Leader-Follower Formation Control of Nonholonomic Mobile Robots

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***Abstract*—This report focuses on the Multi-Agent Control using Leader-Follower Control Approach. We are trying to implement this approach for wheeled mobile robots(non-holonomic). But for that first need to understand kinematics of wheeled mobile robots. Then we will try to implement control strategies. We are focusing on Lyapunov based control Strategy.**

I. INTRODUCTION

Let's begin by delving into the realm of Multi-agent Control and exploring the reasons for its significance. In a multi-agent system, there are numerous agents, each capable of making decisions autonomously, all striving towards a common goal. Employing multiple agents allows us to tackle intricate problems, enhancing robustness and efficiency. Consider a courier warehouse scenario where multiple parcels need to be picked and placed at various locations. Utilizing a network of interconnected agents can create a fast and accurate system to address such complexities. Now, the pivotal question arises: how do we control these agents?

There are primarily two approaches to multi-agent control when we want that agents to coordinate :

Flocking Control draws inspiration from the collective behavior observed in nature among groups of entities such as birds, fish, or insects. Three key principles govern this approach:

- Alignment: Agents align their movement with the average direction of their neighbors, maintaining a consistent heading.
- Cohesion: Agents are attracted to stay close to their neighbors, fostering a cohesive group formation to prevent dispersion.
- Separation: Agents avoid collisions by maintaining a certain distance from their neighbors, preventing overcrowding.

However, Flocking Control has its disadvantages.

Formation Control, on the other hand, involves the coordinated control of a fleet of robots following a predefined trajectory while maintaining a specific spatial pattern. It addresses problems where agents need to stabilize at a given distance from one another. Applications range from vehicle control to UAVs, consensus and formation control of robots, industrial robots, and more.

Let us redefine Formation : A number of individual physical agents often interact locally with the global behavior desired. Agents display spatial patterns, implying inter-agent interaction. Formation should be autonomous or controlled through a single leader vehicle. We do not want 10 vehicles having 10 different controllers and struggling to maintain a formation. This means every agent acts locally, but the effect is global. To acheive this localization there are basically two ideas:-

- Acquiring a certain shape
- Maintaining a certain shape

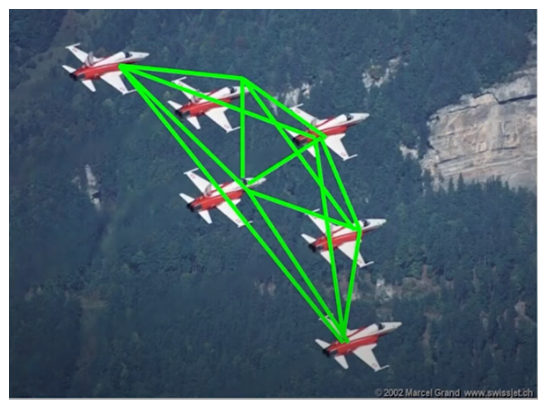

Fig. 1. Formation as rigid structure

Certaintly these formations act like rigid frames. As we donot want that distance between pairs does not change. In rigid formation,some distances need to be explicitly maintained with the rest being consequentially maintained.
Now question is do all agents need to maintain distance or only few need to maintain distance. Like If the distance between agents X and Y is maintained,this may be:

- A task jointly shared by X and Y or
- Something that X does and Y is unconscious about or conversely)meaning one agent agent will be responsible to maintain the distance and other will just follow its path).

Fig. 2. Both responsible to maintain formation

Fig. 3. Follower to maintain formation

Certainly second one is better approach as it is easier to control and less costly. Let us now understand these kind of few approaches:

Leader-Follower Structure:
In this method, one agent is designated as the leader, responsible for generating the desired trajectory, while the others act as followers, tracking the leader's motion and maintaining a specific formation shape and distance.
Advantages include ease of implementation and adaptability to dynamic environments.
Challenges include sensitivity to the leader's performance, potential collapse if the leader deviates, and scalability issues.

Virtual Structure:
Agents maintain positions relative to their neighbors, forming a virtual structure defined by the leader's position.

Behavior-Based Structure:
Agents follow predefined behaviors, contributing to the overall formation pattern.

Formation Control can be further classified based on control strategies:
Centralized Control: A single entity controls the entire group, making decisions for each agent.
Decentralized Control: Agents make decisions independently but consider information from nearby agents.
Distributed Control: Agents have limited information but communicate and coordinate with neighboring agents.

