Simplified Approximations to Centrifugal Particle Mass Analyser Performance

K StJ Reavell, J P R Symonds and M G Rushton

Cambustion Ltd, J6 The Paddocks, 347 Cherry Hinton Road, Cambridge CB1 8DH, UK

Introduction

The Centrifugal Particle Mass Analyser, CPMA, [2] is an instrument for classifying aerosol particles on the basis of their mass:charge ratio. This paper proposes an approximation to the parametrisation of the operation of the CPMA to relate the geometry and operating conditions with the resulting selected mass:charge ratio and resolution of the particle transfer function.

The Centrifugal Particle Mass Analyser

The CPMA can be considered as a generalisation of the APM (Aerosol Particle Mass analyser, [1]). In this instrument the aerosol flows along an essentially annular channel which is rotated around its axis while a voltage is applied between the inner and outer walls. The suspended particles in this aerosol are therefore exposed to centrifugal and electrical forces and for only one mass:charge ratio are these forces in equilibrium: these particles are carried axially along the classifier by the flow while others are precipitated on the walls.

The electrical force on a particle at a distance r from the axis of rotation is:

$$F_E = q\frac{V}{r\ln\left(r_2/r_1\right)} \quad (1)$$

where r_1 and r_2 are respectively the inner and outer radii of the channel, V is the applied voltage and q is the charge on the particle.

The centrifugal force on the particle is

$$F_C = m\omega^2 r \quad (2)$$

Equating these two formulae, we can see that for the APM the equilibrium charge : mass ratio varies with the radius:

$$\left(\frac{m}{q}\right)_{eqm} = \frac{V}{\omega^2 r^2 \ln\left(r_2/r_1\right)} \quad (3)$$

The consequence of this variation is that if a particle in the channel with these forces in equilibrium is radially perturbed, it will no longer be in equilibrium. If perturbed outwards, it will be in a region where it has a higher mass : charge ratio than the local equilibrium and thus will experience greater centrifugal than electrical forces. Therefore the particle will continue to drift outwards and therefore we refer to the system as unstable. The passage of particles along the channel is therefore as modelled below for possible conditions of an instrument, assuming one elementary charge per particle and a simplifying assumption of axial plug flow.

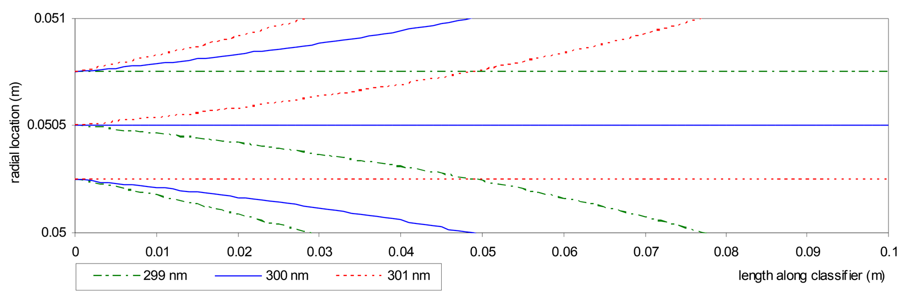

Figure 1. Particle Trajectories in an APM

The field system can be made stable if the centrifugal force can be arranged to reduce with radial location, to match the variation of electrical force. This is achieved in the CPMA by rotating the inner wall of the channel at a higher angular velocity than the outer, with the intention of imposing a desired velocity profile on the fluid as shown below.

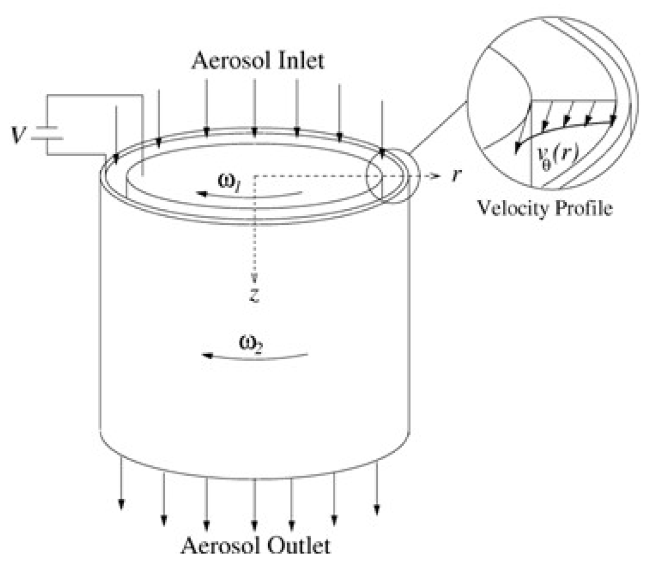

Figure 2. CPMA Classifier (from ref [2])

