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TURN ON

NOTIFICATIONS

REAL NUMBERS

Rational

$\frac{p}{q}$; $q \neq 0$ ✓

- If $q = 2^m \times 5^n$
  - Decimal expansion
  - **Terminating** (END)
  - e.g $\frac{9}{10} = 0.9$
  - e.g $\frac{4}{5} = 0.2$
- If $q \neq 2^m \times 5^n$
  - Decimal expansion
  - **Non term. repeating**
  - e.g $0.12121212\ldots$

Irrational

$\frac{p}{q}$; $q \neq 0$ ✗

- Non Terminatin + Non Repeating
  - e.g $\sqrt{2} = 1.414213\ldots$
  - $\sqrt{3} = 1.73205\ldots$

$$\boxed{\text{Composite number} = \text{Product of primes}}$$

Euclid's Division Lemma

$a = bq + r$, where $[0 \le r < b]$

Dividend, divisor, quotient, remainder

INTEGERS
- Positive
  - Even $2m$
  - Odd $2m+1$
- Negative
  - Even $-2m$
  - odd $1-2m$

[where m is +ve]

HCF or LCM ?

If To Find < Given values
→ Use H.C.F.

If To Find > Given values
→ Use L.C.M.

$$\boxed{HCF \times LCM = a \times b}$$

Chapter-1 Real Number

Very Short Question

Q. 4. Explain whether $3 \times 12 \times 101 + 4$ is a prime number or a composite number. [CBSE 2015]

Sol. $3 \times 12 \times 101 + 4 = 4(3 \times 3 \times 101 + 1)$
$= 4(909 + 1)$
$= 4(910) =$ a composite number
[$\because$ Product of more than two factors]

Q. 5. Find after how many places of decimal the decimal form of the number $\frac{27}{2^3.5^4.3^2}$ will terminate.

Sol.
$$\frac{27}{2^3 5^4 3^2} = \frac{3^3}{3^2 \cdot (2.5)^3 \cdot 5}$$
$$= \frac{3}{5 \cdot 1000} = \frac{6}{1000} = 0.0006$$

Q. 5. Find LCM of numbers whose prime factorisation are expressible as 3×5^2 and $3^2 \times 7^2$. [CBSE 2014]

Sol. LCM of given two numbers $= 3^2 \times 5^2 \times 7^2$
$= 9 \times 25 \times 49 = 11025.$

Q. 1. How many irrational numbers lie between $\sqrt{2}$ and $\sqrt{3}$? Write any two of them. [CBSE 2014]

Sol. ~~Infinite irrational numbers lie between~~ $\sqrt{2}$ and $\sqrt{3}$. We know that $\sqrt{2} = 1.414....$ and $\sqrt{3} = 1.732...$

Thus, two irrational numbers lie between $\sqrt{2}$ and $\sqrt{3}$ are 1.4242242224.... and 1.5050050005...

Q. 6. Given that HCF (306, 1314) = 18. Find LCM (306, 1314). [CBSE 2013]

Sol. Given HCF $(306, 1314) = 18$; LCM $(306, 1314) = ?$

Let $a = 306$, $b = 1314$

We know that

$$a \times b = \text{LCM } (a, b) \times \text{HCF } (a, b)$$
$$\Rightarrow 306 \times 1314 = \text{LCM } (a, b) \times 18$$
$$\Rightarrow \text{LCM } (a, b) = \frac{306 \times 1314}{18}$$
$$\therefore \text{LCM } (306, 1314) = 22338$$

Q.2. Show that $5\sqrt{6}$ is an irrational number. [CBSE 2015]

Sol. Let $5\sqrt{6}$ be a rational number, which can be put in the from $\frac{a}{b}$, where $b \neq 0$; a and b are co-prime.

$$\therefore \quad 5\sqrt{6} = \frac{a}{b}$$

Q.3. Write the denominator of the rational number $\frac{257}{500}$ in the form $2^m \times 5^n$, where m and n are non-negative integers. Hence, write its decimal expansion without actual division. [CBSE 2012]

Sol. Denominator $= 500 = 2^2 \times 5^3$

Decimal expansion,
$$\frac{257}{500} = \frac{257 \times 2}{2 \times 2^2 \times 5^3} = \frac{514}{10^3} = 0.514$$

So, the prime factors of q are 2 and 5.

Q. 1. Is $(\pi - \frac{22}{7})$ a rational number, an irrational number or zero? [CBSE 2014]

Sol. We know that π is an irrational number and $\frac{22}{7}$ is a rational number.

