

Let's construct the system of equations based on the diagrams of forces below.

$$F_1 - f_1 - N_{0x} = M_1 a_1$$

$$T - f_2 = M_2 a_2$$

$$-F_1 = M_3 a_{3x}$$

$$2f_3 + T - M_3 g = M_3 a_{3y}$$

$$N_{0x} - T = M_0 a_0 = 0$$

Constraints(the derivation can be found in the solution of the quiz):

$$a_1 - a_2 - a_{3y} = 0$$

$$a_1 = a_{3x}$$



Let's note that

$$f_1 = \mu_1 N_1$$

$$f_2 = \mu_2 N_2$$

$$f_3 = \mu_3 F_1$$

are forces emerged because of friction.

From the system of equations above, we can find $F_1(t)$ and $x_1(t), x_2(t), y_3(t)$. Having the mentioned functions. We will be able to calculate their values at a given time t .

```
In [1]: import numpy as np
```

```
In [186]: M1 = 10
M2 = 5
M3 = 3
mu1 = 0.09
mu2 = 0.3
mu3 = 0.4
g = 10
```

General solution of the system

```
In [187]: # The system of linear equations (see images calcs1.jpeg and calcs2.jpeg for more detail)
A = np.array([[-(M1 + M3), -M2], [-(2*mu3*M3 + M3), M2 + M3]])
b = np.array([[M2*mu2*g + M1*mu1*g + M2*mu1*g, M3*g - M2*mu2*g]])
```

```
In [188]: # Solve
a = np.linalg.inv(A).dot(b.T)
a = np.squeeze(a)
```

```
In [189]: a1 = [a[0], 0]
a2 = [a[1], 0]
a3 = [a1[0], a[0] - a2[0]]

a1, a2, a3
```

```
Out[189]: ([-2.312977099236641, 0],
[0.3137404580152672, 0],
[-2.312977099236641, -2.6267175572519084])
```

Our problem

```
In [190]: # I don't really get why we need to get F as an input if we can find it from the
system of linear equations I presented above
# but anyways, in that case a1 = -F/M3 and a2 = (T - mu2*M2*g)/M2, where T = F -
mu1*g*(M1 + M2) - M1*a1
def compute_accelerations(F):
    a1 = -F/M3
    T = F - mu1*g*(M1 + M2) - M1*a1
    a2 = (T - mu2*M2*g)/M2

    a1 = [a1, 0]
    a2 = [a2, 0]
    a3 = [a1[0], a1[0] - a2[0]]

    return a1, a2, a3
```

```
In [191]: # Just an example of t and F, it should be changed by the data on which the program should work
t = np.linspace(1, 20, 30)
F = np.random.random(30)*(-1) + 300
```

```
In [192]: t
```

```
Out[192]: array([ 1.          ,  1.65517241,  2.31034483,  2.96551724,  3.62068966,
 4.27586207,  4.93103448,  5.5862069 ,  6.24137931,  6.89655172,
 7.55172414,  8.20689655,  8.86206897,  9.51724138, 10.17241379,
10.82758621, 11.48275862, 12.13793103, 12.79310345, 13.44827586,
14.10344828, 14.75862069, 15.4137931 , 16.06896552, 16.72413793,
17.37931034, 18.03448276, 18.68965517, 19.34482759, 20.          ])
```

```
In [193]: v1 = 0; v2 = 0; v3 = 0
x1 = [0, 0]
x2 = [0, 0]
x3 = [0, 0]
```

```
In [194]: positions = []
for i in range(len(t)):
    a1, a2, a3 = compute_accelerations(F[i])
    x1[0] += v1*t[i] + a1[0]*(t[i]**2)/2
    v1 += a1[0]*t[i]
    x2[0] += v2*t[i] + a2[0]*(t[i]**2)/2
    v2 += a2[0]*t[i]
    x3[0] = x1[0]
    x3[1] += v3*t[i] + a3[1]*(t[i]**2)/2
    v3 += a3[1]*t[i]
    positions.append([x1.copy(), x2.copy(), x3.copy()])
```

```
In [195]: import matplotlib.pyplot as plt
```

