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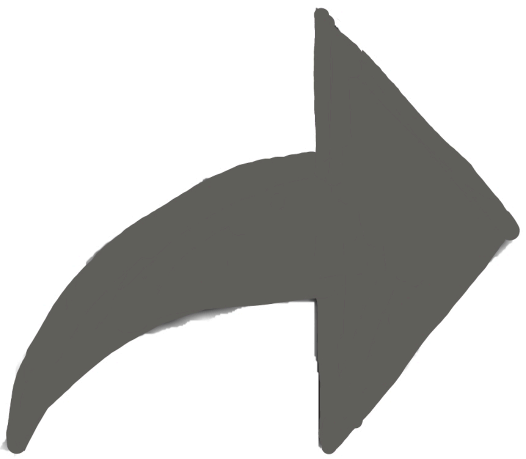

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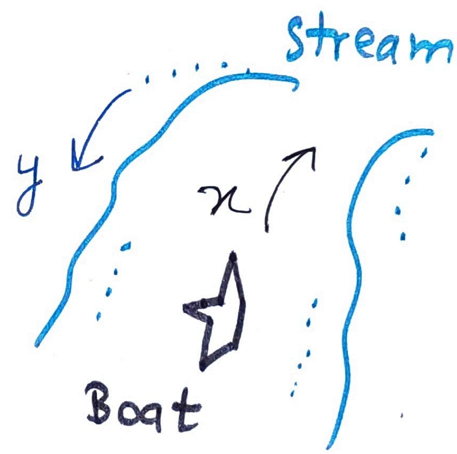
NOTIFICATIONS

PAIR OF LINEAR Eq^n in 2 V^s

Pair of Linear Equations	Algebraic Condition	Graphical Interpretation	Algebraic Interpretation	Consistency
$a_1x + b_1y + c_1 = 0$ $a_2x + b_2y + c_2 = 0$	$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$	Intersecting lines	Unique solution	Consistent
	$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$	Coincident lines	Infinitely many solutions	Dependent Consistent
	$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$	Parallel lines	No solution	Inconsistent

ALERT TIPS

- **Upstream** : $(x - y)\ ms^{-1}$
- **Downstream** : $(x + y)\ ms^{-1}$



- **Taking a number** :-
  - One's digit ⇒ y
  - Ten's digit ⇒ x
  - ↳ Number format : $10x + y$

METHODS TO SOLVE
- Substitution
- Elimination
- Reduction

Chapter-3 Pair of Linear Equation in Two variable

Very Short Question

Q. 1. For what value of k, the pair of linear equations $kx - 4y = 3$, $6x - 12y = 9$ has an infinite number of solutions? [CBSE 2012, 17]

Sol. Pair of linear equations

$$kx - 4y - 3 = 0$$
$$6x - 12y - 9 = 0$$

Condition for infinite solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\frac{k}{6} = \frac{-4}{-12} = \frac{3}{9} \Rightarrow k = 2.$$

Q. 2. Determine the value of y satisfying both the equations $x + 6 = y$ and $2x - y = 4$. [CBSE 2014]

Sol. Here, $x + 6 = y$ ...(1)

and $2x - y = 4$ ...(2)

From eq.(1), substituting the value of y in eq. (2), we obtain

$$2x - (x + 6) = 4$$

$\Rightarrow$ $2x - x - 6 = 4 \Rightarrow x = 10$

From eq. (1), we obtain $y = 10 + 6 = 16$

Q.9. For which values of p and q will the following system of linear equations have infinitely many solutions?

$4x + 5y = 2;$

$(2p + 7q)x + (p + 8q)y = 2q - p + 1$

[CBSE 2011]

Sol. For infinitely many solutions, we have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$\Rightarrow$ $\frac{4}{2p + 7q} = \frac{5}{p + 8q} = \frac{2}{2q - p + 1}$

$\Rightarrow$ $4p + 32q = 10p + 35q$

and $10q - 5p + 5 = 2p + 16q$

$\Rightarrow$ $-7p - 6q = -5$

$\Rightarrow$ $q = -2p$

and $7p + 6q = 5$

Now, put $q = -2p$ in $7p + 6q = 5$, we have

$7p + 6(-2p) = 5$

$\Rightarrow$ $7p - 12p = 5$

$\Rightarrow$ $-5p = 5$

$\Rightarrow$ $p = -1$

$\therefore$ $q = -2p = -2(-1) = 2$

Thus, we have $p = -1$ and $q = 2$.