Hence, $(\pi - \frac{22}{7})$ is an irrational number.

Short Question

Q.18. The LCM of two numbers is 14 times their HCF. The sum of LCM and HCF is 600. If one number is 280, then find the other number. [CBSE 2012]

Sol. According to the statement of the question, we have

LCM of two numbers $= 14 \times$ HCF of two numbers

Also, $\text{LCM} + \text{HCF} = 600$

$\Rightarrow \quad 14 \times \text{HCF} + \text{HCF} = 600$

$\Rightarrow \quad 5\ \text{HCF} = 600$

$\Rightarrow \quad \text{HCF} = 40$

$\therefore \quad \text{LCM} = 14 \times 40 = 560$

Now, one number is 280

$\therefore\ 280 \times \text{Other number} = 40 \times 560$

$\Rightarrow \quad \text{Other number} = \dfrac{40 \times 560}{280} = 80$

Q. 6. The HCF of 65 and 117 is expressible in the form of $65m - 117$. Find the value of m.

m. Also, find the LCM of 65 and 117 using prime factorisation method. [CBSE 2012,13,17]

Sol. Let us find HCF of 65 and 117.

Here, $117 > 65$

$\therefore \quad 117 = 1 \times 65 + 52;$

$65 = 1 \times 52 + 13; \quad 52 = 4 \times 13 + 0$

Thus, HCF of 65 and 117 is 13.

Now, HCF (65,117) *i.e.*, $13 = 65 \times m - 117$

$\Rightarrow \quad 65 \times m = 13 + 117$

$\Rightarrow \quad 65 \times m = 130$

$\Rightarrow \quad m = \dfrac{130}{65} = 2$

Again, $\text{LCM} \times \text{HCF} = 65 \times 117;$

$\text{LCM} \times 13 = 65 \times 117$

$\text{LCM} = \dfrac{65 \times 117}{13} = 5 \times 1$

Q.8. Prove that $3 + \sqrt{5}$ is an irrational number. [CBSE 2011]

Sol. Let $3 + \sqrt{5}$ is a rational number.

$\therefore \quad 3 + \sqrt{5} = \dfrac{p}{q}, q \neq 0; 3 + \sqrt{5} = \dfrac{p}{q}$

$\sqrt{5} = \dfrac{p}{q} - 3; \quad \sqrt{5} = \dfrac{p - 3q}{q}$

$\sqrt{5}$ is irrational and $\dfrac{p - 3q}{q}$ is rational.

But rational number cannot be equal to an irrational number.

$\therefore\ 3 + \sqrt{5}$ is an irrational number.

Q.8. Show that the square of any positive integer is of the form $4m$ or $4m + 1$, where m is any integer. [CBSE 2012]

Sol. Let $a = 4q + r, 0 \le r < 4$

$\Rightarrow \quad a = 4q, 4q + 1, 4q + 2$ or $4q + 3$

Case I : $a^2 = (4q)^2 = 16q^2 = 4(4q^2) = 4m$

where, $m = 4q^2$

Case II : $a^2 = (4q + 1)^2 = 16q^2 + 8q + 1$

$= 4(4q^2 + 2q) + 1 = 4m + 1,$

where, $m = 4q^2 + 2q$

Case III : $a^2 = (4q + 2)^2 = 16q^2 + 16q + 4$

$= 4(4q^2 + 4q + 1) = 4m,$

where, $m = 4q^2 + 4q + 1$

Case IV : $a^2 = (4q + 3)^2 = 16q^2 + 24q + 9$

$= 16q^2 + 24q + 8 + 1$

$= 4(4q^2 + 6q + 2) + 1 = 4m + 1,$

where, $m = 4q^2 + 6q + 2$

From cases I, II, III and IV, we conclude that the square of any +ve integer is of the form $4m$ or $4m + 1$.

Q.9. Prove that $\sqrt{2}$ is an irrational number. [CBSE 2011]

Sol. Let $\sqrt{2}$ be a rational number.

