# Contrast Enhancement using Recursive Mean-Separate Histogram Equalization for Scalable Brightness Preservation

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# Abstract

- Contrast enhancement
  - Histogram equalization
  - Preserving the original brightness to avoid annoying artifacts
- Bright preserving Bi-Histogram Equalization
  - Not handling well to avoid annoying artifacts
- Recursive Mean-Separate Histogram Equalization
  - Separating each new histogram further based on respective mean

# Introduction

- Histogram equalization
  - Base on the probability distribution of the input gray levels
  - Significant change of the brightness
  - Not commonly using in consumer electronics such as TV
  - Necessary of the brightness preservation
- Bi-histogram equalization(BBHE)
  - Separating the input image's histogram into two based  on its mean

- Dualistic Sub-Image Histogram Equalization
  - Separating the histogram based on gray level with cumulative probability density equal to 0.5
- Proposed method
  - Separation each new histogram further based on their respective means
  - Output image's mean brightness converging to the input image's

# Histogram equalization

- Typical histogram equalization
  - Given image X, probability density function

$$p(X_k) = \frac{n^k}{n} \qquad (1)$$

For $k = 0,1,\ldots,L-1$

where $n^k$ represents the number of times that the level $X_k$ appears

$n$ is the total number of samples in the input image

  - Cumulative density function

$$c(x) = \sum_{j=0}^{k} p(X_j) \qquad (2)$$

– Define a transform function

$$f(x) = X_0 + (X_{L-1} - X_0)c(x) \qquad (3)$$

– Output image of HE, $\mathrm{Y} = \{Y(i, j)\}$

$$\mathrm{Y} = f(\mathrm{X}) \qquad (4)$$

$$= \{ f(X(i, j)) \mid \forall X(i, j) \in \mathrm{X} \} \qquad (5)$$

– Significant change in brightness

– Unnatural enhancement

**Fig. 1.(a) Original image arctic hare.** **Fig. 1.(b) Result of HE.**

**Fig. 2.(a) Original image girl.**

**Fig. 2.(b) Result of HE.**

Fig. 3.(a) Original image jet.

Fig. 3.(b) Result of HE.

- Brightness preserving Bi-Histogram Equalization
  - Decompose input Image into two sub-image $X_L$ and $X_U$

$$X = X_L \cup X_U \qquad (6)$$

$$X_L = \{X(i,j) \mid X(i,j) \le X_m, \forall X(i,j) \in X\} \qquad (7)$$

$$X_U = \{X(i,j) \mid X(i,j) > X_m, \forall X(i,j) \in X\} \qquad (8)$$

  - Respective PDF

$$p_L(X_k) = \frac{n_L^k}{n_L} \qquad (9)$$

$$p_U(X_k) = \frac{n_U^k}{n_U} \qquad (10)$$

– Respective CDF

$$c_L(x) = \sum_{j=0}^{k} p_L(X_j) \tag{11}$$

$$c_U(x) = \sum_{j=m+1}^{k} p_U(X_j) \tag{12}$$

– Transform function

$$f_L(x) = X_0 + (X_m - X_0)c_L(x) \tag{13}$$

$$f_U(x) = X_{m+1} + (X_{L+1} - X_{m+1})c_U(x) \tag{14}$$

– Output image of BBHE

$$\mathrm{Y} = \{Y\{i, j)\} \qquad (15)$$

$$= f_L(\mathrm{X}_L) \cup f_U(\mathrm{X}_U) \qquad (16)$$

$$f_L(\mathrm{X}_L) = \{ f_L(X(i, j)) \mid \forall X(i, j) \in \mathrm{X}_L \} \qquad (17)$$

$$f_U(\mathrm{X}_U) = \{ f_U(X(i, j)) \mid \forall X(i, j) \in \mathrm{X}_U \} \qquad (18)$$

- Analysis on the brightness change by the BBHE
  - Suppose X and Y to continuous random variable
  - Result image of HE, uniform density

$$p(x) = 1/(X_{L-1} + X_0) \qquad (19)$$

  - Mean brightness of the output image of the HE

$$E(Y) = \sum_{X_0}^{X_{L-1}} x p(x) dx \qquad (20)$$

$$= \sum_{X_0}^{X_{L-1}} \frac{x}{X_{L-1} - X_0} dx \qquad (21)$$

$$= \frac{X_{L-1} + X_0}{2} \qquad (22)$$

– Mean brightness of the output of the BBHE

$$E(\mathrm{Y}) = E(\mathrm{Y}|\mathrm{X} \le X_m)\Pr(\mathrm{X} \le X_m) + E(\mathrm{Y}|\mathrm{X} > X_m)\Pr(\mathrm{X} > X_m)$$
$$= \tfrac{1}{2}\{E(\mathrm{Y}|\mathrm{X} \le X_m) + E(\mathrm{Y}|\mathrm{X} > X_m)\} \qquad (23)$$

