CS 4820, Spring 2018	**Homework 10, Problem 2**
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(2) *(10 points)* Call a Turing machine M *termination-safe* if $M(y) \neq \nearrow$ for all $y \in \Sigma^*$. In other words, a termination-safe machine is one which is guaranteed to terminate on every input string.

Let $T \subset \Sigma^*$ denote the set of all strings x such that x is the description of a termination-safe Turing machine. Prove that T is not recursively enumerable.

Solution:

The problem is aiming to prove that the set T is not recursively enumerable (*r.e.*), where T belongs to Σ^* denote the set of all strings x such that x is the description of a termination-safe Turing Machine. Termination-safe Turing Machine refers to one that is guaranteed to terminate on every input string. To prove that set T is not recursively enumerable, we reducing from the co-halting problem $\bar{H}$ to T since $\bar{H}$ is not recursively enumerable. Given a Turing Machine M_T which is used to solve the termination-safe problem such that $M_T(M)$ outputs YES if and only if Turing Machine M always halt on input y. We need another machine C which is used to solve the co-halting problem such that $C(x;y)$ outputs YES if and only if x does not halt on input y. Then, we can construct machine C which encoding machine x and input y as the input of machine M_T. Assume that machine M_T takes in a machine which is the encapsulating of x and y which represents as $S_{x,y}$. There is a one-to-one relationship which combines machine M_T and machine C that Turing Machine M is termination-safe if and only if x does not halt on input y. Thus, Turing Machine $S_{x,y}$ should operates as following: If x halts on input y, then the machine should loop forever which means that $S_{x,y}$ is not termination-safe. If x is not halt on input y, then the machine halt which means that $S_{x,y}$ is termination-safe. Now we define the operation rule of Turing Machine $M_T(S_{x,y})$. $M_T(S_{x,y})$ outputs YES if and only if $S_{x,y}$ is termination-safe which means that machine x is not halt on input y. $M_T(S_{x,y})$ outputs NO if and only if $S_{x,y}$ is not termination-safe which means that machine x is halt on input y. Therefore, we reduced T from the co-halting problem $\bar{H}$ which means that the set size of T is greater than or equals to the set size of $\bar{H}$. Since co-halting problem $\bar{H}$ is not recursively enumerable, T is not recursively enumerable. Next, we proof by strong induction. $x;y$ belongs to set $\bar{H}$ if and only if M_T rejects $x;y$ which means $S_{x,y}$ rejects $x;y$. Thus, $\bar{H}$ is recursively enumerable which contradict to the fact that $\bar{H}$ is not recursively enumerable. Therefore, T is not recursively enumerable.