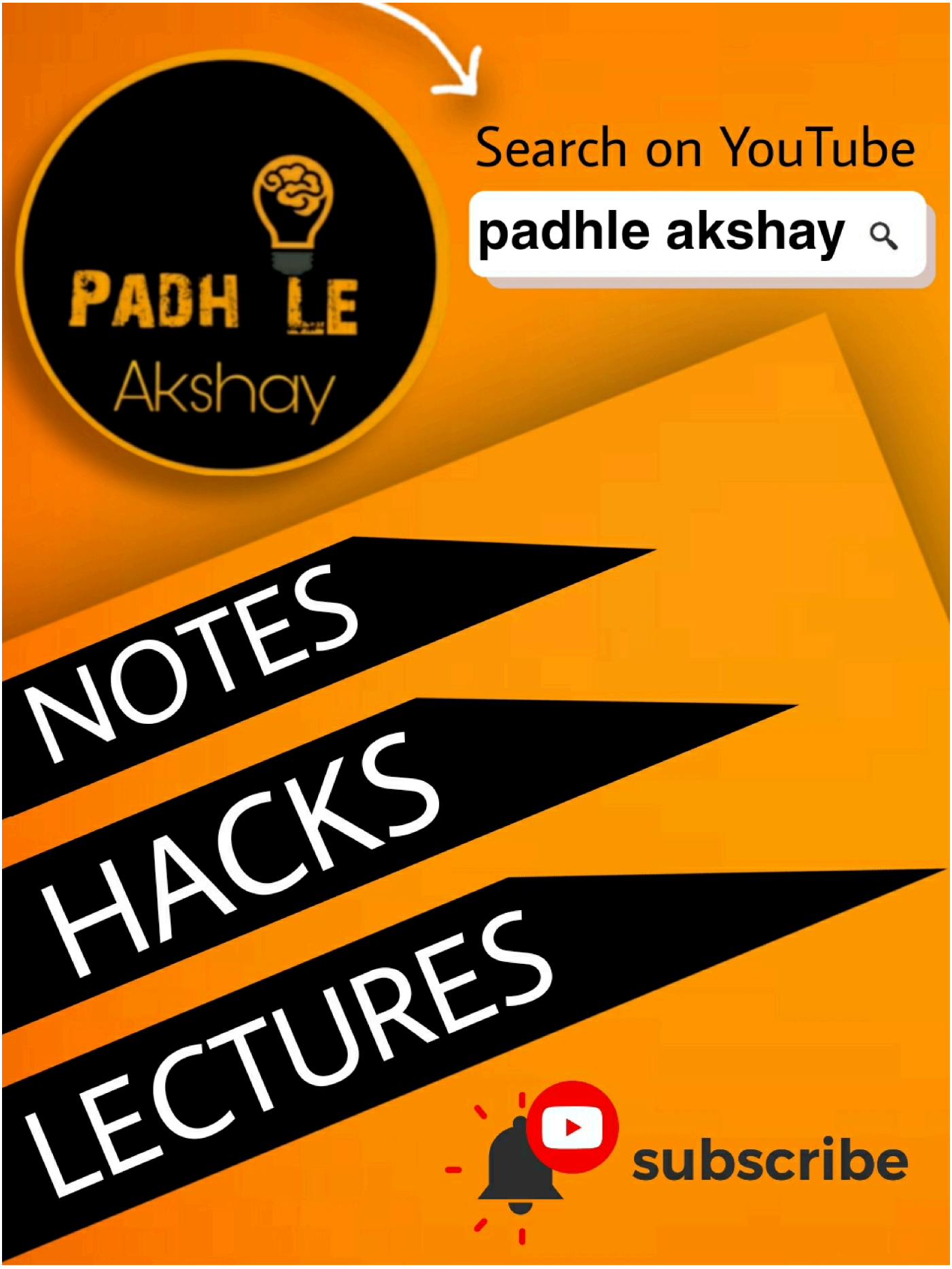
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Chapter-5 Arithmetic Progressions

Very Short Question

Q. 2. Find the 7th term of the sequence whose nth term is given by $a_n = (-1)^{n-1} \cdot n^3$.

[CBSE 2013]

Sol. Given, nth term,

$$a_n = (-1)^{n-1} \cdot n^3$$

$\therefore$ 7th term, $a_7 = (-1)^{7-1} \cdot (7)^3$

$$= (-1)^6 \cdot 7^3$$

$$= 1 \times 343 = 343$$

Q. 3. Is 68 a term of the A.P. 7, 10, 13, ..., ?

[CBSE 2013]

Sol. Given A.P. is 7, 10, 13, ...

