Areas Related to Circles

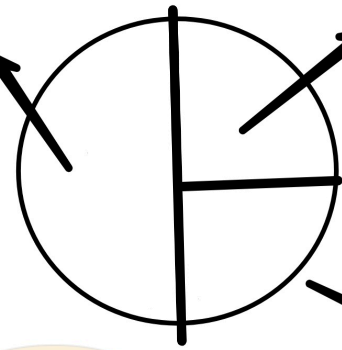
ALERT NOTES!

Area of quadrant

$\frac{\pi R^2}{4}$

Area of semicircle

$\frac{\pi R^2}{2}$



Area of circle

πR^2

Perimeter of circle $2\pi r$

Area enclosed by 2 concentric circles :-

$\pi(R^2 - r^2)$

$= \pi(R-r)(R+r)$

Rotating wheel or planet :-

Distance in 1 revolution

$2\pi R$

Number of Revolutions :-

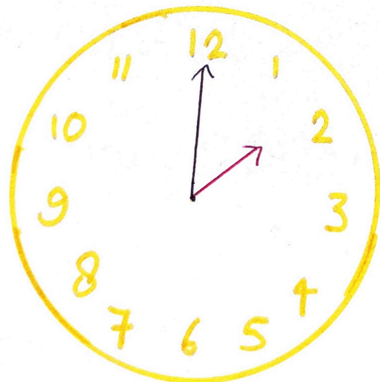
$\nu = \frac{\text{Distance covered}}{2\pi R}$

TIME OF Revolution :-

$t = \frac{2\pi R \ \nu}{\text{Speed}}$

CLOCK

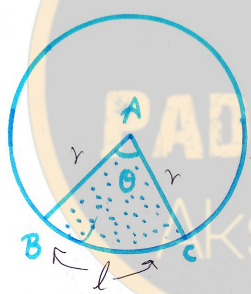
- Area swept by minute hand
  - ↳ in 60 minutes = $360°$
  - ↳ in 1 minute = $6°$



- Area swept by hour hand
  - ↳ in 12 hours = 360
  - ↳ in 1 hour = $30°$
  - ↳ in 1 minute = $0.5°$

Sector Tricks

$$l = \frac{2\pi r\,\theta}{360°}$$

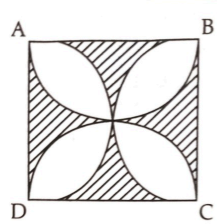


Area

$$\frac{\pi r^2\,\theta}{360°} \qquad \frac{1}{2}\,l r$$

Magic Area Trick

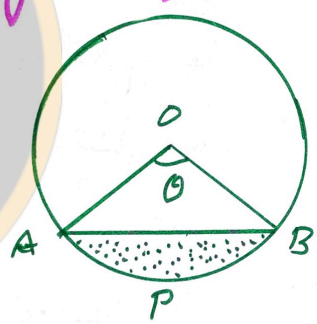
Shaded Area :



ar(sq.) – ar(design in white)

$$\frac{a^2}{2}(\pi - 2)$$

Segment Tricks



> Perimeter = AB + arc $\overparen{APB}$

↳ $\dfrac{2\pi r \sin\theta}{360°} + \dfrac{2\pi r\,\theta}{360°}$

> Minor segment area :

↳ ar (sector) – ar (Δ)

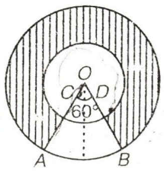
$\theta < 90°$ $\quad \frac{1}{2} r^2 \sin\theta$

$\theta > 90°$ $\quad r^2 \sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)$

Chapter-12 Area Related To Circle

Short Question

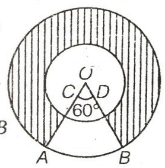
Q.3. In the figure, two concentric circles with centre O, have radii 21 cm and 42 cm. If $\angle AOB = 60°$, find the area of shaded region. [CBSE 2014]



Sol. Area of shaded region = πr^2 – Area of Minor segment

Area of minor segment

= Area of sector – Area of ΔOAB



$= \frac{\theta}{360} \times \pi r^2 - \frac{1}{2} \times (21)^2$

$= \frac{60}{360} \times \frac{22}{7} \times 42 \times 42 - \frac{1}{2} \times 21 \times 21$

$= 924 - 220.5$

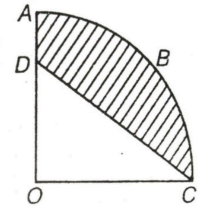
$= 703.5$

$\therefore$ Area of shaded region

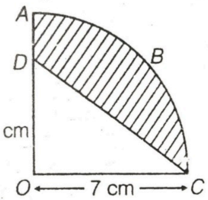
$= \frac{22}{7} \times 42 \times 42 - 703.5$

$= 4840.5\ cm^2$.

