Moon Formation

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Outline

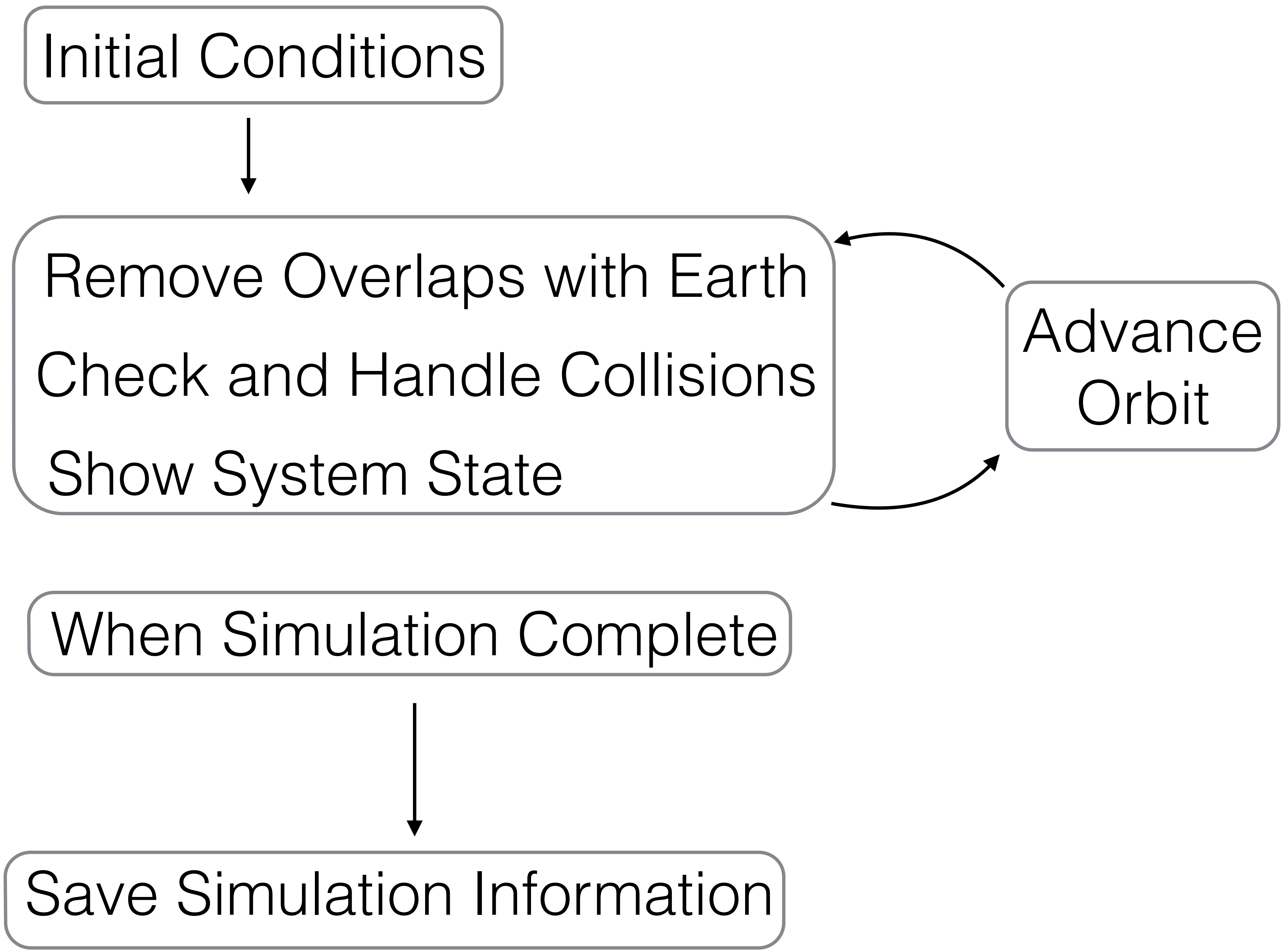
- Background
- Simulation
- Results
- Future Work

Background

Giant Impact Hypothesis

- 4.5 Billion years ago
- Theia (proto-planet) collides with Earth, ejecting debris
- Ejected debris collects under gravity to form Moon

