DH Parameters

Thursday, February 6, 2025 12:40 AM

Homogenous Transformation Matrix

$$H_n^0 = \begin{bmatrix} R_n^0 & d_n^0 \\ 0 & 1 \end{bmatrix}$$

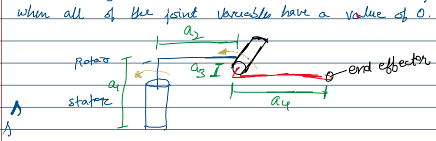
with horns
3 Servo RC ← Futaba S3003
1 Stepper Motor | ® Rack and Pinion
1 DC Motor
screw and standoffs
m3 screws
screw driver
* board
* link to join servo.
(link will be attached to servo horn)
* servo bracket (with m3 nut and bolt) (servo will be mounted onit)

Steps :-

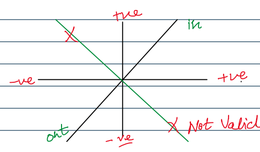
1. Assign frames according to the 4 DH rules
2. Create DH parameter table
3. Plug the table values into the HTM
4. Multiply the matrices together

Kinematic Diagram

Diagram that shows how the links and joints are connected together when all of the joint variables have a value of 0.

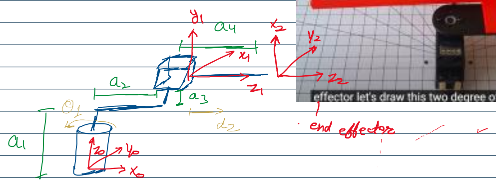


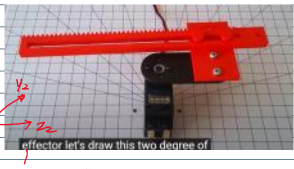
Right hand Rule ↺ the direction CCW



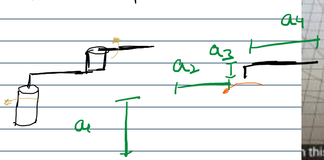
So 3rd axis is valid only when both signs of axis are same.

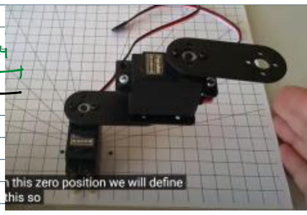
"Workspace" = The area or volume the end effector can reach





Another Example:-

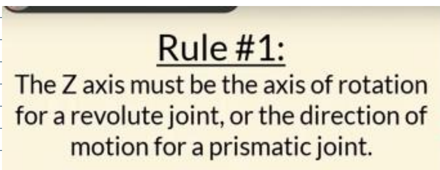




Frames:-

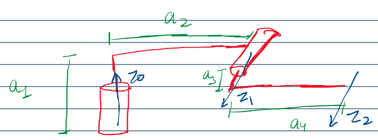
One frame for each joint
One base/world frame
One frame for end effector

4 Rules to follow while assigning frames:-



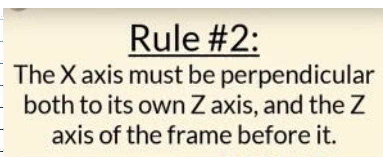
Rule #1:

The Z axis must be the axis of rotation for a revolute joint, or the direction of motion for a prismatic joint.



0 - serves the purpose of base frame and frame of joint 1

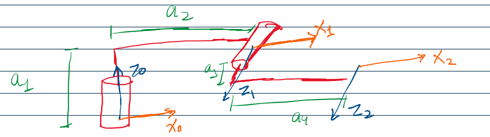
* End effector is no joint. How we decide joint axis z.
Try to place same as previous joint.



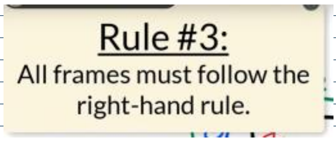
Rule #2:

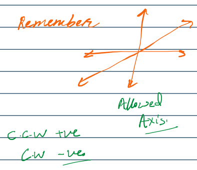
The X axis must be perpendicular both to its own Z axis, and the Z axis of the frame before it.

