Structural Macroeconometrics
Assignment 2

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1 Model specification

In our matlab code we replicate the GRETL script provided for the assignment. The code manually builds the matrices needed for estimation through loops and uses some functions that are user-defined in the homonym folder. Pertaining the treatment of our variables of interest (i.e., tax revenues, government expenditure and GDP), we assume that they follow a VAR process with 4 lags and a linear trend. That is:

$$\begin{cases} w_t := (TAX_t \;\; G_t \;\; GDP_t)' \\ w_t = \mu + \gamma t + \sum_{i=1}^{4}(\Pi_i w_{t-i}) + u_t \\ u_t \sim \text{MDS} \end{cases}$$

We decide not to include the dummy variable "DD75Q2" as we find that it does not significantly alter the results. The model is estimated by exploiting the definition of a "varm" object in matlab, containing a trend, a constant and the pre-defined four matrices Π, which are then used to store the estimated coefficients. The estimates of the VAR model are performed using the "estimate" command which uses maximum likelihood. The "fminunc" command is instead used to minimize the user-defined log likelihood, which is contained in an homonym file inside of the functions folder. It should be noted that this user-defined likelihood is the opposite of the original log-likelihood. In other words, we are maximizing the log-likelihood by minimizing its opposite. Moreover, the numerical method employed is that of the gradient descent maximization. The functional form of the aforementioned log-likelihood is outlined below:

$$\mathcal{L}(A,B) = -const + \frac{T}{2}\log|A^{-1}BB'A^{-1}| + \frac{T}{2}tr(A'B^{-1\prime}B^{-1}A\hat{\Sigma}_u)$$

The matrices to be estimated are identified following Blanchard and Perotti:

$$\begin{pmatrix} 1 & 0 & -2.08 \\ 0 & 1 & 0 \\ a_{31} & a_{32} & 1 \end{pmatrix} u_t = \begin{pmatrix} b_{11} & b_{12} & 0 \\ 0 & b_{22} & 0 \\ 0 & 0 & b_{33} \end{pmatrix} \epsilon_t$$

We then derive the estimate of the companion matrix $\hat{J}$ as:

$$\hat{J} = \begin{pmatrix} \hat{\Pi}_1 & \hat{\Pi}_2 & \hat{\Pi}_3 & \hat{\Pi}_4 \\ I & O & O & O \\ O & I & O & O \\ O & O & I & O \end{pmatrix}$$

where I denotes the identity matrix and O denotes matrix of zeros. Consequently, the estimated impulse response function of variable GDP_t, at time $t+h$, is given by:

$$I\hat{R}F_{*,3}(h) = R\hat{J}^h R'(\hat{A}^{-1})\hat{B}e_3$$

where $e_3 = (0\ \ 0\ \ 1)'$. We now focus on the behaviour of the identified tax and spending multipliers which are the re-scaling of their counterparts in Figure 1 at position (3,1) and (3,2). The formula for the two multipliers is given below:

$$\begin{cases} \hat{M}_{TAX}(h) = \frac{\hat{\phi}_{13}}{\hat{\phi}_{11}} \cdot \frac{1}{T}\sum_{t=1}^{T} \frac{G\tilde{D}P_t}{T\tilde{A}X_t} \\ \hat{M}_G(h) = \frac{\hat{\phi}_{23}}{\hat{\phi}_{22}} \cdot \frac{1}{T}\sum_{t=1}^{T} \frac{G\tilde{D}P_t}{\tilde{G}_t} \end{cases}$$

where tilde denotes non-logged variables.

2 Bootstrap

We now evaluate the uncertainty around these estimates by means of the iid-nonparametric bootstrap procedure. Firstly, we fix the four initial values of each variable to have four initial conditions. We then resample the estimated VAR innovations (i.e., $\hat{u}_t$) to obtain a new process of error terms u_t^*. We employ a recursive algorithm to build the bootstrap sample in the following way:

$$\begin{cases} w_5^* = \hat{\mu} + \hat{\gamma} \cdot 5 + \sum_{i=1}^{4}(\hat{\Pi}_i w_{5-i}) + u_5^* \\ w_6^* = \hat{\mu} + \hat{\gamma} \cdot 6 + \hat{\Pi}_1 w_5^* + \sum_{i=2}^{4}(\hat{\Pi}_i w_{6-i}) + u_6^* \\ .... \\ w_T^* = \hat{\mu} + \hat{\gamma} \cdot T + \sum_{i=1}^{4}(\hat{\Pi}_i w_{T-i}^*) + u_T^* \end{cases}$$