Q. 1. Find the value of $x + y$, if $3x - 2y = 5$ and $3y - 2x = 3$. [CBSE 2014]

Sol. We have, $3x - 2y = 5$ ...(1)

and $-2x + 3y = 3$ ...(2)

On adding eqs.(1) and (2), we get

$$x + y = 8$$

Long Question

Q.12. A father's age is three times the sum of the ages of his two children. After 5 years his age will be two times the sum of their ages. Find the present age of the father. [CBSE 2019]

Sol. Let the present age of father be x years and the sum of present ages of his two children be y years.

According to question,

$$x = 3y$$

$\Rightarrow$ $x - 3y = 0$ ...(i)

After 5 years,

$$x + 5 = 2(y + 10)$$

$\Rightarrow$ $x - 2y = 15$ ...(ii)

On subtracting equation (i) from (ii), we get,

$$\begin{array}{r} x - 2y = 15 \\ x - 3y = 0 \\ - \quad + \quad - \\ \hline y = 15 \end{array}$$

On substituting the value of $y = 15$ in (i), we get,

$$x - 3 \times 15 = 0$$

$\therefore$ $x = 45$

Hence, the present age of father is 45 years.

Q.9. The sum of digits of a two-digit number is 11. The number obtained by interchanging the digits of the given number exceeds that number by 63. Find the number. [CBSE 2011,17]

Sol. Let ten's digit be x and unit digit be y.

$\therefore$ Number $= 10x + y$

According to the statement of the question, we have

$x + y = 11$ ...(1)

and $10x + y + 63 = 10y + x$

$\Rightarrow$ $9x - 9y = -63$

$\Rightarrow$ $x - y = -7$ ...(2)

Solving eqs. (1) and (2), we get

$$2x = 4 \Rightarrow x = 2$$

From eq. (1), we get

$$2 + y = 11 \Rightarrow y = 9$$

$\therefore$ Number $= 10(2) + 9 = 20 + 9 = 29$

Hence, the number is 29.

Q.4. **4 men and 6 boys can finish a piece of work in 5 days, while 3 men and 4 boys can finish it in 7 days. Find the time taken by 1 man alone or that by 1 boy alone.** [*CBSE 2011*]

Sol. Let the man finishes the work in x days and that the boy finishes in y days.

$$\text{One day's work of a man} = \frac{1}{x}$$

and $$\text{one day's work of a boy} = \frac{1}{y}$$

Since, 4 men and 6 boys finish a piece of work in 5 days.

$\therefore$ One day's work of 4 men and 6 boys $= \frac{1}{5}$ part of the work

Now, one day's work of 4 men and 6 boys is $\frac{4}{x} + \frac{6}{y} = \frac{1}{5}$

Similarly, in second case,

One day's work of 3 men and 4 boys $= \frac{1}{7}$ part of the work

$$\Rightarrow \quad \frac{3}{x} + \frac{4}{y} = \frac{1}{7}$$

Thus, we have the following equations

$$\frac{4}{x} + \frac{6}{y} = \frac{1}{5} \quad \text{...(1)}$$

and $$\frac{3}{x} + \frac{4}{y} = \frac{1}{7} \quad \text{...(2)}$$

Here, eqs. (1)and (2) are not in linear form, so we reduce them in linear form by putting

$$\frac{1}{x} = u \text{ and } \frac{1}{y} = v$$

$\therefore$ Eq.(1) becomes $\quad 4u + 6v = \frac{1}{5} \quad$ ...(3)

and eq. (2) becomes $\quad 3u + 4v = \frac{1}{7} \quad$ ...(4)

