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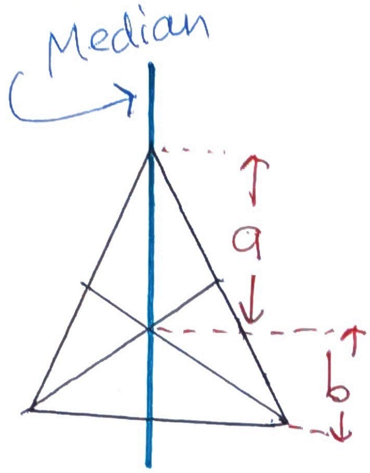
NOTIFICATIONS

COORDINATE GEOMETRY

ALERT TIPS

① Midpoint → $\dfrac{x_1+x_2}{2}, \dfrac{y_1+y_2}{2}$



② Centroid of a Triangle :-

$$\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}$$

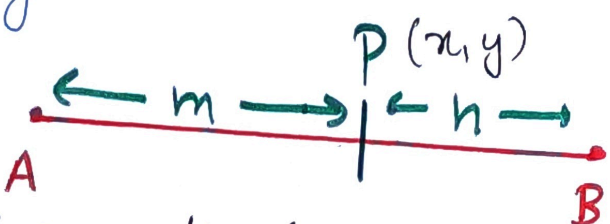
NOTE: Median of Δ is divided by centroid in ratio 2 : 1

i.e. $a:b = 2:1$

③ Distance formula : $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

④ Distance of a point (x, y) from origin : $\sqrt{x^2+y^2}$

⑤ Section formula



↳ AB divided by 'P' in ratio $(m:n)$

$$x = \frac{mx_2+nx_1}{m+n}$$

$$y = \frac{my_2+ny_1}{m+n}$$

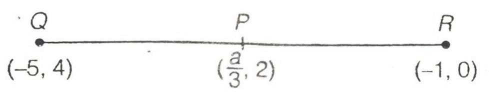
Chapter-7 Coordinate Geometry

Very Short Question

Q. 3. Find the value of a, for which point P $\left(\frac{a}{3}, a\right)$ is the midpoint of the line segment joining the points Q(–5, 4) and R(–1, 0).

[Board Sample Paper, 2016]

Sol.



P is mid-point of QR

or, $\frac{a}{3} = \frac{-5+(-1)}{2}$

or, $\frac{a}{3} = \frac{-6}{2}$

or, $a = -9$

Q. 6. Find the coordinates of the point on y-axis which is nearest to the point (–2, 5).

[Sample Question Paper, 2017]

Sol. The point on y-axis that is nearest to the point (–2, 5) is (0, 5).

Q. 7. Find the distance of a point P(x, y) from the origin. [CBSE 2018]

Sol. Here, $x_1 = 0, y_1 = 0, x_2 = x$ and $y_2 = y$

$\therefore \quad d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(x-0)^2 + (y_2 - 0)^2} = \sqrt{x^2 + y^2}$

Hence, the distance of Point P from the origin is $\sqrt{x^2+y^2}$.

Q. 8. Find the positive value of m for which the distance between the points A(5, – 3) and B(13, m) is 10 units. [CBSE 2019]

Sol. As $AB = 10$, $A \equiv (5, -3)$ and $B \equiv (13, m)$

$\therefore \quad \sqrt{(13-5)^2 + (m+3)^2} = 10$

$\therefore \quad (m+3)^2 = 10^2 - 8^2 = 36$

$\therefore \quad m + 3 = 6$

$\therefore \quad m = 3$

Short Question

Q. 2. The x-coordinate of a point P is twice is y-coordinate. If P is equidistant from Q(2, – 5) and R(–3, 6), find the coordinates of P.

[CBSE 2016]

Sol. Here, x-coordinate of P is twice its y-coordinate

Let coordinates of P be $P(2a, a)$.

Now, P is equidistant from $Q(2, -5)$ and $R(-3, 6)$

$\therefore \quad QP^2 = RP^2$

$\Rightarrow \quad (2-2a)^2 + (-5-a)^2$

$= (-3-2a)^2 + (6-a)^2$

$\Rightarrow 4 + 4a^2 - 8a + 25 + a^2 + 10a$

$= 9 + 4a^2 + 12a + 36 + a^2 - 12a$

$[\because (a-b)^2 = a^2 + b^2 - 2ab]$

$\Rightarrow \quad 5a^2 + 2a + 29 = 5a^2 + 45$

$\Rightarrow \quad 2a = 45 - 29$

$\Rightarrow \quad 2a = 16$

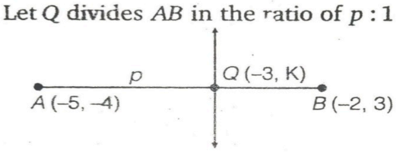
$\Rightarrow \quad a = 8$

Hence, the coordinates of P are $P(16, 8)$.

Q.4. Find the ratio in which the point (–3, k) divides the line segment joining the points (–5, – 4) and (–2, 3). Also find the value of k.