Q.2. In the figure, $OABC$ is a quadrant of a circle of radius 7 cm. If $OD = 4$ cm, find the area of shaded region. [CBSE (F) 2014]



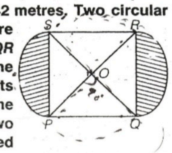
Sol. Area of shaded region = Area of sector $OCBAO$ – Area of ΔODC



$= \frac{90°}{360°} \times \pi \times (7)^2 - \frac{1}{2} \times 7 \times 4$

$= \frac{49\pi}{4} - 14 = 24.46\ cm^2$

Q.1. In the given Figure, $PQRS$ is a square lawn with side $PQ = 42$ metres, Two circular flower beds are there on the sides PS and QR with centre at O, the intersection of its diagonals, Find the total area of the two flower beds (shaded parts). [CBSE (AI) 2011,16]



Sol. $\because$ The diagonals of the square are perpendicular bisector of each other.

$\therefore \quad \angle POS = 90° = \angle QOR$

$\therefore$ Radius of sector OPS

$= \frac{1}{2} \times$ length of diagonal (SQ)

$= \frac{1}{2} \times \sqrt{2} \times \text{Side} = \frac{1}{2} \times \sqrt{2} \times 42$

$\because$ Diagonal of square $= \sqrt{2} \times$ side $= 21\sqrt{2}$ m.

$\therefore$ Also, radius of sector $OQR = 21\sqrt{2}$ m

$\therefore$ Area of (sector $OPSO$ + sector $OQRO$)

$= 2 \times \frac{90°}{360°} \times \frac{22}{7} \times 21\sqrt{2} \times 21\sqrt{2}$

$= 2 \times \frac{1}{4} \times \frac{22}{7} \times 882$

$= \frac{38808}{28} = 1386\ m^2$

Now, area of $(\Delta POQ + \Delta ROS)$

$= \frac{1}{2} \times$ area of square $PQRS$

$= \frac{1}{2} \times 42 \times 42 = 882\ m^2$

Total Area of two flower beds

Q.1. A Farmer has a field in the form of circle. He wants to fencing the field. The field is to be ploughed at the rate of ₹ 0.50 per m^2. If the cost of fencing of a circular field at the rate of ₹ 24 per m is ₹ 5280, then

(i) Find the length of fencing the circular field.

(ii) Find the cost of ploughing the field. (Take, $\pi = \frac{22}{7}$)

(iii) Which value is depicted by the farmer in fencing the field?

Sol. (i) Given, total cost of fencing = ₹ 5280 and rate of fencing per metre = ₹ 24

Length of the fence $= \frac{5280}{24} = 220$ m

(ii) Circumference of the circular field = Length of the fence $2\pi r = 220$

$\Rightarrow \quad 2 \times \frac{22}{7} \times r = 220$

$\Rightarrow \quad r = \frac{220 \times 7}{2 \times 22}$

$\Rightarrow \quad r = 35$ m

i.e., radius of the circular field = 35 m

$\therefore$ Area of the circular field $= \pi r^2$

$= \frac{22}{7} \times (35)^2 = \frac{22}{7} \times 35 \times 35$

$= 22 \times 5 \times 35 = 3850\ m^2$

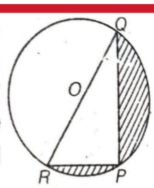
Now, cost of ploughing at the rate of ₹ 0.50 per

m^2 = ₹ 3850 × ₹ 0.50 = ₹ 1925

Hence, total cost of ploughing the field is ₹ 1925.

(iii) Security and separate the boundary of a field.