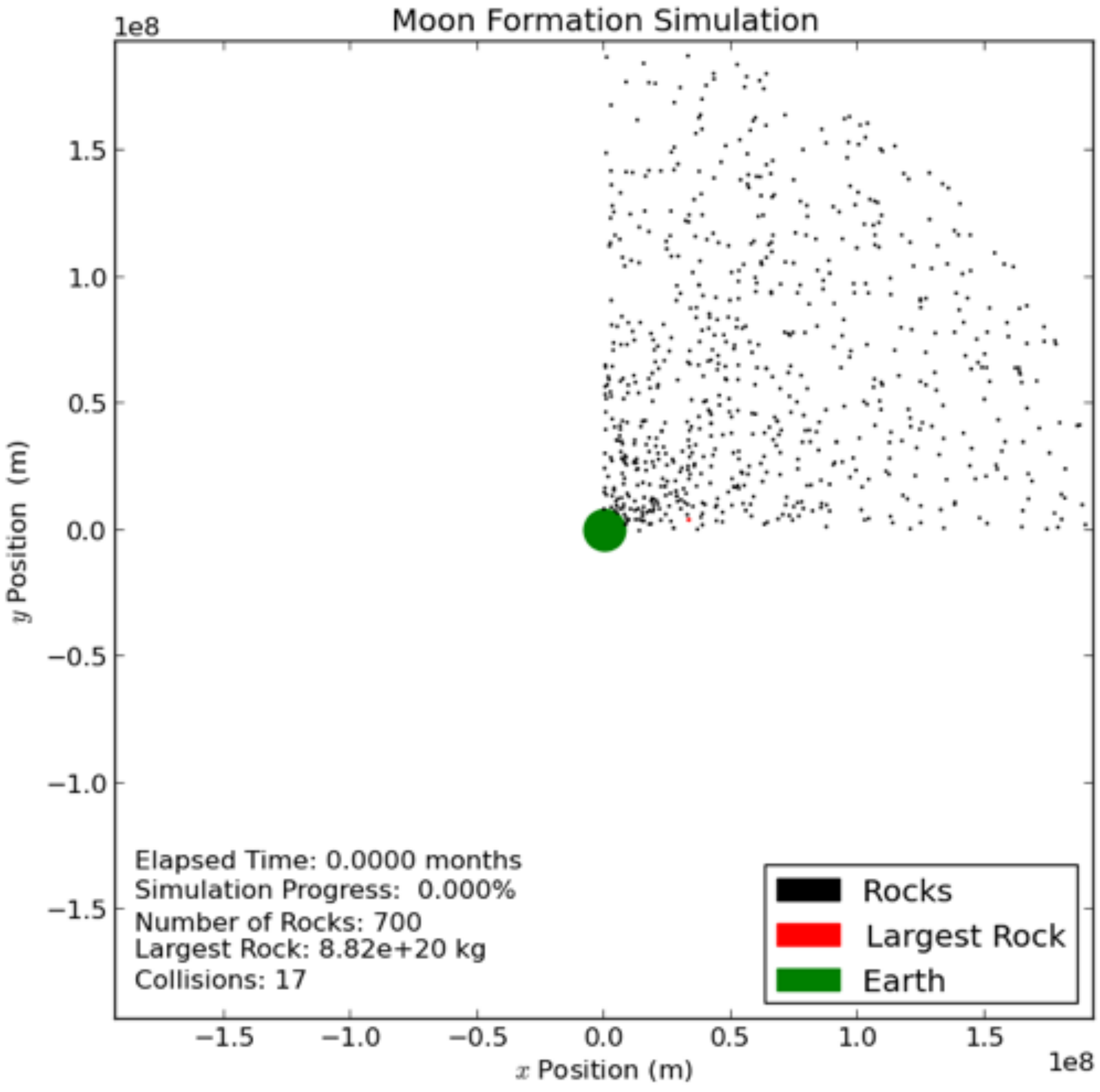
Program Flow



Initial Conditions

Assumed Parameters

- Global Parameters:
  - Number of Rocks
  - Density
  - Planet Mass
- Gaussian Random Parameters:
  - Mass
  - Semi-Major Axes
- Uniform Random Parameters:
  - Eccentricities
  - Ejection Angle