[CBSE (F) 2016]

Sol. Let Q divides AB in the ratio of $p : 1$



$-3 = \frac{-2p - 5}{p+1} \quad \left[\because x = \frac{mx_2 + nx_1}{m+n}\right]$

$\Rightarrow \quad -3(p+1) = -2p - 5$

$\Rightarrow \quad -3p - 3 = -2p - 5$

$\Rightarrow \quad -3p + 2p = -5 + 3$

$\Rightarrow \quad -p = -2$

$\Rightarrow \quad p = 2$

$\therefore$ Required ratio is 2 : 1.

Now, $\quad k = \frac{2 \times 3 - 4}{2+1} = \frac{6-4}{3} = \frac{2}{3}$

$\left[\because y = \frac{my_2 + ny_1}{m+n}\right]$

Q.8. If the points $A(-2, 1)$, $B(a, b)$ and $C(4, -1)$ are collinear and $a - b = 1$, find the values of a and b. **[CBSE 2014]**

Sol. Since, the given points are collinear, then area of $\Delta ABC = 0$

$$\Rightarrow \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$$

Given, $x_1 = -2$, $y_1 = 1$, $x_2 = a$, $y_2 = b$, $x_3 = 4$, $y_3 = -1$

Putting the values,

$$\frac{1}{2}[-2(b + 1) + a(-1 - 1) + 4(1 - b)] = 0$$

$$\Rightarrow \quad -2b - 2 - 2a + 4 - 4b = 0$$

$$\Rightarrow \quad 2a + 6b = 2$$

$$\Rightarrow \quad a + 3b = 1 \quad \ldots(1)$$

Given, $\quad a - b = 1 \quad \ldots(2)$

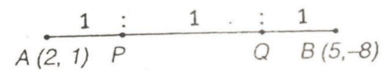
Subtracting eq.(1) from eq.(2), we have

$$4b = 0 \Rightarrow b = 0$$

Substituting the value of b in eq.(2), we have $a = 1$

Q.26. The line segment joining the points $A(2, 1)$ and $B(5, -8)$ is trisected at the points P and Q such that P is nearer to A. If P also lies on the line given by $2x - y + k = 0$, find the value of k. **[CBSE 2019]**

Sol.



Here, $\quad AP : PB = 1 : 2$

$$\therefore \quad P \equiv \left(\frac{1 \times 5 + 2 \times 2}{1 + 2}, \frac{1 \times -8 + 2 \times 1}{1 + 2}\right)$$

$$\Rightarrow \quad P \equiv (3, -2)$$

Since, P lies on the line $2x - y + k = 0$

$$\therefore \quad 2(3) - (-2) + k = 0$$

$$\Rightarrow \quad 6 + 2 + k = 0$$

$$\Rightarrow \quad k = -8$$

Q.16. Find the points on the x-axis which are at a distance of $2\sqrt{5}$ from the point $(7, -4)$. How many such points are there? **[CBSE 2011]**

Sol. Let $P(x, 0)$ be points on x-axis which are at a distance of $2\sqrt{5}$ from the point $A(7, -4)$.

$$\therefore \quad |PA| = 2\sqrt{5}$$

$$\sqrt{(x - 7)^2 + (0 + 4)^2} = 2\sqrt{5}$$

Squaring both sides, we have

$$\Rightarrow \quad x^2 + 49 - 14x + 16 = 20$$

$$\Rightarrow \quad x^2 - 14x + 65 - 20 = 0$$

$$\Rightarrow \quad x^2 - 14x + 45 = 0$$

$$\Rightarrow \quad x^2 - 9x - 5x + 45 = 0$$

$$\Rightarrow \quad x(x - 9) - 5(x - 9) = 0$$

$$\Rightarrow \quad x = 5 \text{ or } x = 9$$

Hence, the required points are $P_1(5, 0)$ and $P_2(9, 0)$.

7

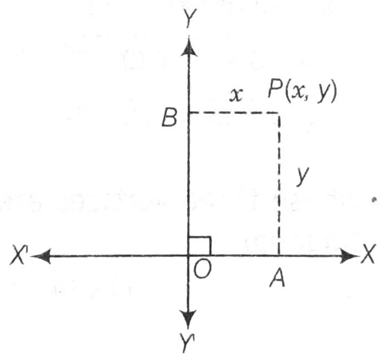
Coordinate Geometry

Exercise 7.1 Multiple Choice Questions (MCQs)

Q. 1 The distance of the point $P(2, 3)$ from the X-axis is

(a) 2 (b) 3 (c) 1 (d) 5

Sol. *(b)* We know that, if (x, y) is any point on the cartesian plane in first quadrant.
Then, x = Perpendicular distance from Y-axis
and y = Perpendicular distance from X-axis



Distance of the point $P(2, 3)$ from the X-axis = Ordinate of a point $P(2, 3) = 3$.

