Very Short Question

Q. 2. Find the values of p for which the quadratic equation $4x^2 + px + 3 = 0$ has equal roots. [CBSE (AI) 2014]

Sol. Given equation is $4x^2 + px + 3 = 0$

Here, $a = 4$, $b = p$ and $c = 3$

For equal roots,

$$D = b^2 - 4ac = 0$$

$$\Rightarrow \quad p^2 - 4 \times 4 \times 3 = 0$$

$$p^2 - 48 = 0$$

$$p^2 = 48$$

$$p = \pm\sqrt{48} = \pm 4\sqrt{3}$$

Hence, the value of p is $4\sqrt{3}$ and $-4\sqrt{3}$.

Q. 1. Solve the quadratic equation $2x^2 + ax - a^2 = 0$ for x. [CBSE 2014]

Sol. Given equation is $2x^2 + ax - a^2 = 0$

Here, $a = 2$, $b = a$ and $c = -a^2$.

Using the formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a},$$

we get $$x = \frac{-a \pm \sqrt{a^2 - 4 \times 2 \times (-a^2)}}{2 \times 2}$$

$$x = \frac{-a \pm \sqrt{9a^2}}{4} = \frac{-a \pm 3a}{4}$$

$$\Rightarrow \quad x = \frac{-a + 3a}{4} = \frac{a}{2}, \; x = \frac{-a - 3a}{4} = -a$$

Hence, $x = \frac{a}{2}, -a$.

Q.7. For what values of k, the roots of the equation $x^2 + 4x + k = 0$ are real? [CBSE 2019]

Sol. $x^2 + 4x + k = 0$

$\therefore$ Roots of given equation are real,

$$D \geq 0$$

$$\Rightarrow \quad (4)^2 - 4 \times k \geq 0$$

$$\Rightarrow \quad -4k \geq -16$$

$$\Rightarrow \quad k \leq 4$$

$\therefore$ k has all real values ≤ 4

Q.5. Write the discriminant of the quadratic equation $(x + 5)^2 = 2(5x - 3)$. [CBSE 2019]

Sol. $(x + 5)^2 = 2(5x - 3)$

$$\therefore \quad x^2 + 10x + 25 = 10x - 6$$

$$x^2 + 0x + 31 = 0$$

$$\therefore \quad D = b^2 - 4ac$$

$$= 0 - 4(1)(31) = -124$$

Q. 5. If $x^2 + 2kx + 4 = 0$ has a root $x = 2$, then find the value of k. [CSBE 2011]

Sol. Since $x = 2$ is a root of given equation.

$$\therefore \quad (2)^2 + 2k\,(2) + 4 = 0$$

$$\Rightarrow \quad 4 + 4k + 4 = 0$$

$$\Rightarrow \quad 4k = -8$$

$$\Rightarrow \quad k = -2$$

Hence, the value of k is -2.

Short Question

Q.5. If the roots of the quadratic equation $(a-b)x^2+(b-c)x+(c-a)=0$ are equal, prove that $2a=b+c$. ***HOTS*** **[CBSE (AI) 2016]**

Sol. Given equation is $(a-b)x^2+(b-c)x+(c-a)=0$

For equal roots

$\therefore \quad D=b^2-4ac=0$

$\therefore \quad (b-c)^2-4(a-b)(c-a)=0$

$\Rightarrow \quad b^2+c^2-2bc-4ac+4a^2+4bc-4ab=0$

$\Rightarrow \quad 4a^2+b^2+c^2-4ab+2bc-4ac=0$

$\Rightarrow (2a)^2+(-b)^2+(-c)^2+2(2a)(-b)+2(-b)(-c)+2(-c)(2a)=0$

$\Rightarrow \quad (2a-b-c)^2=0$

$\Rightarrow \quad 2a-b-c=0$

$\Rightarrow \quad 2a=b+c$

Q.24. Solve the equation $\frac{4}{x}-3=\frac{5}{2x+3}$; $x\neq 0, -\frac{3}{2}$, for x. **[CBSE 2014]**

