Introduction to Algorithms	Problem Set 5
CS 4820, Spring 2018	Due 11:59pm Thursday, March 8

Hand in your solution electronically using CMS. Collaboration is encouraged while solving the problems, but:

1. **list the names of those with whom you collaborated;**
2. **you must write up the solutions in your own words.**

Remember that when a problem asks you to design an algorithm, you must also prove the algorithm's correctness and analyze its running time.

(1) *(Note: This problem is Exercise 5.3 in Kleinberg & Tardos, with an extra "part b" appended.)* Suppose you're consulting for a bank that's concerned about fraud detection, and they come to you with the following problem. They have a collection of n bank cards that they've confiscated, suspecting them of being used in fraud. Each bank card is a small plastic object, containing a magnetic stripe with some encrypted data, and it corresponds to a unique account in the bank. Each account can have many bank cards corresponding to it, and we'll say that two bank cards are *equivalent* if they correspond to the same account.

It's very difficult to read the account number off a bank card directly, but the bank has a high-tech "equivalence tester" that takes two bank cards and, after performing some computations, determines whether they are equivalent.

Their question is the following: among the collection of n cards, is there a set of more than $n/2$ of them that are all equivalent to one another? Assume that the only feasible operations you can do with the cards are to pick two of them and plug them into the equivalence tester.

(a) *(5 points)* Show how to decide the answer to their question with only $O(n \log n)$ invocations of the equivalence tester.

(b) *(5 points)* Now modify the question so that you must decide whether there is a set of more than $n/4$ of the cards that are all equivalent to one another. Show how to decide the answer to this question with only $O(n \log n)$ invocations of the equivalence tester.

If you are confident that you have a correct solution to part (b) of this problem, you can skip part (a) and your score on the problem will be computed by doubling your score for part (b).

(2) *(10 points)* Solve Exercise 5.7 in Kleinberg & Tardos.

(3) *(10 points)* A closed axis-parallel rectangle in the plane is a subset $R \subset \mathbb{R}^2$ that is the Cartesian product of two closed intervals,

$$R = [a, b] \times [c, d] = \{(x, y) \mid a \leq x \leq b \text{ and } c \leq y \leq d\}.$$

We call the 4-tuple (a, b, c, d) the *description* of R. Given the descriptions of n rectangles, design an algorithm to decide whether there exists a point in the plane that belongs to two or more of the rectangles. Your algorithm's running time should be $O(n \log n)$.