Q.11. Find the area of the shaded region in the given figure, if $PQ = 24$ cm, $PR = 7$ cm and O is the centre of the circle. [CBSE 2012,14]



Sol. Hence, $PQ = 24$ cm and $PR = 7$ cm

$\angle RPQ = 90°$ (angle in a semi–circle)

In ΔRPQ, using Pythagoras theorem,

$QR^2 = PQ^2 + PR^2$

$QR^2 = 24^2 + 7^2$

$\Rightarrow \quad QR = 25$ cm

radius $= \frac{QR}{2} = \frac{25}{2}$ cm

$\therefore$ Area of the shaded region

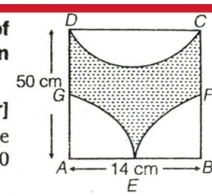
= Area of semi–circle – Area of ΔPOR

$= \frac{22}{7} \times \frac{25}{2} \times \frac{25}{2} \times \frac{1}{2} - \frac{1}{2} \times 7 \times 24$

$= \frac{6875}{28} - 84 = \frac{4523}{28}\ cm^2$

or $161.54\ cm^2$

Q.20. Find the area of the shaded portion from the given figure. [NCERT Exemplar]



Sol. Area of rectangle $ABCD = 50 \times 14 = 700\ cm^2$

Area of two quadrants at A and B

$= 2 \times \frac{1}{4} \times \pi r^2$

$= \frac{1}{2} \times \frac{22}{7} \times 7 \times 7 \quad [\because r = 7$ cm, given]

$= 77\ cm^2$

Area of semi–circle with CD as diameter

$= \frac{1}{2} \times \pi r^2 = \frac{1}{2} \times \frac{22}{7} \times 7 \times 7 = 77\ cm^2$

Now, area of shaded region

= Area of rectangle – (Area of two quadrants AEG and EBF) + Area of semi-circle with CD as diameter)

$= 700 - (77 + 77) = 546\ cm^2$

Hence, the required area of the shaded portion is $546\ cm^2$.

Q.23. Find the area of the minor segment of a circle of radius 14 cm, when its central angle is 60°. Also find the area of the corresponding major segment. [Use $\pi = \frac{22}{7}$]

[CBSE (AI) 2015]

Sol. Given, radius of a circle, $r = 14$ cm and central angle, $\theta = 60°$

$\therefore$ Area of minor segment $= \frac{\pi r^2 \theta}{360°} - \frac{1}{2} r^2 \sin\theta$

$= \frac{22}{7} \times (14)^2 \times \frac{60°}{360°} - \frac{1}{2}(14)^2 \times \sin 60°$

$= \frac{22}{7} \times 14 \times 14 \times \frac{1}{6} - \frac{1}{2} \times 14 \times 14 \times \frac{\sqrt{3}}{2}$

$= \left(\frac{308}{3} - 49\sqrt{3}\right) = \frac{308}{3} - 49 \times 1.73$

$= \frac{308}{3} - 84.77 = 102.67 - 84.77$

$= 17.9 \text{ cm}^2$ (approx.)

and area of major segment = Area of circle – Area of minor segment

$= \pi r^2 - \left[\frac{308}{3} - 49\sqrt{3}\right]$

$= \frac{22}{7} \times 14 \times 14 - \left[\frac{308}{3} - 49\sqrt{3}\right]$

$= 616 - \frac{308}{3} + 49\sqrt{3}$

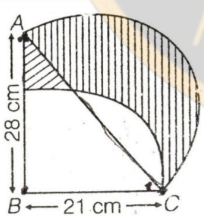
$= \frac{1848 - 308}{3} + 49\sqrt{3}$

$= \frac{1540}{3} + 49\sqrt{3} = 513.33 + 49 \times 1.73$

$= 513.33 + 84.77 = 598.1 \text{ cm}^2$

$= 598 \text{ cm}^2$ (approx.)