Sol. Given that, $\frac{4}{x}-3=\frac{5}{2x+3}$

$\Rightarrow \quad \frac{4-3x}{x}=\frac{5}{2x+3}$

$\Rightarrow \quad (4-3x)(2x+3)=5x$

$\Rightarrow \quad 8x-6x^2+12-9x=5x$

$\Rightarrow \quad 6x^2+6x-12=0$

$\Rightarrow \quad x^2+x-2=0$

$\Rightarrow \quad x^2+2x-x-2=0$

$\Rightarrow \quad x(x+2)-1(x+2)=0$

$\Rightarrow \quad (x-1)(x+2)=0$

either $x-1=0 \Rightarrow x=1$

or $x+2=0 \Rightarrow x=-2$

Q.10. For what values of k, are the roots of the quadratic equation $(k+4)x^2+(k+1)x+1=0$ equal? ***HOTS*** **[CBSE 2013]**

Sol. Given equation is $(k+4)x^2+(k+1)x+1=0$

Here, $a=k+4$, $b=k+1$, $c=1$

For equal roots,

$\therefore \quad D=b^2-4ac=0$

$\Rightarrow \quad (k+1)^2-4(k+4)(1)=0$

$\Rightarrow \quad k^2+1+2k-4k-16=0$

$\Rightarrow \quad k^2-2k-15=0$

$\Rightarrow \quad k^2-5k+3k-15=0$

$\Rightarrow \quad k(k-5)+3(k-5)=0$

$\Rightarrow \quad (k-5)(k+3)=0$

either $k-5=0 \Rightarrow k=5$

or $k+3=0 \Rightarrow k=-3$

Hence, the values of k are 5 and –3.

Q.18. If $x=\frac{2}{3}$ and $x=-3$ are roots of the quadratic equation $ax^2+7x+b=0$, find the values of a and b. **[CBSE 2016]**

Sol. Since $x=\frac{2}{3}$ and $x=-3$ are roots of the quadratic equation

$ax^2+7x+b=0$

$\therefore \quad a\left(\frac{2}{3}\right)^2+7\left(\frac{2}{3}\right)+b=0$

$\Rightarrow \quad 4a+42+9b=0$

$\Rightarrow \quad 4a+9b=-42 \quad \ldots(1)$

and $\quad a(-3)^2+7(-3)+b=0$

$\Rightarrow \quad 9a-21+b=0$

$\Rightarrow \quad 9a+b=21 \quad \ldots(2)$

From eqs.(1) and (2), we obtain

$81a+9b=189$

$4a+9b=-42$

Subtracting above two equations, we get

$77a=231$

$\Rightarrow \quad a=\frac{231}{77}=3$

Now, from eq.(2), we get

$9(3)+b=21$

$\Rightarrow \quad b=21-27$

$\Rightarrow \quad b=-6$

Hence, the values of a and b are 3 and –6, respectively.

Q.31. Solve for x by using quadratic formula $36x^2-12ax+(a^2-b^2)=0$. **[CBSE 2011]**

Sol. Given equation is

$36x^2-12ax+a^2-b^2=0$

Here, $A=36$, $B=-12a$ and $C=a^2-b^2$

$\therefore \quad D=B^2-4AC$

$=(-12a)^2-4(36)(a^2-b^2)$

$=144a^2-144a^2+144b^2=144b^2$

Now, $\quad x=\frac{-B\pm\sqrt{D}}{2A}$

$=\frac{-(-12a)\pm\sqrt{144b^2}}{2\times 36}$

$=\frac{12a\pm 12b}{72}=\frac{a\pm b}{6}$

Hence, the roots are $\frac{a+b}{6}$ and $\frac{a-b}{6}$.

Chapter 4
Quadrilateral Equations

Exercise No. 4.1

Choose the correct answer from the given four options in the following questions:

1.

Which of the following is a quadratic equation?

