Assignment 4

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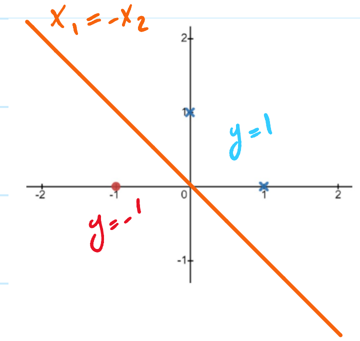

① $(x_n, y_n) \in R^2$

$S_1 = (-1,0) \quad S_2 = (0,1) \quad S_3 = (1,0)$

$S_1' = (-1,0,1) \quad S_2' = (0,1,1) \quad S_3' = (1,0,1)$

$$\Rightarrow \begin{cases} \alpha_1 S_1' \cdot S_1' + \alpha_2 S_2' S_1' + \alpha_3 S_3' S_1' = -1 \\ \alpha_1 S_1' S_2' + \alpha_2 S_2' S_2' + \alpha_3 S_3' S_2' = 1 \\ \alpha_1 S_1' S_3' + \alpha_2 S_2' S_3' + \alpha_3 S_3' S_3' = 1 \end{cases} \longrightarrow \begin{cases} 2\alpha_1 + \alpha_2 = -1 \\ \alpha_1 + 2\alpha_2 + \alpha_3 = 1 \\ \alpha_2 + 2\alpha_3 = 1 \end{cases} \longrightarrow \begin{cases} \alpha_1 = -1 \\ \alpha_2 = 1 \\ \alpha_3 = 0 \end{cases}$$

$$w' = \sum \alpha_i S_i' = (1,0,-1) + (0,1,1) = (1,1,0) \longrightarrow w = (1,1) \quad b = 0$$

$$\longrightarrow wx + b = 0 \longrightarrow \boxed{x_2 = -x_1}$$

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② $K(x_i, x_j) = \exp(-\frac{1}{a}\|x_i - x_j\|^c)$

$$\|\varphi(x_i) - \varphi(x_j)\|^2 = \langle \varphi(x_i), \varphi(x_i) \rangle + \langle \varphi(x_j), \varphi(x_j) \rangle - 2\langle \varphi(x_i), \varphi(x_j) \rangle$$

$$= K(x_i, x_i) + K(x_j, x_j) - 2K(x_i, x_j) = \exp\left(\frac{-1}{a}\|\cancelto{0}{x_i - x_i}\|^c\right) + \exp(\cancelto{0}{\sim}) - 2\exp\left(\frac{-1}{a}\|x_i - x_j\|^c\right)$$

$$= 1 + 1 - 2\exp\left(\frac{-1}{a}\|x_i - x_j\|^c\right) = 2 - \overbrace{2\exp(\sim)}^{\geq 0} \leq 2 \longrightarrow \boxed{k = 2}$$

$$K(x_i,x_j)=\tanh(a x_i^T x_j+b)$$

$$\|\varphi(x_i)-\varphi(x_j)\|^2=\langle\varphi(x_i),\varphi(x_i)\rangle+\langle\varphi(x_j),\varphi(x_j)\rangle-2\langle\varphi(x_i),\varphi(x_j)\rangle$$

$$=K(x_i,x_i)+K(x_j,x_j)-2K(x_i,x_j)=\tanh(a x_i^T x_i+b)+\tanh(a x_j^T x_j+b)-2\tanh(a x_i^T x_j+b)$$

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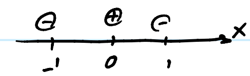
③

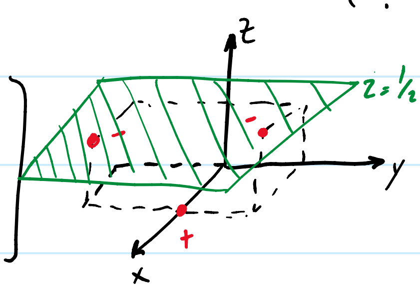


الف) خیر، قابل جداسازی نیست.