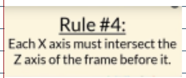
Try to place along link.



Kinematic Diagram





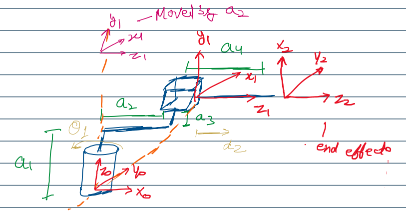


Does not apply to frame zero.

⊗ If it is possible to place next axis in same direction as previous do it. As it makes mathematics easier.

Revision Rules:-

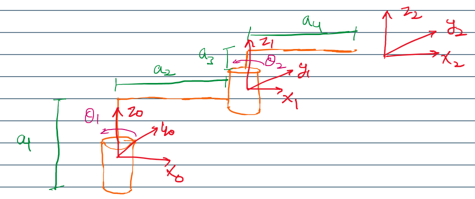
1. Z axis axis of motion
2. X axis must be perpendicular to current z and previous z
3. Right hand rule must be followed.
4. Current x-axis must coincide with previous z



See here:- 4th rule is not followed. z_0 and x_1 are not coinciding. They are passing by.

⇒ So need to shift centre of 1st frame of reference.

Another example:-

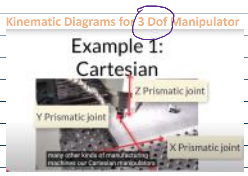


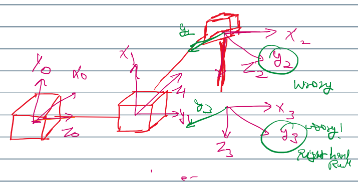
Check if rules are satisfied.

Z-axis of motion ✓
X-axis ⊥ar to Prev Z and current Z ✓
Right hand must be followed ✓
X_{i+1} must coincide Z_0 ✓

⊗ 4 DH parameters

2 link Parameter	2 Joint Parameter
About x_{i-1}	About z_i
Angle, distance	Angle, distance
bet^n z_{i-1} and z_i	bet^n x_{i-1} and x_i

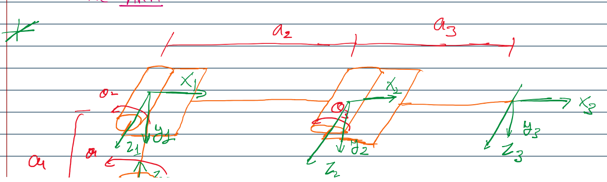


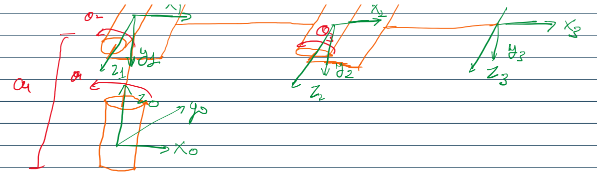


⊗ We can make up-down also

② ARTICULATED

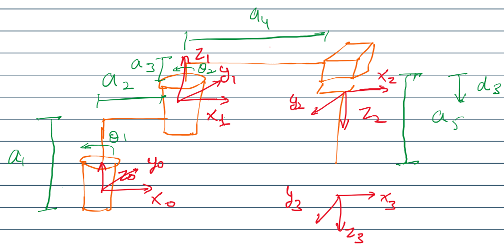
like ARM.





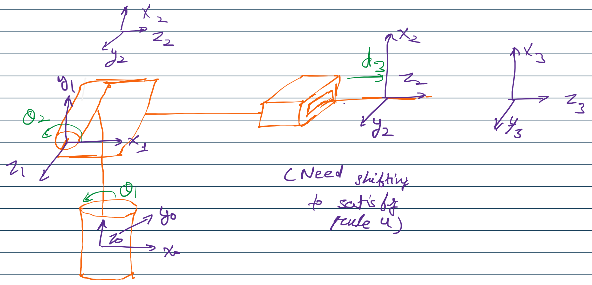
③ SCARA

mostly used for pick and place.



