31 Jan 2018

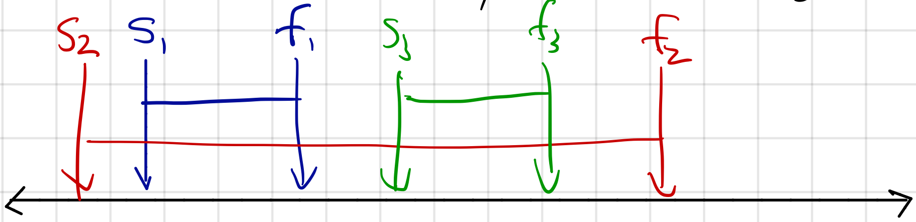
Greedy Algorithms: Chapter 4.

Today: Interval Scheduling, Section 4.1.

__Input.__ A set of intervals $[s_i, f_i]$, $i = 1, \ldots, n$.
Assume all the numbers $s_1, \ldots, s_n, f_1, \ldots, f_n$ are distinct.
Define two intervals to be __conflicting__ if they overlap.

__Output.__ A set of as many non-conflicting intervals as possible.

E.g.

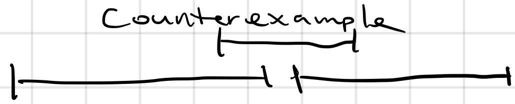


Answer: blue & green intervals.

① Earliest Finishing Time

② Shortest Processing Time

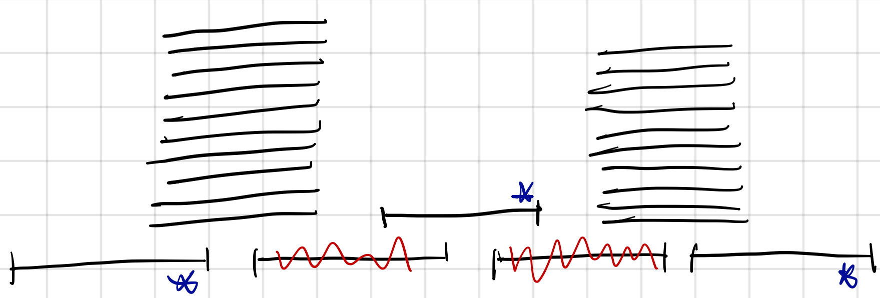
Counterexample



③ Fewest Conflicts

④ Earliest Start Time — counterexample above.

counterexample



Earliest Finish Time (EFT) is a correct algorithm.

How to prove it?
Book proposes "Greedy Stays Ahead".
Instead of trying to prove, in one shot, that the output is optimal, we instead prove, inductively, that the partial solution at the end of every iteration is optimal with respect to some criterion corresponding to what that iteration tried to optimize.

How not to prove it?
"In every iteration the algorithm makes an optimal choice because it chooses the interval with the earliest finish time, which optimizes the finish time and is therefore optimal. Since the choice in each iteration is optimal, the output is optimal."
One logical error here: the proof doesn't acknowledge the distinction between what the algorithm is optimizing (earliness of finish time) and what it was asked to optimize (# of intervals).

Two "greedy stays ahead" arguments.

Let $\{o(1), o(2), \ldots, o(k)\}$ be indices of an optimal set of intervals ordered by increasing finish time.
Let $\{a(1), a(2), \ldots, a(l)\}$ be indices of the intervals that EFT chose.
BOOK'S INDUCTION HYPOTHESIS: $f_{a(j)} \leq f_{o(j)} \ \forall j$.

Let us assume the intervals are numbered so that $f_1 < f_2 < \cdots < f_n$.

THIS LECTURE'S INDUCTION HYPOTHESIS:
The number of intervals with finish time $\leq f_j$ chosen by our algorithm is at least as great as the # of int. with f.t. $\leq f_j$ in any other set non-conflicting set.

Base case $j = 1$.

EFT chooses 1 interval that finishes $\leq f_1$.
There is only one such interval, so no other non-conflicting set contains more than one.

Induction step. $j > 1$. Assume induction hypothesis true for $1 \leq i \leq j-1$. ("strong induction")
Consider any non-conflicting interval set.