We will be following Leader Follower approach and will further illustrate this approach on whelled mobile robots.The leader-follower approach can be implemented as both a centralized and decentralized control method. In a centralized control system, the environment is known and static, and the followers adhere to instructions from a fixed leader or specific sources. Conversely, in a decentralized control system, the leader moves forward and is followed by the followers, each of which operates autonomously and may only require local information. This flexibility allows the leader-follower approach to be adapted to various scenarios, including those where robustness and simplicity are needed, making it suitable for decentralized control applications.For that we will be considering both distance and angle between two agents as parameters that needed to be controlled.

II. PROBLEM FORMULATION

Before we try to control robots, we need to know their kinematics,how different comopenents are resulting in motion, and which are under our control. We will use a Differential Drive WMR to implement control strategies.

A. Kinematics of Differential Drive WMR

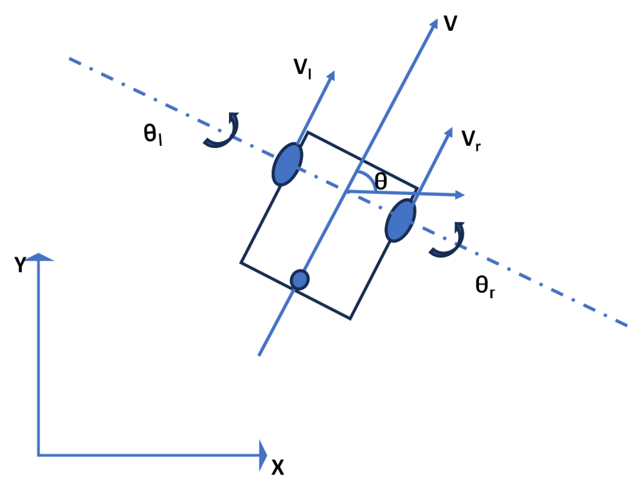

Fig. 4. Diiferrential Drive WMR

A differential drive wheeled mobile robot is a type of non-holonomic system, which means it has constraints on its motion that cannot be integrated into constraints on its position. This is due to the robot's wheels being directly linked to its control inputs, allowing for forward and backward movement and in-place rotation, but not lateral movement.There are two powered wheels and a third wheel for balancing.The difference of speed between these two wheels is responsible to make robot turn.Both wheel speeds are controlled independently.
Assumptions:
1. Point Q coincides with center of gravity of robot
2. There is no slip.
3. Velocity of right wheel is more than the left wheel
Parameters used:
V= Net linear velocity of the robot, V_L= Linear velocity of Left Wheel, V_R= Linear velocity of Right Wheel, $2a$ = Wheel

base, ϕ= Robot orientation, $\dot{\theta}_L$ = angular velocity of left wheel, $\dot{\theta}_R$ = angular velocity of right wheel, r= radius of wheel (both have equal radius), $\dot{\phi}$= Angular velocity of robot

$$V_r = V + a\dot{\phi}$$

$$V_r = V - a\dot{\phi}$$

By the addition of both

$$V = (V_r + V_l)/2$$

By the subtraction of both

$$\dot{\phi} = (V_r - V_l)/2a$$

Assuming no-slip condition

$$V_r = r\dot{\theta}_r$$

$$V_l = r\dot{\theta}_l$$

$$\dot{x} = V cos\phi$$

$$\dot{y} = V sin\phi$$

$$\dot{x} = \frac{(V_r + V_l)cos\phi}{2}$$

$$\dot{x} = \frac{(V_r + V_l)sin\phi}{2}$$

$$\dot{x} = \frac{r(\dot{\theta}_r + \dot{\theta}_l)cos\phi}{2}$$

$$\dot{y} = \frac{r(\dot{\theta}_r + \dot{\theta}_l)sin\phi}{2}$$

$$\dot{\phi} = \frac{r(\dot{\theta}_r - \dot{\theta}_l)}{2a}$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} \frac{r}{2}(cos\phi) & \frac{r}{2}(cos\phi) \\ \frac{r}{2}(sin\phi) & \frac{r}{2}(sin\phi) \\ \frac{r}{2a} & -\frac{r}{2a} \end{bmatrix} \begin{bmatrix} \dot{\theta}_r \\ \dot{\theta}_l \end{bmatrix}$$