Q. 2 The distance between the points $A(0, 6)$ and $B(0, -2)$ is

(a) 6 (b) 8 (c) 4 (d) 2

Thinking Process

The distance between two points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. Use this formula and simplify it.

Sol. *(b)* $\because$ Distance between the points (x_1, y_1) and (x_2, y_2),

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Here, $x_1 = 0, y_1 = 6$ and $x_2 = 0, y_2 = -2$

$\therefore$ Distance between $A(0, 6)$ and $B(0, -2)$,

$$AB = \sqrt{(0 - 0)^2 + (-2 - 6)^2}$$

$$= \sqrt{0 + (-8)^2} = \sqrt{8^2} = 8$$

Q. 3 The distance of the point $P(-6, 8)$ from the origin is

(a) 8 (b) $2\sqrt{7}$ (c) 10 (d) 6

Thinking Process

Coordinate of origin is (0, 0)

Sol. *(c)* $\because$ Distance between the points (x_1, y_1) and (x_2, y_2)

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Here, $x_1 = -6$, $y_1 = 8$ and $x_2 = 0$, $y_2 = 0$

$\therefore$ Distance between $P(-6, 8)$ and origin *i.e.*, $O(0, 0)$,

$$PO = \sqrt{[0 - (-6)]^2 + (0 - 8)^2}$$
$$= \sqrt{(6)^2 + (-8)^2}$$
$$= \sqrt{36 + 64} = \sqrt{100} = 10$$

Q. 4 The distance between the points (0, 5) and (– 5, 0) is

(a) 5 (b) $5\sqrt{2}$ (c) $2\sqrt{5}$ (d) 10

Sol. *(b)* $\because$ Distance between the points (x_1, y_1) and (x_2, y_2),

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Here, $x_1 = 0$, $y_1 = 5$ and $x_2 = -5$, $y_2 = 0$

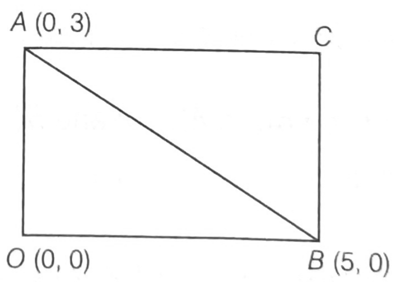
$\therefore$ Distance between the points (0, 5) and (–5, 0)

$$= \sqrt{(-5 - 0)^2 + (0 - 5)^2}$$
$$= \sqrt{25 + 25} = \sqrt{50} = 5\sqrt{2}$$

Q. 5 If *AOBC* is a rectangle whose three vertices are $A(0, 3)$, $O(0, 0)$ and $B(5, 0)$, then the length of its diagonal is

(a) 5 (b) 3 (c) $\sqrt{34}$ (d) 4

Sol. *(c)*



Now, length of the diagonal AB = Distance between the points $A(0, 3)$ and $B(5, 0)$.

$\because$ Distance between the points (x_1, y_1) and (x_2, y_2),

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Here, $x_1 = 0$, $y_1 = 3$ and $x_2 = 5$, $y_2 = 0$

$\therefore$ Distance between the points $A(0, 3)$ and $B(5, 0)$

$$AB = \sqrt{(5 - 0)^2 + (0 - 3)^2}$$
$$= \sqrt{25 + 9} = \sqrt{34}$$

Hence, the required length of its diagonal is $\sqrt{34}$.

Q. 6 The perimeter of a triangle with vertices (0, 4), (0, 0) and (3, 0) is

(a) 5 (b) 12 (c) 11 (d) $7 + \sqrt{5}$

Thinking Process

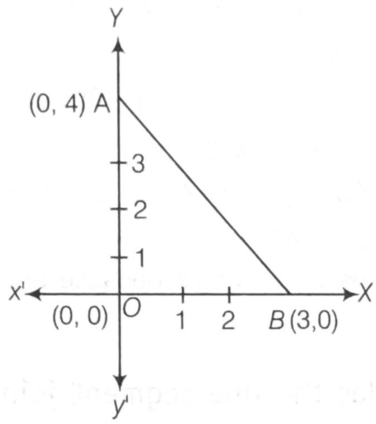
(i) Firstly, plot the given points on a graph paper and join them to get a triangle.

(ii) Secondly, determine the length of the each sides by using the distance formula,

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

(iii) Further, adding all the distance of a triangle to get the perimeter of a triangle.

Sol. (b) We plot the vertices of a triangle *i.e.*, (0, 4), (0, 0) and (3, 0) on the paper shown as given below



Now, perimeter of ΔAOB = Sum of the length of all its sides = $d(AO) + d(OB) + d(AB)$

$\because$ Distance between the points (x_1, y_1) and (x_2, y_2),

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

= Distance between $A(0, 4)$ and $O(0, 0)$ + Distance between $O(0, 0)$ and $B(3, 0)$ + Distance between $A(0, 4)$ and $B(3, 0)$

$= \sqrt{(0-0)^2 + (0-4)^2} + \sqrt{(3-0)^2 + (0-0)^2} + \sqrt{(3-0)^2 + (0-4)^2}$

$= \sqrt{0 + 16} + \sqrt{9 + 0} + \sqrt{(3)^2 + (4)^2} = 4 + 3 + \sqrt{9 + 16}$

$= 7 + \sqrt{25} = 7 + 5 = 12$

Hence, the required perimeter of triangle is 12.