On multiplying eq.(3) by 3 and eq.(4) by 4 and then subtracting eq.(4) from eq.(3), we get

$$18v - 16v = \frac{3}{5} - \frac{4}{7}$$

$$\Rightarrow \quad 2v = \frac{21-20}{35}$$

$$\Rightarrow \quad 2v = \frac{1}{35}$$

$$\Rightarrow \quad v = \frac{1}{70}$$

Put $v = \frac{1}{70}$ in eq.(4), we get, $3u + \frac{4}{70} = \frac{1}{7}$

$$\Rightarrow \quad 3u = \frac{1}{7} - \frac{4}{70}$$

$$\Rightarrow \quad 3u = \frac{6}{70}$$

$$\Rightarrow \quad u = \frac{1}{35}$$

$$\therefore \quad u = \frac{1}{35} \text{ and } v = \frac{1}{70}$$

$$\Rightarrow \quad \frac{1}{x} = \frac{1}{35} \text{ and } \frac{1}{y} = \frac{1}{70}$$

$$\left[\text{put } u = \frac{1}{x} \text{ and } v = \frac{1}{y}\right]$$

$$\Rightarrow \quad x = 35 \text{ and } y = 70$$

Q.8. Raghav scored 70 marks in a test, getting 4 marks for each right answer and losing 1 mark for each wrong answer. Had 5 marks been awarded for each correct answer and 2 marks been deducted for each wrong answer, then Raghav would have scored 80 marks. How many questions were there in the test?

Which values would have Raghav violated if he resorted to unfair means? [*CBSE 2015*]

Sol. Let number of right answers be x.

Let number of wrong answers be y.

As per question

$$4x - y = 70 \quad \text{...(1)}$$

$$5x - 2y = 80 \quad \text{...(2)}$$

On multiplying eq.(1) by 2 then subtract eq.(2) from eq.(1), we get

$$8x - 2y = 140$$
$$5x - 2y = 80$$
$$\underline{-\quad +\qquad -}$$
$$3x = 60$$

$$\Rightarrow \quad x = 20$$

Substituting the value of x in eq.(1) to get value of y

$$4(20) - y = 70$$

$$\Rightarrow \quad 80 - y = 70 \Rightarrow y = 10$$

Hence, total number of questions are $= 20 + 10 = 30$

Value : Honesty would have been violated.

Q.5. If 2 is subtracted from the numerator and 1 is added to the denominator, a fraction becomes $\frac{1}{2}$, but when 4 is added to the numerator and 3 is subtracted from the denominator, it becomes $\frac{3}{2}$. Find the fraction. ***HOTS*** **[CBSE 2012]**

Sol. Let the fraction be $\frac{x}{y}$.

$$\frac{x-2}{y+1} = \frac{1}{2}$$

$$\Rightarrow \quad 2x - 4 = y + 1 \quad \text{...(1)}$$

$$\Rightarrow \quad 2x - y = 5$$

Also, $$\frac{x+4}{y-3} = \frac{3}{2}$$

$$\Rightarrow \quad 2x + 8 = 3y - 9$$

$$2x - 3y = -17 \quad \text{...(2)}$$

Subtracting eq.(2) from eq.(1), we get

$$2y = 22 \quad \Rightarrow \quad y = 11$$

Substituting this value of y in eq.(1), we get

$$2x - 11 = 5 \quad \Rightarrow \quad x = 8$$

$$x = 8, y = 11$$

Hence, $\quad$ Fraction $= \frac{8}{11}$

Q.11. A motor boat whose speed is 18 km/hr in still water 1 hr more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream. **[CBSE 2018]**

Sol. Let the speed of stream be x km/hr

Now, for upstream : speed $= (18 - x)$ km/hr

$$\therefore \quad \text{Time taken} = \left(\frac{24}{18-x}\right)\text{hr}$$

Now, for downstream : speed $= (18 + x)$ km/hr

$$\therefore \quad \text{Time taken} = \left(\frac{24}{18+x}\right)\text{hr}$$

Given that,

$$\frac{24}{18-x} = \frac{24}{18+x} + 1$$

$$-1 = \frac{24}{18+x} - \frac{24}{18-x}$$

$$-1 = \frac{24[(18-x)-(18+x)]}{(18)^2 - x}$$

$$-1 = \frac{24[-2x]}{324 - x^2}$$

$$324 + x^2 = -48x$$

$$x^2 + 48x - 324 = 0$$

$$x^2 + 54x - 6x - 324 = 0$$

$$(x + 54)(x - 6) = 0$$

$$x = -54 \quad \text{or} \quad x = 6$$

$$x = 54 \text{ km/hr (not possible)}$$

Therefore, speed of the stream = 6 km/hr.