$$E(\mathrm{Y}|X \le X_m) = (X_0 + X_m)/2 \qquad (24)$$
$$E(\mathrm{Y}|X > X_m) = (X_m + X_{L-1})/2 \qquad (25)$$

$$E(\mathrm{Y}) = (X_m + X_G)/2 \qquad (26)$$

$$\text{where} \quad X_G = (X_0 + X_{L-1})/2 \qquad (27)$$

– Function of the input mean brightness
– Preserve the brightness

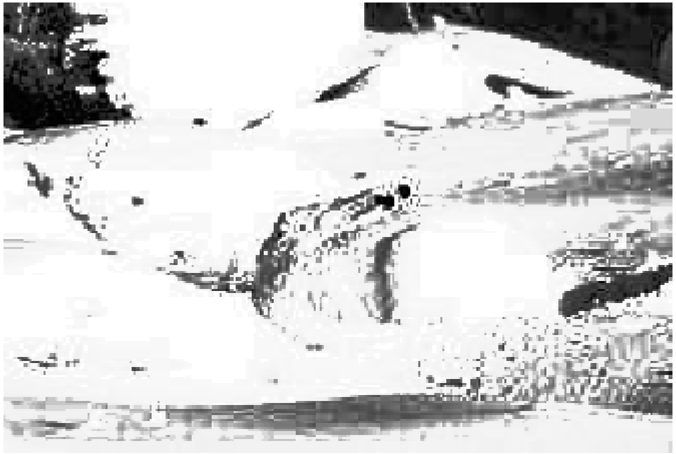


**Fig. 4.(a) Result of BBHE of image arctic hare.**

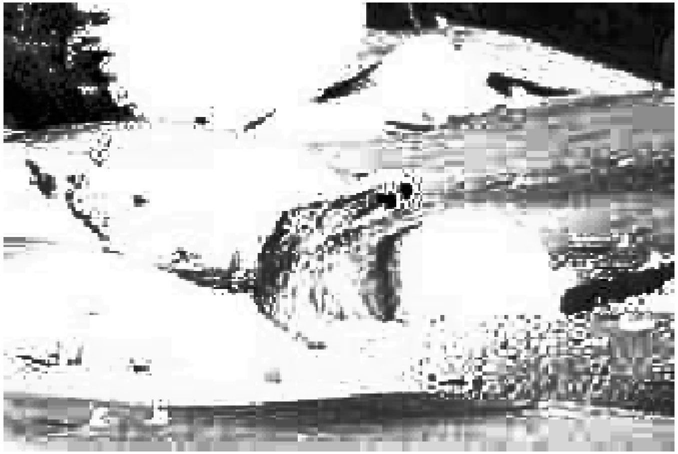


**Fig. 4.(b) Result of DSIHE of image arctic hare**

**Fig. 4.(a) Result of BBHE of image girl.** **Fig. 4.(b) Result of DSIHE of image girl.**

Fig. 4.(a) Result of BBHE of image jet.

Fig. 4.(b) Result of DSIHE of image jet.

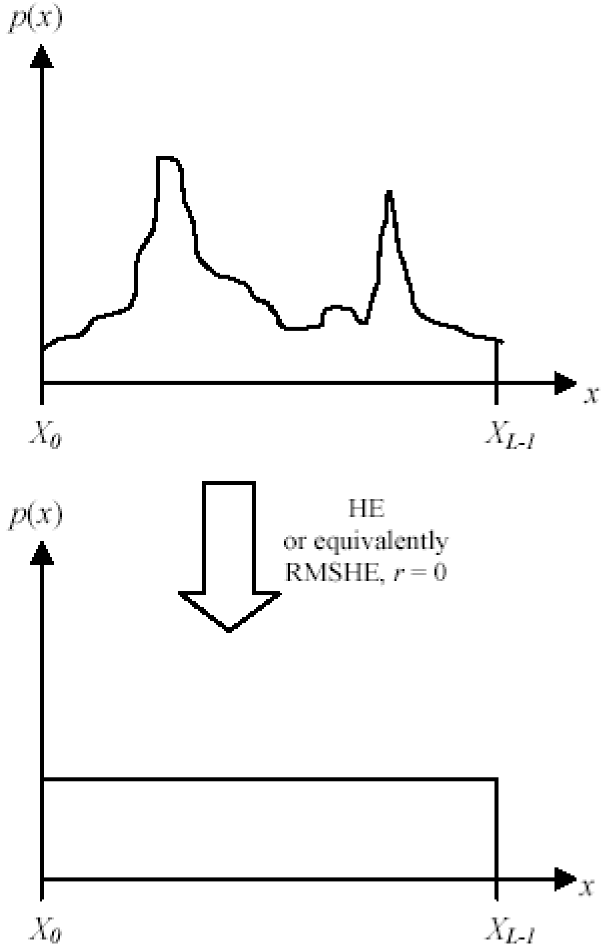
# Recursive mean-separate histogram equalization