$\therefore \quad \sqrt{2} = \dfrac{p}{q}$

Where p and q are co-prime integers and $q \neq 0$

$\Rightarrow \quad 2 = \dfrac{p^2}{q^2} \Rightarrow p = 2q^2$

$\Rightarrow \quad p^2$ is divisible by 2

$\therefore \quad p$ is divisible by 2 ...(1)

Let $p = 2r$ for some positive integer r

$\Rightarrow \quad p^2 = 4r^2$ [from eq. (1)]

$\Rightarrow \quad 2q^2 = 4r^2 \Rightarrow q^2 = 2r^2$

$\therefore \quad q^2$ is divisible by 2

$\Rightarrow \quad q$ is divisible by 2 ...(2)

From (1) and (2), p and q are divisible by 2, which contradicts the fact that p and q are co-primes.

Hence, our assumption is false.

$\therefore \quad \sqrt{2}$ is irrational.

Q.14. Find HCF and LCM of 404 and 96 and verify that HCF × LCM = Product of the two given numbers. **[CBSE 2018]**

Sol. Using the factor three for the prime factorization of 404 and 96, we have

$$404 = 2^2 \times 101 \quad \text{and} \quad 96 = 2^5 \times 3 \ldots(1)$$

To find the HCF, we list common prime factor and their smallest exponent in 404 and 96 as under :

Common prime factor = 2

Least exponent = 2

$\therefore \quad HCF = 2^3 = 4$

To find the LCM, we list all prime factors of 404 and 96 and their greatest exponent as follows :

$$y^3 = (3q+2)^3$$

Prime factors of 404 and 96	Greatest Exponent
2	5
3	1
101	1

$\therefore \quad LCM = 2^5 \times 3^1 \times 101^1$

$= 2^5 \times 3^1 \times 100^1 = 9696$

Now, HCF × LCM = 9696 × 4 = 38784

Product of two numbers = 404 × 96 = 38784

Therefoe HCF × LCM = Product of two numbers

Q.20. Find the greatest number that will divide 445, 572 and 699 leaving remainders 4, 5 and 6 respectively. **[CBSE 2017]**

Sol. Since, the remainders are 4, 5 and 6 respectively.

$\therefore$ We have to find the HCF of 445 – 4, 572 – 5 and 699 – 6, *i.e.*, 441, 567 and 693.

For the HCF of 441, 567 and 693, we have

$$441 = 3 \times 3 \times 7 \times 7 = 3^2 \times 7^2$$

$$567 = 3 \times 3 \times 3 \times 3 \times 7 = 3^4 \times 7$$

$$693 = 3 \times 3 \times 7 \times 11 = 3^2 \times 7 \times 11$$

$\therefore \quad$ HCF of 441, 567 and 693 $= 3^2 \times 7$

$= 9 \times 7 = 63$

$\therefore \quad$ The required number = 63

Q.2. Prove that $\sqrt{3}$ is an irrational number. Hence, show that $7 + 2\sqrt{3}$ is also an irrational number. **[Board Term-1, 2012, Set-DDE-M]**

Sol. If possible, let $\sqrt{3}$ be a rational number.

(i) $\therefore \sqrt{3} = \frac{a}{b}$, where a and b are integers and co-primes and $b \neq 0$.

Squaring both sides, we have

$$\frac{a^2}{b^2} = 3$$

or, $\quad a^2 = 3b^2$

$\because$ a^2 is divisible by 3

$\because$ a is divisible by 3. ...(1)

Let $a = 3c$ for any integer c

$$(3c)^2 = 3b^2$$

$$9c^2 = 3b^2$$

$$b^2 = 3c^2$$

Since, b^2 is divisible by 3, so b is divisible by 3. ...(2)

For equations (1) and (2), we have 3 is a factor of a and b which is contradicting the fact that a and b are co-primes.

Thus, our assumption that $\sqrt{3}$ is rational number is wrong.

Hence, $\sqrt{3}$ is an irrational number.

(ii) Let us assume to contrary that $7 + 2\sqrt{3}$ is a rational number.

$\therefore \quad 7 + 2\sqrt{3} = \frac{p}{q},$

$q \neq 0$ and p and q are co-primes

or, $\quad 2\sqrt{3} = \frac{p}{q} - 7$

or, $\quad 2\sqrt{3} = \frac{p - 7q}{q}$

or, $\quad \sqrt{3} = \frac{p - 7q}{2q}$

$p - 7q$ and $2q$ both are integers, hence $\sqrt{3}$ is a rational number.

But this contradicts the fact that $\sqrt{3}$ is an irrational number. Hence, $7 + 2\sqrt{3}$ is irrational.

1

Real Numbers

Exercise 1.1 Multiple Choice Questions (MCQs)

Q. 1 For some integer m, every even integer is of the form

(a) m (b) $m+1$ (c) $2m$ (d) $2m+1$

Sol. *(c)* We know that, even integers are 2, 4, 6,...

