Representations of a Rotating Dumbbell

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ABSTRACT

For a simple problem with angular momentum and angular velocity, they are usually aligned so there would be no need to differentiate between time because everything will remain constant. Since they are not aligned, we must run through all times and see how angular momentum and torque change with time. This can be by setting up matrices in python using the NumPy library and calculating L and N in matrix form. Then applying the time dependent basis vector that varies time to see how they would change. Plotting L and N we see it has a periodic motion. Therefore, an external torque needs to be applied in a periodic motion for the dumbbell to rotate with a constant angular velocity.

THEORY

Now that we know what our initial conditions and equations are (time = 0). We must now integrate the time dependent basis vector to the regular axis of a rigid body, this is explained in the section Time Dependent Vector Field. In python, we set up our variables in the form of matrices and integrate the basis vector so we can see how exactly angular momentum and torque change with time. It should be noted that w = pi/4, $m_1 = m_2 = 1$, $r_1 = r_2 = b = 1$.

Time Dependent Vector Field

Defining the initial system and body coordinate system: The e_3-axis is oriented along the shaft, the e_2-axis is oriented perpendicular to the shaft and at the plane/origin and the e_1-axis is perpendicular to the plane. Now, that we defined the rigid body, we can now see how the axis will change with time. This is shown below:

$$e'_1 = \text{array}[\cos(w*t)e_1,\ \cos(x)*\sin(w*t)e_2,\ -\sin(x)*\sin(w*t)e_3]$$
$$e'_2 = \text{array}[-\sin(w*t)e_1, \cos(x)*\cos(w*t)e_2, -\sin(x)*\cos(w*t)e_3]$$
$$e'_3 = \text{array}[\sin(w*t)e_1,\ \cos(x)e_2] \quad (5)$$

INTRODUCTION

A dumbbell connected by two masses (m_1, m_2) being rotated around the origin (Fig 5):

For the problem the masses are rotating counter-clockwise around the circles which also rotates the shaft that traces out two cones. For this to happen the masses are moving at a constant velocity (v) and angular velocity (w). The angular velocity is directed along the axis of rotation while the angular momentum (L) is perpendicular to the shaft and at the origin (O). So, we have a constantly changing angular momentum and torque (N) that oscillates with time.

Now that there's a source term that is varying in position and oscillating in time we must now derive equations that can show how angular momentum and torque change with time.

Starting with a initial system, we solve for moments of inertia (I) using eq(1). Then finding what angular velocity (w) is, we can use eq(3) to find the angular momentum for each axis. Finally using eq(4) we can find torque for each axis.

$$I_{ij} = \sum_n m_n (\delta_{ij} \sum_k x_{n,k}\text{^}2 - x_{n,i} x_{n,j}) \quad (1)$$
$$L = \sum_n m_n r_n \times v_n \quad (2)$$
$$\sum_i L_i = \sum_i I_i w_i \quad (3)$$
$$\dot{L} = N \quad (4)$$