Q.7. In the figure, ABC is a right-angled triangle, $\angle B = 90°$, $AB = 28$ cm and $BC = 21$ cm. With AC as diameter, a semi-circle is drawn and with BC as radius a quarter circle is drawn. Find the area of the shaded region. ***HOTS***



[CBSE 2011, 14]

Sol. In right ΔABC, right angled at B,

$AC^2 = AB^2 + BC^2 = 28^2 + 21^2$

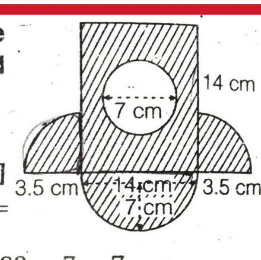
$AC = 35$ cm

Area of shaded region = area of ΔABC + area of semi-circle with diameter AC – area of quadrant with radius BC

$= \frac{1}{2}(21 \times 28) + \frac{1}{2} \times \frac{22}{7} \times \frac{35}{2} \times \frac{35}{2} - \frac{1}{4} \times \frac{22}{7} \times 21 \times 21$

$= \frac{1715}{4} = 428.75 \text{ cm}^2$

Q.9. In figure, find the area of the shaded region. $\left[\text{Use } \pi = \frac{22}{7}\right]$



[CBSE 2011]

Sol. Area of square $= (14)^2 \text{ cm}^2 = 196 \text{ cm}^2$

Area of internal circle $= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \text{ cm}^2$

$= \frac{77}{2} \text{ cm}^2$

Area of semi–circle with 14 cm diameter

$= \frac{1}{2} \times \frac{22}{7} \times 7^2 \text{ cm}^2 = 77 \text{ cm}^2$

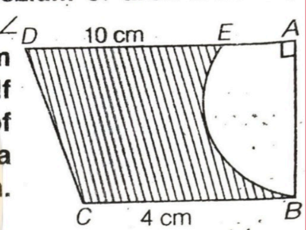
Area of two quarter circles of radius $\frac{7}{2}$ cm

$= 2 \times \frac{1}{4} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} = \frac{77}{4} \text{ cm}^2$

Area of shaded region = 196 – 38.5 + 77 + 19.25

$= 292.25 - 38.5 = 253.75 \text{ cm}^2$

Q.8. $ABCD$ is a trapezium of area 24.5 sq. cm. In it, $AD \parallel BC$, $\angle DAB = 90°$, $AD = 10$ cm and $BC = 4$ cm. If $ABEA$ is a quadrant of a circle, find the area of shaded region. $\left[\text{Use } \pi = \frac{22}{7}\right]$



[CBSE 2014]

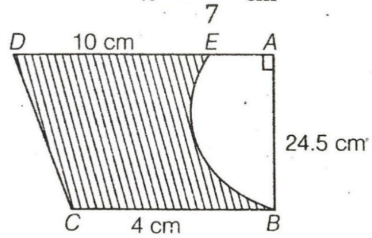
Sol. Area of shaded region = Area of trapezium – (Area of sector ABE)

Given, Area of trapezium $= 24.5 \text{ cm}^2$

$\frac{1}{2} h(a + b) = 24.5$

$\frac{1}{2} h(10 + 4) = 24.5$

$h = \frac{24.5}{7}$ cm



Area of shaded region $= 24.5 - \frac{90°}{360°} \times \frac{22}{7} \times \frac{24.5}{7} \times \frac{24.5}{7}$

$= 24.5 - 0.9625$

Required Area $= 23.54 \text{ cm}^2$.

11

Areas Related *to* Circle

Exercise 11.1 Multiple Choice Questions (MCQs)

Q. 1 If the sum of the areas of two circles with radii R_1 and R_2 is equal to the area of a circle of radius R, then

(a) $R_1 + R_2 = R$
(b) $R_1^2 + R_2^2 = R^2$
(c) $R_1 + R_2 < R$
(d) $R_1^2 + R_2^2 < R^2$

Sol. ***(b)*** According to the given condition,

Area of circle = Area of first circle + Area of second circle

$$\therefore \quad \pi R^2 = \pi R_1^2 + \pi R_2^2$$

$$\Rightarrow \quad R^2 = R_1^2 + R_2^2$$

Q. 2 If the sum of the circumferences of two circles with radii R_1 and R_2 is equal to the circumference of a circle of radius R, then

(a) $R_1 + R_2 = R$
(b) $R_1 + R_2 > R$
(c) $R_1 + R_2 < R$
(d) Nothing definite can be said about the relation among R_1, R_2 and R.