④ SPHERICAL

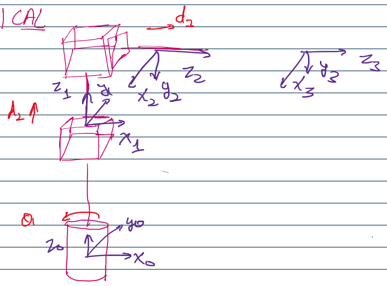
* spherical co-ordinates.
  (Any point in space as combination of two angles and a radius)



(Need shifting to satisfy rule 4)

⑤ CYLINDRICAL

(cylindrical coordinates)

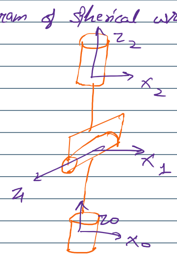


⑦ Six degree of Freedom Manipulators :-

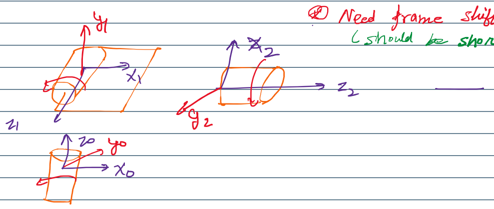
spherical wrist 3DOF

Combine standard with spherical wrist to get 6DoF.

Kinematic diagram of spherical wrist.



① Need frame shifting.
(should be shortest)



⊛ Kinematic Redundant :- like when two joints have same axis of motion such that only one joint was enough to bring that kind of motion.

Homogenous Transformation Matrix.

$$H_n^0 = \begin{bmatrix} R_n^0 & d_n^0 \\ 0\ 0\ 0 & 1 \end{bmatrix}$$

Denavit - Hartenberg (DH) Parameter Table :-

→ It records all the rotational and displacement rel^n bet^n frames.

* Where we can apply D-H Parameters :-
- Prismatic - spherical joints
- serial chain

Alternative :-
- Screw Theory representations
- Hayati - Roberts Method

One Row for each Pair of frames.

Four Parameters.

θ (Joint Parameter) :- Rotation about Z_{n-1} such that X_{n-1} becomes parallel to X_n + (plus)
Joint Rotation Revolute joint $= \theta$, Prismatic $= 0$)

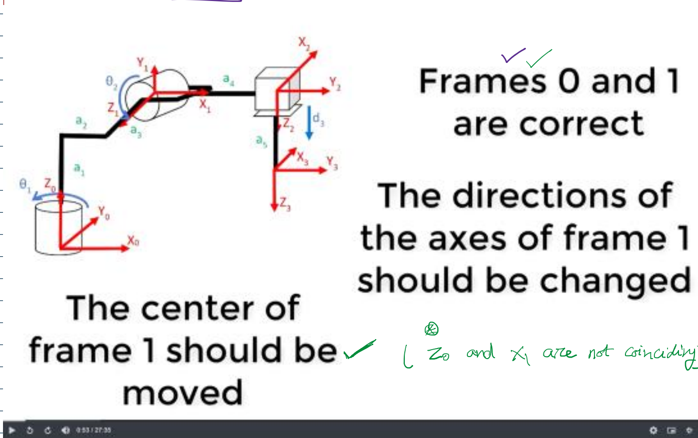
α (link parameter) :- Rotation about X_n such that Z_{n-1} becomes parallel to Z_n

r (link parameter) :- distance bet^n center $n-1$ to center n along X_n.