ب)

$$\varphi(x)=\begin{bmatrix}1\\ \sqrt{2}x\\ x^2\end{bmatrix}$$

$$\longrightarrow A_1^{+}=\begin{bmatrix}1\\0\\0\end{bmatrix}\quad A_2^{-}=\begin{bmatrix}1\\-1\\1\end{bmatrix}\quad A_3^{-}=\begin{bmatrix}1\\1\\1\end{bmatrix}$$



بله، توسط صفحات $0<a<1$ $z=a$ قابل جداسازی است.

ج)

$$W=\begin{bmatrix}w_1\\w_2\\w_3\end{bmatrix}\longrightarrow w^T\varphi(x_i)=(w_1\ w_2\ w_3)\begin{bmatrix}1\\ \sqrt{2}x\\ x^2\end{bmatrix}=w_1+\sqrt{2}w_2x+w_3x^3$$

$$\min_{w,b}\ \frac{1}{2}\|W\|_2^2\ \text{s.t.}\quad y_i\left(w^T\varphi(x_i)+b\right)\geq 1$$

$$L(w,b,\lambda)=\frac{1}{2}ww^T+\sum\lambda_i\left(1-y_i\left(w^T\varphi(x_i)+b\right)\right)$$

$$\nabla_w L=0\longrightarrow w-\sum\lambda y_i\varphi(x_i)=0\longrightarrow w=\sum\lambda_i y_i\varphi(x_i)$$

$$\nabla_b L = 0 \longrightarrow -\sum \lambda_i y_i = 0 \longrightarrow \lambda_2 = \lambda_1 + \lambda_3$$

$$\longrightarrow \min_{\lambda} \frac{1}{2} \sum \lambda_i \lambda_j y_i y_j \varphi^T(x_i)\varphi(x_j) + \sum \lambda_i = \min \frac{1}{2}\left[4\lambda_1^2 + \lambda_2^2 + 4\lambda_3^2 - \lambda_1\lambda_2 - \lambda_{21} - \lambda_{23} - \lambda_{32}\right] - (\lambda_1 + \lambda_2 + \lambda_3)$$

$$= \min_{\lambda} 2\lambda_1^2 + \frac{1}{2}\lambda_2^2 + 2\lambda_3^2 + -\lambda_1\lambda_2 - \lambda_2\lambda_3 - \lambda_1 - \lambda_2 - \lambda_3 = \min_{\lambda} 2\lambda_1^2 - 4\lambda_1 \longrightarrow \boxed{\begin{array}{l}\lambda_1 = \lambda_3 = 1 \\ \lambda_2 = 2\end{array}}$$

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④ a)

$$\begin{cases} x_i w + b \geq +1 - \xi_i & y = +1 \\ x_i w + b \leq -1 + \xi_i & y = -1 \end{cases}$$

≥ 0

$$\text{I}: y = 1 \text{ and } x_i w + b < 0 \longrightarrow y(x_i w + b) < 0 \longrightarrow 1 - \xi_i < 0 \longrightarrow \xi_i > 1$$

$$\text{II}: y = -1 \text{ and } x_i w + b > 0 \longrightarrow y(x_i w + b) < 0 \longrightarrow 1 - \xi_i < 0 \longrightarrow \xi_i > 1$$

$$\text{error} = \text{I} + \text{II} \longrightarrow \boxed{\text{error} \leq \sum_{i}^{m} \xi_i}$$

b) $K(x,y)^2 \leq K(x,x)K(y,y)$

$$K(x,y) = \begin{pmatrix} K(x,x) & K(x,y) \\ K(y,x) & K(y,y) \end{pmatrix} \longrightarrow \det(K(x,y)) = \begin{vmatrix} K(x,x) & K(x,y) \\ K(y,x) & K(y,y) \end{vmatrix} \geq 0$$

$$* \; K(x,y) = K(y,x) \longrightarrow K(x,x)K(y,y) - K^2(x,y) \geq 0 \longrightarrow \boxed{K^2(x,y) \geq K(x,x)K(y,y)}$$