Q. 7 The area of a triangle with vertices $A(3, 0)$, $B(7, 0)$ and $C(8, 4)$ is

(a) 14 (b) 28 (c) 8 (d) 6

Thinking Process

The area of triangle, whose vertices are $A(x_1, y_1)$, $B(x_2, y_2)$ and (x_3, y_3) is given by $\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$. Use this formula and simplify it to get the result.

Sol. (c) Area of ΔABC whose Vertices $A \equiv (x_1, y_1)$, $B \equiv (x_2, y_2)$ and $C \equiv (x_3, y_3)$ are given by

$$\Delta = \left|\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]\right|$$

Here, $x_1 = 3$, $y_1 = 0$, $x_2 = 7$, $y_2 = 0$, $x_3 = 8$ and $y_3 = 4$

$\therefore \quad \Delta = \left|\frac{1}{2}[3(0-4) + 7(4-0) + 8(0-0)]\right| = \left|\frac{1}{2}(-12 + 28 + 0)\right| = \left|\frac{1}{2}(16)\right| = 8$

Hence, the required area of ΔABC is 8.

Q. 8 The points $(-4, 0)$, $(4, 0)$ and $(0, 3)$ are the vertices of a

(a) right angled triangle
(b) isosceles triangle
(c) equilateral triangle
(d) scalene triangle

Sol. ***(b)*** Let $A(-4, 0)$, $B(4, 0)$, $C(0, 3)$ are the given vertices.

Now, distance between $A(-4, 0)$ and $B(4, 0)$,

$$AB = \sqrt{[4-(-4)]^2 + (0-0)^2}$$

$$\left[\because \text{distance between two points } (x_1, y_1) \text{ and } (x_2, y_2), d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\right]$$

$$= \sqrt{(4+4)^2} = \sqrt{8^2} = 8$$

Distance between $B(4, 0)$ and $C(0, 3)$,

$$BC = \sqrt{(0-4)^2 + (3-0)^2} = \sqrt{16+9} = \sqrt{25} = 5$$

Distance between $A(-4, 0)$ and $C(0, 3)$,

$$AC = \sqrt{[0-(-4)]^2 + (3-0)^2} = \sqrt{16+9} = \sqrt{25} = 5$$

$\because$ $BC = AC$

Hence, ΔABC is an isosceles triangle because an isosceles triangle has two sides equal.

Q. 9 The point which divides the line segment joining the points $(7, -6)$ and $(3, 4)$ in ratio $1 : 2$ internally lies in the

(a) I quadrant (b) II quadrant (c) III quadrant (d) IV quadrant

Sol. ***(d)*** If $P(x, y)$ divides the line segment joining $A(x_1, y_1)$ and $B(x_2, y_2)$ internally in the ratio $m : n$, then $x = \dfrac{mx_2 + nx_1}{m+n}$ and $y = \dfrac{my_2 + ny_1}{m+n}$

Given that, $x_1 = 7$, $y_1 = -6$, $x_2 = 3$, $y_2 = 4$, $m = 1$ and $n = 2$

$$\therefore \quad x = \frac{1(3) + 2(7)}{1+2}, \; y = \frac{1(4) + 2(-6)}{1+2} \qquad \text{[by section formula]}$$

$$\Rightarrow \quad x = \frac{3+14}{3}, \; y = \frac{4-12}{3}$$

$$\Rightarrow \quad x = \frac{17}{3}, \; y = -\frac{8}{3}$$

So, $(x, y) = \left(\dfrac{17}{3}, -\dfrac{8}{3}\right)$ lies in IV quadrant.

[since, in IV quadrant, x-coordinate is positive and y-coordinate is negative]

Q. 10 The point which lies on the perpendicular bisector of the line segment joining the points $A(-2, -5)$ and $B(2, 5)$ is

(a) (0, 0) (b) (0, 2) (c) (2, 0) (d) (−2, 0)

Sol. ***(a)*** We know that, the perpendicular bisector of the any line segment divides the line segment into two equal parts *i.e.*, the perpendicular bisector of the line segment always passes through the mid-point of the line segment.

$\therefore$ Mid-point of the line segment joining the points $A(-2, -5)$ and $B(2, 5)$

$$= \left(\frac{-2+2}{2}, \frac{-5+5}{2}\right) = (0, 0)$$

$\left[\text{since, mid-point of any line segment which passes through the points } (x_1, y_1) \text{ and } (x_2, y_2) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)\right]$

Hence, (0, 0) is the required point lies on the perpendicular bisector of the lines segment.