- Separate the resulting histograms again based on their respective means
- Generalization of HE and BBHE
  - Output mean E(Y) of typical HE, $r = 0$

$$E(Y) = (X_0 + X_{L-1})/2 = X_G \tag{28}$$

  - Output mean E(Y) of BBHE, $r = 1$

$$E(Y) = (X_m + X_G)/2 \tag{29}$$



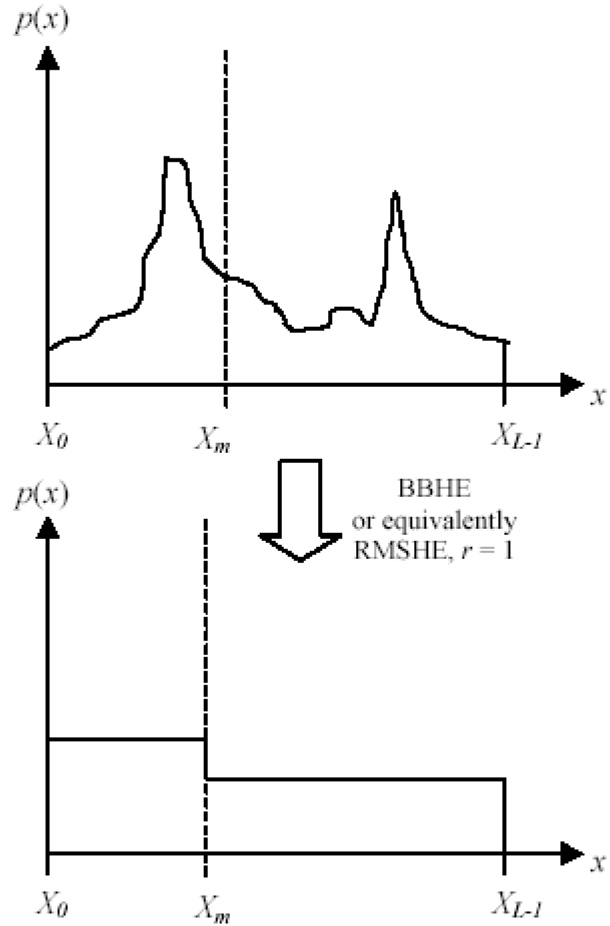




**Fig. 7. Histogram before and after HE or equivalently, RMSHE, $r=0$.**

**Fig. 8. Histogram before and after BBH or equivalently, RMSHE, $r=1$.**

◆ RMSHE with recursion level, $r = 2$

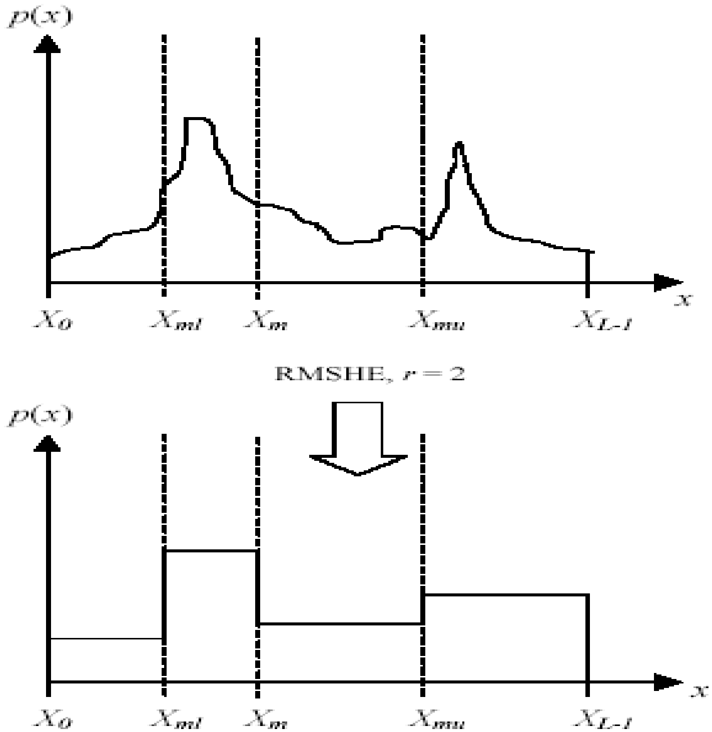



**Fig. 9. Histogram before and after RMSHE, $r = 2$.**

– Mean of the two new histogram, $X_{ml}$ and $X_{mu}$

$$X_{ml} = \frac{\int_{X_0}^{X_m} xp(x)dx}{\int_{X_0}^{X_m} p(x)dx} = 2\int_{X_0}^{X_m} xp(x)dx \tag{30}$$

