# Math 225B Differential Geometry: Homework 7

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**Exercise 1** (Problem 6).

*Proof.* a) Consider the case of an arbitrary 2-form, expressible in a basis as: $\omega_1 e_1 \wedge e_2 + \omega_2 e_2 \wedge e_3 + \omega_3 e_3 \wedge e_1$ Given a solution to the equations

$$\begin{aligned}\omega_1 &= a_2 b_3 - a_3 b_2 \\ \omega_2 &= a_1 b_3 - a_3 b_1 \\ \omega_3 &= a_1 b_2 - a_2 b_1\end{aligned}$$

We have that $(a_1 e_1 + a_2 e_2 + a_3 e_3) \wedge (b_1 e_1 + b_2 e_2 + b_3 e_3) = (a_2 b_3 - a_3 b_2) e_1 \wedge e_2 + (a_1 b_3 - a_3 b_1) e_2 \wedge e_3 + (a_1 b_2 - a_2 b_1) e_3 \wedge e_1 = \omega_1 e_1 \wedge e_2 + \omega_2 e_2 \wedge e_3 + \omega_3 e_3 \wedge e_1$. As given any vector in $\mathbb{R}^3$ we may obtain two vectors that are orthogonal to it and thus that cross product to it, we have that the above system of equations has a solution, and thus any 2-form may be expressed as an indecomposable one. $\square$