CS 4820, Spring 2018 **Homework 1, Problem 1**
Name: Rongguang Wang
NetID: rw564
Collaborators: Siyuao Liu (sl2928); Yihao Chen (yc2288)

(1) *(5 points)*
For any positive integer n, let L_n denote an L-shaped region in the plane obtained by starting with a square of side length 2^n and deleting its upper right quadrant. For example, L_1 is the "L-shaped tromino tile" discussed in class on Wednesday. See Figure 1 for additional examples.

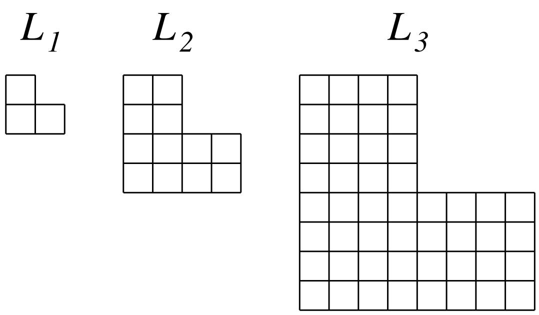

Figure 1: The regions L_1, L_2, L_3

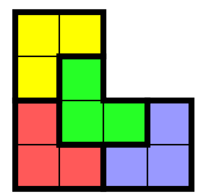

Figure 2: Tiling L_2 with copies of L_1

Prove, **using mathematical induction,** that for every positive integer n it is possible to tile L_n using copies of L_1. In other words, you should prove that L_n can be partitioned into regions, each congruent to L_1. See Figure 2 for an example when $n = 2$.

Try to make your proof as clear and precise as possible. You do not need to describe an algorithm to compute the tiling, nor analyze its running time. You only must prove that such a tiling exists.

Solution:

Let P(n) be the mathematical statement,

For every positive integer n, it is possible to tile L_n using copies of L_1.

Base Case: When n = 1, L_1 can be tiled by it self. So P(1) is correct.

Induction Hypothesis: Assume that P(k) is correct for some positive integer k. That means L_k can be partitioned into regions, each congruent to L_1. Hence, L_k can be tiled by L_m for some integer m (m < k).

Induction Step: Showing that P(k + 1) is correct. Since L_2 can be tiled with copies of L_1, L_3 can be tiled with copies of L_2 which can be tiled with copies of L_1, which means that L_3 can be tiled with copies of L_1. Therefore, L_k can be tiled with copies of L_m, and thereby, L_{k+1} can be tiled with copies of L_m. So P(k + 1) is correct.

Hence, by mathematical induction P(n) is correct for every positive integers n.