Non-holonomic Constraint

$$\dot{y}cos\phi - \dot{x}sin\phi = 0$$

B. Follower Formation Modelling

i represents leader vehicle
j represents follower vehicle
L_{ij} current distance between leader and follower
L_{ij}^d desired distance between leader and follower
θ_i orientation of leader vehicle
$\theta_j$orientation of follower vehicle
ϕ_{ij} represents relative angle between leader and follower
d is distance between rear and front wheel. Here it will work to compensate when different centre is taken for leader and follower.

$$\begin{bmatrix} x_j \\ y_j \\ \phi_j \end{bmatrix} = \begin{bmatrix} x_i - dcos\theta_i + L_{ij}cos(\phi_{ij} + \theta_i) \\ y_i - dcos\theta_i + L_{ij}sin(\phi_{ij} + \theta_i \\ \theta_i + \phi_{ij} \end{bmatrix}$$

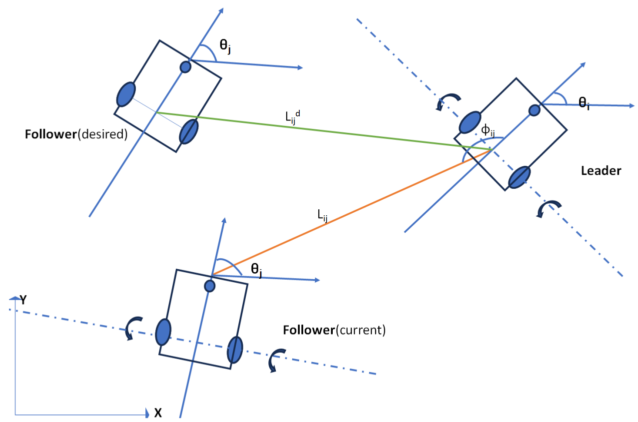

Fig. 5. Leader-Follower Formation

$$L_{ijx} = x_i - x_j - dcos\theta_i$$

$$L_{ijy} = y_i - y_j - dsin\theta_i$$

$$L_{ij} = \sqrt{L_{ijx}^2 + L_{ijy}^2}$$

As we know

$$\dot{x}_i = V_i cos\theta_i$$

$$\dot{y}_i = V_i sin\theta_i$$

$$\dot{\theta}_i = \omega_i$$

Differentiating

$$\dot{L}_{ijx} = \dot{x}_i - \dot{x}_j - dsin\theta_i\dot{\theta}_i$$

$$\dot{L}_{ijx} = V_i cos\theta_i - V_j cos\theta_j + d\omega_i sin\theta_i$$

$$\dot{L}_{ijy} = \dot{y}_i - \dot{y}_j - dcos\theta_i\dot{\theta}_i$$

$$\dot{L}_{ijy} = V_i sin\theta_i - V_j sin\theta_j - d\omega_i cos\theta_i$$

$$L_{ij} = \sqrt{L_{ijx}^2 + L_{ijy}^2}$$

Differentiating

$$\dot{L}_{ij} = \frac{1}{\sqrt{L_{ijx}^2 + L_{ijy}^2}} \left(L_{ijx}\dot{L}_{ijx} + L_{ijy}\dot{L}_{ijy} \right)$$

$$\dot{L}_{ij} = \frac{1}{L_{ij}} \{ L_{ijx} \left(V_i \cos\theta_i - V_j \cos\theta_j + d\omega_i \sin\theta_i \right) + L_{ijx} \left(V_i \cos\theta_i - V_j \cos\theta_j + d\omega_i \sin\theta_i \right) \}$$

$$\dot{L}_{ij} = \frac{1}{L_{ij}} \{ V_i (L_{ijx}cos\theta_i + L_{ijy}sin\theta_i) \} + (-)\frac{1}{L_{ij}} \{ V_j (L_{ijx}cos\theta_j + L_{ijy}sin\theta_j) \} + \frac{1}{L_{ij}} \{ (-L_{ijx}sin\theta_i + L_{ijy}cos\theta_i) - d\omega_i \}$$