Initial Conditions

Derived Parameters

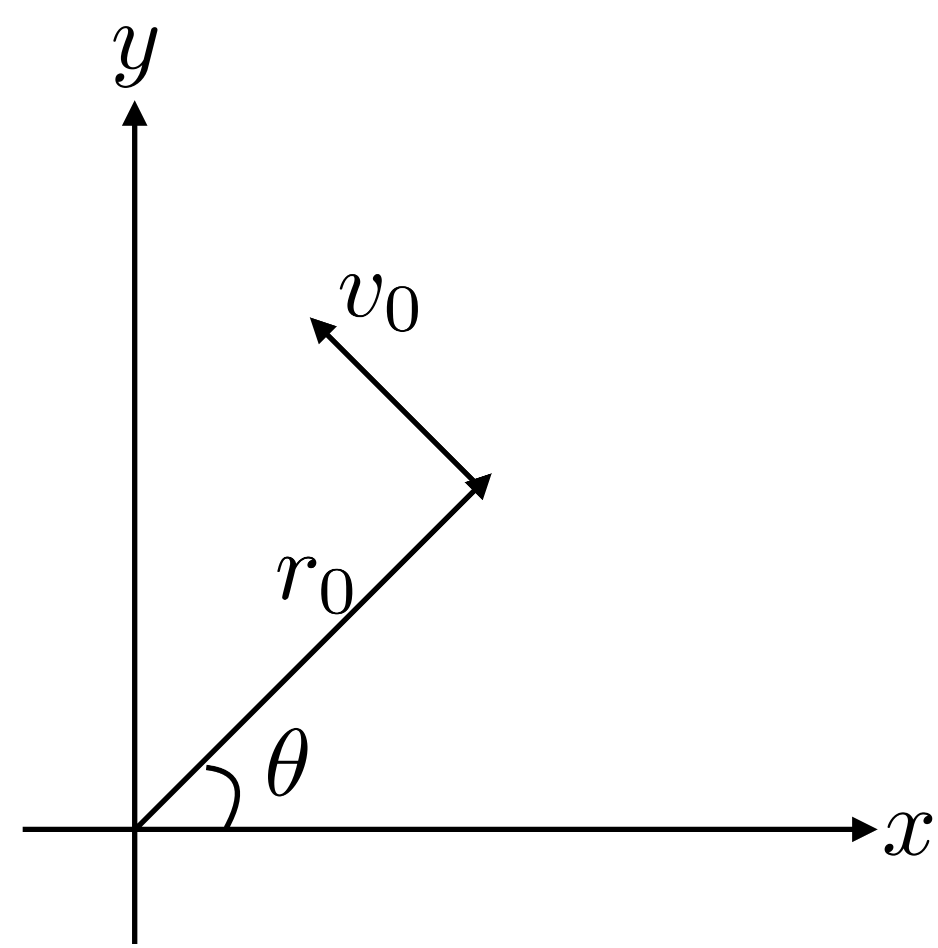
$$r_0 = \frac{1-e}{1+q}a$$

$$v_0 = \frac{1}{1+q}\sqrt{\frac{1+e}{1-e}}\sqrt{\frac{Gm_{total}}{a}}$$

$$x = r_0\cos(\theta) \qquad y = r_0\sin(\theta)$$

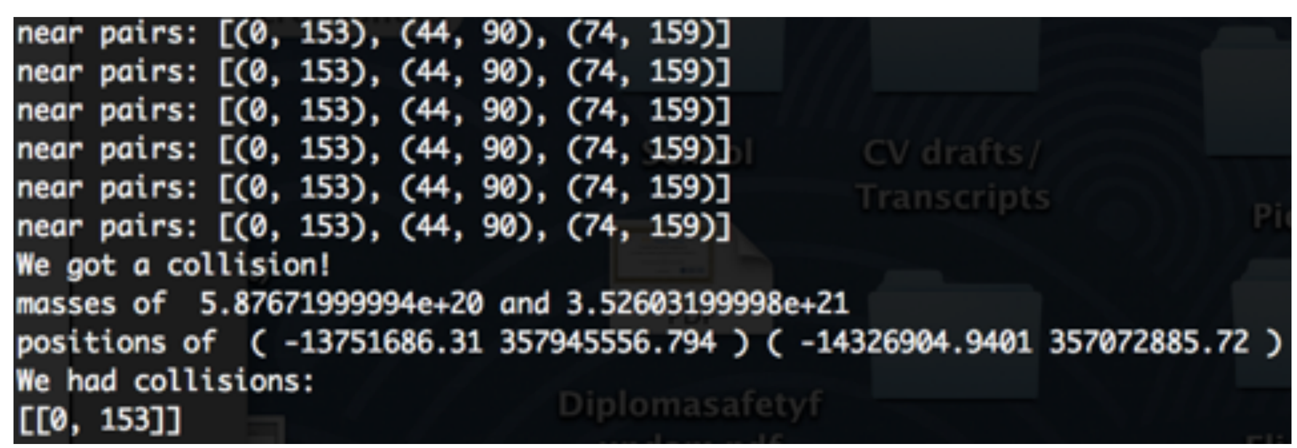
$$v_x = \pm v_0\sin(\theta) \qquad v_y = \pm v_0\cos(\theta)$$

- Earth:
  - Position Fixed at Origin
  - Velocity Fixed to Zero
- Rocks:
  - 80% w/ bulk velocity (-x,+y)