This procedure yields a simulated process $w_t^* = (TAX_t^*\ \ G_t^*\ \ GDP_t^*)'$. We estimate a VAR(4) model with linear trend with these new time series, considering the estimates we derived at the beginning ($\hat{\Pi}_1\ \ \hat{\Pi}_2\ \ \hat{\Pi}_3\ \ \hat{\Pi}_4\ \ \hat{u}_t$) as the unknown in the bootstrap world. The estimation yields new VAR coefficients ($\hat{\Pi}_1^*\ \ \hat{\Pi}_2^*\ \ \hat{\Pi}_3^*\ \ \hat{\Pi}_4^*\ \ \hat{u}_t^*$). The bootstrap companion matrix is defined as:

$$\hat{J}^* = \begin{pmatrix} \hat{\Pi}_1^* & \hat{\Pi}_2^* & \hat{\Pi}_3^* & \hat{\Pi}_4^* \\ I & O & O & O \\ O & I & O & O \\ O & O & I & O \end{pmatrix}$$

Additionally, we compute the bootstrap variance-covariance matrix $\hat{\Sigma}_t^*$ and solve the log-likelihood maximization problem to derive the vector of bootstrap structural parameters $\hat{\gamma}^* = (\hat{a}_{31}^*\ \ \hat{a}_{32}^*\ \ \hat{b}_{11}^*\ \ \hat{b}_{12}^*\ \ \hat{b}_{22}^*\ \ \hat{b}_{33}^*)'$.

The bootstrap impulse response function is given by:

$$I\hat{R}F^{*}(h) = R\hat{J}^{*h}R'(\hat{A}^{*-1})\hat{B}^{*}$$

Ultimately, the bootstrap tax and spending multipliers are respectively:

$$\begin{cases} \hat{M}^{*}_{TAX}(h) = \frac{\hat{\phi}^{*}_{13}}{\hat{\phi}^{*}_{11}} \cdot \frac{1}{T}\sum_{t=1}^{T} \frac{G\tilde{D}P^{*}_{t}}{T\tilde{A}X^{*}_{t}} \\ \hat{M}^{*}_{G}(h) = \frac{\hat{\phi}^{*}_{23}}{\hat{\phi}^{*}_{22}} \cdot \frac{1}{T}\sum_{t=1}^{T} \frac{G\tilde{D}P^{*}_{t}}{\tilde{G}^{*}_{t}} \end{cases}$$

We repeat the bootstrap algorithm $B = 1000$ times to get 1000 matrices of impulse responses, and for each coefficient ϕ^{*}_{ijh} (estimated B times) we consider the 5th and 95th quantiles for the plot of the confidence bands. The same method applies for the bootstrap confidence bands of the two multipliers.

3 Results

The IRFs estimates with the bootstrap confidence intervals are reported in Figure 2. We assume that we have a sufficiently-large sample ($N = 224$) and run a sufficiently-large number of iterations B for the Central Limit Theorem to hold. This guarantees that the bootstrap empirical PDF of coefficients ϕ^{*}_{ijh} and both multipliers $\hat{M}^{*}(h)$ is a good approximation of their correspondent asymptotic PDF. That is, the inference based on the bootstrap quantiles is valid. In terms of Matlab code, the procedure outlined above is implemented through a series of nested for-loops, which return us all the needed variables and confidence intervals, which are extracted from the simulated quantiles obtained through bootstrap. The code could be made more efficient by employing parallel computing or other ways of making the outer-for loop on the bootstrap iterations. However, these solution would require a different coding approach which is beyond the scope of this exercise. As a final note, it is worth mentioning that the computed confidence intervals might underestimate the true uncertainty since they are based upon the assumption of i.i.d. data. In fact, the resampling procedure yields independent bootstrap error terms $\hat{u}^{*}_{t}$. Unfortunately, this assumption is usually not verified in macroeconomic data and indeed the literature has moved towards more robust methods such as the Moving Block Bootstrap.