Q. 11 The fourth vertex D of a parallelogram $ABCD$ whose three vertices are $A(-2, 3)$, $B(6, 7)$ and $C(8, 3)$ is

(a) (0, 1) (b) (0, −1) (c) (−1, 0) (d) (1, 0)

Thinking Process

(i) Firstly, consider the fourth vertex of a parallelogram be $D(x_4, y_4)$.

(ii) Secondly, determine the mid point of AC and BD by using the formula $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

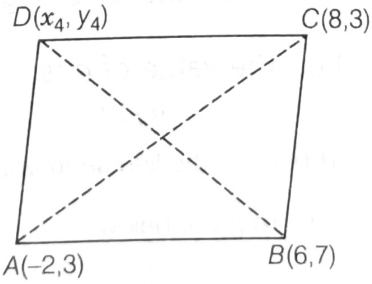
(iii) Further, equating both points and get the required coordinate of fourth vertex.

Sol. ***(b)*** Let the fourth vertex of parallelogram, $D \equiv (x_4, y_4)$ and L, M be the middle points of AC and BD, respectively.

Then, $L \equiv \left(\frac{-2+8}{2}, \frac{3+3}{2}\right) \equiv (3, 3)$

$\left[\text{since, mid-point of a line segment having points } (x_1, y_1) \text{ and } (x_2, y_2) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)\right]$

and $$M \equiv \left(\frac{6+x_4}{2}, \frac{7+y_4}{2}\right)$$



Since, $ABCD$ is a parallelogram, therefore diagonals AC and BD will bisect each other. Hence, L and M are the same points.

$$\therefore \qquad 3 = \frac{6 + x_4}{2} \text{ and } 3 = \frac{7 + y_4}{2}$$

$$\Rightarrow \qquad 6 = 6 + x_4 \text{ and } 6 = 7 + y_4$$

$$\Rightarrow \qquad x_4 = 0 \quad \text{and } y_4 = 6 - 7$$

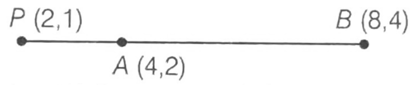
$$\therefore \qquad x_4 = 0 \quad \text{and } y_4 = -1$$

Hence, the fourth vertex of parallelogram is $D \equiv (x_4, y_4) \equiv (0, -1)$.

Q. 12 If the point $P(2, 1)$ lies on the line segment joining points $A(4, 2)$ and $B(8, 4)$, then

(a) $AP = \frac{1}{3} AB$ (b) $AP = PB$ (c) $PB = \frac{1}{3} AB$ (d) $AP = \frac{1}{2} AB$

Sol. ***(d)*** Given that, the point $P(2, 1)$ lies on the line segment joining the points $A(4, 2)$ and $B(8, 4)$, which shows in the figure below:



Now, distance between $A(4, 2)$ and $(2, 1)$, $AP = \sqrt{(2-4)^2 + (1-2)^2}$

$\left[\because \text{ distance between two points two points } (x_1, y_1) \text{ and } B(x_2, y_2), d = \sqrt{(x_2 - x_1)^2 + (x_2 - y_1)^2}\right]$

$$= \sqrt{(-2)^2 + (-1)^2} = \sqrt{4+1} = \sqrt{5}$$

Distance between $A(4, 2)$ and $B(8, 4)$,

$$AB = \sqrt{(8-4)^2 + (4-2)^2}$$

$$= \sqrt{(4)^2 + (2)^2} = \sqrt{16+4} = \sqrt{20} = 2\sqrt{5}$$

Distance between $B(8, 4)$ and $P(2, 1)$, $BP = \sqrt{(8-2)^2 + (4-1)^2}$

$$= \sqrt{6^2 + 3^2} = \sqrt{36+9} = \sqrt{45} = 3\sqrt{5}$$

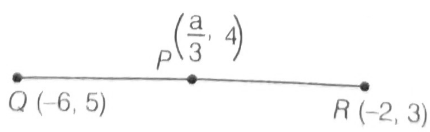
$$\therefore \qquad AB = 2\sqrt{5} = 2AP \Rightarrow AP = \frac{AB}{2}$$

Hence, required condition is $AP = \frac{AB}{2}$.

Q. 13 If $P\left(\frac{a}{3}, 4\right)$ is the mid-point of the line segment joining the points $Q(-6, 5)$ and $R(-2, 3)$, then the value of a is

(a) -4 (b) -12 (c) 12 (d) -6

Sol. ***(b)*** Given that, $P\left(\frac{a}{3}, 4\right)$ is the mid-point of the line segment joining the points $Q(-6, 5)$ and $R(-2, 3)$, which shows in the figure given below



$\therefore$ Mid-point of $QR = P\left(\frac{-6-2}{2}, \frac{5+3}{2}\right) = P(-4, 4)$

$\left[\text{since, mid-point of line segment having points } (x_1, y_1) \text{ and } (x_2, y_2) = \left(\frac{(x_1+x_2)}{2}, \frac{(y_1+y_2)}{2}\right)\right]$