Parametrisation of Operating Conditions

The precise velocity profile, the variation of tangential speed of the flow with radial location, is the solution of the Navier-Stokes equation described in [2]

$$v_\theta = \omega_1 \frac{\hat{r}^2 - \hat{\omega}}{\hat{r}^2 - 1} r + \omega_1 r_1^2 \frac{\hat{\omega} - 1}{\hat{r}^2 - 1}\frac{1}{r} \quad (4)$$

where v_θ is the tangential velocity, ω_1 and ω_2 are the velocities of the inner and outer walls as shown above, and $\hat{\omega} = \omega_2/\omega_1$ and $\hat{r} = r_1/r_2$.

As an alternative to the analysis in [2], it is here proposed to approximate this velocity profile, given a narrow gap, with a power law, and it is more convenient to deal in terms of angular rather than tangential velocity:

$$\omega_r \propto r^{n_\omega} \quad (5)$$

where

$$\frac{\omega_1}{\omega_2} = \left(\frac{r_1}{r_2}\right)^{n_\omega} \quad (6)$$

The variation of equilibrium mass:charge ratio across the channel can then be written:

$$\left(\frac{m}{q}\right)_{eqm} \propto r^{n_m} \quad (7)$$

where $n_m = -2n_\omega - 2$

The relationship for the APM in (3) can then easily be seen as the case of this with $n_m = -2$. The effects of the variation in equilibrium mass : charge ratio across the channel are firstly to widen the range of particle mass to charge ratios (ie. sizes) which pass through the classifier, reducing its resolution, and secondly to increase particle losses due to the unstable nature of the variation, as shown by [3]. This behaviour will be exhibited in varying amount for all conditions where $n_m < 0$ where the field is unstable. These effects can be mitigated by optimisation of the instrument design, such as ensuring a very narrow gap between the inner and outer walls of the channel, but this may compromise other aspects of the performance such as diffusive particle losses.

The CPMA can be operated with $n_m = 0$, achieved by setting the speed ratio $n_\omega = -1$, the inner cylinder rotating faster than the outer. The field variation in the channel is then neutrally stable: a particle of the equilibrium mass : charge ratio perturbed from its initial radial location will remain at the same location. In this case the equilibrium charge : mass ratio is constant across the gap and this gives the narrowest transfer function. The particle trajectories as before can be visualised:

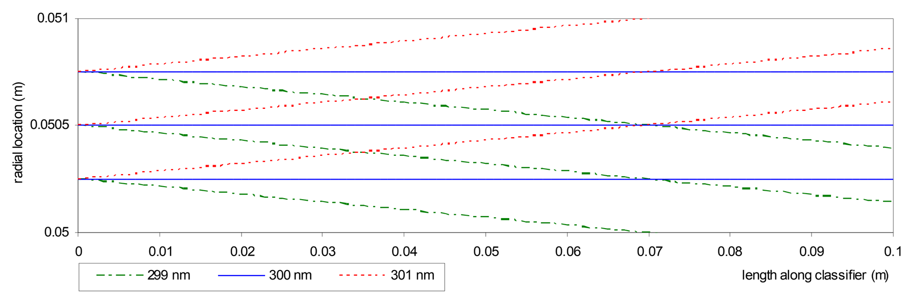

Figure 3. Particle Trajectories in a Neutral CPMA.