As shown in the top panel of Figure 1, the spending multiplier on impact is significant and slightly greater than 1. It then increases until reaching a peak 2 periods after the shock (that is, period 3 in the graph since $h = 0$ is labeled as 1). Afterwards, the multiplier declines steadily, approaching roughly 0.5 at the end of the horizon considered. However, the significance of the effect is lost after two years, that is for $h = 7$.
Pertaining the tax dynamic multiplier, on impact it is around -0.4 and significant. Afterwards, it decreases and becomes non significant, before returning to be significant in the fourth period. Then, the tax multiplier becomes non significant in period 9, that is for $h = 8$. For the reminding periods, the effect is non significant and slowly approaches zero.

Overall, as long as tax and public spending are perturbed in isolation by one-standard-deviation shock, the impact of a perturbation of the former variable is less pronounced than that of the latter. In other words, our model suggest that the economy is less responsive to a tax revenue shock than it is to a public spending one. It should be noticed that this comparison holds regardless of the sign as one could model also a tax cut in the same framework. This could be achieved by simply changing the vector of shocks to be a one-standard-deviation decrease, rather than an increase. In fact, the values and the confidence intervals would be exactly the same but multiplied by minus 1. That said, the dynamic multipliers tend to fade over time, reinforcing the idea that the relationship between the variables behaves as stationary over time. Indeed, an interesting exercise would be to investigate possible co-integration relationships between the variables of interest, but such analysis is beyond the scope of this exercise.

To sum up, it is worth mentioning again that our results are computed in the presence of underestimated uncertainty due to the employed bootstrap method and should therefore be taken with caution as preliminary evidence.

