Sum 3

1) Given :-

cutt off frequency (up) = 1000 Hz

cutt off frequency down = 350 Hz

sampling frequency = 5000 Hz

Range = 1000 Hz to 10000 Hz

$\alpha_s = 10$, $\alpha_p = 3$

formulae.

$$\omega_p = 2\pi \times 1000$$

$$\omega_s = 2\pi \times 350$$

$$\Omega_p = \frac{2}{T} \times \tan\left(\frac{\omega_p T}{2}\right)$$

$$\Omega_s = \frac{2}{T} \tan\left(\frac{\omega_s T}{2}\right)$$

$$T = 1/f_s.$$

solution :-

1) $T = 1/f_s = \frac{1}{5000} = 2 \times 10^{-4}$

2) $\Omega_p = \frac{2}{2 \times 10^{-4}} \tan\left(\frac{2\pi \times 1000 \times 2 \times 10^{-4}}{2}\right)$

$$\Omega_p = 7265.4 \text{ rad/sec}$$

3) $\Omega_s = \frac{2}{2 \times 10^{-4}} \tan\left(\frac{2\pi \times 350 \times 2 \times 10^{-4}}{2}\right)$

$$= 2235.26 \text{ rad/sec}$$

iv) $N = \dfrac{\log \sqrt{\dfrac{10^{(0.1\alpha p)} - 1}{10^{(0.1\alpha s)} - 1}}}{\log\left(\dfrac{\Omega s}{\Omega p}\right)}$

putting values

$N = \dfrac{0.478}{0.511} = 0.935 \quad \therefore N \approx 1$

v) $H(s) = \dfrac{1}{1+s}$ for $N = 1$

$S = \dfrac{\Omega p}{s}$

$H(s) = \cancel{\dfrac{1}{1+7265}} \quad \dfrac{1}{1+\dfrac{7265}{s}}$

$\therefore \boxed{H(s) = \dfrac{s}{s+7265}}$

vi) $H(z) = H(s)\Big|_{s = \frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$

$\therefore H(z) = \dfrac{\dfrac{2}{2\times10^{-4}}\left(\dfrac{1-z^{-1}}{1+z^{+1}}\right)}{\dfrac{2}{2\times10^{-4}}\left(\dfrac{1-z^{-1}}{1+z^{-1}}\right) + 7265}$

$\therefore H(z) = \dfrac{y(n)}{x(n)}$

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$$\frac{y(n)}{x(n)} = \frac{10^4\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}{10^4\left(\frac{1-z^{-1}}{1+z^{-1}}\right) + 7265}$$

$$= \frac{10^4 - 10^4 z^{-1}}{10^4 - 10^4 z^{-1} + 7265 + 7265 z^{-1}}$$

$$\frac{y(n)}{x(n)} = \frac{10^4 - 10^4 z^{-1}}{\cancel{10^4 - 10^4 z^{-1}}.\quad 10^4 - 2735 z^{-1} + 7265}$$

$$= \frac{10^4 - 10^4 z^{-1}}{17265 - 2735 z^{-1}}$$

$$\frac{y(n)}{x(n)} = \frac{10000\,(1 - z^{-1})}{17265\left(1 - \frac{2735}{17265} z^{-1}\right)}$$

$$\frac{y(n)}{x(n)} = \frac{0.579\,(1 - z^{-1})}{1 - 0.158 z^{-1}}$$

$$\boxed{\therefore\ y(n) = 0.158\,y(n-1) + 0.579\,x(n) - 0.579\,x(n-1)}$$

Sum 3

(2) For $F_s = 4000 Hz$

i) $T = \frac{1}{F_s} = 2.5 \times 10^{-4}$

ii) $\Omega_p = \frac{2\tan}{T}\left(\frac{2\pi \times 1000 \times T}{2}\right)$

$= \frac{2}{2.5 \times 10^{-4}} \tan\left(\frac{2000\pi \times \cancel{1/4}\ 2.5 \times 10^{-4}}{2}\right)$

$\Omega_p = 8000$ rad/s

iii) $\Omega_s = \frac{2}{T} \tan\left(\frac{2\pi \times 350 \times 2.5 \times 10^{-4}}{2}\right)$

$\Omega_s = 2256$ rad/s

iv) $N = \frac{\log \sqrt{\frac{10^{(0.1\alpha_p)} - 1}{10^{(0.1\alpha_s)} - 1}}}{\log\left(\frac{\Omega_s}{\Omega_p}\right)}$

$\therefore N = \frac{-0.47}{-0.54} = 0.87$

$N \approx 1$

v) $H(s) = \frac{1}{1+s} = \frac{1}{1 + \frac{8000}{s}} = \frac{s}{s+8000}$

$H(s) = \frac{s}{s+8000}$

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vi) $H(z) = H(s)\Big|_{s = \frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$

$$= \frac{\frac{2}{2.5\times10^{-4}}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}{\frac{2}{2.5\times10^{-4}}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)+8000}$$

$$\therefore H(z) = \frac{8000(1-z^{-1})}{8000(1-z^{-1})+8000(1+z^{-1})}$$

$$= \frac{8000(1-z^{-1})}{16000}$$

$$\therefore H(z) = \frac{y(n)}{x(n)} = 0.5 - 0.5z^{-1}$$

$$\boxed{y(n) = 0.5x(n) - 0.5x(n-1)}$$

③ For $F_s = 2500\,Hz$

i) $T = \frac{1}{f_s} = \frac{1}{2500} = 4 \times 10^{-4}$

ii) $\Omega_p = \frac{2}{T} \tan\left(\frac{2\pi \times 1000 \times T}{2}\right)$