(A) $x^2 + 2x + 1 = (4 - x)^2 + 3$ **(B)** $-2x^2 = (5 - x)(2x-(2/5))$

(C) $(k + 1)\, x^2 + (3/2)\, x = 7$, **where** $k = -1$ **(D)** $x^3 - x^2 = (x - 1)^3$

Solution:

Correct answer is (D) $x^3 - x^2 = (x - 1)^3$

Justification:
We have quadratic equation:
$ax^2 + bx + c = 0$,

(A)
$x^2 + 2x + 1 = (4 - x)^2 + 3$
$x^2 + 2x + 1 = 16 - 8x + x^2 + 3$
$10x - 18 = 0$
This is linear equation.

(B)
$-2x^2 = (5 - x)\,(2x - 2/5)$
$-2x^2 = 10x - 2x^2 - 2 + 2/5x$
$52x - 10 = 0$
This is also a linear equation.

(C)
$(k + 1)\, x^2 + 3/2\, x = 7$,
As $k = -1$
$(-1 + 1)\, x^2 + 3/2\, x = 7$
$3x - 14 = 0$
This is a linear equation.

(D)
$x^3 - x^2 = (x - 1)^3$
$x^3 - x^2 = x^3 - 3x^2 + 3x - 1$
$2x^2 - 3x + 1 = 0$
Above equation represents a quadratic equation.

2.

Which of the following is not a quadratic equation?

(A) $2(x - 1)^2 = 4x^2 - 2x + 1$ (B) $2x - x^2 = x^2 + 5$

(C) $(\sqrt{2}x + \sqrt{3})^2 + x^2 = 3x^2 - 5x$ (D) $(x^2 + 2x)^2 = x^4 + 3 + 4x^3$

Solution:

Correct answer is (D) $(x^2 + 2x)^2 = x^4 + 3 + 4x^3$

Equation to be quadratic, it should be in the form,

$ax^2 + bx + c = 0, a \neq 0$

(A)

$2(x - 1)^2 = 4x^2 - 2x + 1$

$2(x^2 - 2x + 1) = 4x^2 - 2x + 1$

$2x^2 + 2x - 1 = 0$

This equation represents a quadratic equation.

(B)

$2x - x^2 = x^2 + 5$

$2x^2 - 2x + 5 = 0$

This equation represents a quadratic equation.

(C)

$(\sqrt{2}x + \sqrt{3})^2 = 3x^2 - 5x$

$2x^2 + 2\sqrt{6}x + 3 = 3x^2 - 5x$

$x^2 - (5 + 2\sqrt{6})x - 3 = 0$

This equation represents a quadratic equation.

(D)

$(x^2 + 2x)^2 = x^4 + 3 + 4x^2$

$x^4 + 4x^3 + 4x^2 = x^4 + 3 + 4x^2$

$4x^3 - 3 = 0$

This equation represents a cubic equation.

3.

Which of the following equations has 2 as a root?

(A) $x^2 - 4x + 5 = 0$ (B) $x^2 + 3x - 12 = 0$

(C) $2x^2 - 7x + 6 = 0$ (D) $3x^2 - 6x - 2 = 0$

Solution:

Correct answer is (C) $2x^2 - 7x + 6 = 0$

As 2 is a root then putting value 2 in place of x, we should get zero.

(A)

$x^2 - 4x + 5 = 0$

$(2)^2 - 4(2) + 5 = 1$

$1 \neq 0$

Therefore, $x = 2$ is not a root of $x^2 - 4x + 5 = 0$

(B)

$x^2 + 3x - 12 = 0$

$(2)^2 + 3(2) - 12 = -2 \neq 0$

Therefore, $x = 2$ is not a root of $x^2 + 3x - 12 = 0$

(C)

$2x^2 - 7x + 6 = 0$

$2(2)^2 - 7(2) + 6 = 0$

So, $x = 2$ is a root of $2x^2 - 7x + 6 = 0$

(D)

$3x^2 - 6x - 2 = 0$

$3(2)^2 - 6(2) - 2 = -2$

$-2 \neq 0$

Therefore, $x = 2$ is not a root of $3x^2 – 6x – 2 = 0$

4.

If ½ is a root of the equation $x^2 + kx – 5/4 = 0$, then the value of k is

(A) 2 (B) – 2

(C) ¼ (D) ½

Solution:

Correct answer is (A) 2.

