Leader-Follower Formation Control for Non-Holonomic WMR

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Introduction

- Leader-Follower

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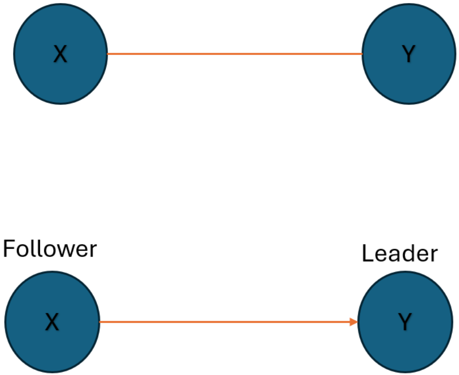

Formation Control

- Formation Control

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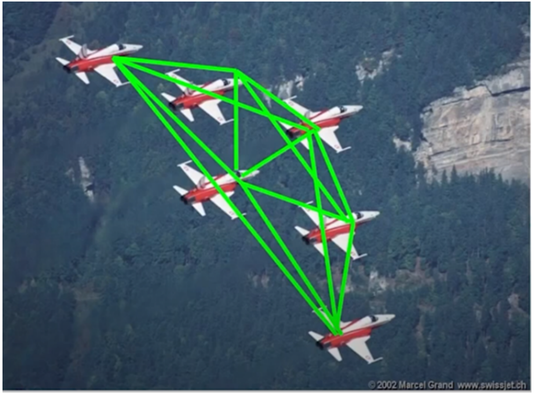

Differential Drive WMR

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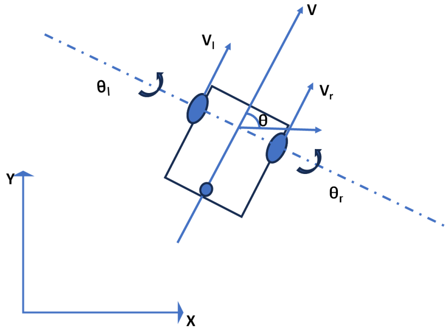
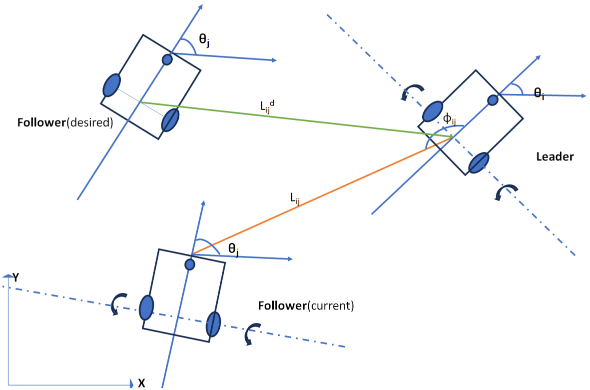


Leader-Follower Relation



Relation

Distance between Leader and Follower

$$L_{ijx} = x_i - x_j - d\cos\theta_i$$

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$$L_{ijy} = y_i - y_j - d\sin\theta_i$$

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Distance between Leader and Follower

$$L_{ijx} = x_i - x_j - d\cos\theta_i$$

$$L_{ijy} = y_i - y_j - d\sin\theta_i$$

$$L_{ij} = \sqrt{L_{ijx}^2 + L_{ijy}^2}$$

Relation

Relative Bearing

$$\phi_{ij} = \mathrm{atan2}\left(\frac{L_{ijy}}{L_{ijx}}\right) - \theta_i + \pi$$

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Relative Bearing

$$\phi_{ij} = \mathrm{atan2}\left(\frac{L_{ijy}}{L_{ijx}}\right) - \theta_i + \pi$$

$$\gamma_{ij} = \theta_i + \phi_{ij} - \theta_j$$

Control Law

Assumptions:
Distance between rear and front wheel is negligible.

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Both Leader and Follower have same orientation.

$$\theta_i = \theta_j$$

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Distance between rear and front wheel is negligible.
Both Leader and Follower have same orientation.

$$\theta_i = \theta_j$$

Relative Bearing is also assumed to be zero.

$$\phi_{ij} = 0$$

Control Law

Using Lyapunov Approach

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$$V = \frac{1}{2}(L_{dx} - L_{ijx})^2 + \frac{1}{2}(L_{dy} - L_{ijy})^2$$

Control Law

Using Lyapunov Approach

$$V = \frac{1}{2}(L_{dx} - L_{ijx})^2 + \frac{1}{2}(L_{dy} - L_{ijy})^2$$

$$\dot{L}_{ijx} = V_i \cos\theta_i - V_j \cos\theta_j$$

$$\dot{L}_{ijy} = V_i \sin\theta_i - V_j \sin\theta_j$$

Control Law

$$\dot{V} = (L_{dx} - x_i + x_j)(u_x - V_i \cos\theta_i) + (L_{dy} - y_i + y_j)(u_y - V_i \sin\theta_i)$$

Control Law

$$\dot{V} = (L_{dx} - x_i + x_j)(u_x - V_i \cos\theta_i) + (L_{dy} - y_i + y_j)(u_y - V_i \sin\theta_i)$$

$$u_x = -K_x(L_{dx} - x_i + x_j) + V_i \cos\theta_i$$
$$u_y = -K_y(L_{dy} - y_i + y_j) + V_i \sin\theta_i$$