The final condition is when $n_m > 0$. In this case the field is stable: a particle perturbed from its equilibrium radius will tend to return towards it. The trajectories in the channel in this case are as shown below:

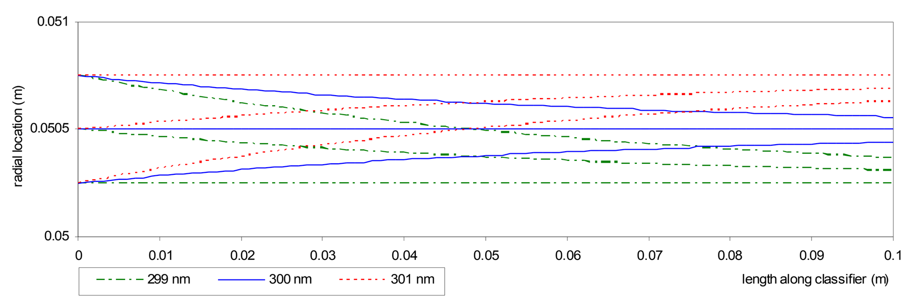

Figure 4. Particle Trajectories in a Stable CPMA.

The variation of equilibrium mass : charge ratio across the gap is seen here: this will lead again to a broadening of the transfer function and theoretically poorer resolution than the neutrally stable case. However, the tendency to concentrate the selected particles in the centre of the channel may reduce diffusive losses.

This positively stable situation is achieved by $n_\omega < -1$. In practice n_ω cannot be lower than -2 as this is also the Rayleigh criterion for stability against Taylor vortices forming in the channel which would destroy any classification of particles.

Non-Diffusive Resolution of the CPMA

Drift Limited Broadening

In most situations the main determinant of the resolution of the CPMA is particle drift.

For the neutrally stable case, the theoretical non-diffusive transfer function approximates a triangle, peaking at unity at the equilibrium mass : charge ratio and dropping to zero at the mass : charge ratio which only just passes the channel:

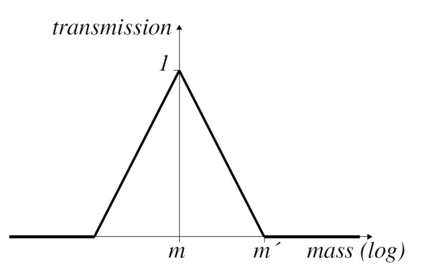

Figure 6. Theoretical Drift Limited Transfer Function

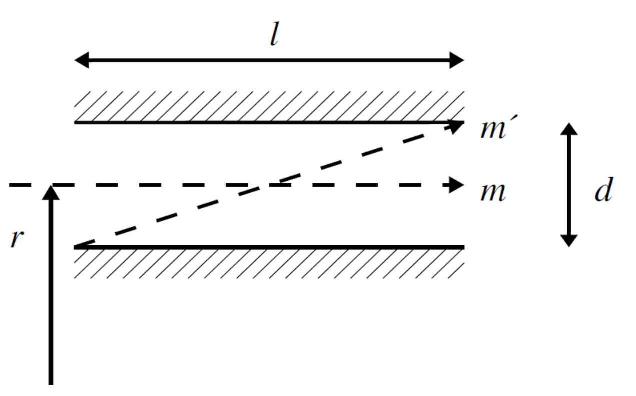

Figure 5. Limiting Particle Mass

For the calculation of this limiting mass, the approximation is made that the channel is narrow enough that variation of the forces in the gap can be ignored. The axial velocity profile is here assumed to be plug flow and the all particles are assumed to have the same charge.

Considering drift across the channel gives us:

$$F = \frac{Q\,d}{B'\,l\,A} \quad (8)$$

where d is the classifier gap, F is the net force on the particle ($F_C - F_E$), B is the mobility of the particle, l is the length of the channel, A is the flow cross sectional area, which can be approximated as $2\pi r d$, and Q is the volumetric flow rate.

Substituting in the formulae for the forces,

$$m'\omega^2 r - qE = \frac{Q}{2\pi B' l r} \quad (9)$$

referring this to the equilibrium particle:

$$\left(\frac{m'}{m}\right)^{(n_B+1)} - \left(\frac{m'}{m}\right)^{n_B} = \frac{Q}{m B 2\pi r^2 l \omega^2} \quad (10)$$

where n_B is the index of variation of mobility with particle mass: this includes the effects of slip correction and fractal dimensions of some particle types.

