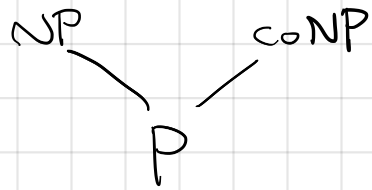
30 March 2018

Panorama of complexity classes.

A decision problem Π belongs to coNP if $\exists$ a verifier V such that

$\Pi(x) = \text{no} \Rightarrow \exists y \; V(x,y) = \text{yes}$

$\Pi(x) = \text{yes} \longrightarrow \nexists y \; V(x,y) = \text{yes}$



Equivalently $\Pi \in \text{coNP}$ iff $\neg\Pi \in \text{NP}$ where $\neg\Pi$ is the decision problem "output the negation of $\Pi(x)$."

E.g. Unsatisfiability = "Does this formula have no satisfying truth assignments?"

Does every unsatisfiable formula have a short, efficiently verifiable proof of unsatisfiability? Does NP equal coNP?

If $P = NP$ then $NP = coNP$.

Most computer scientists believe $NP \neq coNP$ and $P \neq NP \cap coNP$.

A problem in $NP \cap coNP$ but not known to be in P.

"Parity Games": given a directed graph with distinct integer labels on vertices. Alice & Bob play a game where they start at node s, make alternating moves along directed edges, Alice moves first, winner is Alice if the lowest-numbered node that gets visited ∞ often is even-numbered, Bob wins if it's odd.

Given a parity game, can Alice force a win?

Fastest known algorithm is "quasi-polynomial" time

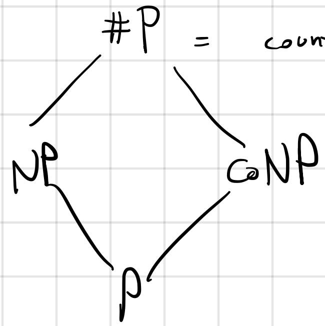
$$n^{\log^c(n)} \text{ for some constant } c < \infty.$$

Another $NP \cap coNP$ problem: knot triviality.

Given a diagram like this...

determine if the 3-D curve represented by the diagram can be "unknotted" without breaking the loop.

Another problem that is $NP \cap coNP$: PRIMALITY. As of 2003, known to be in P.



#P = counting problems. Given a verifier V for an NP problem and given some input x, how many y such that $V(x,y) = \text{yes}$?

E.g. How many satisfying assignments does this CNF-SAT formula have?

#P believed to be much harder than NP.

Interesting facts: counting # of perfect matchings in a bipartite graph is #P-complete.

Whereas counting spanning trees of a graph has a polytime algorithm.

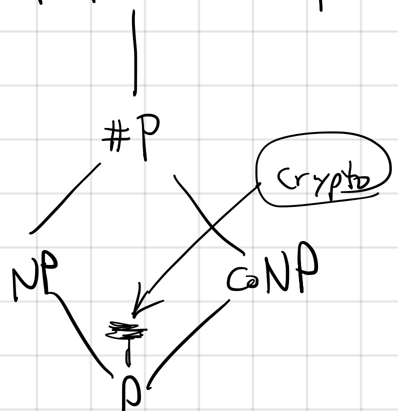
Importance: Computing probabilities of complex events.

Given a network whose edges "fail" independently with failure probability $p(e)$ on edge e, what is the probability of a network partition? (Set of failures that disconnect the graph.)

Called "network reliability" problem. #P-complete.

Many other such examples from machine learning.
(E.g. marginalization in graphical models.)

PSPACE = problems that can be solved in polynomial space.



Importance: Complexity of solving 2-player combinatorial games.
Complexity of many "dynamic programming" problems of importance in OR.

Below P:

Immerman – Szelepszenyi Theorem

P NL = coNL

NC

L = decision problems solvable in $O(\log n)$

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problems that can be solved in parallel in $O(\log^c n)$ time and $O(n^c)$ work.