

$$\phi_t = \frac{\langle w_t, u \rangle}{\|w_t\| \|u\|} \quad (1)$$

$$\langle w_{t+1}, u \rangle = \langle w_t + y_t x_t, u \rangle = \langle w_t, u \rangle + y_t \underbrace{\langle x_t, u \rangle}_{\geq \delta}$$

$$\leadsto \langle w_{T+1}, u \rangle \geq T\delta \quad (*)$$

$$\begin{aligned} \|w_{T+1}\|^2 &= \langle w_t + y_t x_t, w_t + y_t x_t \rangle \\ &= \|w_t\|^2 + \|x_t\|^2 + 2y_t \underbrace{\langle x_t, w_t \rangle}_{\leq 0} \rightarrow \|w_{T+1}\|^2 \leq T \end{aligned}$$

$$1 \geq \phi_{T+1} = \frac{\langle w_{T+1}, u \rangle}{\|w_{T+1}\|} \geq \frac{T\delta}{\sqrt{T}} \leadsto T \leq 1/\delta^2$$

$$\begin{aligned} D_{t+1}(i) &= \frac{D_t(i)}{Z_t} e^{-\alpha_t y_i h_t(x_i)} \\ D_{T+1}(i) &= D_1(i) \frac{e^{-\alpha_1 y_1 h_1(x_1)}}{Z_1} \frac{e^{-\alpha_2 y_2 h_2(x_2)}}{Z_2} \dots \frac{e^{-\alpha_T y_T h_T(x_T)}}{Z_T} \\ &= \frac{1}{m} \frac{e^{-y_i g_T(x_i)}}{\prod_t Z_t} \end{aligned} \quad (2)$$

$$\begin{aligned} \hat{err}(g) &= \frac{1}{m} \sum_i \mathbb{1}\{g(x_i) \neq y_i\} \\ &= \frac{1}{m} \sum_i \mathbb{1}\{y_i g_T(x_i) \leq 0\} \\ &\leq \frac{1}{m} \sum_i e^{-y_i g_T(x_i)} \\ &\leq \frac{1}{m} \sum_i D_{T+1}(i) m \prod_t Z_t \end{aligned}$$

$$= \prod_t z_t \sum_i \mathcal{D}_{T+1}(i)$$

$$= \prod_t z_b$$