So, it can be written in the form of $2m$.

where, $m = \text{Integer} = Z$ [since, integer is represented by Z]

or $m = \cdots, -1, 0, 1, 2, 3, \ldots$

$\therefore$ $2m = \cdots, -2, 0, 2, 4, 6..$

Alternate Method

Let 'a' be a positive integer. On dividing 'a' by 2, let m be the quotient and r be the remainder. Then, by Euclid's division algorithm, we have

$$a = 2m + r, \text{ where } a \le r < 2 \text{ i.e., } r = 0 \text{ and } r = 1.$$

$$\Rightarrow \quad a = 2m \text{ or } a = 2m + 1$$

when, $a = 2m$ for some integer m, then clearly a is even.

Q. 2 For some integer q, every odd integer is of the form

(a) q (b) $q+1$ (c) $2q$ (d) $2q+1$

Sol. *(d)* We know that, odd integers are 1, 3, 5, ⋯

So, it can be written in the form of $2q + 1$.

where, $q = \text{integer} = Z$

or $q = \cdots, -1, 0, 1, 2, 3, \ldots$

$\therefore$ $2q + 1 = \cdots, -3, -1, 1, 3, 5, \ldots$

Alternate Method

Let 'a' be given positive integer. On dividing 'a' by 2, let q be the quotient and r be the remainder. Then, by Euclid's division algorithm, we have

$$a = 2q + r, \text{ where } 0 \le r < 2$$

$$\Rightarrow \quad a = 2q + r, \text{ where } r = 0 \text{ or } r = 1$$

$$\Rightarrow \quad a = 2q \text{ or } 2q + 1$$

when $a = 2q + 1$ for some integer q, then clearly a is odd.

Q. 3 $n^2 - 1$ is divisible by 8, if n is

(a) an integer (b) a natural number

(c) an odd integer (d) an even integer

Sol. ***(c)*** Let $a = n^2 - 1$

Here n can be ever or odd.

Case I n = Even *i.e.*, $n = 2k$, where k is an integer.

$$\Rightarrow \quad a = (2k)^2 - 1$$

$$\Rightarrow \quad a = 4k^2 - 1$$

At $k = -1, = 4(-1)^2 - 1 = 4 - 1 = 3$, which is not divisible by 8.

At $k = 0$, $a = 4(0)^2 - 1 = 0 - 1 = -1$, which is not divisible by 8, which is not

Case II n = Odd *i.e.*, $n = 2k + 1$, where k is an odd integer.

$$\Rightarrow \quad a = 2k + 1$$

$$\Rightarrow \quad a = (2k + 1)^2 - 1$$

$$\Rightarrow \quad a = 4K^2 + 4k + 1 - 1$$

$$\Rightarrow \quad a = 4k^2 + 4k$$

$$\Rightarrow \quad a = 4k(k + 1)$$

At $k = -1$, $a = 4(-1)(-1 + 1) = 0$ which is divisible by 8.

At $k = 0$, $a = 4(0)(0 + 1) = 4$ which is divisible by 8.

At $k = 1$, $a = 4(1)(1 + 1) = 8$ which is divisible by 8.

Hence, we can conclude from above two cases, if n is odd, then $n^2 - 1$ is divisible by 8.

Q. 4 If the HCF of 65 and 117 is expressible in the form $65m - 117$, then the value of m is

(a) 4 (b) 2 (c) 1 (d) 3

Thinking Process

Apply Euclid's division algorithm until the remainder is 0. Finally we get divisor, which is the required HCF of 65 and 117. Now, put 65 m − 117 = HCF (65, 117) and get the value of m.

Sol. ***(b)*** By Euclid's division algorithm,

$$b = aq + r, 0 \le r < a \quad [\because \text{dividend} = \text{divisor} \times \text{quotient} + \text{remainder}]$$

$$\Rightarrow \quad 117 = 65 \times 1 + 52$$

$$\Rightarrow \quad 65 = 52 \times 1 + 13$$

$$\Rightarrow \quad 52 = 13 \times 4 + 0$$

$$\therefore \quad \text{HCF}(65, 117) = 13 \quad \ldots(\text{i})$$

Also, given that, HCF (65, 117) = $65m - 117$ ...(ii)

From Eqs. (i) and (ii),

$$65m - 117 = 13$$

$$\Rightarrow \quad 65m = 130$$

$$\Rightarrow \quad m = 2$$

Q. 5 The largest number which divides 70 and 125, leaving remainders 5 and 8 respectively, is

(a) 13 (b) 65 (c) 875 (d) 1750

Thinking Process

First, we subtract the remainders 5 and 8 from corresponding numbers respectively and then get HCF of resulting numbers by using Euclid's division algorithm, which is the required largest number.