But mid-point $P\left(\frac{a}{3}, 4\right)$ is given.

$$\therefore \quad \left(\frac{a}{3}, 4\right) = (-4, 4)$$

On comparing the coordinates, we get

$$\frac{a}{3} = -4$$

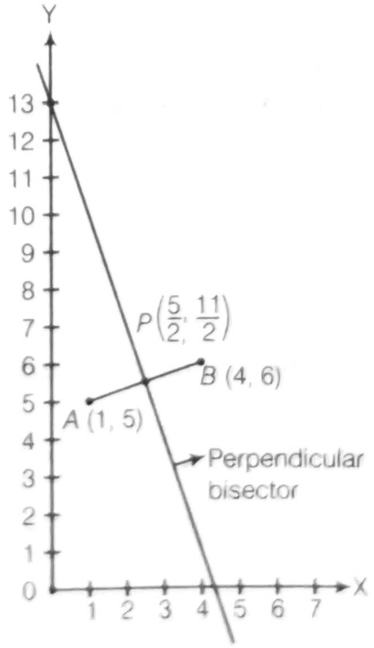
$$\therefore \quad a = -12$$

Hence, the required value of a is -12.

Q. 14 The perpendicular bisector of the line segment joining the points $A(1, 5)$ and $B(4, 6)$ cuts the Y-axis at

(a) (0, 13) (b) (0, −13) (c) (0, 12) (d) (13, 0)

Sol. *(a)* Firstly, we plot the points of the line segment on the paper and join them.



We know that, the perpendicular bisector of the line segment AB bisect the segment AB, i.e., perpendicular bisector of line segment AB passes through the mid-point of AB.

$$\therefore \quad \text{Mid-point of } AB = \left(\frac{1+4}{2}, \frac{5+6}{2}\right)$$

$$\Rightarrow \quad P = \left(\frac{5}{2}, \frac{11}{2}\right)$$

$\left[\because \text{mid-point of line segment passes through the points } (x_1, y_1) \text{ and } (x_2, y_2) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)\right]$

Now, we draw a straight line on paper passes through the mid-point P. We see that the perpendicular bisector cuts the Y-axis at the point (0, 13).

Hence, the required point is (0, 13).

Alternate Method

We know that, the equation of line which passes through the points (x_1, y_1) and (x_2, y_2) is

$$(y - y_1) = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1) \quad \text{...(i)}$$

Here, $x_1 = 1, y_1 = 5$ and $x_2 = 4, y_2 = 6$

So, the equation of line segment joining the points $A(1, 5)$ and $B(4, 6)$ is

$$(y - 5) = \frac{6 - 5}{4 - 1}(x - 1)$$

$$\Rightarrow \quad (y - 5) = \frac{1}{3}(x - 1)$$

$$\Rightarrow \quad 3y - 15 = x - 1$$

$$\Rightarrow \quad 3y = x - 14 \Rightarrow y = \frac{1}{3}x - \frac{14}{3} \quad \text{...(ii)}$$

$\therefore$ Slope of the line segment, $m_1 = \frac{1}{3}$

If two lines are perpendicular to each other, then the relation between its slopes is

$$m_1 \cdot m_2 = -1 \quad \text{...(iii)}$$

where, m_1 = Slope of line 1

and = Slope of line 2

Also, we know that the perpendicular bisector of the line segment is perpendicular on the line segment.

Let slope of line segment is m_2.

From Eq. (iii),

$$m_1 \cdot m_2 = \frac{1}{3} \cdot m_2 = -1$$

$$\Rightarrow \quad m_2 = -3$$

Also we know that the perpendicular bisector is passes through the mid-point of line segment.

$\therefore$ Mid-point of line segment $= \left(\frac{1+4}{2}, \frac{5+6}{2}\right) = \left(\frac{5}{2}, \frac{11}{2}\right)$

Equation of perpendicular bisector, which has slope (–3) and passes through the point $\left(\frac{5}{2}, \frac{11}{2}\right)$, is

$$\left(y - \frac{11}{2}\right) = (-3)\left(x - \frac{5}{2}\right)$$

[since, equation of line passes through the point (x_1, y_1) and having slope m
$(y - y_1) = m(x - x_1)$]

$$\Rightarrow \quad (2y - 11) = -3(2x - 5)$$

$$\Rightarrow \quad 2y - 11 = -6x + 15$$

$$\Rightarrow \quad 6x + 2y = 26$$

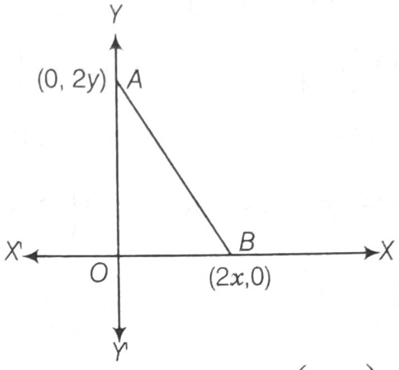
$$\Rightarrow \quad 3x + y = 13 \quad \text{...(iv)}$$

If the perpendicular bisector cuts the Y-axis, then put $x = 0$ in Eq. (iv),

$$3 \times 0 + y = 13 \Rightarrow y = 13$$

So, the required point is (0, 13).