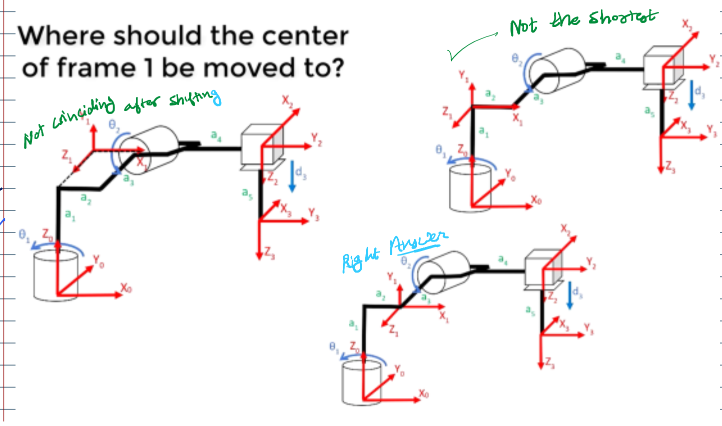
d (Joint parameter) :- distance bet^n center $n-1$ to centre n along Z_{n-1}.
(plus joint vareable)
(prismatic $= d$, revolute $= 0$)

$$H_n^{n-1} = \begin{bmatrix} C(\theta_n) & -S(\theta_n)\,C(\alpha_n) & S(\theta_n)\,S(\alpha_n) & r_n\,C(\theta_n) \\ S(\theta_n) & C(\theta_n)\,C(\alpha_n) & -C(\theta_n)\,S(\alpha_n) & r_n\,S(\theta_n) \\ 0 & S(\alpha_n) & C(\alpha_n) & d_n \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

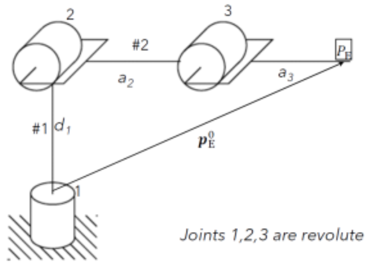
PRACTICE PROBLEM :-



⊛ (Z_0 and X_1 are not coinciding)

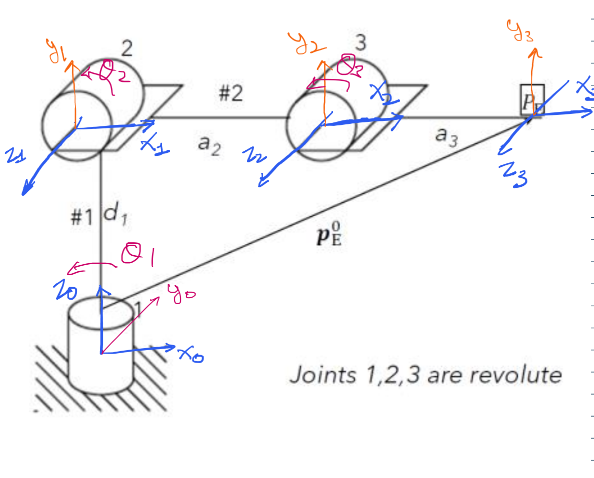


Exercise 2



Task

1. Assign frames as per DH convention
2. Tabulate DH parameters
3. Find T_1^0, T_2^1, T_3^2
4. Find T_3^0
5. Find R_E^0 , p_E^0



DH - TABLE

	θ_n	α_n	r_n	d_n
1	θ_1	$\pi/2$	0	d_1
2	θ_2	0	a_2	0
3	θ_3	0	a_3	0

मेरा Global frame तो है Joint 1 पे ही frame है so Find T_2^0 frame w.r.t frame zero.