Sol. (a) Since, 5 and 8 are the remainders of 70 and 125, respectively. Thus, after subtracting these remainders from the numbers, we have the numbers $65 = (70 - 5)$, $117 = (125 - 8)$, which is divisible by the required number.

Now, required number = HCF of 65, 117 [for the largest number]

For this, $117 = 65 \times 1 + 52$ [$\because$ dividend = divisor × quotient + remainder]

$\Rightarrow$ $65 = 52 \times 1 + 13$

$\Rightarrow$ $52 = 13 \times 4 + 0$

$\therefore$ HCF = 13

Hence, 13 is the largest number which divides 70 and 125, leaving remainders 5 and 8.

Q. 6 If two positive integers a and b are written as $a = x^3y^2$ and $b = xy^3$, where x, y are prime numbers, then HCF (a, b) is

(a) xy (b) xy^2 (c) x^3y^3 (d) x^2y^2

Sol. (b) Given that, $a = x^3 y^2 = x \times x \times x \times y \times y$

and $b = xy^3 = x \times y \times y \times y$

$\therefore$ HCF of a and b $= \text{HCF}(x^3y^2, xy^3) = x \times y \times y = xy^2$

[since, HCF is the product of the smallest power of each common prime facter involved in the numbers]

Q. 7 If two positive integers p and q can be expressed as $p = ab^2$ and $q = a^3b$; where a, b being prime numbers, then LCM (p, q) is equal to

(a) ab (b) a^2b^2 (c) a^3b^2 (d) a^3b^3

Sol. (c) Given that, $p = ab^2 = a \times b \times b$

and $q = a^3b = a \times a \times a \times b$

$\therefore$ LCM of p and q $= \text{LCM}(ab^2, a^3b) = a \times b \times b \times a \times a = a^3b^2$

[since, LCM is the product of the greatest power of each prime factor involved in the numbers]

Q. 8 The product of a non-zero rational and an irrational number is

(a) always irrational (b) always rational

(c) rational or irrational (d) one

Sol. (a) Product of a non-zero rational and an irrational number is always irrational *i.e.*, $\frac{3}{4} \times \sqrt{2} = \frac{3\sqrt{2}}{4}$ (irrational).

Q. 9 The least number that is divisible by all the numbers from 1 to 10 (both inclusive)

(a) 10 (b) 100 (c) 504 (d) 2520

Sol. ***(d)*** Factors of 1 to 10 numbers

$$
\begin{aligned}
1 &= 1\\
2 &= 1\times 2\\
3 &= 1\times 3\\
4 &= 1\times 2\times 2\\
5 &= 1\times 5\\
6 &= 1\times 2\times 3\\
7 &= 1\times 7\\
8 &= 1\times 2\times 2\times 2\\
9 &= 1\times 3\times 3\\
10 &= 1\times 2\times 5
\end{aligned}
$$

$\therefore$ LCM of number 1 to 10 $=$ LCM (1, 2, 3, 4, 5, 6, 7, 8, 9, 10)

$$= 1\times 2\times 2\times 2\times 3\times 3\times 5\times 7 = 2520$$

Q. 10 The decimal expansion of the rational number $\dfrac{14587}{1250}$ will terminate after

(a) one decimal place (b) two decimal places
(c) three decimal places (d) four decimal places

Thinking Process

In terminating rational number the denominator always have the form $2^m \times 5^n$.

Sol. ***(d)*** Rational number $= \dfrac{14587}{1250} = \dfrac{14587}{2^1\times 5^4}$

$$= \frac{14587}{10\times 5^3}\times\frac{(2)^3}{(2)^3}$$

$$= \frac{14587\times 8}{10\times 1000}$$

$$= \frac{116696}{10000} = 11.6696$$

$$
\begin{array}{r|l}
2 & 1250 \\ \hline
5 & 625 \\ \hline
5 & 125 \\ \hline
5 & 25 \\ \hline
5 & 5 \\ \hline
 & 1
\end{array}
$$

Hence, given rational number will terminate after four decimal places.