13 April 2018 Universal Turing Machine

Key features of a model of computation.

Monday
- [1] Finite internal state / memory
- [2] Infinite external storage
- [3] Finite length program specification

today
- [4] Self-reference: a program description can be the input to another program.
- [5] Universality: there is a program to simulate other programs, given their description as input.

<u>Turing machine descriptions</u>: are written using the symbols

$$\emptyset \quad 1 \quad (\quad) \quad ,$$

For a Turing machine M with alphabet Σ, state set K, transition rule δ, the description of M is

$$m, n, \underbrace{\delta_1, \delta_2, \ldots, \delta_N}$$

m: binary encoding of $|\Sigma|$

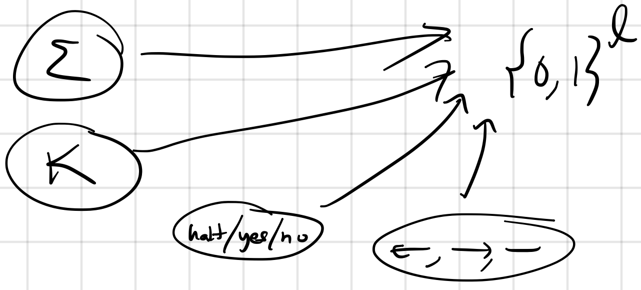
n: binary encoding of $|K|$

each of these is an encoding of one (input, output) pair of δ.

$$\delta_i = (\underline{q}, \underline{\sigma}, \underline{q'}, \underline{\sigma'}, \underline{d})$$

binary string in $\{0,1\}^l$ where $l = \lceil \log_2(|\Sigma| + |K| + 6) \rceil$

There's a one-to-one function



$\triangleright, \sqcup \in \Sigma$

$s \in K$

halt, yes, no, $\leftarrow, \rightarrow, -$

These have pre-defined meanings.

Encode them using the binary encodings of $\emptyset$ thru 8.

So a fact like $\delta(q, \sigma) = (q', \sigma', d)$ is encoded as a five-tuple of binary strings $(q, \sigma, q', \sigma', d)$ in the list $\delta_1, \ldots, \delta_N$.

<u>Multi-Tape TMs.</u> Have a fixed finite # of tapes instead of just one. Each tape is infinite, and has its own read-write head that moves independently of the others.

But the machine has just one set of internal states (not one per tape).

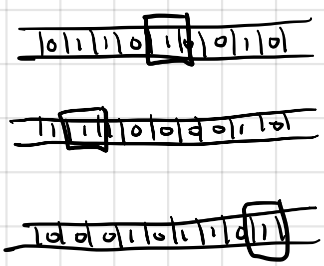
If t denotes # of tapes, transition function is

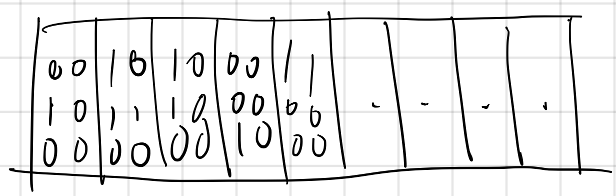
$$\delta : K \times \Sigma^t \longrightarrow \left(K \cup \{\text{halt}, \text{yes}, \text{no}\}\right) \times \Sigma^t \times \{\leftarrow, \rightarrow, -\}^t$$

<u>Single tape can simulate multi-tape.</u>

For a multitape machine M with alph Σ, state set K, t tapes, create a machine $\hat{M}$ with alph $\hat{\Sigma} = \Sigma^t \times \{0,1\}^t$ state set $\hat{K}$, and only one tape.

Ex. $t=3$ Configuration of M Corresponding tape of $\hat{M}$





$\hat{M}$ runs a sequence of simulation rounds. Each round starts with tape head on $\rhd$ and makes a left-to-right pass reading symbols in $\hat{\Sigma}$ and accumulating (in internal storage of $\hat{M}$) a t-tuple of elements of Σ denoting the t symbols that M is currently reading with its read-write head.

$\hat{K}$ must be big enough that an element of $\hat{K}$ can encode an element of $K \times \Sigma^t$.

Once finished with this left-right pass, we return to $\rhd$, and make another left-right pass updating the tape according to δ of M. Return to $\rhd$ to start next round.