$$X_{mu} = \frac{\int_{X_m}^{X_{L-1}} xp(x)dx}{\int_{X_m}^{X_{L-1}} p(x)dx} = 2\int_{X_m}^{X_{L-1}} xp(x)dx \tag{31}$$

$$\text{where } \int_{X_0}^{X_m} p(x)dx = \int_{X_m}^{X_{L-1}} p(x)dx = 1/2 \tag{32}$$

– Formulation of the output mean

$$\begin{aligned} E(\mathrm{Y}) = {} & E(\mathrm{Y} | \mathrm{X} \le X_{ml}) \Pr(\mathrm{X} \ge X_m) \\ & + E(\mathrm{Y} | X_{ml} \le \mathrm{X} \le X_m) \Pr(X_{ml} < \mathrm{X} \le X_m) \\ & + E(\mathrm{Y} | X_m \le \mathrm{X} \le X_{mu}) \Pr(X_m < \mathrm{X} \le X_{mu}) \\ & + E(\mathrm{Y} | \mathrm{X} > X_{mu}) \Pr(\mathrm{X} > X_{mu}) \\ = {} & \tfrac{1}{4} \{ E(\mathrm{Y} | \mathrm{X} \le X_{ml}) + E(\mathrm{Y} | X_{ml} \le \mathrm{X} \le X_m) \\ & + E(\mathrm{Y} | X_m \le \mathrm{X} \le X_{mu}) + E(\mathrm{Y} | \mathrm{X} > X_{mu}) \} \end{aligned} \tag{33}$$

– Similar discussion to obtain (22)

$$E(\mathrm{Y}) = \tfrac{1}{4}\{[(X_0 + X_{ml})/2] + [(X_{ml} + X_m)/2]$$
$$+[(X_m + X_{mu})/2] + [(X_{mu} + X_{L-1})/2]\}$$
$$= \tfrac{1}{4}\{[(X_0 + X_{L-1})/2] + [2(X_{ml} + X_{mu})/2] + X_m\}$$
$$= \tfrac{1}{4}\{X_G + 2X_m + X_m\}$$
$$= \tfrac{1}{4}\{X_G + 3X_m\} \qquad (34)$$

- From (30) and (31)

$$\frac{X_{mu}+X_{ml}}{2}=\frac{2\int_0^{X_m} xp(x)dx+2\int_{X_m}^{X_{L-1}} xp(x)dx}{2}$$

$$=\int_0^{X_m} xp(x)dx+\int_{X_m}^{X_{L-1}} xp(x)dx$$

$$=\int_{X_0}^{X_{L-1}} xp(x)dx$$

$$=X_m \qquad (35)$$

- Increasing $X_m$ to three times as much as the weight of middle gray level, $X_G$

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– Output mean $E(Y)$ for RMSHE recursion level $r = n$

$$r = 0,\quad E(\mathrm{Y}) = X_G$$

$$r = 1,\quad E(\mathrm{Y}) = (X_m + X_G)/2$$

$$r = 2,\quad E(\mathrm{Y}) = (3X_m + X_G)/4$$

$$\ldots$$

$$\begin{aligned} r = n,\quad E(\mathrm{Y}) &= ((2^n - 1)X_m + X_G)/2^n \\ &= X_m + [(X_G - X_m)/2^n] \end{aligned} \tag{35}$$

– Larger $n$, $E(Y)$ converge to the input mean, $X_m$

# Results and discussions

- Result from RMSHE with $r = 2$
  - Increasing the brightness preservation
  - More natural enhancement

**Fig. 10. Result of RMSHE r=2 of Image arctic hare.**

Fig. 11. Result of RMSHE r=2 of Image of girl

Fig. 12. Result of RMSHE r=2 of Image of jet

# Conclusion

- Recursive mean-separate histogram equalization
  - Generalization of HE and BBHE in term of brightness preservation
  - Recursively separating the input histogram based on the mean
- Future work
  - Proper mechanism to automate the selection of the recursion level, r that give optimum output