3

Pair *of* Linear Equations *in* Two Variables

Exercise 3.1 Multiple Choice Questions (MCQs)

Q. 1 Graphically, the pair of equations

$$6x - 3y + 10 = 0$$
$$2x - y + 9 = 0$$

represents two lines which are

(a) intersecting at exactly one point
(b) intersecting exactly two points
(c) coincident
(d) parallel

Sol. *(d)* The given equations are

$$6x - 3y + 10 = 0$$

$\Rightarrow$ $$2x - y + \frac{10}{3} = 0 \quad \text{[dividing by 3]} \ldots \text{(i)}$$

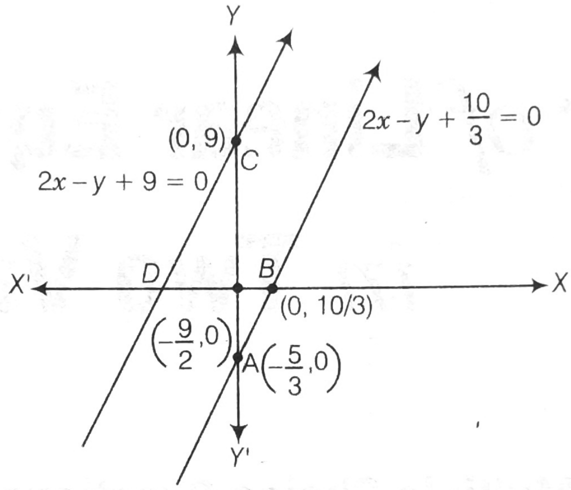
and $$2x - y + 9 = 0 \quad \ldots \text{(ii)}$$

Now, table for $2x - y + \frac{10}{3} = 0$,

x	0	$-\frac{5}{3}$
$y = 2x + \frac{10}{3}$	$\frac{10}{3}$	0
Points	A	B

and table for $2x - y + 9 = 0$,

x	0	$-\frac{9}{2}$
$y = 2x + 9$	9	0
Points	C	D



Hence, the pair of equations represents two parallel lines.

Q. 2 The pair of equations $x + 2y + 5 = 0$ and $-3x - 6y + 1 = 0$ has

(a) a unique solution (b) exactly two solutions
(c) infinitely many solutions (d) no solution

Sol. *(d)* Given, equations are $x + 2y + 5 = 0$ and $-3x - 6y + 1 = 0$

Here, $a_1 = 1, b_1 = 2, c_1 = 5$ and $a_2 = -3, b_2 = -6, c_2 = 1$

$$\therefore \quad \frac{a_1}{a_2} = -\frac{1}{3}, \frac{b_1}{b_2} = -\frac{2}{6} = -\frac{1}{3},$$

$$\frac{c_1}{c_2} = \frac{5}{1}$$

$$\therefore \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Hence, the pair of equations has no solution.

Q. 3 If a pair of linear equations is consistent, then the lines will be

(a) parallel (b) always coincident
(c) intersecting or coincident (d) always intersecting

Sol. *(c)* Condition for a consistent pair of linear equations

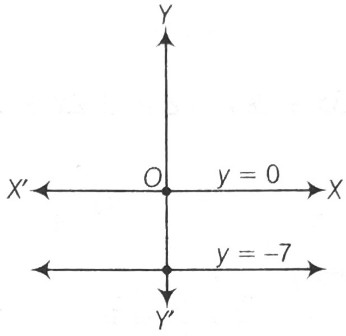
$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \quad \text{[intersecting lines having unique solution]}$$

and $$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \quad \text{[coincident or dependent]}$$

Q. 4 The pair of equations $y = 0$ and $y = -7$ has

(a) one solution (b) two solutions
(c) infinitely many solutions (d) no solution

Sol. *(d)* The given pair of equations are $y = 0$ and $y = -7$.