  - 10% w/ bulk velocity (+x,+y)
  - 10% w/ bulk velocity (+x,-y)



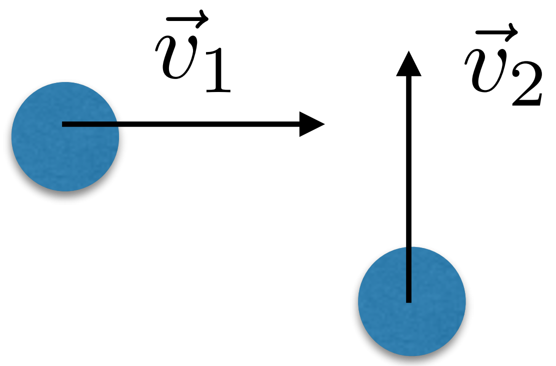
Collision Detection & Handling

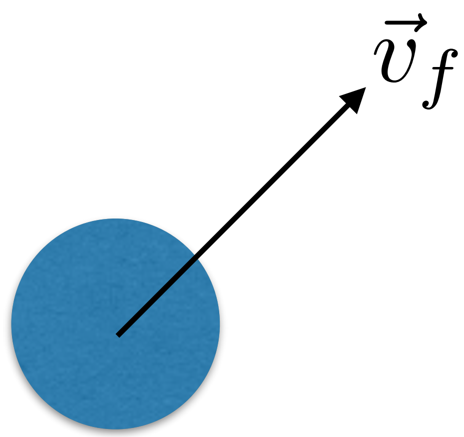
- Take in rock_list
- Using KDTree, search for nearest pairs
- Determine which of the nearest pairs collide
- Check for multi-body collisions
- Calculate post-collision position/velocity using momentum conservation
- Remove old rocks
- Return new rock_list