Let nth term of the A.P. be 68.

Here, $a = 7$, $d = 10 - 7 = 3$

$\therefore \quad a_n = 68$

$\Rightarrow \quad a + (n-1)d = 68$

$\Rightarrow \quad 7 + (n-1)3 = 68$

$\Rightarrow \quad (n-1)3 = 68 - 7$

$\Rightarrow \quad (n-1)3 = 61$

$\Rightarrow \quad n - 1 = \frac{61}{3}$

$\Rightarrow \quad n = \frac{61}{3} + 1 = \frac{61+3}{3}$

As $n = \frac{64}{3}$ is not a natural number.

So, 68 is not a term of given A.P.

Q. 7. If the numbers $x - 2$, $4x - 1$ and $5x + 2$ are in A.P. Find the value of x.

[NECRT Exemplar; CBSE 2011]

Sol. Given, $x - 2$, $4x - 1$ and $5x + 2$ are in A.P.

$\therefore$ Second term – First term = Third term – Second term

$\Rightarrow (4x - 1) - (x - 2) = (5x + 2) - (4x - 1)$

$\Rightarrow \quad 3x + 1 = x + 3$

$\Rightarrow \quad 2x = 2 \Rightarrow x = 1$

Hence, the value of x is 1.

Q.17. Find, how many two digit natural numbers are divisible by 7. [CBSE 2019]

Sol. The two digits natural numbers which is divisible by 7 are

14, 21, 28 98

This is an AP series with first term $a = 21$, common difference $d = 7$ and last term $l = 8$

Let n be the numbers of terms in the AP series

Now,

$$T_n = l = a + (n-1)d$$

$\Rightarrow \quad 98 = 21 + (n-1) \times 7$

$\Rightarrow \quad n = 13$

$\therefore$ Number of terms (numbers) = 13

Q. 4. Find the natural numbers between 101 and 999 which are divisible by both 2 and 5.

[CBSE 2014]

Sol. Given, $a_1 = 110$, $d = 10$, $a_n = 990$

We know that, $a_n = a_1 + (n-1)d$

$$990 = 110 + (n-1)10$$

$$(n-1) = \frac{990 - 110}{10}$$

$$n = 88 + 1$$

$$n = 89$$

Hence, the natural numbers between 101 and 999 divisible by 2 and 5 are 89.

Chapter-5 Arithmetic Progressions

Short Question

Q.13. The digits of a positive number of three digits are in A.P. and their sum is 15. The number obtained by reversing the digits is 594 less than the original number. Find the number. *HOTS* [CBSE 2016]

Sol. Let $a-d$, a and $a+d$ be the three positive integers of a three digit number, which are in A.P.

Now, sum of the digits $= 15$ [given]

$\therefore \quad a-d+a+a+d=15$

$\Rightarrow \quad 3a=15$

$\Rightarrow \quad a=5$

According to the question, we get

$(5+d)100+5\times10+5-d$

$=(5-d)100+5\times10+5+d-594$

$\Rightarrow \quad 500+100d+50+5-d$

$=500-100d+50+5+d-594$

$\Rightarrow \quad 198d=-594$

$\Rightarrow \quad d=-3$

Hence, the three digits are $5-(-3)$, 5 and $5-3$, (i.e., 8, 5 and 2)

Hence, the number is 852.

Q.14. The 16th term of an A.P. is 1 more than twice its 8th term. If the 12th term of the A.P. is 47, then find its nth term. [CBSE 2012]

Sol. Here, $a_{12}=47$

$\Rightarrow \quad a+11d=47 \quad \ldots(1)$

and $\quad a_{16}=1+2a_8$

$\Rightarrow \quad a+15d=1+2(a+7d)$

$\Rightarrow 2a-a+14d-15d=-1$

$a-d=-1 \quad \ldots(2)$

Subtracting eq.(2) from eq.(1), we get

$\Rightarrow \quad 12d=48$

$\Rightarrow \quad d=4$

Putting the value of d in equation (2),

$a-4=-1$

$\Rightarrow \quad a=3$

$a_n=3+(n-1)4$

$[\because a_n=a+(n-1)d]$

$=4n-1$

Hence, the nth term is $4n-1$.