By graphically, both lines are parallel and having no solution.

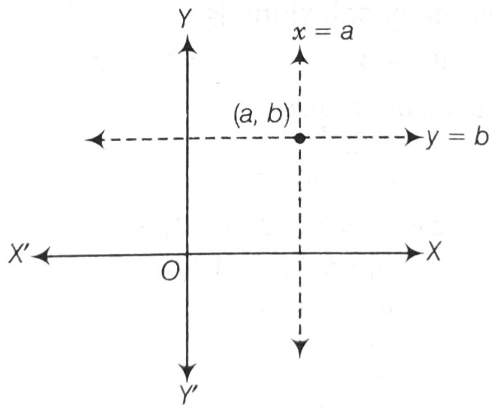
Q. 5 The pair of equations $x = a$ and $y = b$ graphically represents lines which are

(a) parallel (b) intersecting at (b, a)
(c) coincident (d) intersecting at (a, b)

Sol. *(d)* By graphically in every condition, if $a, b > 0$; $a, b < 0$, $a > 0$, $b < 0$; $a < 0$, $b > 0$ but $a = b \neq 0$.

The pair of equations $x = a$ and $y = b$ graphically represents lines which are intersecting at (a, b).

If $a, b > 0$



Similarly, in all cases two lines intersect at (a, b).

Q. 6 For what value of k, do the equations $3x - y + 8 = 0$ and $6x - ky = -16$ represent coincident lines?

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$ (c) 2 (d) -2

Sol. *(c)* Condition for coincident lines is

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \quad \ldots(i)$$

Given lines, $3x - y + 8 = 0$
and $6x - ky + 16 = 0$

Here, $a_1 = 3, b_1 = -1, c_1 = 8$

and $a_2 = 6, b_2 = -k, c_2 = 16$

From Eq. (i), $\frac{3}{6} = \frac{-1}{-k} = \frac{8}{16}$

$\Rightarrow \frac{1}{k} = \frac{1}{2}$

$\therefore k = 2$

Q. 7 If the lines given by $3x + 2ky = 2$ and $2x + 5y = 1$ are parallel, then the value of k is

(a) $-\frac{5}{4}$ (b) $\frac{2}{5}$ (c) $\frac{15}{4}$ (d) $\frac{3}{2}$

Sol. *(c)* Condition for parallel lines is

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \quad \ldots(i)$$

Given lines, $3x + 2ky - 2 = 0$

and $2x + 5y - 1 = 0$

Here, $a_1 = 3, b_1 = 2k, c_1 = -2$

and $a_2 = 2, b_2 = 5, c_2 = -1$

From Eq. (i), $\frac{3}{2} = \frac{2k}{5}$

$\therefore k = \frac{15}{4}$

Q. 8 The value of c for which the pair of equations $cx - y = 2$ and $6x - 2y = 3$ will have infinitely many solutions is

(a) 3 (b) -3 (c) -12 (d) no value

Sol. *(d)* Condition for infinitely many solutions

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \quad \ldots(i)$$

The given lines are $cx - y = 2$ and $6x - 2y = 3$

Here, $a_1 = c, b_1 = -1, c_1 = -2$

and $a_2 = 6, b_2 = -2, c_2 = -3$

From Eq. (i), $\frac{c}{6} = \frac{-1}{-2} = \frac{-2}{-3}$

Here, $\frac{c}{6} = \frac{1}{2}$ and $\frac{c}{6} = \frac{2}{3}$

$\Rightarrow c = 3$ and $c = 4$

Since, c has different values.

Hence, for no value of c the pair of equations will have infinitely many solutions.