As, ½ is a root of the equation$x^2 + kx – 5/4 = 0$.

Putting the value of ½ in place of x gives us the value of k.

As,

$x = ½$

$(½)^2 + k (½) – (5/4) = 0$

$(k/2) = (5/4) – ¼$

So, $k = 2$

5.

Which of the following equations has the sum of its roots as 3?

(A) $2x^2 – 3x + 6 = 0$ (B) $–x^2 + 3x – 3 = 0$

(C) $\sqrt{2}x^2 – 3/\sqrt{2}x+1= 0$ (D) $3x^2 – 3x + 3 = 0$

Solution:

Correct answer is (B) $–x^2 + 3x – 3 = 0$.

The sum of the roots of a quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$ is given by,

Coefficient of x / coefficient of $x^2 = – (b/a)$

(A) We have,

$2x^2 – 3x + 6 = 0$

Sum of the roots $= – b/a$

$= - (-3/2)$

Sum of the roots = 3/2

(B) We have,

$-x^2 + 3x - 3 = 0$

Sum of the roots = – b/a

= -(3/-1)

= 3

(C) We have,

$\sqrt{2}x^2 - 3/\sqrt{2}x+1=0$

$2x^2 - 3x + \sqrt{2} = 0$

Sum of the roots = – b/a

= -(-3/2)

= 3/2

(D) We have,

$3x^2 - 3x + 3 = 0$

Sum of the roots = – b/a

= -(-3/3)

= 1

6.

Value(s) of k for which the quadratic equation $2x^2 - kx + k = 0$ has equal roots is

(a) 0 only
(b) 4
(c) 8 only
(d) 0, 8

Solution:

(d)
The condition for equal roots of quadratic equation $ax^2 + bx + c= 0$ is
$b^2 - 4ac = 0$.

We have:
$2x^2 - kx + k = 0$

Condition for equal roots,
$b^2 - 4ac = 0$

$(-k)^2 - 4(2)(k) = 0$ (As, $a = 2$, $b = -k$, $c = +k$)
$k^2 - 8k = 0$
$k(k - 8) = 0$
$k = 0$
Or
$k - 8 = 0$
$k = 8$
As, the values of k are 0 and 8.
The answer is (d).

7.

Which constant must be added and subtracted to solve the quadratic equation

$$9x^2 + \frac{3}{4}x - \sqrt{2} = 0$$

by the method of completing the square?

(a) $\frac{1}{8}$

(b) $\frac{1}{64}$

(c) $\frac{1}{4}$

(d) $\frac{9}{64}$

Solution:

(b) $\frac{1}{64}$

The given equation is

$$9x^2 + \frac{3}{4}x - \sqrt{2} = 0$$

So, to make the expression a complete square, we have to subtract $\frac{1}{64}$.

$$9x^2 + \frac{3}{4}x + \frac{1}{64} - \sqrt{2} - \frac{1}{64} = 0$$

$$\left(3x+\frac{1}{8}\right)^2=\sqrt{2}+\frac{1}{64}$$

8.

The quadratic equation has:
$2x^2-\sqrt{5}x+1=0$

(a) two distinct real roots
(b) two equal real roots
(c) no real roots
(d) more than two real roots

Solution:
(c)no real roots

We have,
$2x^2-\sqrt{5}x+1=0$

Now,
D = b^2– 4ac,

Checking the following conditions:

(i) for no real rootsD < 0
(ii) for two equal roots D = 0
(iii) for two distinct roots D > 0 and any quadratic equation must have only roots.

The equation is:
$2x^2-\sqrt{5}x+1=0$

So,
D = b^2– 4ac

Where,
a = 2,
b $=-\sqrt{5}$
c = 1

D= 5 – 8
D = –3

As D < 0 so, the given equations has no real roots.

9.

Which of the following equations has two distinct real roots?