Sol. ***(a)*** According to the given condition,

Circumference of circle = Circumference of first circle + Circumference of second circle

$$\therefore \quad 2\pi R = 2\pi R_1 + 2\pi R_2$$

$$\Rightarrow \quad R = R_1 + R_2$$

Q. 3 If the circumference of a circle and the perimeter of a square are equal, then

(a) Area of the circle = Area of the square
(b) Area of the circle > Area of the square
(c) Area of the circle < Area of the square
(d) Nothing definite can be said about the relation between the areas of the circle and square

Sol. *(b)* According to the given condition,

Circumference of a circle = Perimeter of square

$$2\pi r = 4a$$

[where, r and a are radius of circle and side of square respectively]

$$\Rightarrow \quad \frac{22}{7} r = 2a \Rightarrow 11r = 7a$$

$$\Rightarrow \quad a = \frac{11}{7} r \Rightarrow r = \frac{7a}{11} \quad \text{...(i)}$$

Now, area of circle, $A_1 = \pi r^2$

$$= \pi \left(\frac{7a}{11}\right)^2 = \frac{22}{7} \times \frac{49a^2}{121} \quad \text{[from Eq. (i)]}$$

$$= \frac{14a^2}{11} \quad \text{...(ii)}$$

and area of square, $A_2 = (a)^2$ ...(iii)

From Eqs. (ii) and (iii), $A_1 = \frac{14}{11} A_2$

$\therefore \quad A_1 > A_2$

Hence, Area of the circle > Area of the square.

Q. 4 **Area of the largest triangle that can be inscribed in a semi-circle of radius r units is**

(a) r^2 sq units (b) $\frac{1}{2} r^2$ sq units (c) $2r^2$ sq units (d) $\sqrt{2}\, r^2$ sq units

Sol. *(a)* Take a point C on the circumference of the semi-circle and join it by the end points of diameter A and B.

$\therefore \quad \angle C = 90°$ [by property of circle]

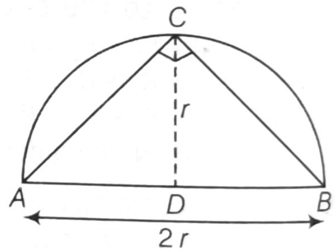
[angle in a semi-circle are right angle]

So, ΔABC is right angled triangle.

$$\therefore \quad \text{Area of largest } \Delta ABC = \frac{1}{2} \times AB \times CD$$

$$= \frac{1}{2} \times 2r \times r$$

$$= r^2 \text{ sq units}$$



Q. 5 **If the perimeter of a circle is equal to that of a square, then the ratio of their areas is**

(a) 22 : 7 (b) 14 : 11 (c) 7 : 22 (d) 11 : 14

Sol. *(b)* Let radius of circle be r and side of a square be a.

According to the given condition,

Perimeter of a circle = Perimeter of a square

$$\therefore \quad 2\pi r = 4a \Rightarrow a = \frac{\pi r}{2} \quad \text{...(i)}$$

Now,

$$\frac{\text{Area of circle}}{\text{Area of square}} = \frac{\pi r^2}{(a)^2} = \frac{\pi r^2}{\left(\frac{\pi r}{2}\right)^2} \quad \text{[from Eq. (i)]}$$

$$= \frac{\pi r^2}{\pi^2 r^2 / 4} = \frac{4}{\pi} = \frac{4}{22/7} = \frac{28}{22} = \frac{14}{11}$$

Q. 6 It is proposed to build a single circular park equal in area to the sum of areas of two circular parks of diameters 16 m and 12 m in a locality. The radius of the new park would be

(a) 10 m (b) 15 m (c) 20 m (d) 24 m

Sol. (a) Area of first circular park, whose diameter is 16 m

$$= \pi r^2 = \pi\left(\frac{16}{2}\right)^2 = 64\,\pi\text{ m}^2 \qquad \left[\because r = \frac{d}{2} = \frac{16}{2} = 8\text{ m}\right]$$

Area of second circular park, whose diameter is 12 m

$$= \pi\left(\frac{12}{2}\right)^2 = \pi\,(6)^2 = 36\,\pi\text{m}^2 \qquad \left[\because r = \frac{d}{2} = \frac{12}{2} = 6\text{ m}\right]$$