$$T_2^0 = T_1^0 \, T_2^1$$

$$T = \begin{bmatrix} C\theta & -S\theta & 0 & r \\ S\theta C\alpha & C\theta C\alpha & -S\alpha & -dS\alpha \\ S\theta S\alpha & C\theta S\alpha & C\alpha & dC\alpha \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_1^0 = \begin{bmatrix} C\theta_1 & -S\theta_1 & 0 & 0 \\ S\theta_1 C(\pi/2) & C\theta_1 C(\pi/2) & -S(\pi/2) & -d_1 S(\pi/2) \\ S\theta_1 S(\pi/2) & C\theta_1 S(\pi/2) & C(\pi/2) & d_1 \cos(\pi/2) \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C\theta_1 & -S\theta_1 & 0 & 0 \\ 0 & 0 & -1 & -d_1 \\ S\theta_1 & C\theta_1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2^1 = \begin{bmatrix} C\theta_2 & -S\theta_2 & 0 & a_2 \\ S\theta_2 S(0) & C\theta_2 C(0) & -S(0) & -\cancel{0} \\ S\theta_2 S(0) & C\theta_2 S(0) & C(0) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C\theta_2 & -S\theta_2 & 0 & a_2 \\ 0 & C\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} C\theta_1 & -S\theta_1 & 0 & 0 \\ 0 & 0 & -1 & -d_1 \\ S\theta_1 & C\theta_1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C\theta_2 & -S\theta_2 & 0 & a_2 \\ 0 & C\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} C\theta_1 C\theta_2 & -C\theta_1 S\theta_2 - S\theta_1 C\theta_2 & 0 & a_2 C\theta_1 \\ 0 & -1 & -1 & -d_1 \\ S\theta_1 C\theta_2 & -S\theta_1 S\theta_2 + C\theta_1 C\theta_2 & 0 & a_2 S\theta_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Taking Joint Variable $\theta_1 = \theta_2 = 0$

$$\begin{bmatrix} 1 & 0 & 0 & a_2 \\ 0 & -1 & -1 & -d_1 \end{bmatrix} \checkmark$$

* Used traditional transformation Matrix

Taking Joint Variable $\theta_1 = \theta_2 = 0$

$$\begin{bmatrix} 1 & 0 & 0 & a_2 \\ 0 & -1 & -1 & -d_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

* Used traditional transformation Matrix with Modern DH Parameter

$$H_n^{n-1} = \begin{bmatrix} C(\theta_n) & -S(\theta_n)\,C(\alpha_n) & S(\theta_n)\,S(\alpha_n) & r_n\,C(\theta_n) \\ S(\theta_n) & C(\theta_n)\,C(\alpha_n) & -C(\theta_n)\,S(\alpha_n) & r_n\,S(\theta_n) \\ 0 & S(\alpha_n) & C(\alpha_n) & d_n \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

DH - TABLE

	θ_n	α_n	r_n	d_n
1	θ_1	$\pi/2$	0	d_1
2	θ_2	0	a_2	0
3	θ_3	0	a_3	0

$$T_1^0 = \begin{bmatrix} C\theta_1 & -S\theta_1\,C(\pi/2) & S\theta_1\,S(\pi/2) & 0 \\ S\theta_1 & C\theta_1\,C(\pi/2) & -C\theta_1\,S(\pi/2) & 0 \\ 0 & S(\pi/2) & C(\pi/2) & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} C\theta_1 & 0 & S\theta_1 & 0 \\ S\theta_1 & 0 & -C\theta_1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2^1 = \begin{bmatrix} C\theta_2 & -S\theta_2\,C(0) & S\theta_2\,S(0) & a_2\,C\theta_2 \\ S\theta_2 & C\theta_2\,C(0) & -C(\theta_2)\,S(0) & a_2\,S\theta_2 \\ 0 & S(0) & C(0) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} C\theta_2 & -S\theta_2 & 0 & a_2 C\theta_2 \\ S\theta_2 & C\theta_2 & 0 & a_2 S\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & a_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Taking $\theta_1 = \theta_2 = 0$

$$T_2^0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & a_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & a_2 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Correct

$\theta_3 = 0$

$$T_3^2 = \begin{bmatrix} C\theta_3 & -S\theta_3\,C(0) & S\theta_3\,S(0) & a_3\,C\theta_3 \\ S\theta_3 & C\theta_3\,C(0) & -C\theta_3\,S(0) & a_3\,S\theta_3 \\ 0 & S(0) & C(0) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} C\theta_3 & -S\theta_3 & 0 & a_3\,C\theta_3 \\ S\theta_3 & C\theta_3 & 0 & a_3\,S\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$T_3^0 = \begin{bmatrix} 1 & 0 & 0 & a_2 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & a_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & a_3 + a_2 \\ 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\left. \begin{matrix} 0 & a_3 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{matrix} \right]$$