

Quiz 1 (Version 2)

CAS CS 132: *Geometric Algorithms*

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- ▷ You will have approximately 35 minutes to complete this exam.
- ▷ Your final solution must appear in the solution boxes for each problem. **Only include your final solution in the solution box. You must show your work outside of the solution box.** You will not receive credit if you don't show your work.

1 Row Operations

Apply the row operations:

$$R_3 \leftarrow R_3 - 5R_1$$

$$R_3 \leftrightarrow R_1$$

$$R_1 \leftarrow -5R_1$$

from top to bottom to the following matrix. You must write down the intermediate matrices and row operations you used in your calculation.

$$\begin{array}{c}
 \left[\begin{array}{ccc} -5 & 3 & -3 \\ 10 & 0 & 9 \\ 2 & 5 & 3 \end{array} \right] \\
 \xrightarrow{\substack{R_3 \leftarrow R_3 - 5R_1}} \left[\begin{array}{ccc} -5 & 3 & -3 \\ 10 & 0 & 9 \\ 27 & -10 & 18 \end{array} \right] \\
 \xrightarrow{R_3 \leftrightarrow R_1} \left[\begin{array}{ccc} 27 & -10 & 18 \\ 10 & 0 & 9 \\ -5 & 3 & -3 \end{array} \right] \\
 \xrightarrow{R_1 \leftarrow -5R_1} \left[\begin{array}{ccc} -135 & 50 & -90 \\ 10 & 0 & 9 \\ -5 & 3 & -3 \end{array} \right]
 \end{array}$$

$$\begin{array}{r}
 3 \quad 27 \\
 \underline{-} \quad \underline{5} \\
 \hline 135
 \end{array}$$

$$\begin{array}{r}
 4 \quad 18 \\
 \underline{-} \quad \underline{5} \\
 \hline 9 \quad 0
 \end{array}$$

Solution.

$$\left[\begin{array}{ccc} -135 & 50 & -90 \\ 10 & 0 & 9 \\ -5 & 3 & -3 \end{array} \right]$$

2 Reduced Echelon Forms

Determine the reduced echelon form of the following matrix. You must write down the intermediate matrices and row operations you used in your calculation.

$$\begin{bmatrix} 1 & 0 & 6 & -4 \\ -3 & -1 & -13 & 14 \\ 0 & -1 & 5 & 2 \end{bmatrix}$$

$$\left[\begin{array}{cccc} 1 & 0 & 6 & -4 \\ -3 & -1 & -13 & 14 \\ 0 & -1 & 5 & 2 \end{array} \right] \xrightarrow{R_2 \leftarrow R_2 + 3R_1} \left[\begin{array}{cccc} 1 & 0 & 6 & -4 \\ 0 & -1 & 5 & 2 \\ 0 & -1 & 5 & 2 \end{array} \right]$$

$$\xrightarrow{R_3 \leftarrow R_3 - R_2} \left[\begin{array}{cccc} 1 & 0 & 6 & -4 \\ 0 & -1 & 5 & 2 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$\xrightarrow{R_2 \leftarrow -R_2} \left[\begin{array}{cccc} 1 & 0 & 6 & -4 \\ 0 & 1 & -5 & -2 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

Solution.

$$\left[\begin{array}{cccc} 1 & 0 & 6 & -4 \\ 0 & 1 & -5 & -2 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

3 General Form Solutions

Determine a general form solution for a linear system whose augmented matrix is row equivalent to the following matrix.

$$\begin{bmatrix} 1 & 3 & 0 & 0 & 0 & -5 \\ 0 & 0 & 1 & -4 & 0 & -3 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Solution.

$$x_1 = -5 - 3x_7$$

x_7 is free

$$x_3 = -3 + 4x_4$$

x_4 is free

$$x_5 = -1$$