Q. 15 The coordinates of the point which is equidistant from the three vertices of the ΔAOB as shown in the figure is



(a) (x, y) (b) (y, x) (c) $\left(\frac{x}{2}, \frac{y}{2}\right)$ (d) $\left(\frac{y}{2}, \frac{x}{2}\right)$

Thinking Process

(i) Firstly consider the new point be $P(h,k)$.

(ii) Secondly, determine the distance PO, PA and PB by using the formula, $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ and equating them i. e., $PO = PA = PB$.

(iii) Further, solving two-two terms at a time and solving them to get required point.

Sol. (a) Let the coordinate of the point which is equidistant from the three vertices $O(0, 0)$, $A(0, 2y)$ and $B(2x, 0)$ is $P(h,k)$.

Then, $$PO = PA = PB$$

$$\Rightarrow \quad (PO)^2 = (PA)^2 = (PB)^2 \quad \ldots \text{(i)}$$

By distance formula,

$$\left[\sqrt{(h-0)^2+(k-0)^2}\right]^2 = \left[\sqrt{(h-0)^2+(k-2y)^2}\right]^2 = \left[\sqrt{(h-2x)^2+(k-0)^2}\right]^2$$

$$\Rightarrow \quad h^2+k^2 = h^2+(k-2y)^2 = (h-2x)^2+k^2 \quad \ldots \text{(ii)}$$

Taking first two equations, we get

$$h^2+k^2 = h^2+(k-2y)^2$$

$$\Rightarrow \quad k^2 = k^2+4y^2-4yk \Rightarrow 4y(y-k)=0$$

$$\Rightarrow \quad y = k \quad [\because y \neq 0]$$

Taking first and third equations, we get

$$h^2+k^2 = (h-2x)^2+k^2$$

$$\Rightarrow \quad h^2 = h^2+4x^2-4xh$$

$$\Rightarrow \quad 4x(x-h)=0$$

$$\Rightarrow \quad x = h \quad [\because x \neq 0]$$

$$\therefore \quad \text{Required points} = (h, k) = (x, y)$$

Q. 16 If a circle drawn with origin as the centre passes through $\left(\frac{13}{2}, 0\right)$, then the point which does not lie in the interior of the circle is

(a) $\left(\frac{-3}{4}, 1\right)$ (b) $\left(2, \frac{7}{3}\right)$ (c) $\left(5, \frac{-1}{2}\right)$ (d) $\left(-6, \frac{5}{2}\right)$

Sol. ***(d)*** It is given that, centre of circle in (0,0) and passes through the point $\left(\frac{13}{2}, 0\right)$

$\therefore$ Radius of circle = Distance between (0, 0) and $\left(\frac{13}{2}, 0\right)$

$$= \sqrt{\left(\frac{13}{2} - 0\right)^2 + (0-0)^2} = \sqrt{\left(\frac{13}{2}\right)^2} = \frac{13}{2} = 6.5$$

A point lie outside on or inside the circles of the distance of it from the centre of the circle is greater than equal to or less than radius of the circle.

Now, to get the correct option we have to check the option one by one.

(a) Distance between (0,0) and $\left(\frac{-3}{4}, 1\right) = \sqrt{\left(\frac{-3}{4} - 0\right)^2 + (1-0)^2}$

$$= \sqrt{\frac{9}{16} + 1} = \sqrt{\frac{25}{16}} = \frac{5}{4} = 1.25 < 6.5$$

So, the point $(-\frac{3}{4}, 1)$ lies interior to the circle.

(b) Distance between (0,0) and $\left(2, \frac{7}{3}\right) = \sqrt{(2-0)^2 + \left(\frac{7}{3} - 0\right)^2}$

$$= \sqrt{4 + \frac{49}{9}} = \sqrt{\frac{36+49}{9}}$$

$$= \sqrt{\frac{85}{9}} = \frac{9.22}{3} = 3.1 < 6.5$$

So, the point $\left(2, \frac{7}{3}\right)$ lies inside the circle.

(c) Distance between (0,0) and $\left(5, \frac{-1}{2}\right) = \sqrt{(5-0)^2 + \left(-\frac{1}{2} - 0\right)^2}$

$$= \sqrt{25 + \frac{1}{4}} = \sqrt{\frac{101}{4}} = \frac{10.04}{2}$$

$\Rightarrow$ $= 5.02 < 6.5$

So, the point $\left(5, -\frac{1}{2}\right)$ lies inside the circle.