Line-Formation Simulation

Parameters Taken

Agent	**Coordinates**	**Desired Distance**
Leader	$(-13, -10)$	
Follower 1	$(6, 6)$	$(6, 0)$
Follower 2	$(-9, 6)$	$(8, 0)$
Follower 3	$(12, 12)$	$(6, 0)$
Follower 4	$(-18, -12)$	$(8, 0)$

Rectangle-Formation Simulation

Parameters Taken

Agent	**Coordinates**	**Desired Distance**
Leader	$(120, 0)$	
Follower 1	$(6, 6)$	$(4, 5)$
Follower 2	$(-12, -16)$	$(4, -5)$
Follower 3	$(30, 30)$	$(7, 0)$
Follower 4	$(-18, -12)$	$(7, 0)$
Follower 5	$(-20, 10)$	$(4, 0)$
Follower 6	$(24, 30)$	$(11, 0)$

Complete Control Law

Considering error in current and desired orientation and position error

$$V = \frac{1}{2}(L_{dx} - L_{ijx})^2 + \frac{1}{2}(L_{dy} - L_{ijy})^2 + \frac{1}{2}(\theta_d - \theta_c)^2$$

$$\theta_c = \theta_i - \theta_j$$

Complete Control Law

Considering error in current and desired orientation and position error

$$V = \frac{1}{2}(L_{dx} - L_{ijx})^2 + \frac{1}{2}(L_{dy} - L_{ijy})^2 + \frac{1}{2}(\theta_d - \theta_c)^2$$

$$\theta_c = \theta_i - \theta_j$$

Differentiating with time

$$\begin{aligned}\dot{V} = {} & (L_{dx} - L_{ijx})(\dot{L}_{dx} - \dot{L}_{ijx}) \\ & + (L_{dy} - L_{ijy})(\dot{L}_{dy} - \dot{L}_{ijy}) \\ & + (\theta_d - \theta_i + \theta_j)(\dot{\theta}_d - \dot{\theta}_i + \dot{\theta}_j)\end{aligned}$$

Complete Control Law

As we already know

$$L_{ijx} = x_i - x_j - dcos\theta_i$$

$$L_{ijy} = y_i - y_j - dsin\theta_i$$

$$\dot{L}_{ijx} = V_i cos\theta_i - V_j cos\theta_j + d\omega_i sin\theta_i$$

$$\dot{L}_{ijy} = V_i sin\theta_i - V_j sin\theta_j - d\omega_i cos\theta_i$$

Controllers

Controllers for follower velocity

$$u_x = V_j cos\theta_j$$

$$u_y = V_j sin\theta_j$$

$$\omega_j = \dot{\theta}_j$$

Differentation of Lyapunov Function

So $\dot{V}$ comes out to be

$$\begin{aligned}\dot{V} = &(L_{dx} - L_{ijx})(u_x - V_i \cos\theta_i) \\ &+ (L_{dy} - L_{ijy})(u_y - V_i \sin\theta_i) \\ &+ (\theta_d - \theta_i + \theta_j)(-\omega_i + \omega_j)\end{aligned}$$

Complete Control Law

Choosing

$$u_x = -K(L_{dx} - x_i + x_j) + V_i cos\theta_i$$

$$u_y = -K(L_{dy} - y_i + y_j) + V_i sin\theta_i$$

$$\omega_j = (\omega_i) + (-\theta_d + \theta_i - \theta_j)$$

Stability

$$\dot{V} = -K(L_{dx} - x_i + x_j)^2 - K(L_{dy} - y_i + y_j)^2 - (\theta_d - \theta_i + \theta_j)^2$$

Stability

$$\dot{V} = -K(L_{dx} - x_i + x_j)^2 - K(L_{dy} - y_i + y_j)^2 - (\theta_d - \theta_i + \theta_j)^2$$

which is negative semi-definite.

Stability

$$\dot{V} = -K(L_{dx} - x_i + x_j)^2 - K(L_{dy} - y_i + y_j)^2 - (\theta_d - \theta_i + \theta_j)^2$$

which is negative semi-definite.
According to Lasalle's Principle

$$L_{dx} = x_i - x_j$$

$$L_{dy} = y_i - y_j$$

$$\theta_d = \theta_i - \theta_j$$

Stability

Therefore, according to LaSalle's invariance principle, trajectories of the system will converge to the equilibrium point as time progresses.

Simulation

Parameters Taken

Agent	Coordinates	Orientation	Desired Distance	Desired Orientation
Leader	$(-13, -10)$	$\pi/4$		
Follower 1	$(6, 6)$	$\pi/2$	$(6, 0)$	$\pi/12$
Follower 2	$(-9, 6)$	π	$(8, 0)$	$\pi/12$
Follower 3	$(12, 12)$	$\pi/3$	$(6, 0)$	$\pi/12$

Issues

- Sharp Trajectory

Issues

- Sharp Trajectory
- Collision Avoidance

Work Distribution

- Pranav Deshpande
  - Velocity Control
  - Simulation
  - Presentation
- Arashdeep Singh
  - Relation between Leader-Follower
  - Orientation Control
  - Simulation

References

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