$$
\begin{aligned}
\dot{L}_{ij} = &\{V_i(-L_{ij}cos(\theta_i + \phi_i j)cos\theta_i\} + \\
&\{V_i(-L_{ij}sin(\theta_i + \phi_i j)sin\theta_i\} \\
&- \{V_j(-L_{ij}cos(\theta_i + \phi_i j)cos\theta_j\} - \\
&\{V_j(-L_{ij}sin(\theta_i + \phi_i j)sin\theta_j\} \\
&- d\omega_i \{L_{ij}cos(\theta_i + \phi_i j)sin\theta_j \\
&- L_{ij}sin(\theta_i + \phi_i j)cos\theta_j
\end{aligned}
$$

$$
\begin{aligned}
\dot{L}_{ij} = &-v_i \ cos(\theta_i + \phi_{ij})cos\theta_i + sin(\theta_i + \phi_{ij})sin\theta_i) \\
&+v_j \ cos(\theta_i + \phi_{ij})cos\theta_j + sin(\theta_i + \phi_{ij})sin\theta_j) \\
&-d\omega_i \ cos(\theta_i + \phi_{ij})sin\theta_j - sin(\theta_i + \phi_{ij})cos\theta_j)
\end{aligned}
$$

$$\theta_i + \phi_{ij} - \theta_j = \gamma_i j$$

$$\dot{L}_{ij} = -v_i cos\phi_i j + v_j cos\gamma_i j + d\omega_j sin_i j$$

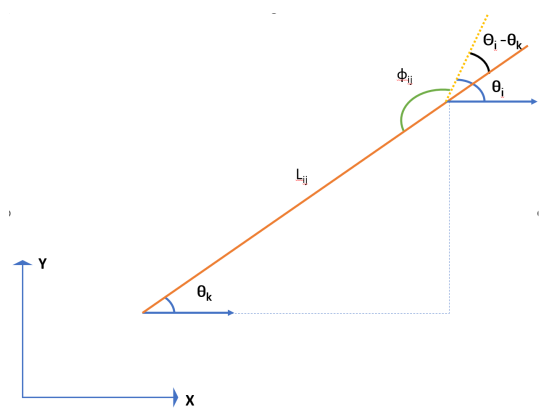

Fig. 6. Calculation of relative bearing

$$\phi_{ij} = atan2\frac{L_{ijy}}{L_{ijx}} - \theta_i + \pi$$

III. Lyapunov Control

A. Position only

Considering error in x and y directions.

Let

$$V = \frac{1}{2}(L_{dx} - L_{ijx})^2 + \frac{1}{2}(L_{dy} - L_{ijy})^2$$

Differentiating with time

$$\dot{V} = (L_{dx} - L_{ijx})(\dot{L}_{dx} - \dot{L}_{ijx}) + (L_{dy} - L_{ijy})(\dot{L}_{dy} - \dot{L}_{ijy})$$

As L_d is not changing with time so $\dot{L}_d = 0$

$$\dot{V} = (L_{dx} - L_{ijx})(-\dot{L}_{ijx}) + (L_{dy} - L_{ijy})(-\dot{L}_{ijy})$$

$$L_{ijx} = x_i - x_j - dcos\theta_i$$

$$L_{ijy} = y_i - y_j - dsin\theta_i$$

$$\dot{L}_{ijx} = \dot{x}_i - \dot{x}_j - dsin\theta_i\dot{\theta}_i$$

$$\dot{L}_{ijx} = V_i cos\theta_i - V_j cos\theta_j + d\omega_i sin\theta_i$$

$$\dot{L}_{ijy} = V_i sin\theta_i - V_j sin\theta_j - d\omega_i cos\theta_i$$

Let d=0,distance between rear and front wheel

$$\theta_i = \theta_j$$

$$\phi_{ij} = 0$$

$$\dot{L}_{ijx} = V_i cos\theta_i - V_j cos\theta_j$$

$$\dot{L}_{ijy} = V_i sin\theta_i - V_j sin\theta_j$$

Controllers for follower velocity

$$u_x = V_j cos\theta_j$$

$$u_y = V_j sin\theta_j$$

$$\dot{V} = (L_{dx} - x_i + x_j)(u_x - V_i cos\theta_i) + (L_{dy} - y_i + y_j)(u_y - V_i sin\theta_i)$$

Now need to find u$_x and u_y$

$$u_x = -K(L_{dx} - x_i + x_j) + V_i cos\theta_i$$

$$u_y = -K(L_{dy} - y_i + y_j) + V_i sin\theta_i$$

checking $\dot{V}$

$$\dot{V} = -K(L_{dx} - x_i + x_j)^2 - K(L_{dy} - y_i + y_j)^2$$

which is negative semi-definite.

