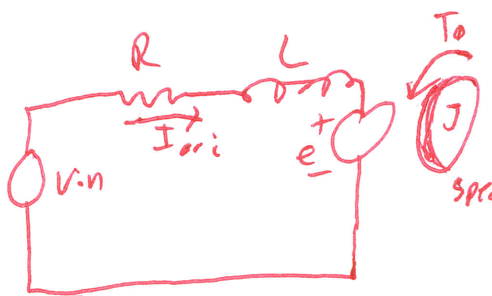


← This will have friction = $b\dot{\phi}$

speed = $\frac{d\phi}{dt}$ = rate of change of The shaft angle

$T = K_t I$

$e = K_e\dot{\phi}$

T = torque

I is The armature current.

e is The back EmF

J is The moment of Inertia

Ke is Eletromotive force Constant

Kt is motor torque constant

R motor Resistance

L motor Inductance

$\dot{\phi}$ speed of shaft

There is a Law That says motor torque and Back EmF are Equal whendone in SI units so:

$$K_t = K_e = K$$

from Newton and Kirchhoff Laws → we want to find The speed Vs. input Voltage Relationship or $\frac{\dot{\phi}}{V_{in}}$

$$J\ddot{\phi} + b\dot{\phi} = Ki$$

$$L\frac{di}{dt} + Ri + K\dot{\phi} = V_{in}$$

e

$J\ddot{\phi} + b\dot{\phi} = kI \Rightarrow J\ddot{\phi} + b\dot{\phi} - kI = 0$

$L\dot{I} + RI + k\dot{\phi} = V_{in}$

$\ddot{\phi} = \frac{kI}{J} - \frac{b}{J}\dot{\phi}$

$\dot{I} = -\frac{RI}{L} - \frac{k\dot{\phi}}{L} + \frac{V_{in}}{L}$

or

$$\Rightarrow \begin{bmatrix} \ddot{\phi} \\ \dot{I} \end{bmatrix} = \underbrace{\begin{bmatrix} -\frac{b}{J} & \frac{k}{J} \\ -\frac{k}{L} & -\frac{R}{L} \end{bmatrix}}_{A} \begin{bmatrix} \dot{\phi} \\ I \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix}}_{B} V_{in}$$

$$y = \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{C} \begin{bmatrix} \dot{\phi} \\ I \end{bmatrix} \Leftarrow \text{out put is your speed}$$

This is ~~a~~ dynamic Equation

motor_ss(A,B,C,0)
step(motor_ss)

do it Laplace style:

$Js^2\phi + bs\phi = kI$ and $LsI + RI = V_{in} + ks\phi$

$\cancel{s^2\phi = \frac{k}{J}I - \frac{b}{J}s\phi} \Rightarrow$

factor ϕ's

$\cancel{s^2\phi + \frac{b}{J}s\phi}$

$\phi(Js^2 + bs) = kI$ and $(Ls+R)I = V_{in} + ks\phi$

get rid of I

$\Downarrow$

$I = \frac{V_{in} + ks\phi}{(Ls+R)}$

$\phi(Js^2 + bs) = k\left(\frac{V_{in} + ks\phi}{Ls+R}\right)$

Now move all the ϕ on one side and V_{in} on the other

$\phi(Js^2 + bs) - \frac{k^2 s\phi}{Ls+R} = \frac{kV_{in}}{Ls+R}$

$\phi\left(Js^2 - \left(\frac{k^2}{Ls+R}\right) + b s\right) = \frac{kV_{in}}{Ls+R} \Rightarrow \frac{\dot{\phi}}{V_{in}} = \frac{K}{(Js+b)(Ls+R) + k^2} \Rightarrow \boxed{\frac{K}{JLs^2 + (bL+JR)s + k^2}}$

s=tf('s')

motor_TF = $\frac{k}{(JLs\cdot s + (bL+JR)s + k^2)}$

step(motor_TF)

J = Moment of Inertia for The Rotor $\Rightarrow$ .1 $kg \cdot m^2$

b = motor friction $\Rightarrow$ 1 $N \cdot m \cdot s$

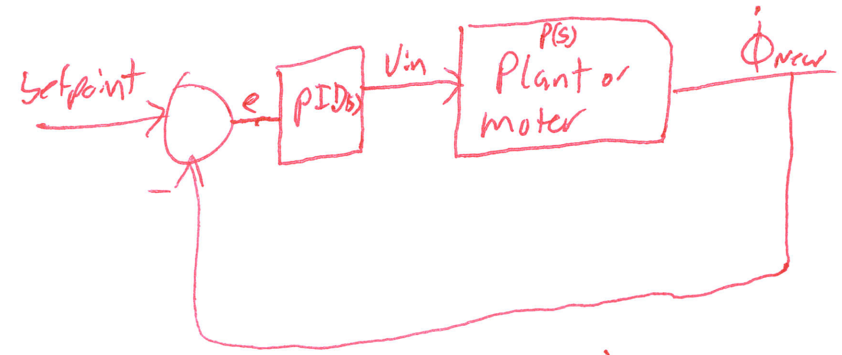
K_e = electromotive force $\Rightarrow$ .01 $\frac{V}{rad/sec}$

K_t = motor torque constant $\Rightarrow$ .01 $\frac{N \cdot m}{Amp}$

R = Resistance $\Rightarrow$ $5\,\Omega$

L = Inductance $\Rightarrow$ .5 H

for Pid system Look Like



Transfer function = $P(s) = \frac{\dot{\theta}}{Vin} = \frac{K}{JL\,s^2 + (bL + JR)s + K^2}$

$\frac{\dot{\theta}_{new}}{setpoint} \Rightarrow$ $e = setpoint - \dot{\theta}_{new}$

$\dot{\theta}_{new} = e \cdot PID(s) \cdot P(s) = \dot{\theta}_{new} = (setpoint - \dot{\theta}_{new})\,pid \cdot P(s)$

$\dot{\theta}_{new}\left(1 - Pid(s) \cdot P(s)\right) = Setpoint\ Pid(s)\,P(s) \Rightarrow$

$\frac{\dot{\theta}_{new}}{Setpoint} = \frac{Pid(s)\,P(s)}{1 - Pid(s)\,P(s)}$

we control PID(s) to control The Dynamic Equation.