Collision Detection & Handling

Inelastic Collision
Momentum Conservation





$$\vec{p}_i = m_1\vec{v}_1 + m_2\vec{v}_2$$

$$\vec{p}_f = (m_1 + m_2)\vec{v}_f$$

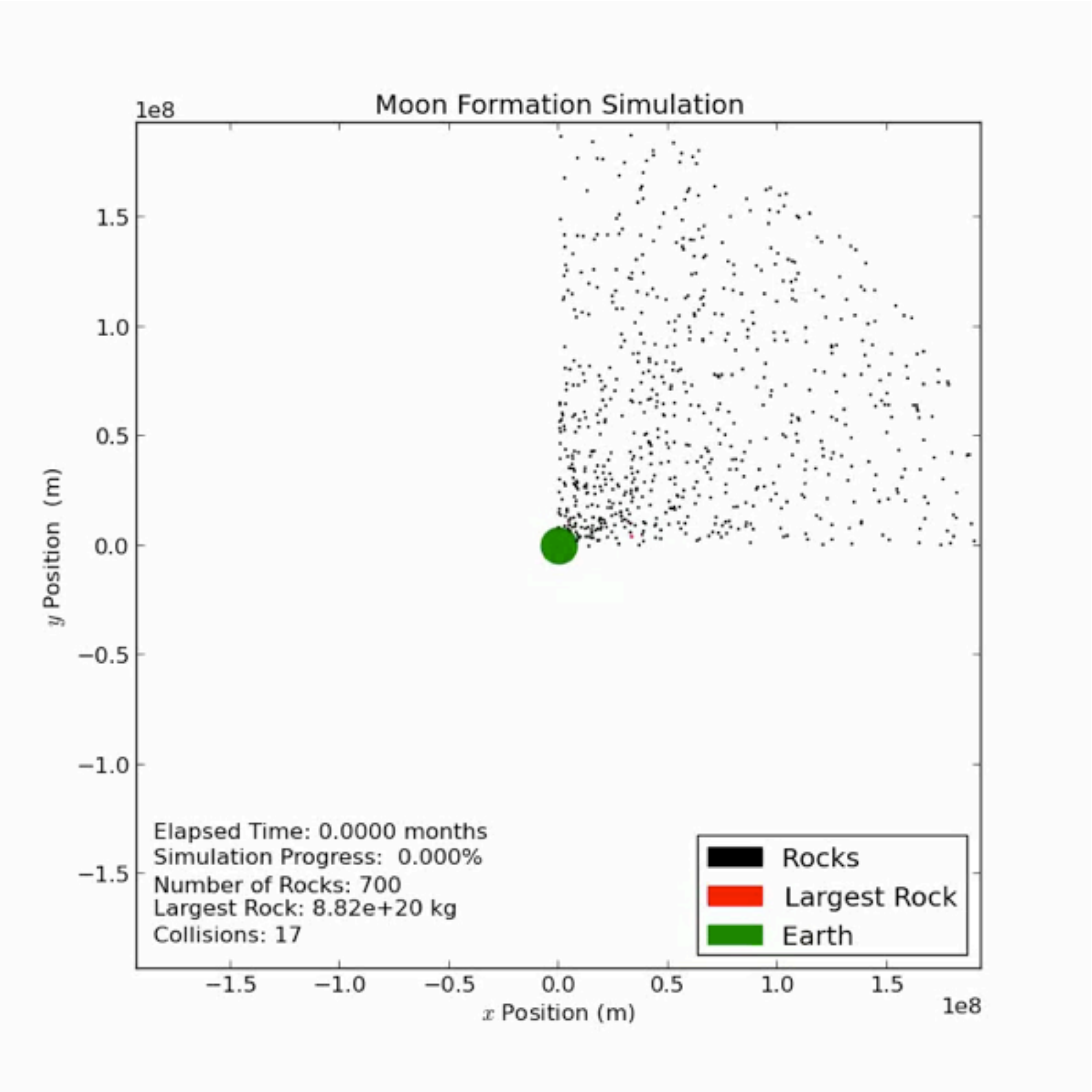
Advance Orbit

- ‘Soften’ Gravity Around Earth Surface

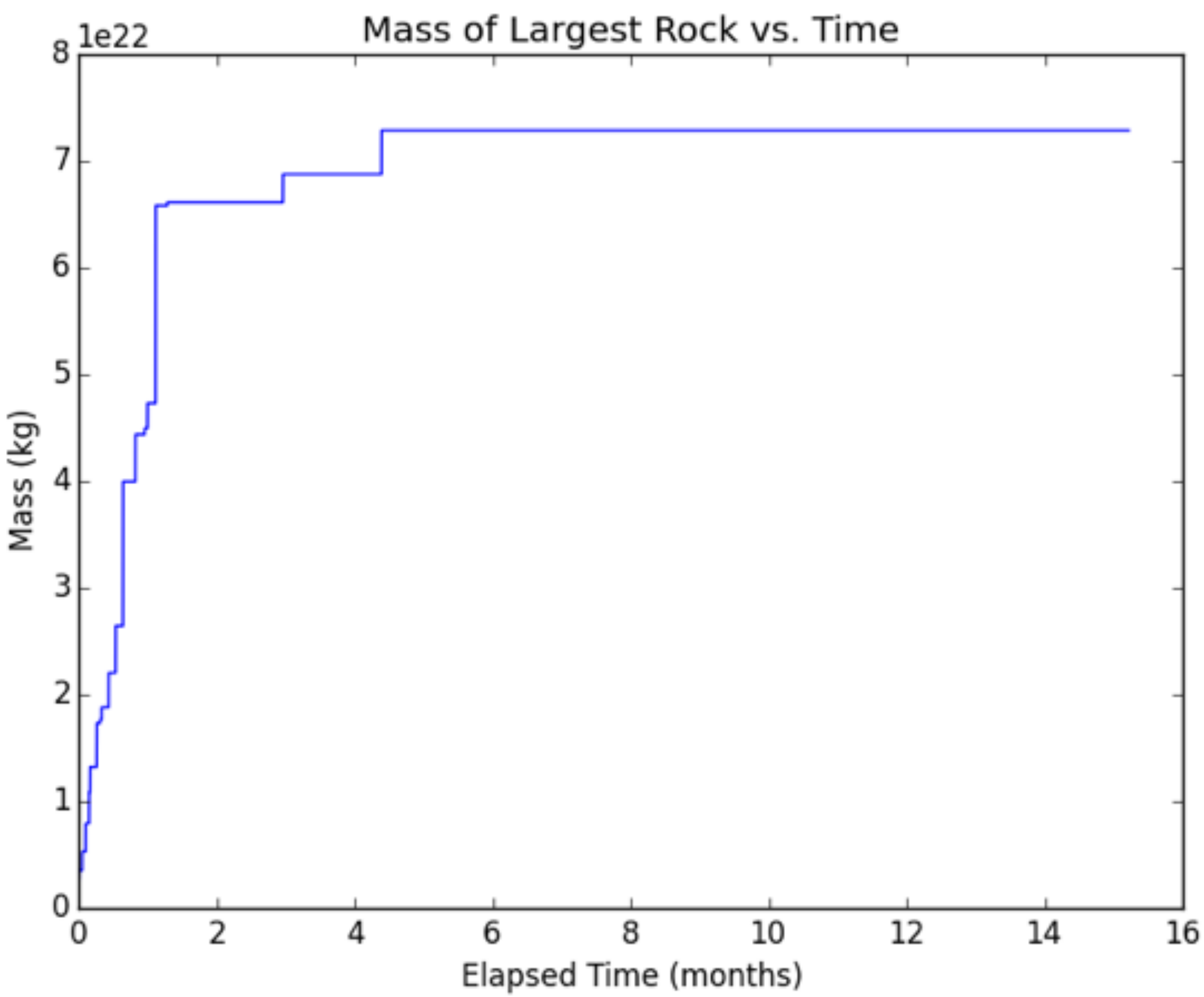
$$\frac{d\vec{v}}{dt} = -\frac{GM_{planet}}{r_{rock}^2 + r_{soft}^2}\hat{r}_{rock}$$

- Change ODEint tolerances to speed up integration

Simulation


Moon Formation Simulation
1e8
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
y Position (m)
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
1e8
x Position (m)
Elapsed Time: 0.0000 months
Simulation Progress: 0.000%
Number of Rocks: 700
Largest Rock: 8.82e+20 kg
Collisions: 17
Rocks
Largest Rock
Earth

Results



Initial number of rocks: 750
Initial mean distance: 1.932e+8 m
Initial total mass: 2.20e+23 kg
Simulation time step: 56.31 s
Plotting interval: 100.0
Simulation run time: 4.0e+7 s
Moon mass: 7.28e+22 kg
Remaining mass in rocks: 1.81e+23 kg
Remaining number of rocks: 15
Earth-Moon Distance: 6.0e+7 m

Future Work

- Include Self-Gravity
- Upgrade to 3 Dimensions
- Earth Orbiting Center of Mass