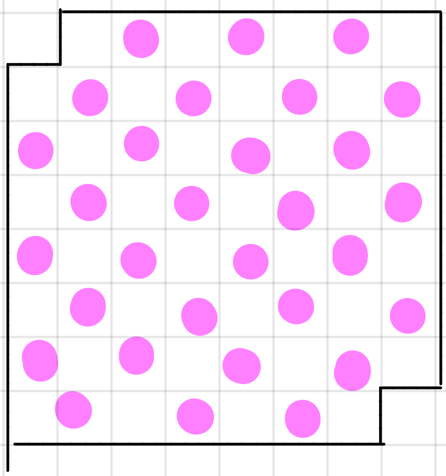
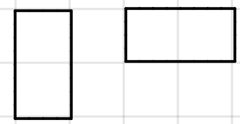
CS 4820: Algorithms

http://www.cs.cornell.edu/courses/cs4820/2018sp/

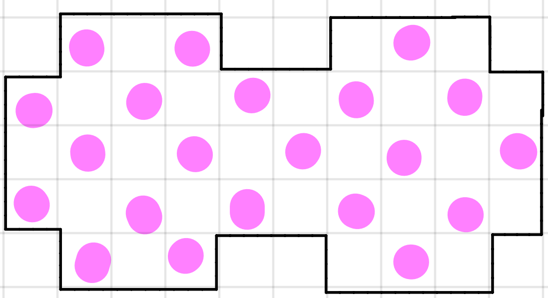
$O(n \log n)$ sorting algorithm. §5.1.



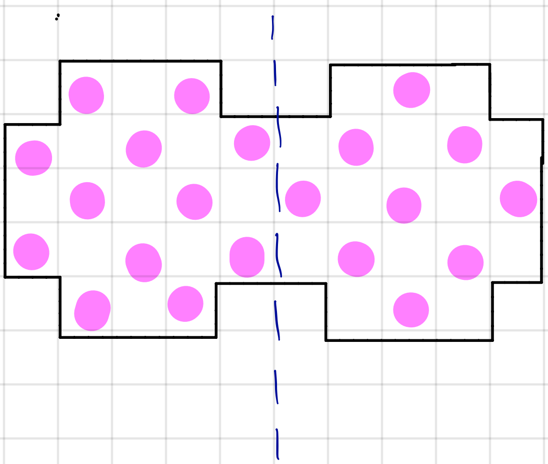
Can you cover this shape with 31 non-overlapping dominoes?



No! Every domino covers exactly one pink & one white square. But there are 32 white and 30 pink squares.

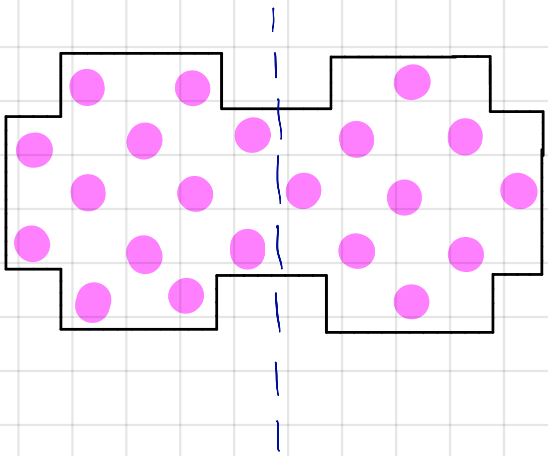


Can you cover this shape with non-overlapping dominoes?



Jorge: "No. There are 12 pink squares on the left half of the figure with only 11 white squares adjacent to them."

<u>Def</u>: A Jorge <u>obstruction</u> to domino tiling a shape is a set of k squares, all the same color (in the standard 2-coloring) that have $< k$ oppositely colored neighbors.

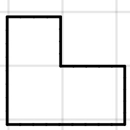


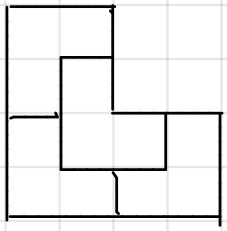
Jorge: "No. There are 12 pink squares on the left half of the figure with only 11 white squares adjacent to them."

Def: A Jorge obstruction to domino tiling a shape is a set of k squares, all the same color (in the standard 2-coloring) that have $< k$ oppositely colored neighbors.

Easy Pigeonhole Principle argument: if a shape has a Jorge obstruction then it has no domino tiling.

Fact (wait until March): If it is impossible to tile a shape with dominoes then the shape has a Jorge obstruction. Furthermore there is an efficient algorithm that either finds a domino tiling or a Jorge obstruction.

Tromino tiling: Using L-shaped trominoes  is it possible to tile this shape?



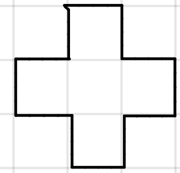
(Answer: yes.)

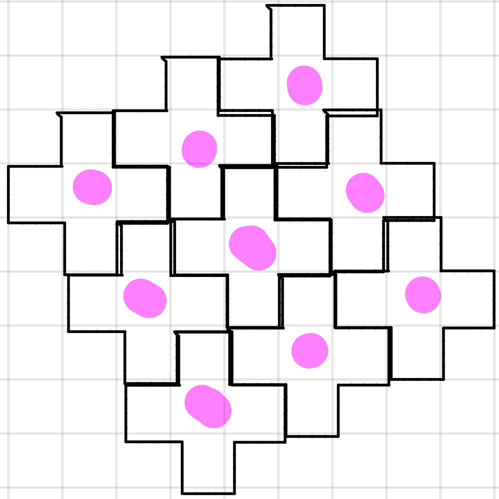
to be defined in March

Tromino tiling is NP-Complete: it is believed that there is no efficient algorithm.

Tiling the infinite plane.

Can you cover the entire plane using an infinite number of plus-shaped tiles.





Generalized Tiling Question: Given a finite list of tile shapes, if we have an infinite supply of each kind, can we cover the entire plane with non-overlapping tiles?

Complexity: Undecidable.

There is no algorithm that gives the correct answer to every possible instance of this problem.

How can you prove $\nexists$ an algorithm to solve some problem? To be discussed in April.