}{PD} = \frac{6}{5} \quad \ldots(i)$$

and $$\frac{BP}{CP} = \frac{3}{2.5} = \frac{6}{5} \quad \ldots(ii)$$

From Eqs. (i) and (ii)

$$\frac{AP}{PD} = \frac{BP}{CP}$$

$\therefore$ $\Delta APB \sim \Delta DPC$ [by SAS similarity criterion]

$\therefore$ $\angle A = \angle D = 30°$ [corresponding angles of similar triangles]

In ΔAPB, $\angle A + \angle B + \angle APB = 180°$ [sum of angles of a triangle = 180°]

$\Rightarrow$ $30° + \angle B + 50° = 180°$

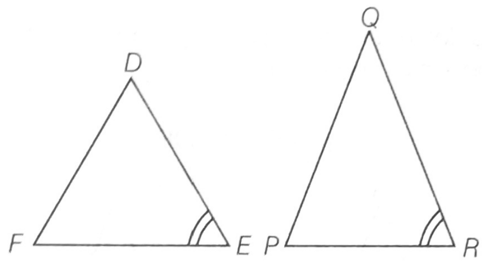
$\therefore$ $\angle B = 180° - (50° + 30°) = 100°$

i.e., $\angle PBA = 100°$

Q. 6 If in two ΔDEF and ΔPQR, $\angle D = \angle Q$ and $\angle R = \angle E$, then which of the following is not true?

(a) $\frac{EF}{PR} = \frac{DF}{PQ}$ (b) $\frac{DE}{PQ} = \frac{EF}{RP}$ (c) $\frac{DE}{QR} = \frac{DF}{PQ}$ (d) $\frac{EF}{RP} = \frac{DE}{QR}$

Sol. *(b)* Given, in ΔDEF and ΔPQR, $\angle D = \angle Q$, $\angle R = \angle E$



$\therefore$ $\Delta DEF \sim \Delta QRP$ [by AAA similarity criterion]

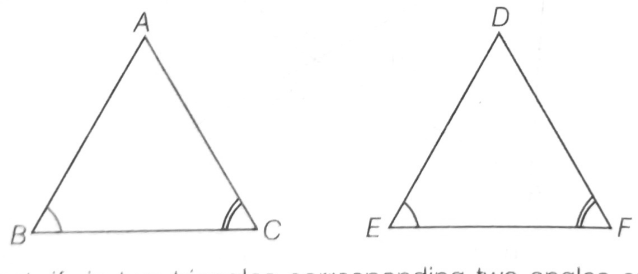
$\Rightarrow$ $\angle F = \angle P$ [corresponding angles of similar triangles]

$\therefore$ $\frac{DF}{QP} = \frac{ED}{RQ} = \frac{FE}{PR}$

Q. 7 In ΔABC and ΔDEF, $\angle B = \angle E$, $\angle F = \angle C$ and $AB = 3DE$. Then, the two triangles are

(a) congruent but not similar (b) similar but not congruent
(c) neither congruent nor similar (d) congruent as well as similar

Sol. *(b)* In ΔABC and ΔDEF, $\angle B = \angle E$, $\angle F = \angle C$ and $AB = 3DE$



We know that, if in two triangles corresponding two angles are same, then they are similar by AAA similarity criterion. Also, ΔABC and ΔDEF do not satisfy any rule of congruency, (SAS, ASA, SSS), so both are not congruent.

Q. 8 If $\Delta ABC \sim \Delta PQR$ with $\frac{BC}{QR} = \frac{1}{3}$, then $\frac{\text{ar}(\Delta PRQ)}{\text{ar}(\Delta BCA)}$ is equal to

(a) 9 (b) 3 (c) $\frac{1}{3}$ (d) $\frac{1}{9}$

Thinking Process

Use the property of area of similar triangle.

Sol. *(a)* Given, $\Delta ABC \sim \Delta PQR$ and $\frac{BC}{QR} = \frac{1}{3}$

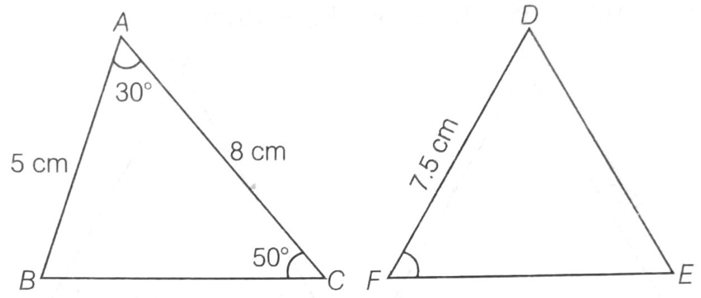
We know that, the ratio of the areas of two similar triangles is equal to square of the ratio of their corresponding sides.

$$\therefore \quad \frac{\text{ar}(\Delta PRQ)}{\text{ar}(\Delta BCA)} = \frac{(QR)^2}{(BC)^2} = \left(\frac{QR}{BC}\right)^2 = \left(\frac{3}{1}\right)^2 = \frac{9}{1} = 9$$

Q. 9 If $\Delta ABC \sim \Delta DFE$, $\angle A = 30^\circ$, $\angle C = 50^\circ$, $AB = 5$ cm, $AC = 8$ cm and $DF = 7.5$ cm. Then, which of the following is true?

