Quiz 1 (Version 1)

CAS CS 132: *Geometric Algorithms*

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- ▷ You will have approximately 35 minutes to complete this exam.
- ▷ Your final solution must appear in the solution boxes for each problem. **Only include your final solution in the solution box. You must show your work outside of the solution box.** You will not recieve credit it you don't show your work.

1 Row Operations

Apply the row operations:

$$R_1 \leftarrow R_1 + 2R_2$$
$$R_1 \leftrightarrow R_2$$
$$R_2 \leftarrow -3R_2$$

from top to bottom to the following matrix. You must write down the intermediate matrices and row operations you used in your calculation.

$$\begin{bmatrix} -1 & -3 & 10 \\ 1 & -1 & -2 \\ -9 & 10 & -4 \end{bmatrix} \xrightarrow{R_1 \leftarrow R_1 + 2R_2} \begin{bmatrix} 1 & -5 & 6 \\ 1 & -1 & -2 \\ -9 & 10 & -4 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{bmatrix} 1 & -1 & -2 \\ 1 & -5 & 6 \\ -9 & 10 & -4 \end{bmatrix} \xrightarrow{R_2 \leftarrow -3R_2} \begin{bmatrix} 1 & -1 & -2 \\ -3 & 15 & -18 \\ -9 & 10 & -4 \end{bmatrix}$$

Solution.

$$\begin{bmatrix} 1 & -1 & -2 \\ -3 & 15 & -18 \\ -9 & 10 & -4 \end{bmatrix}$$

2 Reduced Echelon Froms

Determine the reduced echelon form of the following matrix. You must write down the intermediate matrices and row operations you used in your calculation.

$$\begin{bmatrix} 1 & -5 & 2 & -6 \\ -2 & 10 & -3 & 10 \\ -3 & 15 & -3 & 12 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -5 & 2 & -6 \\ -2 & 10 & -3 & 10 \\ -3 & 15 & -3 & 12 \end{bmatrix} \xrightarrow{R_2 \leftarrow R_2 + 2R_1} \begin{bmatrix} 1 & -5 & 2 & -6 \\ 0 & 0 & 1 & -2 \\ -3 & 15 & -3 & 12 \end{bmatrix}$$

$$\xrightarrow{R_3 \leftarrow R_3 + 3R_1} \begin{bmatrix} 1 & -5 & 2 & -6 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 3 & -6 \end{bmatrix}$$

$$\xrightarrow{R_3 \leftarrow R_3 - 3R_2} \begin{bmatrix} 1 & -5 & 2 & -6 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\xrightarrow{R_1 \leftarrow R_1 - 2R_2} \begin{bmatrix} 1 & -5 & 0 & -2 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Solution.

$$\begin{bmatrix} 1 & -5 & 0 & -2 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

3 General Form Solutions

Determine a general form solution for a linear system whose augmented matrix is row equivalent to the following matrix.

$$\begin{bmatrix} 1 & 5 & 0 & 0 & -5 & 0 \\ 0 & 0 & 1 & 0 & 4 & 4 \\ 0 & 0 & 0 & 1 & -1 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Solution.

$x_1 = -5x_2 + 5x_5$

x_2 is free

$x_3 = 4 - 4x_5$

$x_4 = 5 + x_5$

x_5 is free