*Hitting Set* is in NP: Given an instance of the problem, and a proposed set $H$, we can check in polynomial time whether $H$ has size at most $k$, and whether some member of each set $S_i$ belongs to $H$.

*Hitting Set* looks like a covering problem, since we are trying to choose at most $k$ objects subject to some constraints. We show that *Vertex Cover* $\leq_P$ *Hitting Set*. Thus, we begin with an instance of *Vertex Cover*, specified by a graph $G = (V, E)$ and a number $k$. We must construct an equivalent instance of *Hitting Set*. In *Vertex Cover*, we are trying to choose at most $k$ nodes to form a vertex cover. In *Hitting Set*, we are trying to choose at most $k$ elements to form a hitting set. This suggests that we define the set $A$ in the *Hitting Set* instance to be the $V$ of nodes in the *Vertex Cover* instance. For each edge $e_i = (u_i, v_i) \in E$, we define a set $S_i = \{u_i, v_i\}$ in the *Hitting Set* instance.

Now we claim that there is a hitting set of size at most $k$ for this instance, if and only if the original graph had a vertex cover of size at most $k$. For if we consider a hitting set $H$ of size at most $k$ as a subset of the nodes of $G$, we see that every set is "hit," and hence every edge has at least one end in $H$: $H$ is a vertex cover of $G$. Conversely, if we consider a vertex cover $C$ of $G$, and consider $C$ as a subset of $A$, we see that each of the sets $S_i$ is "hit" by $C$.

[1] ex635.897.959