Q.15. The 8th term of an A.P. is equal to three times its third term. If its 6th term is 22, find the A.P. [CBSE 2013]

Sol. Let a and d be the first term and common difference of the required A.P.

$\therefore \quad a_6=a+5d=22 \quad \ldots(1)$

and $\quad a_8=3(a_3)$

$\Rightarrow \quad a+7d=3(a+2d)=3a+6d$

$\Rightarrow \quad 2a-d=0$

$\Rightarrow \quad d=2a \quad \ldots(2)$

From eqs.(1) and (2), we get

$a+5(2a)=22$

$11a=22$

$\Rightarrow \quad a=2$

Putting the value of a in eq.(2)

$d=2\times2=4$

Hence, the required A.P. is 2, 6, 10, 14,...

Q.16. Which term of the A.P.: 121, 117, 113, ..., is its first negative term?

[Hint: Find n for $a_n<0$] [NCERT, CBSE 2012]

Sol. Given A.P. is 121, 117, 113 .....

Here, $a=121$ and $d=117-121=-4$

nth term of A.P. $\quad a_n=a+(n-1)d$

$=121+(n-1)\times(-4)$

$=121-4n+4=125-4n$

For the first negative term, we get

$a_n<0$

$\Rightarrow \quad (125-4n)<0$

$\Rightarrow \quad 125<4n \quad \Rightarrow \quad \frac{125}{4}<n$

$\Rightarrow \quad 31\frac{1}{4}<n \quad$ or $\quad n>31\frac{1}{4}$

Least integral value of $n=32$

Hence, 32nd term of the given A.P. is the first negative term.

Q.17. The 4th term of an A.P. is zero. Prove that the 25th term of the A.P. is three times its 11th term. [CBSE 2016]

Sol. Let a and d be the first term and common difference of given A.P.

Given, 4th term, $a_4=0$

$\Rightarrow \quad a+3d=0 \quad \ldots(1)$

Now, $\quad a_{25}=a+24d$

Also, $\quad 3a_{11}=3(a+10d)=3a+30d$

$=a+24d+2a+6d$

$=a+24d+2(a+3d)$

$=a+24d+0$

[using eq.(1)]

$=a+24d=a_{25}$

Long Question

Q. 1. The 17th term of an A.P. is 5 more than twice its 8th term. If the 11th term of the A.P. is 43, then find its nth term. [CBSE 2012]

Sol. Here, $a_{11} = 43$

$$a + 10d = 43 \quad [\because a_n = a + (n-1)d] \quad \ldots(1)$$

and

$$a_{17} = 5 + 2a_8$$

$$a + 16d = 5 + 2[a + 7d] \quad [\because a_n = a + (n-1)d]$$

$$a + 16d = 5 + 2a + 14d$$

$$\Rightarrow \quad a - 2d = -5 \quad \ldots(2)$$

Subtracting eq.(2) from eq.(1), we get

$$a + 10d = 43$$

$$a - 2d = -5$$

$$- \quad + \quad +$$

$$12d = 48$$

$$\Rightarrow \quad d = 4 \quad \ldots$$

From eqs.(3) and (1), we get

$$a + 40 = 43$$

$$\Rightarrow \quad a = 3$$

We know that, $a_n = a + (n-1)d$

$$a_n = 3 + (n-1)4$$

$$= 3 + 4n - 4$$

$$= 4n - 1$$

Hence, nth term of given A.P. is $4n - 1$.

Chapter 5
Arithmetic Progressions

Exercise No. 5.1

Multiple Choice Questions:

Question: 1
Choose the correct answer from the given four options in the following questions:

1. In an AP, if $d = -4, n = 7, a_n = 4$, then a is
A. 6
B. 7
C. 20
D. 28

Solution:

(D) 28

In an A.P,
$a_n = a + (n - 1)d$
(a = first term, an is nth term and d is the common difference)

$4 = a + (7 - 1)(-4)$
$4 = a - 24$
$a = 24 + 4$
$= 28$

2. In an AP, if $a = 3.5, d = 0, n = 101$, then a_n will be
A. 0
B. 3.5
C. 103.5
D. 104.5

Solution:

(B) 3.5

In an A.P,
$a_n = a + (n - 1)d$
(a = first term, an is nth term and d is the common difference)

$a_n = 3.5 + (101 - 1)0$
$= 3.5$
(Since, d = 0, it's a constant A.P)

3. The list of numbers – 10, – 6, – 2, 2, … is

A. an AP with d = – 16

B. an AP with d = 4

C. an AP with d = – 4

D. not an AP

Solution:

In the given A.P,

$a_1 = -10$

$a_2 = -6$

$a_3 = -2$

$a_4 = 2$

$a_2 - a_1 = 4$

$a_3 - a_2 = 4$

$a_4 - a_3 = 4$

$a_2 - a_1 = a_3 - a_2$

$= a_4 - a_3$

$= 4$

So, it is an A.P with d = 4.