(d) Distance between (0,0) and $\left(-6, \frac{5}{2}\right) = \sqrt{(-6-0)^2 + \left(\frac{5}{2} - 0\right)^2}$

$$= \sqrt{36 + \frac{25}{4}} = \sqrt{\frac{144+25}{4}}$$

$$= \sqrt{\frac{169}{4}} = \frac{13}{2} = 6.5$$

So, the point $\left(-6, \frac{5}{2}\right)$ lis an the circle *i.e.*, does not lie interior to the circle.

Q. 17 A line intersects the Y-axis and X-axis at the points P and Q, respectively. If $(2, -5)$ is the mid-point of PQ, then the coordinates of P and Q are, respectively

(a) $(0, -5)$ and $(2, 0)$ (b) $(0, 10)$ and $(-4, 0)$

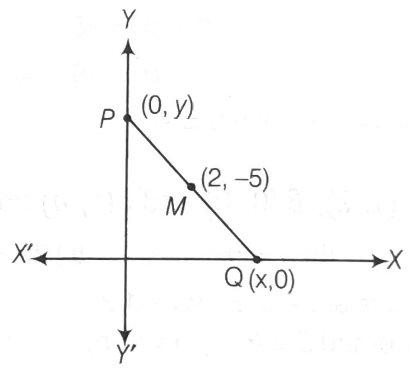
(c) $(0, 4)$ and $(-10, 0)$ (d) $(0, -10)$ and $(4, 0)$

Sol. *(d)* Let the coordinates of P and Q $(0, y)$ and $(x, 0)$, respectively.

So, the mid-point of $P(0, y)$ and $Q(x, 0)$ is $M\left(\frac{0+x}{2}, \frac{y+0}{2}\right)$

$\left[\because \text{mid-point of a line segment having points } (x_1, y_1) \text{ and } (x_2, y_2) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)\right]$

But it is given that, mid-point of PQ is $(2, -5)$.



$\therefore \quad 2 = \frac{x+0}{2}$

and $\quad -5 = \frac{y+0}{2}$

$\Rightarrow \quad 4 = x$ and $-10 = y$

$\Rightarrow \quad x = 4$ and $y = -10$

So, the coordinates of P and Q are $(0, -10)$ and $(4, 0)$.

Q. 18 The area of a triangle with vertices $(a, b+c)$, $(b, c+a)$ and $(c, a+b)$ is

(a) $(a+b+c)^2$ (b) 0 (c) $(a+b+c)$ (d) abc

Sol. *(b)* Let the vertices of a triangle are, $A \equiv (x_1, y_1) \equiv (a, b+c)$

$B \equiv (x_2, y_2) \equiv (b, c+a)$ and $C \equiv (x_3, y_3) \equiv (c, a+b)$

$\because$ Area of $\Delta ABC = \Delta = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$$\therefore \quad \Delta = \frac{1}{2}[a(c+a-a-b) + b(a+b-b-c) + c(b+c-c-a)]$$

$$= \frac{1}{2}[a(c-b) + b(a-c) + c(b-a)]$$

$$= \frac{1}{2}(ac - ab + ab - bc + bc - ac) = \frac{1}{2}(0) = 0$$

Hence, the required area of triangle is 0.

Q. 19 If the distance between the points $(4, p)$ and $(1, 0)$ is 5, then the value of p is

(a) 4 only (b) ± 4 (c) -4 only (d) 0

Sol. *(b)* According to the question, the distance between the points $(4, p)$ and $(1, 0) = 5$

i.e., $$\sqrt{(1-4)^2 + (0-p)^2} = 5$$

$$\left[\because \text{distance between the points } (x_1, y_1) \text{ and } (x_2, y_2), d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\right]$$

$$\Rightarrow \sqrt{(-3)^2 + p^2} = 5$$

$$\Rightarrow \sqrt{9 + p^2} = 5$$

On squaring both the sides, we get

$$9 + p^2 = 25$$

$$\Rightarrow p^2 = 16 \Rightarrow p = \pm 4$$

Hence, the required value of p is ± 4.

Q. 20 If the points $A(1, 2)$, $B(0, 0)$ and $C(a, b)$ are collinear, then

(a) $a = b$ (b) $a = 2b$ (c) $2a = b$ (d) $a = -b$

Sol. *(c)* Let the given points are $A \equiv (x_1, y_1) \equiv (1, 2)$,
$B \equiv (x_2, y_2) \equiv (0, 0)$ and $C \equiv (x_3, y_3) \equiv (a, b)$.

$\because$ Area of ΔABC $\Delta = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$$\therefore \Delta = \frac{1}{2}[1(0-b) + 0(b-2) + a(2-0)]$$

$$= \frac{1}{2}(-b + 0 + 2a) = \frac{1}{2}(2a - b)$$

Since, the points $A(1, 2)$, $B(0, 0)$ and $C(a, b)$ are collinear, then area of ΔABC should be equal to zero.

i.e., area of $\Delta ABC = 0$

$$\Rightarrow \frac{1}{2}(2a - b) = 0$$

$$\Rightarrow 2a - b = 0$$

$$\Rightarrow 2a = b$$

Hence, the required relation is $2a = b$.