As when $L_{dx} = 0$, then $x_i = x_j$, similarly when $L_{dy} = 0$, then $y_i = y_j$.
$\dot{V} = 0$, when L_{dx} and $L_{dy} = 0$

Using Lassale's principle:-

$$L_{dx} - x_i + x_j = 0$$

$$L_{dx} = x_i - x_j$$

$$L_{dy} - y_i + y_j = 0$$

$$L_{dy} = y_i - y_j$$

This represents the equilibrium point of the system. Therefore, according to LaSalle's invariance principle, trajectories of the system will converge to the equilibrium point as time progresses.

B. Velocity and Orientation Control

Let us define a combined lyapunov function considering error in x,y axis and orientation .
Let

$$V = \frac{1}{2}(L_{dx} - L_{ijx})^2 + \frac{1}{2}(L_{dy} - L_{ijy})^2 + \frac{1}{2}(\theta_d - \theta_c)^2$$

$$\theta_c = \theta_i - \theta_j$$

Differentiating with time

$$\begin{aligned}\dot{V} &= (L_{dx} - L_{ijx})(\dot{L}_{dx} - \dot{L}_{ijx}) \\ &+ (L_{dy} - L_{ijy})(\dot{L}_{dy} - \dot{L}_{ijy}) \\ &+ (\theta_d - \theta_i + \theta_j)(\dot{\theta}_d - \dot{\theta}_i + \dot{\theta}_j)\end{aligned}$$

As L_{dx} is not changing with time, $\dot{L}_{dx} = 0$
As L_{dy} is not changing with time, $\dot{L}_{dy} = 0$

$$\begin{aligned}\dot{V} &= (L_{dx} - L_{ijx})(-\dot{L}_{ijx}) \\ &+ (L_{dy} - L_{ijy})(-\dot{L}_{ijy}) \\ &+ (\theta_d - \theta_i + \theta_j)(\dot{\theta}_d - \dot{\theta}_i + \dot{\theta}_j)\end{aligned}$$

$$L_{ijx} = x_i - x_j - dcos\theta_i$$

$$L_{ijy} = y_i - y_j - dsin\theta_i$$

$$\dot{L}_{ijx} = \dot{x}_i - \dot{x}_j - dsin\theta_i\dot{\theta}_i$$

$$\dot{L}_{ijx} = V_i cos\theta_i - V_j cos\theta_j + d\omega_i sin\theta_i$$

$$\dot{L}_{ijy} = V_i sin\theta_i - V_j sin\theta_j - d\omega_i cos\theta_i$$

$$\dot{\theta}_i = \text{angular velocity of leader}$$
$$\dot{\theta}_j = \text{angular velocity of follower}$$

Let $d = 0$,Distance between rear and front wheel

$$\dot{L}_{ijx} = V_i cos\theta_i - V_j cos\theta_j$$

$$\dot{L}_{ijy} = V_i sin\theta_i - V_j sin\theta_j$$

Controllers for follower velocity

$$u_x = V_j cos\theta_j$$

$$u_y = V_j sin\theta_j$$

$$\omega_j = \dot{\theta}_j$$

$$\begin{aligned}\dot{V} &= (L_{dx} - L_{ijx})(u_x - V_i \cos\theta_i) \\ &+ (L_{dy} - L_{ijy})(u_y - V_i \sin\theta_i) \\ &+ (\theta_d - \theta_i + \theta_j)(-\omega_i + \omega_j)\end{aligned}$$

Now need to find u_x and u_y and ω_j
choosing

$$u_x = -K(L_{dx} - x_i + x_j) + V_i cos\theta_i$$

$$u_y = -K(L_{dy} - y_i + y_j) + V_i sin\theta_i$$

$$\omega_j = (\omega_i) + (-\theta_d + \theta_i - \theta_j)$$

Checking $\dot{V}$

$$\dot{V} = -K(L_{dx} - x_i + x_j)^2 - K(L_{dy} - y_i + y_j)^2 - (\theta_d - \theta_i + \theta_j)^2$$

which is negative semi-definite.
As when $L_{dx} = 0$, then $x_i = x_j$, similarly when $L_{dy} = 0$, then $y_i = y_j$.
also when $\theta_d = 0$ then $\theta_i = \theta_j$
when L_{dx} and $L_{dy} = 0$ and $\theta_d = 0$
Then $\dot{V} = 0$,
Using Lassale's principle:-

$$L_{dx} - x_i + x_j = 0$$

$$L_{dx} = x_i - x_j$$

$$L_{dy} - y_i + y_j = 0$$

$$L_{dy} = y_i - y_j$$

$$\theta_d - \theta_i + \theta_j = 0$$

$$\theta_d = \theta_i - \theta_j$$

This represents the equilibrium point of the system. Therefore, according to LaSalle's invariance principle, trajectories of the system will converge to the equilibrium point as time progresses.