4. The 11th term of the AP: $-5, \frac{-5}{2}, 0, \frac{5}{2}, \ldots$ is

A. –20

B. 20

C. –30

D. 30

Solution:

According to the given A.P.

$a = -5$

$d = 5 - (-\frac{5}{2})$

$= 5/2$

$n = 11$

Also,

$an = a + (n - 1)d$

Here, (a = first term, an is nth term and d is the common difference)

$a_{11} = -5 + (11 - 1)(\frac{5}{2})$

$a_{11} = -5 + 25$

$= 20$

5. The first four terms of an AP, whose first term is –2 and the common difference is –2, are

A. – 2, 0, 2, 4

B. – 2, 4, – 8, 16

C. – 2, – 4, – 6, – 8

D. – 2, – 4, – 8, –16

Solution:

First term,

$a = -2$

Second Term,

$d = -2$

$a_1 = a$

$= -2$

Also,

$a_n = a + (n - 1)d$

Where,

a = first term, a_n is nth term, d is the common difference

Therefore,

$a_2 = a + d$

$= -2 + (-2)$

$= -4$

Similarly,

$a_3 = -6$

$a_4 = -8$

So the A.P is– 2, – 4, – 6, – 8.

6. The 21st term of the AP whose first two terms are –3 and 4 is

A. 17

B. 137

C. 143

D. –143

Solution:

First two terms of an AP are $a = -3$ and $a_2 = 4$.

We know, nth term of an AP is

$a_n = a + (n - 1)d$

Here, a = first term, a_n is nth term, d is the common difference

$a_2 = a + d$

$4 = -3 + d$

$d = 7$

Common difference,

$d = 7$

$a_{21} = a + 20d$

$= -3 + (20)(7)$

$= 137$

7. If the 2^{nd} term of an AP is 13 and the 5^{th}term is 25, what is its 7^{th} term?

A. 30
B. 33
C. 37
D. 38

Solution:

In an A.P.

$a_n = a + (n - 1)d$

Here, a = first term, a_n is nth term, d is the common difference

$a_2 = a + d$

$= 13$ …..(i)

$a_5 = a + 4d$

$= 25$ … (ii)

From equation (i),

$a = 13 - d$

Using this in equation (ii),

$13 - d + 4d = 25$

$13 + 3d = 25$

$3d = 12$

$d = 4$

$a = 13 - 4$

$= 9$

$a_7 = a + 6d$

$= 9 + 6(4)$

$= 9 + 24$

= 33

8. Which term of the AP: 21, 42, 63, 84… is 210?

A. 9^{th}

B. 10^{th}

C. 11^{th}

D. 12^{th}

Solution:

Let nth term of the given AP be 210.

According to question,

First term,

a = 21

Common difference,

d = 42 – 21

= 21

$a_n = 210$

We know that the nth term of an AP is $a_n = a + (n – 1)d$

Where, a = first term, a_n is nth term, d is the common difference

210 = 21 + (n – 1)21

189 = (n – 1)21

n – 1 = 9

n = 10

So, 10th term of an AP is 210.

9. If the common difference of an AP is 5, then what is $a_{18} - a_{13}$?

A. 5

B. 20

C. 25

D. 30

Solution:

Given, d = 5

Now,

As we know, nth term of an AP is

$a_n = a + (n - 1)d$

Here, a = first term, a_n is nth term, d is the common difference

$a_{18} - a_{13} = a + 17d - (a + 12d)$

$= 5d$

$= 5(5)$

$= 25$

10. What is the common difference of an AP in which $a_{18} - a_{14} = 32$?

A. 8
B. – 8
C. – 4
D. 4

Solution:

(a)

$a_{18} - a_{14} = 32$

$[a + (18 - 1)d] - [a + (14 - 1)d] = 32$ $[a_n = a + (n - 1)d]$

$a + 17d - a - 13d = 32$

$4d = 32$

$d = 8$

So, (a) is the correct answer.

