23 April 2018. - Reducing from the co-halting problem to show sets are not rie. - Rice's Theorem "the co-halting problem" = Z\* \H. We've seen that H is r.e. but not decidable.

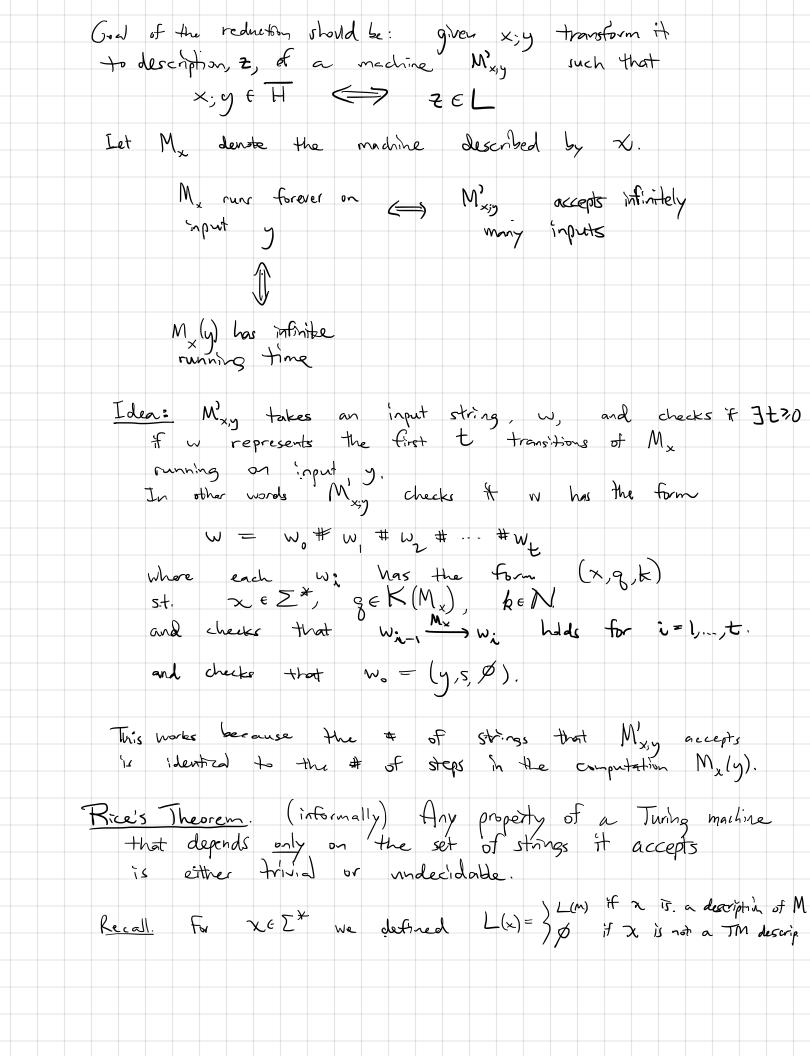
Recall: a set & decidable () it and its complement are r.e. is not re. This makes II a useful storting point for reductions showing other sets are not me. Example. Let  $L = \{x \mid x \text{ is a discription of a Turky machine M and } | L(M) | = \infty \}$ . Prove that L'is not r.e. (We should suspect L is not r.e. because how would you give a finite amount of oridence that proves that M occepts or many strings?)

Proof. To prove L is not rie, we reduce from H. In other words given a hypothetical machine ML that accepts strings iff they belong to L, we construct a nachine M that accepts strings if and only if they belong to H, using M, as a submartine. God of the reduction should be: given x; y transform it to description, Z, of a mechine M'x; y such that ×, y f H => ZEL If we can do this, M- can operate as follows.

1. Transform x,y into z.

2. Execute M on input z.

As long as step 1 is guaranteed to run in finite time, x; y & H >> 7 & Copts x; y : H= L(MH) &



RICE'S THM
Let P be a set of strings, $P = \Sigma^*$ , such that
$\forall x,y \qquad (L(x)=L(y)) \Longrightarrow (x \in P \iff y \in P),$
Then either () P = 0 (2) P = 5* (3) P is undecidable. 3 Non-trivial.
Proof. See leeture notes.
Applications. All of the following are undecidable.  (1) Given x is  L(x)  = 00? (We already showed this isn't re.)
Undecadable is a weaker conclusion.)
(z) Is $L(x) \neq \emptyset$ ? i.e. does a specified Turing machine accept at least one string?
(3) Does L(x) contain the empty strong?
(4) Does L(x) contain at least one string of length k for every kcW?
Examples of publems not covered by Rize's Theorem:
(A) Does the madine described by X run for more than 100 steps on every input string?
more than 100 steps on every input sings: