26 March 2018

Recap from end of last lecture...

HAMILTONIAN CYCLE. Given directed graph G, does it have a cycle that visits every vertex exactly once?

verifier is given graph & cycle, checks that every 2 consecutive vertices of cycle are joined by an edge & that every vertex is used exactly once.

$\therefore$ HAM CYCLE $\in$ NP.

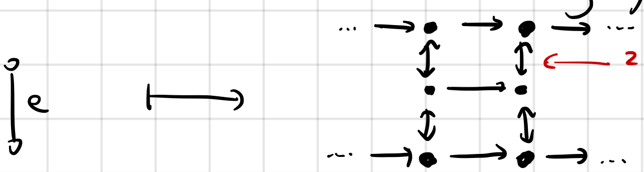
Next step: reduce some other NP-Complete problem to HAM CYCLE.

3SAT $\le_p$ HAM CYCLE: see book.

VERTEX COVER $\le_p$ HAM CYCLE.

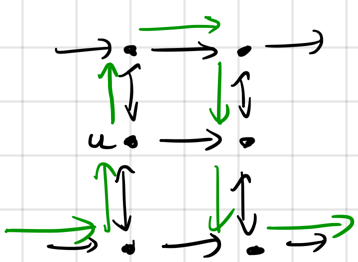
$(V,E,k) \longmapsto$ directed graph.

Each edge e transforms into gadget G_e:

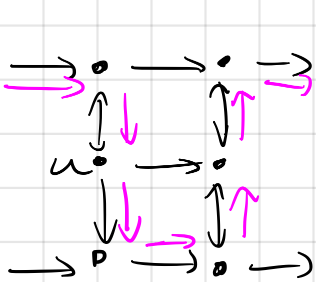


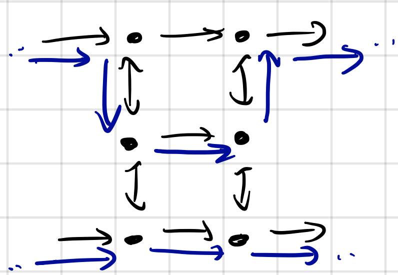
$\leftarrow$ 2 dir edges, one up, one down

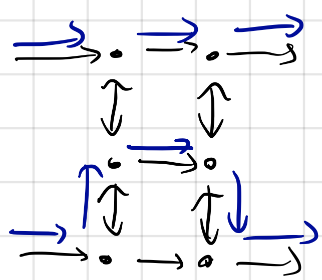
The 6 nodes in this gadget have no edges other than the ones shown.



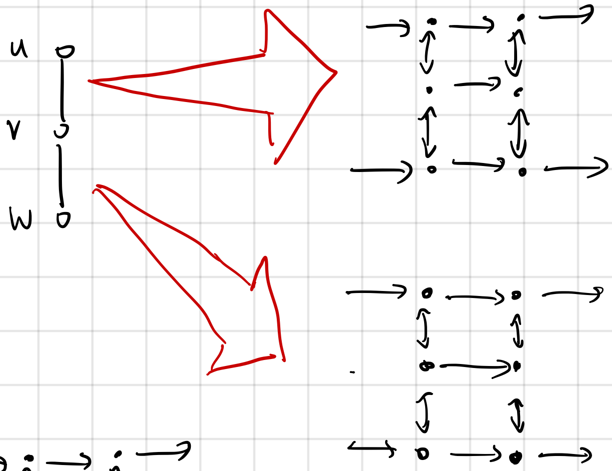
what edge of the cycle exits u?



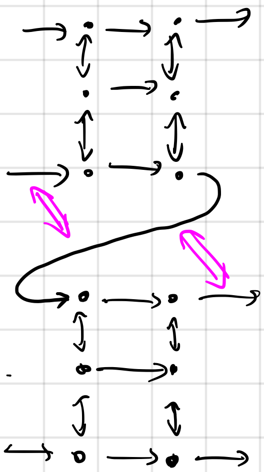


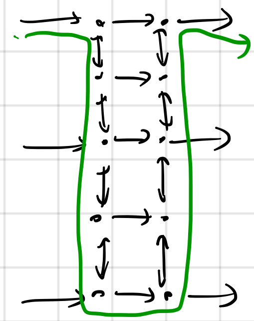


How to represent the constraint that you only choose k vertices?



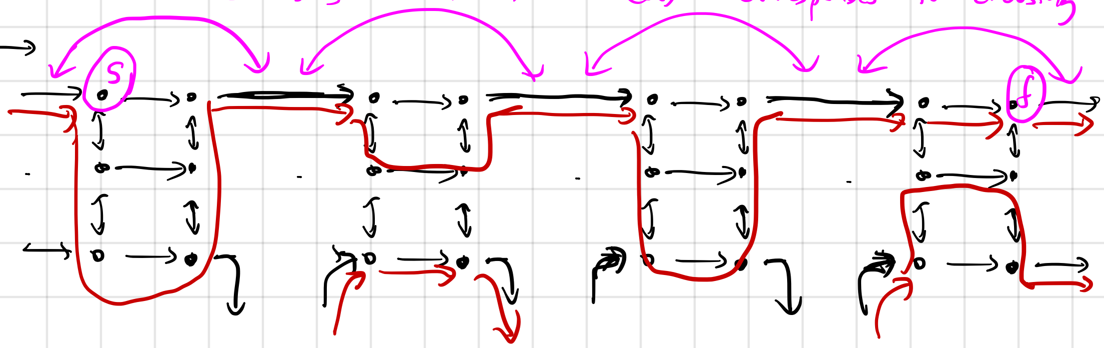
How to join the gadgets together to constrain them to make the same choice w.r.t. vertex v.





