

Assignment 1 Solutions

1.3

$$\begin{bmatrix} 1 & -2 & -2 & 2 \\ 2 & -3 & -5 & 2 \\ -2 & 2 & 7 & -1 \end{bmatrix}$$

augmented

$$\begin{bmatrix} 1 & -2 & -2 \\ 2 & -3 & -5 \\ -2 & 2 & 7 \end{bmatrix}$$

coefficient

1.6

$$2(10) - 2(8) - 6(2) + 4(-3) = 20 - 16 - 12 - 12 = -20$$

$$10 + 2(8) - 6(2) - 4(-3) = 10 + 16 - 12 + 12 = 26$$

$$10 - 4(2) - (-3) = 10 - 8 + 3 = 5$$

✓

✓

1.7

$$\begin{bmatrix} 3 & 6 & -15 \\ -1 & 1 & 5 \end{bmatrix}$$

$R_1 \leftrightarrow R_2$

$$\begin{bmatrix} -1 & 1 & 5 \\ 3 & 6 & -15 \end{bmatrix}$$

$R_2 \leftarrow R_2 + 3R_1$

$$\begin{bmatrix} -1 & 1 & 5 \\ 0 & 9 & 0 \end{bmatrix}$$

$R_2 \leftarrow R_2 / 9$

$$\begin{bmatrix} -1 & 1 & 5 \\ 0 & 1 & 0 \end{bmatrix}$$

$R_1 \leftarrow R_1 - R_2$

$$\begin{bmatrix} -1 & 0 & 5 \\ 0 & 1 & 0 \end{bmatrix}$$

$R_1 \leftarrow -R_1$

$$\begin{bmatrix} 1 & 0 & -5 \\ 0 & 1 & 0 \end{bmatrix}$$

$x_1 = -5$

$x_2 = 0$

1.11

$$\left[\begin{array}{cccc} -3 & 0 & 6 & 12 \\ 3 & 3 & -12 & -21 \\ 2 & -6 & 10 & 16 \end{array} \right]$$

$$\begin{aligned} R_1 &\leftarrow R_1 / -3 \\ R_2 &\leftarrow R_2 / 3 \\ R_3 &\leftarrow R_3 / 2 \end{aligned}$$

$$\left[\begin{array}{cccc} 1 & 0 & -2 & -4 \\ -1 & 1 & -4 & -7 \\ 1 & -3 & 5 & 8 \end{array} \right]$$

$$\overrightarrow{R_2 \leftarrow R_2 - R_1}$$

$$\left[\begin{array}{cccc} 1 & 0 & -2 & -4 \\ 0 & 1 & -2 & -3 \\ -1 & -3 & 5 & 8 \end{array} \right]$$

$$x_1 = 4$$

$$x_2 = 5$$

$$x_3 = 4$$

$$\overrightarrow{R_3 \leftarrow R_3 - R_1}$$

$$\left[\begin{array}{cccc} 1 & 0 & -2 & -4 \\ 0 & 1 & -2 & -3 \\ 0 & -3 & 7 & 12 \end{array} \right]$$

$$x_2 + x_3$$

$$\overrightarrow{R_3 \leftarrow R_3 + 3R_2}$$

$$\left[\begin{array}{cccc} 1 & 0 & -2 & -4 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 1 & 4 \end{array} \right]$$

$$\overrightarrow{R_2 \leftarrow R_2 + 2R_3}$$

$$\overrightarrow{R_1 \leftarrow R_1 + 2R_3}$$

$$\left[\begin{array}{cccc} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 4 \end{array} \right]$$

1.16

$$\left[\begin{array}{ccc|c} -4 & 1 & 3 \\ -7 & 0 & 5 \\ -4 & 3 & 1 \\ \hline +21 & & -15 \end{array} \right] \xrightarrow{R_3 \leftarrow R_3 - 3R_2} \left[\begin{array}{ccc|c} -4 & 1 & 3 \\ -7 & 0 & 5 \\ 17 & -12 & 3 & -14 \\ \hline +9 & & & \end{array} \right]$$

$$\xrightarrow{R_2 \leftarrow R_2 + 3R_1} \left[\begin{array}{ccc|c} -4 & 1 & 3 \\ -7 & 0 & 5 \\ 5 & 6 & -5 \\ \hline & & & \end{array} \right]$$

$$\xrightarrow{R_1 \leftrightarrow R_3} \left[\begin{array}{ccc|c} 5 & 6 & -5 \\ -7 & 0 & 5 \\ -4 & 1 & 3 \\ \hline & & & \end{array} \right]$$