(a) $DE = 12$ cm, $\angle F = 50^\circ$
(b) $DE = 12$ cm, $\angle F = 100^\circ$
(c) $EF = 12$ cm, $\angle D = 100^\circ$
(d) $EF = 12$ cm, $\angle D = 30^\circ$

Sol. *(b)* Given, $\Delta ABC \sim \Delta DFE$, then $\angle A = \angle D = 30^\circ$, $\angle C = \angle E = 50^\circ$



$\therefore$ $\angle B = \angle F = 180^\circ - (30^\circ + 50^\circ) = 100^\circ$

Also, $AB = 5$ cm, $AC = 8$ cm and $DF = 7.5$ cm

$\therefore$ $\dfrac{AB}{DF} = \dfrac{AC}{DE}$

$\Rightarrow$ $\dfrac{5}{7.5} = \dfrac{8}{DE}$

$\therefore$ $DE = \dfrac{8 \times 7.5}{5} = 12$ cm

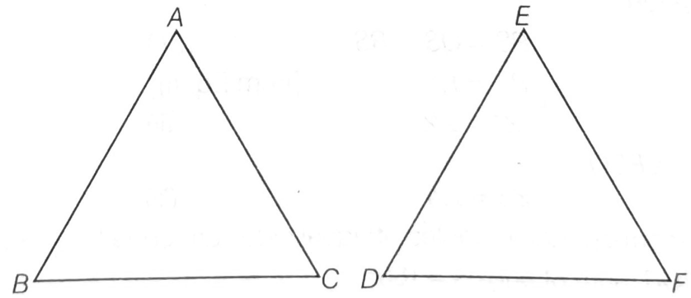
Hence, $DE = 12$ cm, $\angle F = 100^\circ$

Q. 10 If in ΔABC and ΔDEF, $\dfrac{AB}{DE} = \dfrac{BC}{FD}$, then they will be similar, when

(a) $\angle B = \angle E$
(b) $\angle A = \angle D$
(c) $\angle B = \angle D$
(d) $\angle A = \angle F$

Sol. *(c)* Given, in ΔABC and ΔEDF,

$$\frac{AB}{DE} = \frac{BC}{FD}$$



By converse of basic proportionality theorem,

$\Delta ABC \sim \Delta EDF$

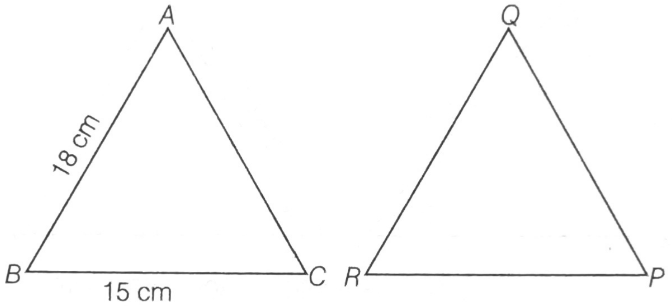
Then, $\angle B = \angle D$, $\angle A = \angle E$

and $\angle C = \angle F$

Q. 11 If $\Delta ABC \sim \Delta QRP$, $\dfrac{ar(\Delta ABC)}{ar(\Delta PQR)} = \dfrac{9}{4}$, $AB = 18$ cm and $BC = 15$ cm, then PR is equal to

(a) 10 cm (b) 12 cm (c) $\dfrac{20}{3}$ cm (d) 8 cm

Sol. ***(a)*** Given, $\Delta ABC \sim \Delta QRP$, $AB = 18$ cm and $BC = 15$ cm



We know that, the ratio of area of two similar triangles is equal to the ratio of square of their corresponding sides.

$\therefore \quad \dfrac{ar(\Delta ABC)}{ar(\Delta QRP)} = \dfrac{(BC)^2}{(RP)^2}$

But given, $\quad \dfrac{ar(\Delta ABC)}{ar(\Delta PQR)} = \dfrac{9}{4}$ [given]

$\Rightarrow \quad \dfrac{(15)^2}{(RP)^2} = \dfrac{9}{4}$ [$\because BC = 15$ cm, given]

$\Rightarrow \quad (RP)^2 = \dfrac{225 \times 4}{9} = 100$

$\therefore \quad RP = 10$ cm

Q. 12 If S is a point on side PQ of a ΔPQR such that $PS = QS = RS$, then

(a) $PR \cdot QR = RS^2$ (b) $QS^2 + RS^2 = QR^2$
(c) $PR^2 + QR^2 = PQ^2$ (d) $PS^2 + RS^2 = PR^2$

Sol. ***(c)*** Given, in ΔPQR,

$PS = QS = RS$ ...(i)

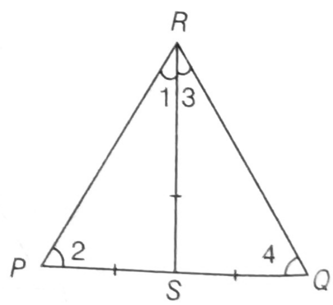
In ΔPSR, $\quad PS = RS$ [from Eq. (i)]

$\Rightarrow \quad \angle 1 = \angle 2$ ...(ii)

Similarly, in ΔRSQ,

$\Rightarrow \quad \angle 3 = \angle 4$ ...(iii)

[corresponding angles of equal sides are equal]



Now, in ΔPQR, sum of angles = 180°

$\Rightarrow \quad \angle P + \angle Q + \angle R = 180°$

$\Rightarrow \quad \angle 2 + \angle 4 + \angle 1 + \angle 3 = 180°$

$\Rightarrow \quad \angle 1 + \angle 3 + \angle 1 + \angle 3 = 180°$ [using Eqs. (ii) and (iii)]

$\Rightarrow \quad 2(\angle 1 + \angle 3) = 180°$

$\Rightarrow \quad \angle 1 + \angle 3 = \dfrac{180°}{2} = 90°$

$\therefore \quad \angle R = 90°$

In ΔPQR, by Pythagoras theorem,

$$PR^2 + QR^2 = PQ^2$$