The right hand side of this relationship is a non-dimensional group which governs the transfer function broadening due to drift limiting. Comparing this with (3) it is clear that the presence of both ω^2 and m in the denominator imply that both variation in V and ω is required to select a different mass of particle while maintaining the same transfer function width.

$\left(\frac{m'}{m}\right)$ is thus the half width of the triangular transfer function. We can define a resolution parameter R_m analogously to that used widely for the DMA [5] except based on mass rather than mobility:

$$\frac{1}{R_m} = \left(\frac{m'}{m}\right) - 1 \quad (11)$$

Broadening due to Positive Stability

As discussed above, variation in the equilibrium mass : charge ratio across the channel in the positively stable situation can contribute to broadening of the transfer function. Due to the concentration of different mass : charge ratios at different radii across the channel, this results in a range of particles being transmitted with the theoretical 100% efficiency. This can be considered a rectangular transfer function which is convolved with the triangular drift limited function above to approximate the whole transfer function.

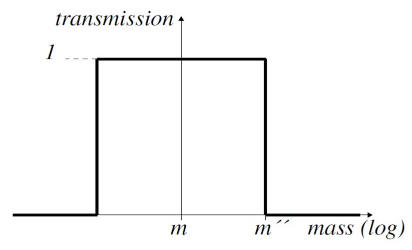

Figure 7. Stability Induced Transfer Function

The width of this transfer function is calculated by comparing the equilibrium mass at the centre and edge of the channel.

$$\frac{m''/q''}{m/q} = \frac{E''/\omega''^2 r}{E/\omega^2 r} \quad (12)$$

Assuming constant charge, and substituting $E \propto 1/r$ and (5):

$$\left(\frac{m''}{m}\right) = \left(\frac{r''}{r}\right)^{(-2n_w - 2)} \quad (13)$$

For the neutrally stable case, $n_\omega = -1$, the index here is equal to zero, and so there is no broadening by this mechanism. Note that this relationship does not hold for the unstable case.

The overall transfer function can be obtained by convolution of these elements but in some circumstances it may be useful to use the geometric standard deviation of these functions. By integration of the moments of the distributions we can obtain:

$$\sigma_{g,drift} = \left(\frac{m'}{m}\right)^{\sqrt{1/6}} \quad (14)$$

$$\sigma_{g,stable} = \left(\frac{m''}{m}\right)^{\sqrt{1/3}} \quad (15)$$

Experimental Validation

Direct measurement of the transfer function of the CPMA is very difficult because it is difficult to obtain a narrow enough size distribution to resolve the CPMA transfer function. In these experiments, a PSL aerosol was passed through a DMA and then through the CPMA (described in [4]) which is step scanned across a range of masses. The relationship in (10) is used to calculate the combination of electric field and rotational speed required to achieve the desired equilibrium mass and resolution parameter. The aerosol concentration was measured upstream and downstream of the CPMA with CPCs to produce the measured input/output ratio plotted. This is repeated for scans up and down the mass range.

This input/output ratio was predicted by multiplying the assumed PSL size spectrum (from the sample specified GSD), the normal triangular DMA transfer function and the CPMA transfer function from the relations above across the size range of interest. This produces the red lines on the graphs. Due to the other functions in the chain, the overall transfer function does not reach 100% as for the theoretical CPMA function itself.

