

Test for divergence

Does $\lim_{n \rightarrow \infty} a_n = 0$?

No → $\sum a_n$ Diverges

Yes

p-series

Does $a_n = n^{-p}$, $n \geq 1$?

Yes → Is $p > 1$?

Yes → $\sum a_n$ Converges

No → $\sum a_n$ Diverges

No

Geometric series

Does $a_n = ar^{n-1}$, $n \geq 1$?

Yes → Is $|r| < 1$?

Yes → $\sum_{n=1}^{\infty} a_n = \frac{a}{1-r}$

No → $\sum a_n$ Diverges

No

Alternating series

Does $a_n = (-1)^n b_n$ or $a_n = (-1)^{n-1} b_n$, $b_n \geq 0$?

Yes → Is $b_{n+1} \leq b_n$ & $\lim_{n \rightarrow \infty} b_n = 0$?

Yes → $\sum a_n$ Converges

No

No

Telescoping series

Do pairs of terms cancel?
(May have to use partial fractions or logs to see!)

Yes → Does $\lim_{n \rightarrow \infty} s_n = s$, with finite s ?

Yes → $\sum_{n=1}^{\infty} a_n = s$

No → $\sum a_n$ Diverges

No

Taylor series

Does $a_n = \frac{f^{(n)}(a)}{n!} (x-a)^n$?

Yes → Is x in interval of convergence?

Yes → $\sum_{n=1}^{\infty} a_n = f(x)$

No → $\sum a_n$ Diverges

No

Try these:

Limit comparison test

Pick $\{b_n\}$. Does $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = c > 0$ with c finite & $a_n, b_n > 0$?

Yes → Does $\sum_{n=1}^{\infty} b_n$ converge?

Yes → $\sum a_n$ Converges

No → $\sum a_n$ Diverges

Integral test

Does $a_n = f(n)$, $f(n)$ is positive, continuous & decreasing on $[a, \infty)$?

Yes → Does $\int_a^{\infty} f(x)$ converge?

Yes → $\sum a_n$ Converges

No → $\sum a_n$ Diverges

Ratio test

Is $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| \neq 1$?

Yes → Is $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| < 1$?

Yes → $\sum a_n$ Abs Conv.

No → $\sum a_n$ Diverges

Root test

Is $\lim_{n \rightarrow \infty} \sqrt[n]{|a_n|} \neq 1$?

Yes → Is $\lim_{n \rightarrow \infty} \sqrt[n]{|a_n|} < 1$?

Yes → $\sum a_n$ Abs Conv.

No → $\sum a_n$ Diverges

Comparison test

Pick $\{b_n\}$. Does $\sum b_n$ converge?

Yes → Is $0 \leq a_n \leq b_n$?

Yes → $\sum a_n$ Converges

No (try again)

No → Is $0 \leq b_n \leq a_n$?

Yes → $\sum a_n$ Diverges