(a) $2x^2 - 3\sqrt{2}x + \frac{9}{4} = 0$

(b) $x^2 + x - 5 = 0$

(c) $x^2 + 3x + 2\sqrt{2} = 0$

(d) $5x^2 - 3x + 1 = 0$

Solution:

Correct answer is (b)$x^2 + x - 5 = 0$

We have,

For real distinct roots $D > 0$

(a) Equation is $2x^2 - 3\sqrt{2}x + \frac{9}{4} = 0$

$D = b^2 - 4ac$

$D = 9 \times 2 - 18$

$D = 0$

For, $D = 0$, the given equation has two real equal roots.

(b) Equation is $x^2 + x - 5 = 0$

$D = b^2 - 4ac$

$D = (1)^2 - 4(1)(-5)$ (where, $a = 1$, $b = 1$, $c = -5$)

$D = 1 + 20$

$D = 21$

For, $D > 0$, the given equation has two distinct real roots.

(c) Equation is $x^2 + 3x + 2\sqrt{2} = 0$

$D = b^2 - 4ac$

$D = (3)2 - 4(1)\,2\sqrt{2}$ (Where, $a = 1$, $b = 3$, $c = 2\sqrt{2}$)

$D = 9 - 11.312$

$D = -2.312$

As $D < 0$, so the given equation has no real roots.

(d) Equation is $5x^2 - 3x + 1 = 0$

$D = b2 - 4ac$

$D = (-3)2 - 4(5)(1)$ (Where, $a = 5$, $b = -3$, $c = 1$)

$D = 9 - 20$

$D = -11$

As $D < 0$, so the given equation has no real roots.

10.

Which of the following equations has no real roots.

(a) $x^2 - 4x + 3\sqrt{2} = 0$

(b) $x^2 + 4x - 3\sqrt{2} = 0$

(c) $x^2 - 4x - 3\sqrt{2} = 0$

(d) $3x^2 + 4\sqrt{3}x + 4 = 0$

Solution:

Correct answer is (a) $x^2 - 4x + 3\sqrt{2} = 0$.

Given equation is $x^2 - 4x + 3\sqrt{2} = 0$

D = $b^2 - 4ac$

D= $(-4)^2 - 4(1)(3\sqrt{2})$ (a = 1, b = –4, c = $3\sqrt{2}$)

D= 16 – 12 × 1.414

D= 16 – 16.968

D = –0.968

As D < 0, so the given equation has no real roots.

(b)

D = $b^2 - 4ac$

D= $(4)^2 - 4(1)3\sqrt{2}$ (a = 1, b = 4, c = $3\sqrt{2}$)

D = 16 +$12\sqrt{2}$

Here,

D > 0

Hence, the given equation has two distinct real roots.

(c)

D = $b^2 - 4ac$

D = $(-4)^2 - 4(1)(-3\sqrt{2})$ (where, a = 1, b = –4, c = $-3\sqrt{2}$)

D = 16 +$12\sqrt{2}$

Here,

D > 0

So, the given equation has two real distinct roots.

(d)

D = $b^2 - 4ac$

D= $12\sqrt{2} - 4(3)(4)$(where, a = 3, b =$12\sqrt{2}$, c = 4)

D= 16 × 3 – 48 = 48 – 48

D = 0

So, the given equation has two real and equal roots.

11.

$(x^2 + 1)^2 - x^2 = 0$ has

(a) four real roots
(b) two real roots
(c) no real roots
(d) one real root

Solution:
(c)no real roots

We have,
$(x^2+1)^2 - x^2 = 0$
$(x^2)^2 + (1)^2 + 2(x^2)(1) - x^2 = 0$
$(x^2)^2 + 1x^2 + 1 = 0$

Taking,
$x^2 = y$so,$y^2 + 1y + 1 = 0$

Now,
$D = b^2 - 4ac$

$D = (1)^2 - 4(1)(1)$
$D = 1 - 4$ (Where, $a = 1, b = 1, c = 1$)
$D = -3$
$D < 0$
So, the given equation $y^2 + y + 1 = 0$ has no values of y in equation $y^2 + 1y + 1 = 0$ or if y is not real then x^2will not be real so no values of x are real or the given equation has no real roots.