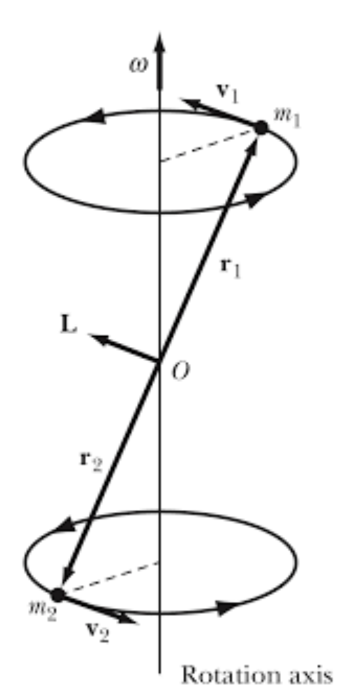
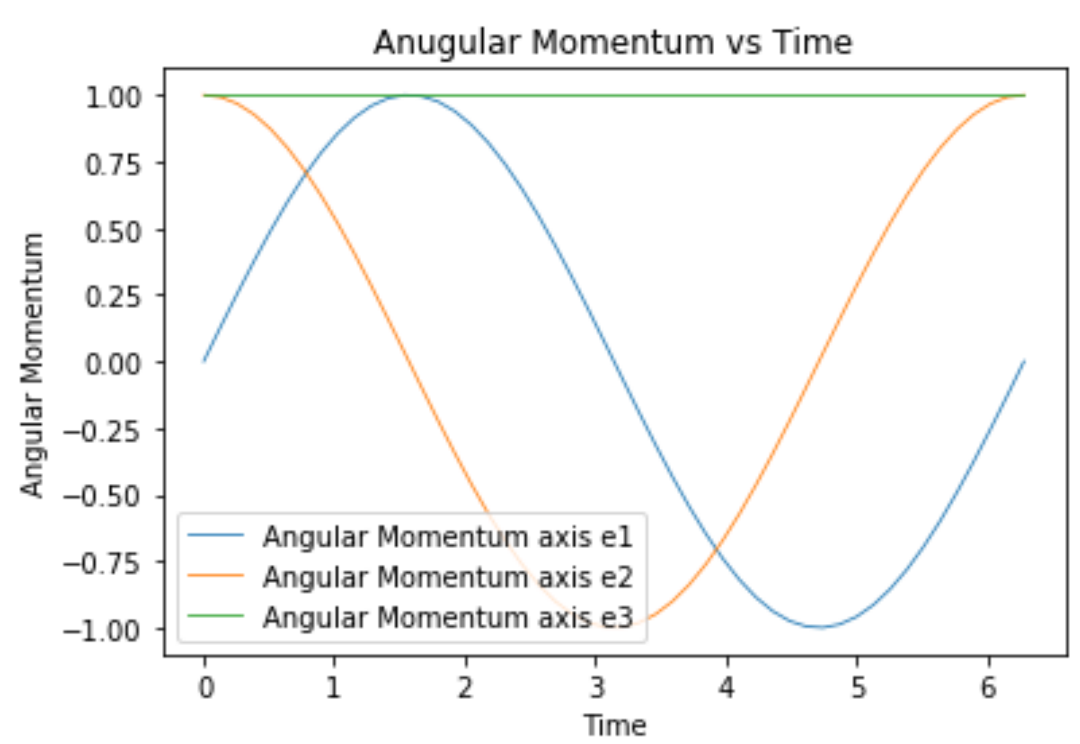
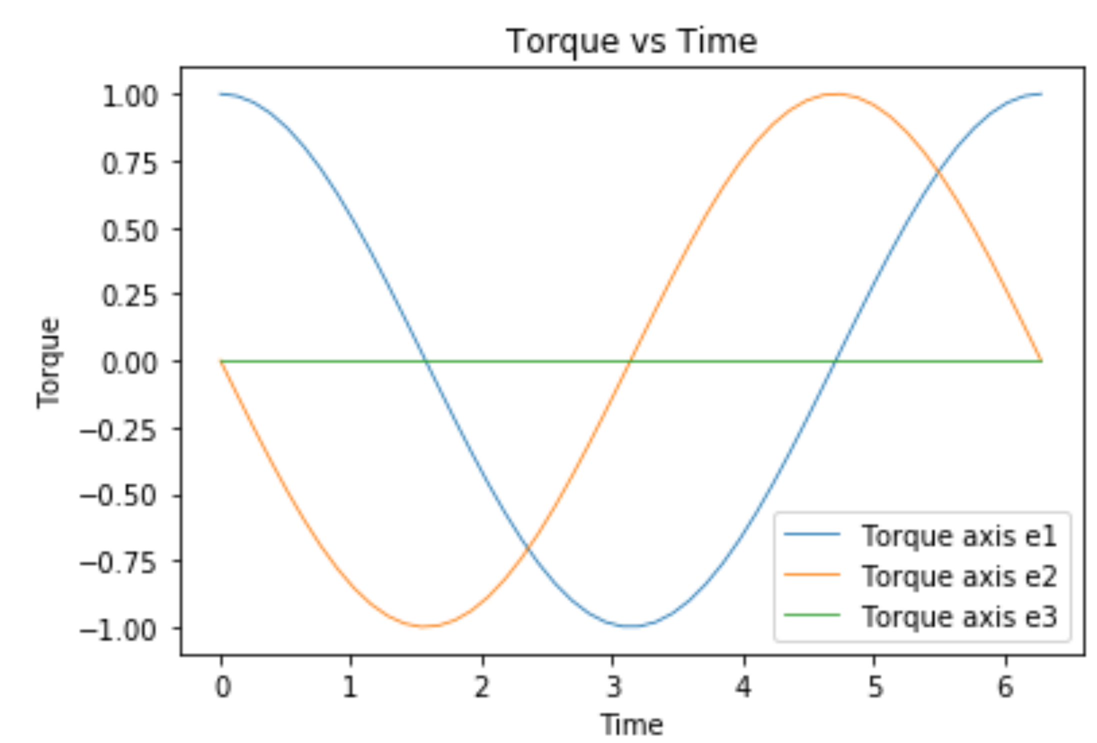