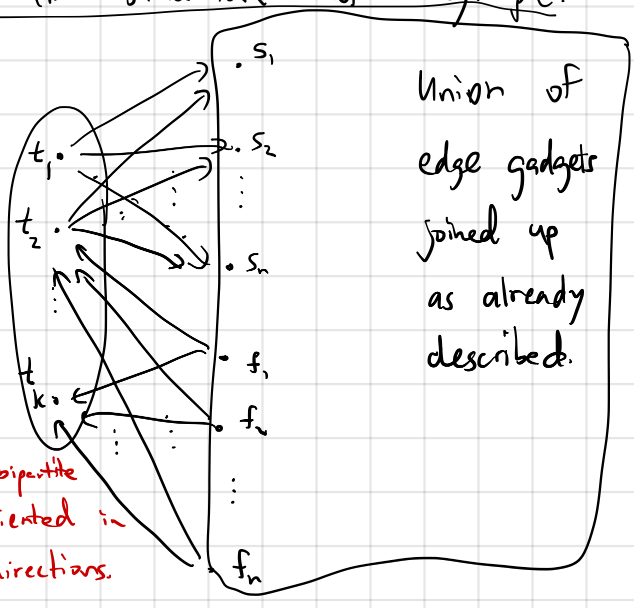
Doesn't work: has "unintended" solutions that don't correspond to satisfying the edge constraints.

Choosing all of these edges corresponds to choosing v.

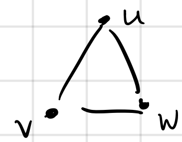


Overall structure of graph.

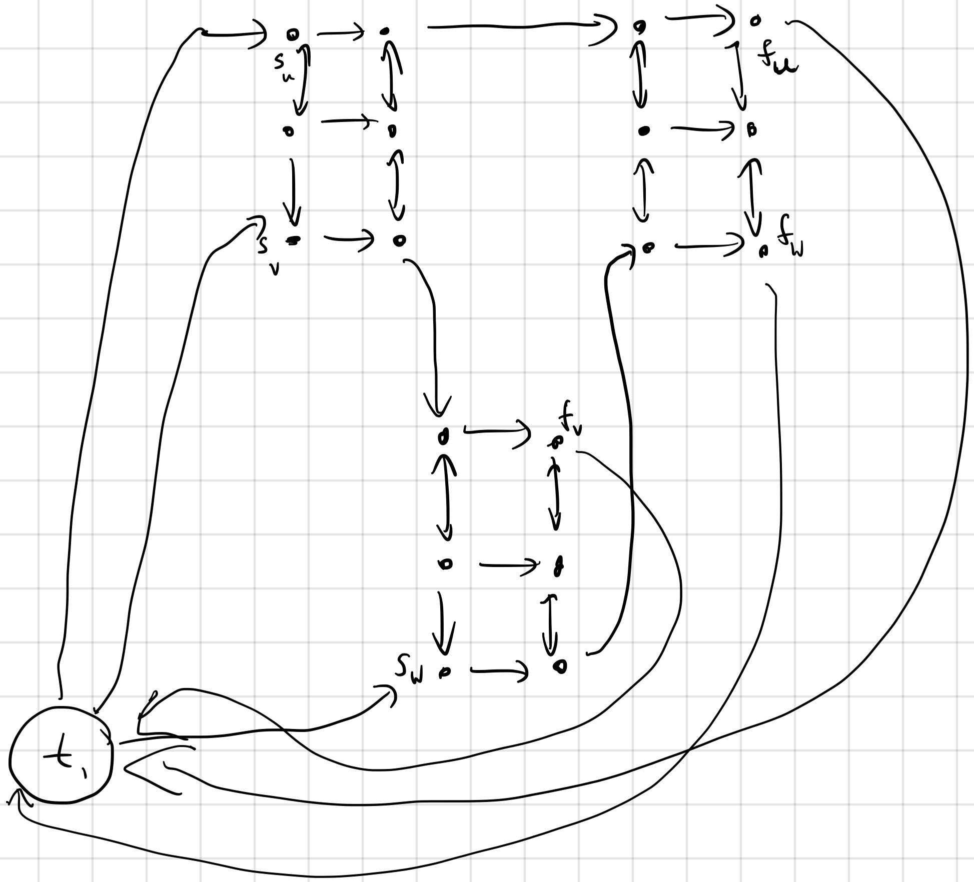
s_i, f_i are starting & finishing points in the "train of edge gadgets" for vertex v_i.

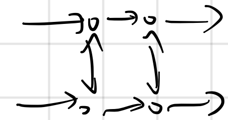


2 complete bipartite graphs oriented in opposite directions.

Applying the reduction to 

, k=1





Simpler edge gadget that also would have worked!

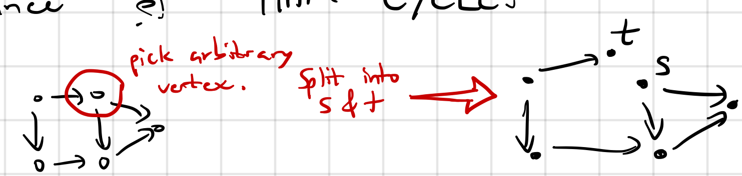
Problems that are NP-Complete by reduction from HAMILTONIAN CYCLE:

1. Hamiltonian path: given $G=(V,E)$ directed, given s,t, does G contain a path from s to t that visits every vertex exactly once?

   To show HAM PATH is NP-complete we must reduce

   HAM CYCLE $\leq_p$ HAM PATH

   meaning use HAM PATH to solve an arbitrary instance of HAM CYCLE.



2. Undirected Hamiltonian path & cycle.
3. Traveling Salesman Problem: given a graph with edge costs, find the Hamiltonian cycle of minimum total cost.