1.18

$$x_1 = -5 + 3x_2$$

x_2 is free

$$x_3 = 8$$

$$x_4 = 5$$

1.23

$$x_1 = -6 + 3x_4$$

$$x_2 = 5 - 5x_4$$

$$x_3 = -4$$

x_4 is free

$$s_1 = (-6, 5, -4, 0)$$

$$s_2 = (-3, 0, -4, 1)$$

$$s_3 = (-9, 10, -4, -1)$$

1.28

$$\left[\begin{array}{ccccc} 1 & -1 & -2 & 1 \\ -1 & 2 & 4 & 0 \\ 2 & -3 & -6 & 2 \\ -2 & 1 & 2 & -1 \end{array} \right] \xrightarrow{R_2 \leftarrow R_2 + R_1} \left[\begin{array}{ccccc} 1 & -1 & -2 & 1 \\ 0 & 1 & 2 & 1 \\ 2 & -3 & -6 & 2 \\ -2 & 1 & 2 & -1 \end{array} \right]$$

$$\xrightarrow{R_3 \leftarrow R_3 - 2R_1} \left[\begin{array}{ccccc} 1 & -1 & -2 & 1 \\ 0 & 1 & 2 & 1 \\ 0 & -1 & -2 & 0 \\ -2 & 1 & 2 & -1 \end{array} \right]$$

$$\xrightarrow{R_4 \leftarrow R_4 + 2R_1} \left[\begin{array}{ccccc} 1 & -1 & -2 & 1 \\ 0 & 1 & 2 & 1 \\ 0 & -1 & -2 & 0 \\ 0 & -1 & -2 & 1 \end{array} \right]$$

$$\xrightarrow{R_3 \leftarrow R_3 + R_2} \left[\begin{array}{ccccc} 1 & -1 & -2 & 1 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & -1 & -2 & 1 \end{array} \right]$$

$$\xrightarrow{R_4 \leftarrow R_4 + R_2} \left[\begin{array}{ccccc} 1 & -1 & -2 & 1 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 2 \end{array} \right]$$

$$R_4 \leftarrow R_4 - 2R_3 \rightarrow \begin{bmatrix} 1 & -1 & -2 & 1 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$R_2 \leftarrow R_2 + R_3 \rightarrow \begin{bmatrix} 1 & -1 & -2 & 1 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$R_1 \leftarrow R_1 + R_2 \rightarrow \begin{bmatrix} 1 & \cancel{-1}^{\cancel{+1}} & \cancel{-2}^{\cancel{+2}} & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\boxed{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}}$$

1.29

$$\left[\begin{array}{cccc|c} 1 & -2 & -7 & -3 & 1 \\ 2 & 1 & 6 & 14 & 7 \\ -1 & 1 & 5 & 9 & 1 \end{array} \right] \xrightarrow{\quad} \left[\begin{array}{cccc|c} 1 & -2 & -7 & -3 & 1 \\ 0 & 5 & 20 & 20 & 7 \\ 0 & 3 & 12 & 12 & 1 \end{array} \right]$$

$$x_1 = 5 - x_3$$

$$x_2 = 4 - 4x_3$$

x_3 is free

$$\xrightarrow{\quad} \left[\begin{array}{cccc|c} 1 & -2 & -7 & -3 & 1 \\ 0 & 1 & 4 & 4 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{array} \right]$$

$$\xrightarrow{\quad} \left[\begin{array}{cccc|c} 1 & 0 & 1 & 5 & 1 \\ 0 & 1 & 4 & 4 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

1.41 True (we proved this in lecture)

1.45 True (RREFs are unique)

1.47 True (we proved this in lecture)

1.49 False $\left[\begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix} \right] \sim \left[\begin{matrix} 1 & 1 \\ 0 & 1 \end{matrix} \right] \sim \left[\begin{matrix} 1 & 2 \\ 0 & 1 \end{matrix} \right]$ 1.50 False $x + y + z = 1$
 $x + y + z = 2$

1.54 False

$$\begin{aligned}x &= 5 \\y &= 6\end{aligned}\quad \left[\begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array} \right]$$

1.55 True $\left(\Rightarrow 0 = 1 \text{ in an equation in system represented by echelon form} \right)$

1.56 False

$$\begin{aligned}x + y &= 1 \\2x + 2y &= 2\end{aligned}$$

1.57 True

$$\left[\begin{array}{cc} a & 2a \\ b & 3b \end{array} \right] \xrightarrow{\begin{array}{l} R_1 \leftarrow R_1/a \\ R_2 \leftarrow R_2/b \end{array}} \left[\begin{array}{cc} 1 & 2 \\ 1 & 3 \end{array} \right] \sim \left[\begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array} \right]$$

1.59

$$\left[\begin{array}{cccc} 1 & 0 & -1/3 & 1 \\ 0 & 1 & 1/4 & 4 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$x_1 = 1 + \frac{1}{3}x_3 \quad 1 - \frac{1}{3} = 1 - 4 = -3$$

$$x_2 = 4 - \frac{1}{4}x_3 \quad 4 - \frac{1}{4} = 4 - 3 = 1$$

x_3 is free

$$s = (-3, 1, 12)$$

1.60

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

1.64

$$\left[\begin{array}{ccc} 1 & 4 & -1 \\ -2 & -12 & +3 \\ 3 & -h & 7 \end{array} \right] \sim \left[\begin{array}{ccc} 1 & 4 & -1 \\ 0 & -(h+12) & 10 \end{array} \right]$$

$$h = -12$$

↑
must be 0
to be inconsistent