According to the given condition,

Area of single circular park = Area of first circular park + Area of second circular park

$$\pi R^2 = 64\,\pi + 36\,\pi \qquad [\because R \text{ be the radius of single circular park}]$$

$$\Rightarrow \qquad \pi R^2 = 100\pi \Rightarrow R^2 = 100$$

$$\therefore \qquad R = 10\text{ m}$$

Q. 7 The area of the circle that can be inscribed in a square of side 6 cm is

(a) 36π cm^2 (b) 18π cm^2 (c) 12π cm^2 (d) 9π cm^2

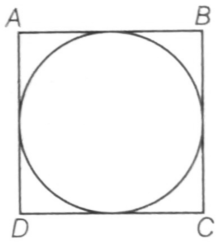
Sol. (d) Given, side of square = 6 cm

$\therefore$ Diameter of a circle, (d) = Side of square = 6 cm

$$\therefore \qquad \text{Radius of a circle } (r) = \frac{d}{2} = \frac{6}{2} = 3\text{ cm}$$

$$\therefore \qquad \text{Area of circle} = \pi\,(r)^2$$

$$= \pi\,(3)^2 = 9\,\pi\text{ cm}^2$$



Q. 8 The area of the square that can be inscribed in a circle of radius 8 cm is

(a) 256 cm^2 (b) 128 cm^2 (c) $64\sqrt{2}$ cm^2 (d) 64 cm^2

Sol. (b) Given, radius of circle, $r = OC = 8$ cm.

$\therefore$ Diameter of the circle = $AC = 2 \times OC = 2 \times 8 = 16$ cm

which is equal to the diagonal of a square.

Let side of square be x.

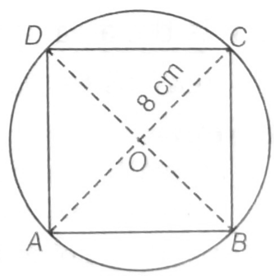
In right angled ΔABC, $AC^2 = AB^2 + BC^2$ [by Pythagoras theorem]

$$\Rightarrow \qquad (16)^2 = x^2 + x^2$$

$$\Rightarrow \qquad 256 = 2x^2$$

$$\Rightarrow \qquad x^2 = 128$$

$$\therefore \qquad \text{Area of square} = x^2 = 128\text{ cm}^2$$



Alternate Method

Radius of circle (r) = 8 cm

Diameter of circle (d) = $2r = 2 \times 8 = 16$ cm

Since, square inscribed in circle.

$\therefore$ Diagonal of the squre = Diameter of circle

$$\text{Now,} \qquad \text{Area of square} = \frac{(\text{Diagonal})^2}{2} = \frac{(16)^2}{2} = \frac{256}{2} = 128\text{ cm}^2$$

Q. 9 The radius of a circle whose circumference is equal to the sum of the circumferences of the two circles of diameters 36 cm and 20 cm is

(a) 56 cm (b) 42 cm (c) 28 cm (d) 16 cm

Sol. (c) $\because$ Circumference of first circle $= 2\pi r = \pi d_1 = 36\pi$ cm [given, $d_1 = 36$ cm]

and circumference of second circle $= \pi d_2 = 20\pi$ cm [given, $d_2 = 20$ cm]

According to the given condition,

Circumference of circle = Circumference of first circle + Circumference of second circle

$\Rightarrow \quad \pi D = 36\pi + 20\pi$ [where, D is diameter of a circle]

$\Rightarrow \quad D = 56$ cm

So, diameter of a circle is 56 cm.

$\therefore$ Required radius of circle $= \dfrac{56}{2} = 28$ cm

Q. 10 The diameter of a circle whose area is equal to the sum of the areas of the two circles of radii 24 cm and 7 cm is

(a) 31 cm (b) 25 cm (c) 62 cm (d) 50 cm

Sol. (d) Let $r_1 = 24$ cm and $r_2 = 7$ cm

$\therefore$ Area of first circle $= \pi r_1^2 = \pi (24)^2 = 576\pi$ cm^2

and area of second circle $= \pi r_2^2 = \pi (7)^2 = 49\pi$ cm^2

According to the given condition,

Area of circle = Area of first circle + Area of second circle

$\therefore \quad \pi R^2 = 576\pi + 49\pi$ [where, R be radius of circle]

$\Rightarrow \quad R^2 = 625 \Rightarrow R = 25$ cm

$\therefore$ Diameter of a circle $= 2R = 2 \times 25 = 50$ cm