11. Two APs have the same common difference. The first term of one of these is –1 and that of the other is – 8. Then the difference between their 4^{th} terms is

A. -1
B. -8
C. 7
D. -9

Solution:

(c)

According to question,

a_1 (for the first AP)= –1 and a_1 (for the second AP)

= - 8

Let d be the same common difference of two A.Ps.

$d_1 = d$,

$a_n = a + (n - 1)d$

Now,

a_4 of first AP - a_4 of second AP

$(-1 + 3d) - [-8 + 3d]$

$-1 + 3d + 8 - 3d = 7$

So, the required answer is (c).

12. If 7 times the 7^{th} term of an AP is equal to 11 times its 11^{th} term, then its 18^{th} term will be

A. 7
B. 11
C. 18
D. 0

Solution:

(d)

$a_{18} = a + (18 - 1)d$

$= a + 17d$

Also,

$7a_7 = 11a_{11}$ (Given)

$7[a + (7 - 1)d] = 11[a + (11 - 1)d]$

$7[a + 6d] = 11[a + 10d]$

$7a + 42d = 11a + 110d$

$0 = 11a - 7a + 110d - 42d$

$0 = 4a + 68d$

$0 = a + 17d$

$a_{18} = 0$

So, (d) is the correct answer.

13. The 4th term from the end of the AP: –11, –8, –5, ..., 49 is
A. 37
B. 40
C. 43
D. 58

Solution:

(b)
Reversing the A.P., we get
49... –5, –8, and –11

$d = -8 - (-5)$
$= -8 + 5$
$= -3$

$a = 49$ and $n = 4$
$a_n = a + (n - 1)d$
$a_4 = 49 + (4 - 1)(-3)$
$a_4 = 49 + 3(-3)$
$= 49 - 9$

$a_4 = 40$
So, the required value of a_4 is 40 and answer is (b).

14. The famous mathematician associated with finding the sum of the first 100 natural numbers is

A. Pythagoras
B. Newton
C. Gauss
D. Euclid

Solution:

(c)

Gauss is the famous mathematician associated with finding the sum of first 100 natural numbers, i.e., 1 + 2 + 3 + 4 + 5 + ... + 100

a = 1, d = 1, n = 100

As, $S_n = \frac{n}{2}$ [2a + (n – 1)d]

$S_{100} = \frac{100}{2}$ [2(1) + (100 – 1)1]

$= \frac{100}{2}$ [2 + 99]

$= 50 \times 101$

$= 5050$

15. If the first term of an AP is –5 and the common difference is 2, then the sum ofthe first 6 terms is

A. 0
B. 5
C. 6
D. 15

Solution:

(a)

a = –5,

d = 2,

n = 6

We have,

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_6 = \frac{6}{2}[2(-5) + (6-1)2]$

$= 3[-10 + 5 \times 2]$

$= 3[-10 + 10]$

$= 3[0]$

$S_6 = 0$

So, (a) is the correct answer.

16. The sum of first 16 terms of the AP: 10, 6, 2,... is

A. –320

B. 320

C. –352

D. –400

Solution:

(a)

a = 10,

n = 16,

d = 6 – 10

= –4

$S_n = [2a + (n-1)d]$

$S_{16} = [2 \times 10 + (16-1)(-4)]$

$=8[20 + 15(-4)]$

$= 8[20 - 60]$

$= 8 \times (-40)$

$S_{16} = -320$

So, the required answer is (a).

17. In an AP if $a = 1, a_n = 20$ and $S_n = 399$, then *n* is

A. 19
B. 21
C. 38
D. 42

Solution:

(c)

$S_n = [2a + (n - 1)d]$

$S_n = [a + a + (n - 1)d]$

$399 = [a + a_n]$ (a_n = last term)

$399 = [1 + 20]$

$n = 38$

So, (c) is the correct answer.

18. The sum of first five multiples of 3 is

A. 45
B. 55
C. 65
D. 75

Solution:

(a)

1st five multiples of 3 are 3, 6, 9, 12, 15, …..
$a = 3,$
$n = 5,$
$d = 6 - 3$
$= 3$

$S_5 = \frac{5}{2} [2 \times 3 + (5 - 1)3]$

$S_5 = \frac{5}{2} [6 + 12]$

$$= \frac{5}{2} \times 18$$
$$= 45$$

(a) is the correct answer.