Fig 5: A dumbbell connected by masses m_1 and m_2 at the ends of its shaft.

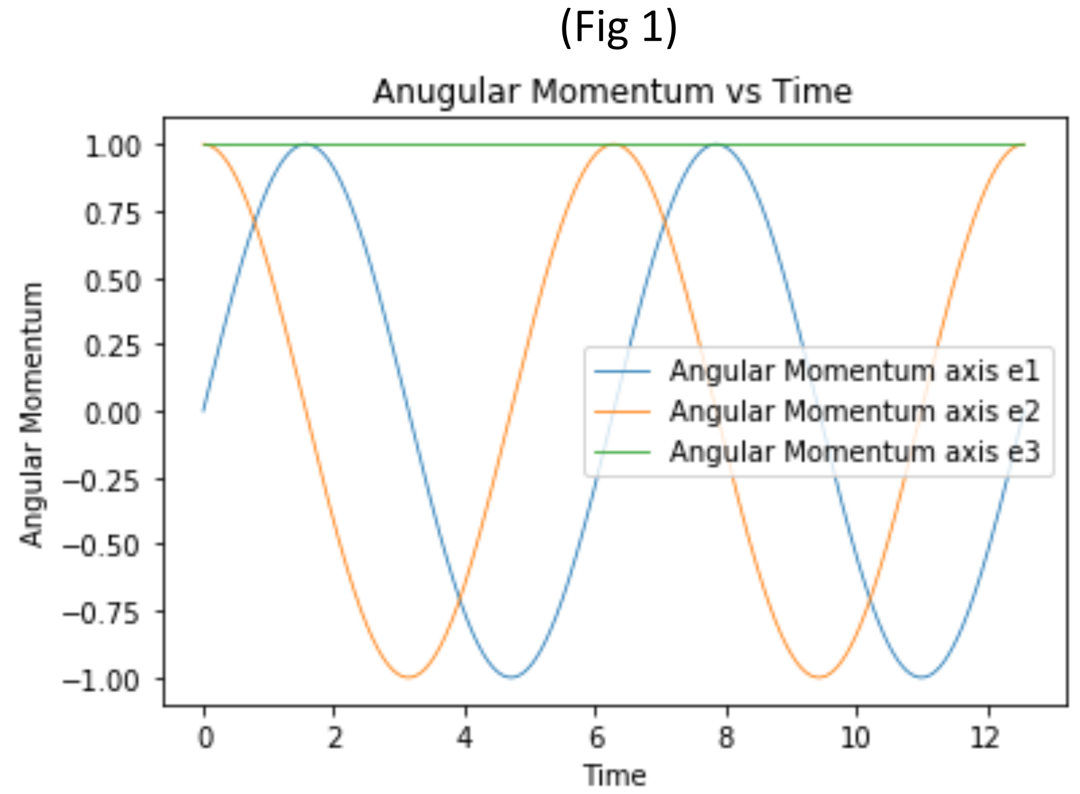
RESULTS



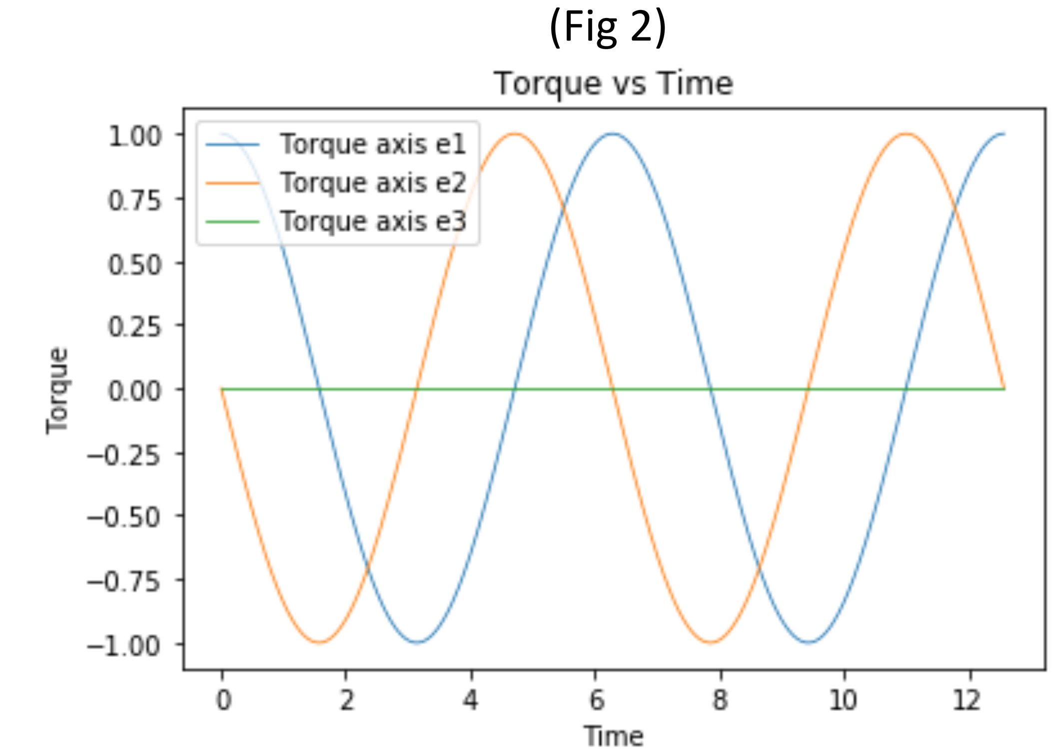
(Fig 1)



(Fig 2)



(Fig 3)



(Fig 4)

Fig 1: The shaft's angular momentum demonstrates periodic motion. As the shaft rotates around the origin to keep the angular velocity constant, the angular momentum in the e1 and e2 axis must continuously change while the e3-axis must stay constant at 1.

Fig2: The shaft's torque demonstrates periodic motion. The torque in the e1 and e2 axis must continuously change and the e3-axis must be 0. Analytically, $\dot{L}_1 = N_1$, so since L_1 is a sine function, N_1 will be a cosine function. L_2 is a cosine function so N_2 will be a sine function. Finally, since L_3 is a constant N_3 is 0.

DISCUSSION:

We plot the results of external torque needed to keep the shaft rotating about the origin and connected to the masses (m_1 and m_2). This makes it so that the masses are moving at a constant angular velocity which we denote as w = 1. This gives us a periodic motion for angular momentum and torque for e1-axis and e2-axis with a wavelength of 2pi for both.

This makes sense analytically because angular momentum for e1-axis is a sine function and e2-axis is a cosine function. Also, for torque the e1-axis is a cosine function and e2-axis is a sine function.

Physically, we see when angular momentum and angular velocity are not aligned. More specifically angular momentum is directed perpendicular to the shaft and at the plane while angular velocity is directed along the axis of rotation. Consequently, this requires an external torque to keep the dumbbell rotating at a constant angular velocity.

CONCLUSIONS

We numerically solved for angular momentum and torque using python generated matrices which would span through multiple different times to see how L and N change with time. We discovered that there needs to be a constantly changing external torque to keep the dumbbell rotating at a constant velocity centered about the origin.

REFERENCES ST Thornton and JB Marion, Classical Dynamics of Particles and Systems, (5th ed) 2008