So our required control law is

$$u_x = -K(L_{dx} - x_i + x_j) + V_i cos\theta_i$$

$$u_y = -K(L_{dy} - y_i + y_j) + V_i sin\theta_i$$

$$\omega_j = (\omega_i) + (-\theta_d + \theta_i - \theta_j)$$

C. SIMULATION

1) Line Formation using Velocity Control Only: Here robots will try to follow leader by remaining in straight line.

Parameters Taken

Agent	Coordinates	Desired Distance
Leader	$(-13, -10)$	
Follower 1	$(6, 6)$	$(6, 0)$
Follower 2	$(-9, 6)$	$(8, 0)$
Follower 1-1	$(12, 12)$	$(6, 0)$
Follower 2-1	$(-18, -12)$	$(8, 0)$

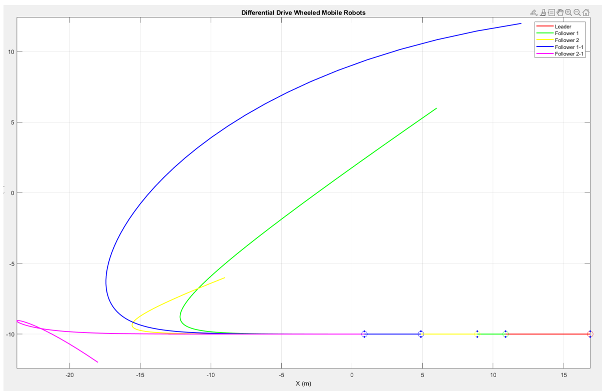

Fig. 7. Line Formation

2) Rectangular Formation using Velocity Control Only: Here robots will follow the leader while maintaining a rectangular shape.

Parameters Taken

Agent	Coordinates	Desired Distance
Leader	$(120, 0)$	
Follower 1	$(6, 6)$	$(4, 5)$
Follower 2	$(-12, -16)$	$(4, -5)$
Follower 1-1	$(30, 30)$	$(7, 0)$
Follower 2-1	$(-18, -12)$	$(7, 0)$
Follower 5	$(-20, 10)$	$(4, 0)$
Follower 6	$(24, 30)$	$(11, 0)$

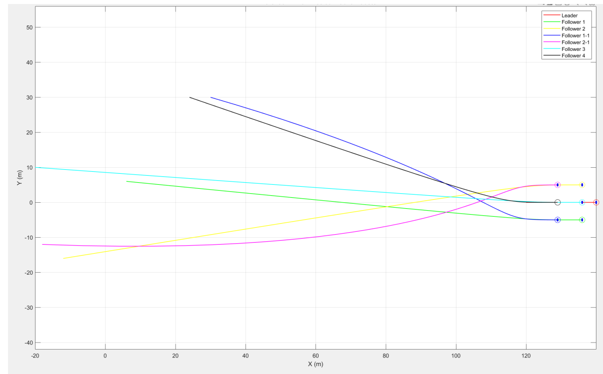

Fig. 8. Rectangular Formation

3) Complex Trajectory with complete Control Law maintaining distance: Here robots will follow the leader while maintaining distance and orientation from leader.

Agent	Coordinates	Orientation	Desired Distance	Desired Orientation
Leader	$(-13, -10)$	$\pi/4$		
Follower 1	$(6, 6)$	$\pi/2$	$(6, 0)$	$\pi/12$
Follower 2	$(-9, 6)$	π	$(8, 0)$	$\pi/12$
Follower 3	$(12, 12)$	$\pi/3$	$(6, 0)$	$\pi/12$