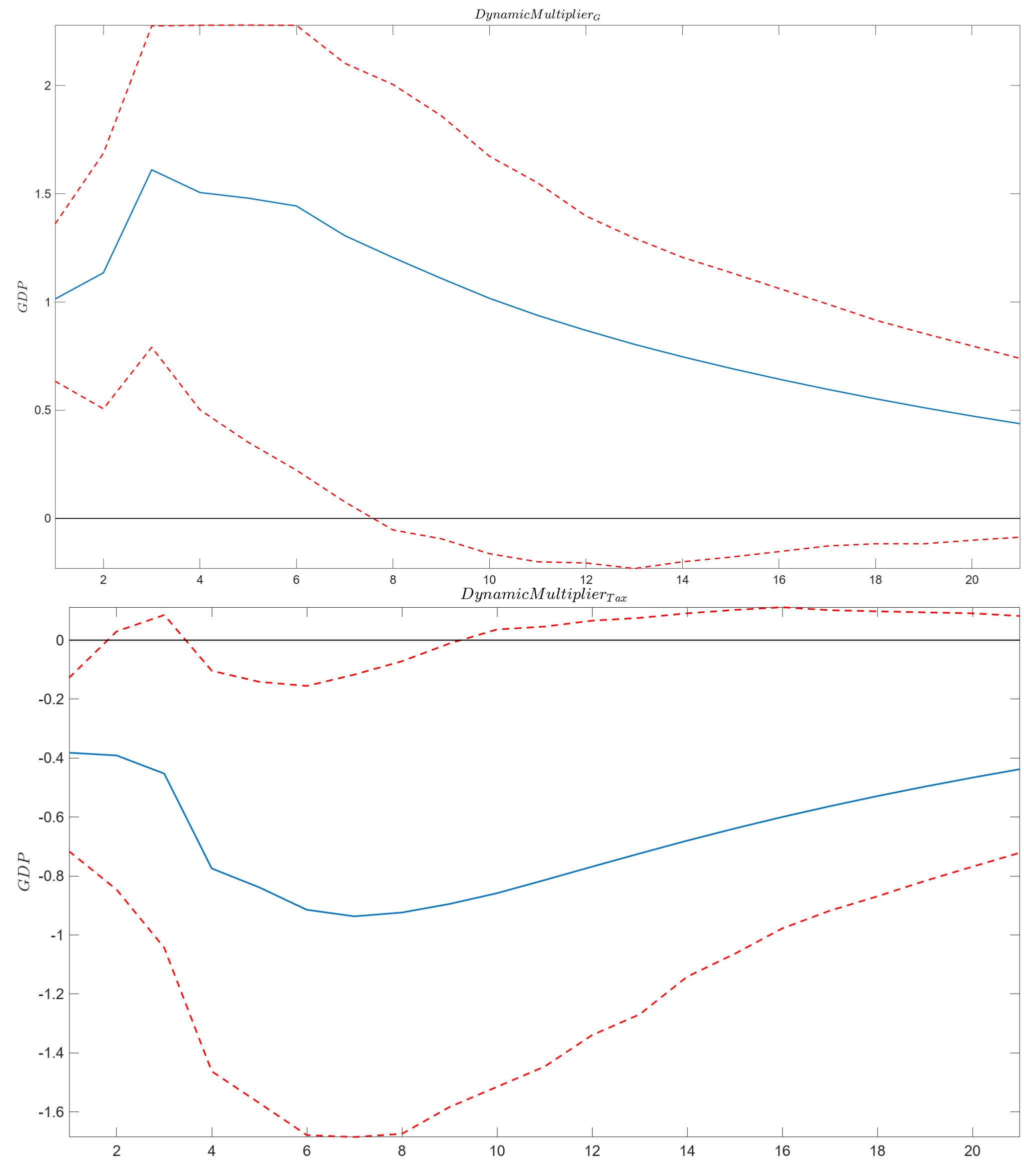

Figure 1: Dynamic Spending Multiplier (top) and Dynamic Tax multiplier (bottom)

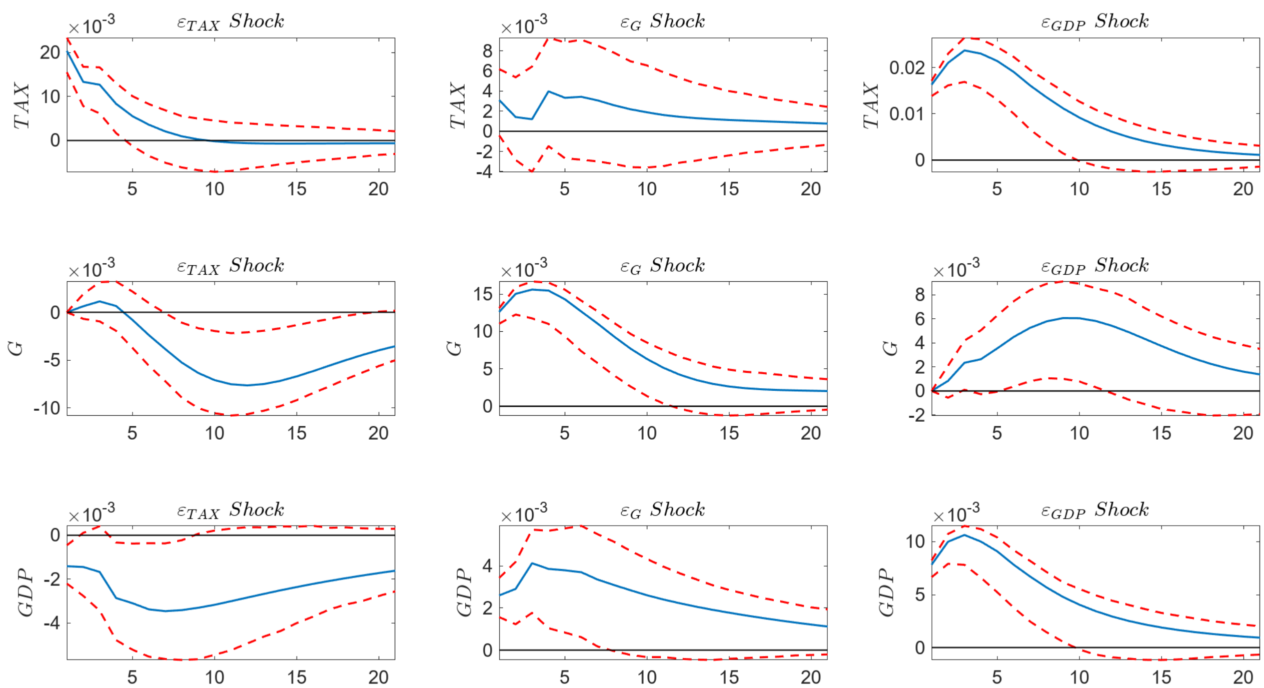

Figure 2: IRFs for all variables