Q. 9 One equation of a pair of dependent linear equations is $-5x + 7y - 2 = 0$. The second equation can be

(a) $10x + 14y + 4 = 0$ (b) $-10x - 14y + 4 = 0$

(c) $-10x + 14y + 4 = 0$ (d) $10x - 14y + 4 = 0$

Sol. *(d)* Condition for dependent linear equations

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \frac{1}{k} \quad \ldots(i)$$

Given equation of line is, $-5x + 7y - 2 = 0$

Here, $a_1 = -5, b_1 = 7, c_1 = -2$

From Eq. (i), $-\frac{5}{a_2} = \frac{7}{b_2} = -\frac{2}{c_2} = \frac{1}{k}$ [say]

$\Rightarrow$ $a_2 = -5k, b_2 = 7k, c_2 = -2k$

where, k is any arbitrary constant.

Putting $k = 2$, then $a_2 = -10, b_2 = 14$

and $c_2 = -4$

$\therefore$ The required equation of line becomes

$$a_2x + b_2y + c_2 = 0$$

$$\Rightarrow \quad -10x + 14y - 4 = 0$$

$$\Rightarrow \quad 10x - 14y + 4 = 0$$

Q. 10 A pair of linear equations which has a unique solution $x = 2$ and $y = -3$ is

(a) $x + y = 1$ and $2x - 3y = -5$
(b) $2x + 5y = -11$ and $4x + 10y = -22$
(c) $2x - y = 1$ and $3x + 2y = 0$
(d) $x - 4y - 14 = 0$ and $5x - y - 13 = 0$

Sol. ***(b)*** If $x = 2, y = -3$ is a unique solution of any pair of equation, then these values must satisfy that pair of equations.

From option (b), LHS $= 2x + 5y = 2(2) + 5(-3) = 4 - 15 = -11 =$ RHS

and LHS $= 4x + 10y = 4(2) + 10(-3) = 8 - 30 = -22 =$ RHS

Q. 11 If $x = a$ and $y = b$ is the solution of the equations $x - y = 2$ and $x + y = 4$, then the values of a and b are, respectively

(a) 3 and 5 (b) 5 and 3 (c) 3 and 1 (d) -1 and -3

Sol. ***(c)*** Since, $x = a$ and $y = b$ is the solution of the equations $x - y = 2$ and $x + y = 4$, then these values will satisfy that equations

$$a - b = 2 \quad \ldots\text{(i)}$$

and $$a + b = 4 \quad \ldots\text{(ii)}$$

On adding Eqs. (i) and (ii), we get

$$2a = 6$$

$$\therefore \quad a = 3 \text{ and } b = 1$$

Q. 12 Aruna has only ₹ 1 and ₹ 2 coins with her. If the total number of coins that she has is 50 and the amount of money with her is ₹ 75, then the number of ₹ 1 and ₹ 2 coins are, respectively

(a) 35 and 15 (b) 35 and 20 (c) 15 and 35 (d) 25 and 25

Sol. ***(d)*** Let number of ₹ 1 coins $= x$

and number of ₹ 2 coins $= y$

Now, by given conditions $x + y = 50$ ...(i)

Also, $x \times 1 + y \times 2 = 75$

$\Rightarrow$ $x + 2y = 75$ ...(ii)

On subtracting Eq. (i) from Eq. (ii), we get

$$(x + 2y) - (x + y) = 75 - 50$$

$$\Rightarrow \quad y = 25$$

When $y = 25$, then $x = 25$

Q. 13 The father's age is six times his son's age. Four years hence, the age of the father will be four times his son's age. The present ages (in year) of the son and the father are, respectively

(a) 4 and 24 (b) 5 and 30

(c) 6 and 36 (d) 3 and 24

Sol. *(c)* Let x yr be the present age of father and y yr be the present age of son.

Four years hence, it has relation by given condition,

$$(x + 4) = 4(y + 4)$$

$$\Rightarrow \quad x - 4y = 12 \quad \dots(i)$$

and

$$x = 6y \quad \dots(ii)$$

On putting the value of x from Eq. (ii) in Eq. (i), we get

$$6y - 4y = 12$$

$$\Rightarrow \quad 2y = 12$$

$$\Rightarrow \quad y = 6$$

When $y = 6$, then $x = 36$

Hence, present age of father is 36 yr and age of son is 6 yr.