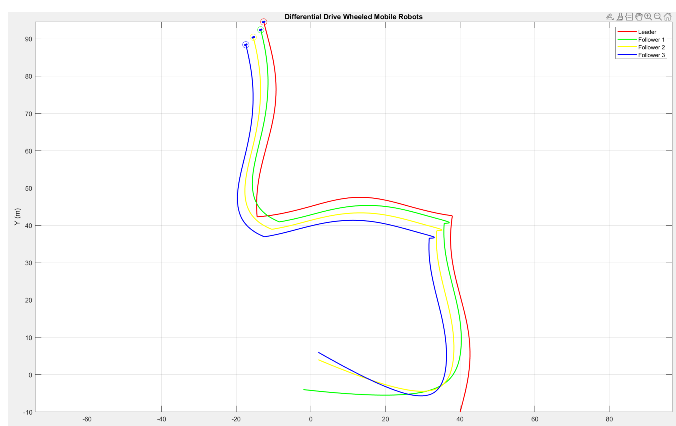

Fig. 9. Complex Trajectory

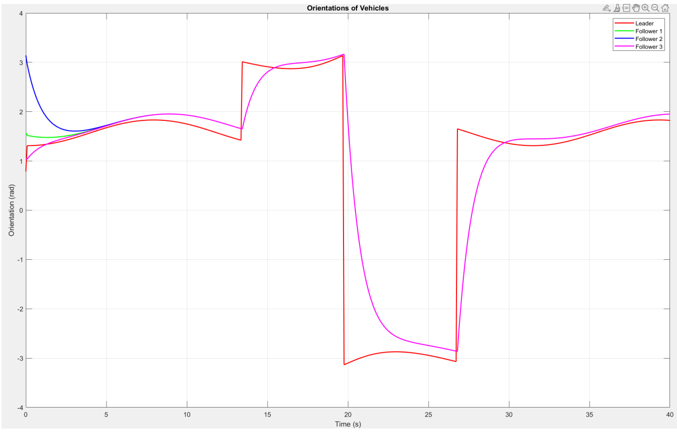

Fig. 10. Orientation

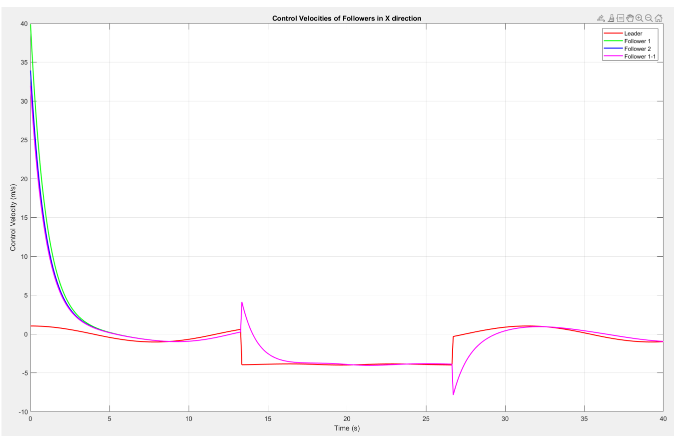

Fig. 11. Velocity along X axis

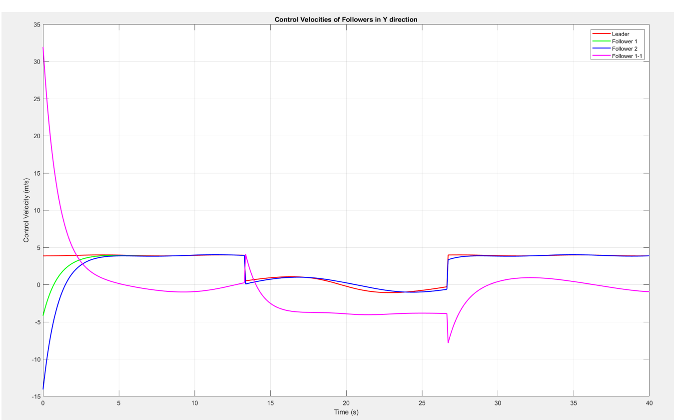

Fig. 12. Velocity along Y axis

IV. Conclusion

We have used lyapunov approach to design controllers for leader follower formation control.However we have used orientation of vehicles instead of relative bearing for designing control. Because as due to non-holonomic constraints both should be same in most cases. We found that our control law is working in simple scenarios.But It needs more some modifications according to trajectory of leader.As when there is sudden changes in orientation of leader, followers may not be able to maintain distance or orientation. Future research endeavors will investigate: implement obstacles avoidance and self-collision avoidance .

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