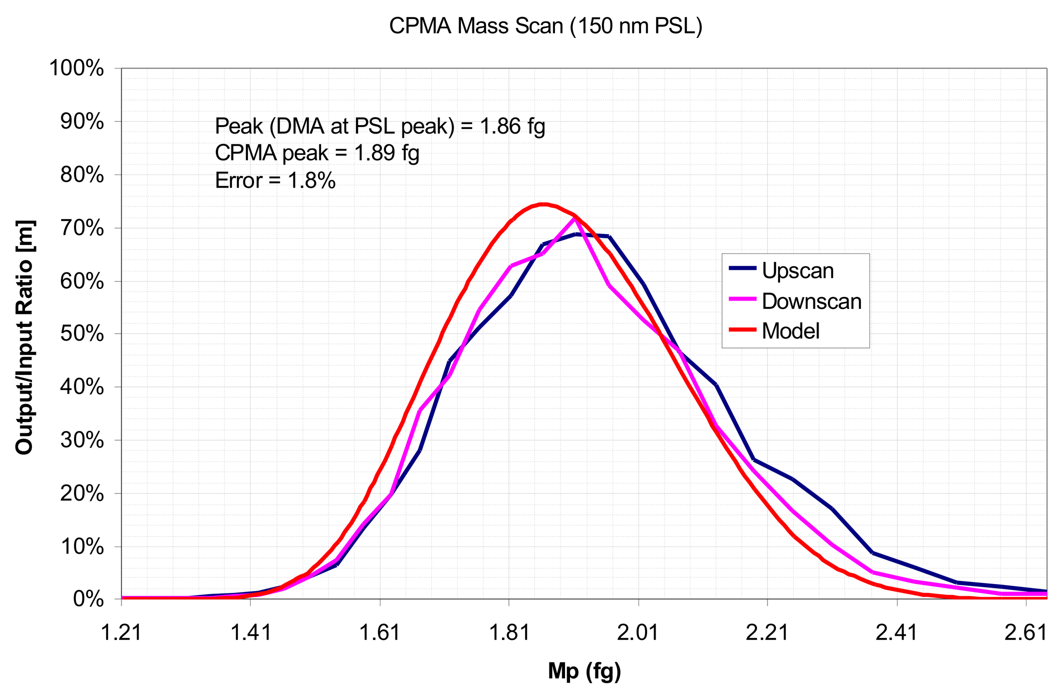

Figure 8. Comparison of Model with Measurements: 150nm PSL, 1/R_m=0.195

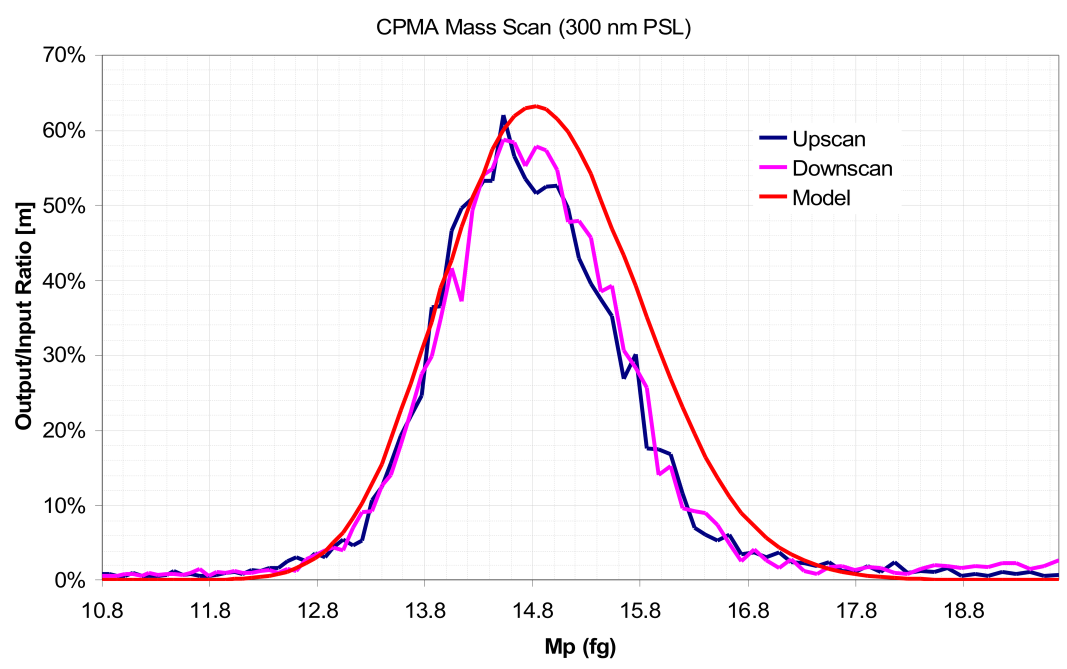

Figure 9. Comparison of Model with measurements: 300 PSL, 1/R_m =0.1

The modelled function is in good agreement with the measured results.

Conclusions

A simplified model for the transfer function of the CPMA has been described. This model can be used to easily establish the operating conditions required to achieve a desired operation in terms of selected mass and resolution.

The model has been validated against measurements with good agreement.

References

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