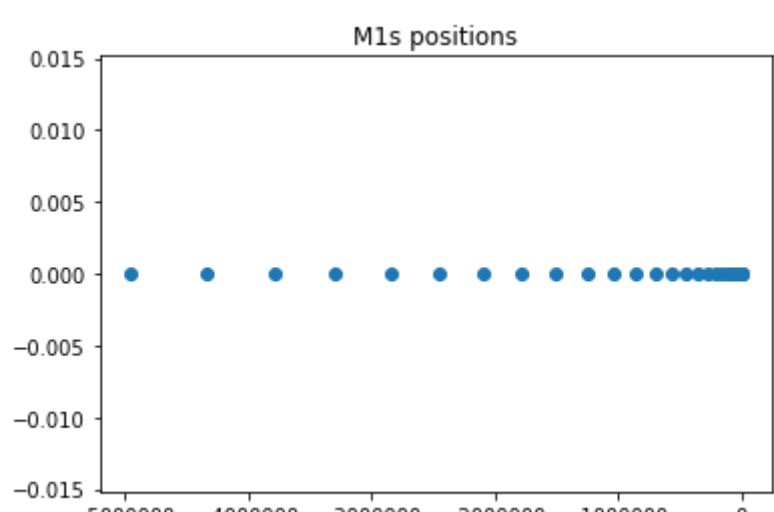
```
In [196]: positions = np.array(positions)
```

```
In [197]: x1s = positions[:,0,:]

x1s_x = x1s[:, 0]
x1s_y = x1s[:, 1]

plt.scatter(x1s_x, x1s_y)
plt.title('M1s positions')
```

```
Out[197]: Text(0.5, 1.0, 'M1s positions')
```

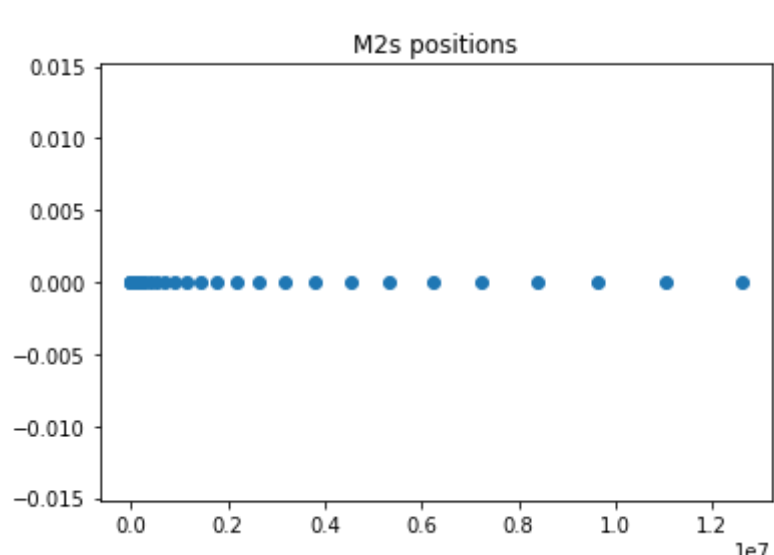


```
In [198]: x1s = positions[:,1,:]

x1s_x = x1s[:, 0]
x1s_y = x1s[:, 1]

plt.scatter(x1s_x, x1s_y)
plt.title('M2s positions')
```

```
Out[198]: Text(0.5, 1.0, 'M2s positions')
```



```
In [199]: x1s = positions[:,2,:]

x1s_x = x1s[:, 0]
x1s_y = x1s[:, 1]

plt.scatter(x1s_x, x1s_y)
plt.title('M3s positions')
```

```
Out[199]: Text(0.5, 1.0, 'M3s positions')
```