$= 15388.41$ rad/sec

iii) $\Omega_s = \frac{2}{T} \tan\left(\frac{2\pi \times 350 \times T}{2}\right)$

$\Omega_s = \frac{2}{4 \times 10^{-4}} \tan\left(\frac{2\pi \times 350 \times 4 \times 10^{-4}}{2}\right)$

$\Omega_s = 2352.82$ rad/sec.

iv) $N = \frac{\log \sqrt{\frac{10^{0.1\alpha p} - 1}{10^{0.1\alpha s} - 1}}}{\log\left(\frac{\Omega_s}{\Omega_p}\right)}$

$N = \frac{-0.85}{-0.815} = N \approx 1$

v) $H(s) = \frac{s}{s + 15388}$

vi) $H(z) = H(s)\Big|_{s = \frac{2}{T}\left(\frac{1 - z^{-1}}{1 + z^{-1}}\right)}$

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$$H(z) = \frac{\frac{2}{4\times10^{-4}}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}{\frac{2}{4\times10^{-4}}\left(\frac{1-z^{-1}}{1+z^{-1}}\right) + 15388}$$

$$H(z) = \frac{5000 - 5000z^{-1}}{5000(1-z^{-1}) + 15388(1+z^{-1})}$$

$$= \frac{5000 - 5000z^{-1}}{20388 + 10388z^{-1}}$$

$$H(z) = \frac{0.24 - 0.24z^{-1}}{1 + 0.509z^{-1}}$$

$$\therefore \frac{y(n)}{x(n)} = \frac{0.24 - 0.24z^{-1}}{1 + 0.509z^{-1}}$$

$$\therefore \boxed{y(n) = 0.24x(n) - 0.24x(n-1) - 0.509y(n-1)}$$

(4) $f_s = 7500\,Hz$

① $T = \frac{1}{f_s} = 1.333\times10^{-4} = \frac{4}{3}\times10^{-4}$

② $\Omega_p = \frac{2}{T}\tan\left(\frac{2\pi\times1000\times T}{2}\right)$

$= \frac{2}{1.33\times10^{-4}}\tan\left(\frac{2\pi\times1000\times1.33\times10^{-4}}{2}\right)$

$$\boxed{\Omega_p = 6678.43\ rad/sec}$$

③ $\Omega_s = \frac{2}{T}\tan\left(\frac{2\pi\times\cancel{30}\times350\times T}{2}\right)$

$\Omega_s = 2211.79\ rad/sec$

④ $N = \frac{-0.47}{-0.479} = N \approx 1$

⑤ $H(s) = \frac{s}{s+6678}$

6) $H(z) = H(s)\Big|_{s=\frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$

$$H(z) = \frac{\frac{2\times3}{4\times10^{-4}}\frac{(1-z^{-1})}{(1+z^{-1})}}{\frac{3}{2\times10^{-4}}\frac{(1-z^{-1})}{(1+z^{-1})}+6678}$$

$$H(z) = \frac{1.5\times10^{-4}\;(1-z^{-1})}{1.5\times10^{-4}\;(1-z^{-1}) + 6678\cdot(1+z^{-1})}$$

$$H(z) = \frac{15000 - 15000z^{-1}}{21659.61 - 8340.39z^{-1}}$$

$$\frac{y(n)}{x(n)} = \frac{0.69 - 0.69z^{-1}}{1 - 0.38z^{-1}}$$

$$\boxed{y(n) = 0.69x(n) - 0.69x(n-1) + 0.38y(n-1)}$$

(5)

for $F_s = 10000\,Hz$

i) $T = \frac{1}{F_s} = 1 \times 10^{-4}$

ii) $\Omega_p = \frac{2}{T} \tan\left(\frac{2\pi \times 1000 \times T}{2}\right)$

$$\boxed{\Omega_p = 6498.39 \text{ rad/sec}}$$

iii) $\Omega_s = \frac{2}{T} \tan\left(\frac{\pi \times 350 \times T}{2}\right)$

$$\boxed{\Omega_s = 2208.02 \text{ rad/sec}}$$

(iv) $N = \frac{-0.47}{-0.46} \approx 1$

vi) $H(s) = \frac{s}{s + 6498}$

v) $H(z) = H(s)\Big|_{s = \frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$

$$H(z) = \frac{\frac{2}{10^{-4}}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}{\frac{2}{10^{-4}}\left(\frac{1-z^{-1}}{1+z^{-1}}\right) + 6498}$$

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$$H(z) = \frac{20000(1-z^{-1})}{2000(1-z^{-1}) + 6498(1-z^{-1})}$$

$$H(z) = \frac{20000(1-z^{-1})}{26498 - 13502 z^{-1}}$$

$$\frac{y(n)}{x(n)} = H(z) = \frac{0.754 - 0.754 z^{-1}}{1 - 0.509 z^{-1}}$$

$$\boxed{\therefore y(n) = 0.754\, x(n) - 0.754\, x(n-1) + 0.509\, y(n-1)}$$