Introduction to Algorithms	**Problem Set 10**
CS 4820, Spring 2018	**Due 11:59pm Thursday, April 26**

Hand in your solution electronically using CMS. Collaboration is encouraged while solving the problems, but:

1. **list the names of those with whom you collaborated;**
2. **you must write up the solutions in your own words.**

On this problem set, you do not need to analyze the running times of algorithms and/or reductions, you only need to prove their correctness.

(1) *(10 points)* Recall that an instance of the halting problem is a string of the form $x; y$ and the goal is to decide if the Turing machine M_x encoded by the string x halts on input y. If M_x halts on y, then $x; y$ is a YES instance. Otherwise, $x; y$ is a NO instance. Let us say that a Turing machine M *fails to solve* the halting problem for instance $x; y$ if it never terminates or if it produces the wrong answer, i.e., it rejects in case that $x; y$ is a YES instance or it accepts in case that $x; y$ is a NO instance.

Because the halting problem is undecidable, we know that for every Turing machine M there exists an instance $x; y$ of the halting problem such that M fails to solve $x; y$. Describe an algorithm to find such an instance. The input to your algorithm is a description of a Turing machine M. For every such input, your algorithm should run for a finite number of steps and output a halting problem instance $x; y$ such that M fails to solve $x; y$.

(2 *(10 points)* Call a Turing machine M *termination-safe* if $M(y) \neq \nearrow$ for all $y \in \Sigma^*$. In other words, a termination-safe machine is one which is guaranteed to terminate on every input string.

Let $T \subset \Sigma^*$ denote the set of all strings x such that x is the description of a termination-safe Turing machine. Prove that T is not recursively enumerable.

(3) *(10 points)* As in question (2), let $T \subset \Sigma^*$ denote the set of all strings x such that x is the description of a termination-safe Turing machine. Prove that $\overline{T}$, the complement of T in Σ^*, is not recursively enumerable.