

Quiz 1 (Version 3)

CAS CS 132: *Geometric Algorithms*

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- ▷ You will have approximately 35 minutes to complete this exam.
- ▷ Your final solution must appear in the solution boxes for each problem. **Only include your final solution in the solution box. You must show your work outside of the solution box.** You will not receive credit if you don't show your work.

1 Row Operations

Apply the row operations:

$$R_2 \leftrightarrow R_1$$

$$R_3 \leftarrow R_3 - 4R_2$$

$$R_1 \leftarrow R_1 - 2R_3$$

from top to bottom to the following matrix. You must write down the intermediate matrices and row operations you used in your calculation.

$$\begin{array}{c}
 \left[\begin{array}{ccc} 8 & 8 & 2 \\ 6 & 6 & -2 \\ -4 & -8 & 2 \end{array} \right] \xrightarrow{R_2 \leftrightarrow R_1} \left[\begin{array}{ccc} 6 & 6 & -2 \\ 8 & 8 & 2 \\ -4 & -8 & 2 \end{array} \right] \\
 \xrightarrow{R_3 \leftarrow R_3 - 4R_2} \left[\begin{array}{ccc} 6 & 6 & -2 \\ 8 & 8 & 2 \\ -36 & -40 & -6 \end{array} \right] \\
 \xrightarrow{R_1 \leftarrow R_1 - 2R_3} \left[\begin{array}{ccc} 78 & 86 & 10 \\ 8 & 8 & 2 \\ -36 & -40 & -6 \end{array} \right]
 \end{array}$$

Solution.

$$\left[\begin{array}{ccc} 78 & 86 & 10 \\ 8 & 8 & 2 \\ -36 & -40 & -6 \end{array} \right]$$

2 Reduced Echelon Forms

Determine the reduced echelon form of the following matrix. You must write down the intermediate matrices and row operations you used in your calculation.

$$\begin{array}{c}
 \left[\begin{array}{cccc} 1 & 2 & -1 & -5 \\ -3 & -6 & 4 & 14 \\ 1 & 2 & -3 & -3 \end{array} \right] \xrightarrow{R_2 \leftarrow R_2 + 3R_1} \left[\begin{array}{cccc} 1 & 2 & -1 & -5 \\ 0 & 0 & 1 & -1 \\ 1 & 2 & -3 & -3 \end{array} \right] \\
 \xrightarrow{R_3 \leftarrow R_3 - R_1} \left[\begin{array}{cccc} 1 & 2 & -1 & -5 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & -2 & 2 \end{array} \right] \\
 \xrightarrow{R_3 \leftarrow R_3 + 2R_2} \left[\begin{array}{cccc} 1 & 2 & -1 & -5 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{array} \right] \\
 \xrightarrow{R_1 \leftarrow R_1 + R_2} \left[\begin{array}{cccc} 1 & 2 & 0 & -6 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{array} \right]
 \end{array}$$

Solution.

$$\left[\begin{array}{cccc} 1 & 2 & 0 & -6 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

3 General Form Solutions

Determine a general form solution for a linear system whose augmented matrix is row equivalent to the following matrix.

$$\begin{bmatrix} 1 & -2 & 0 & 6 & 0 & -4 \\ 0 & 0 & 1 & -4 & 0 & 6 \\ 0 & 0 & 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Solution.

$$x_1 = -4 + 2x_2 - 6x_4$$

x_2 is free

$$x_3 = 6 + 4x_4$$

x_4 is free

$$x_5 = 5$$