Phase space Project

“PHS”

Fateme Dashti - Arash T. Jamshidi

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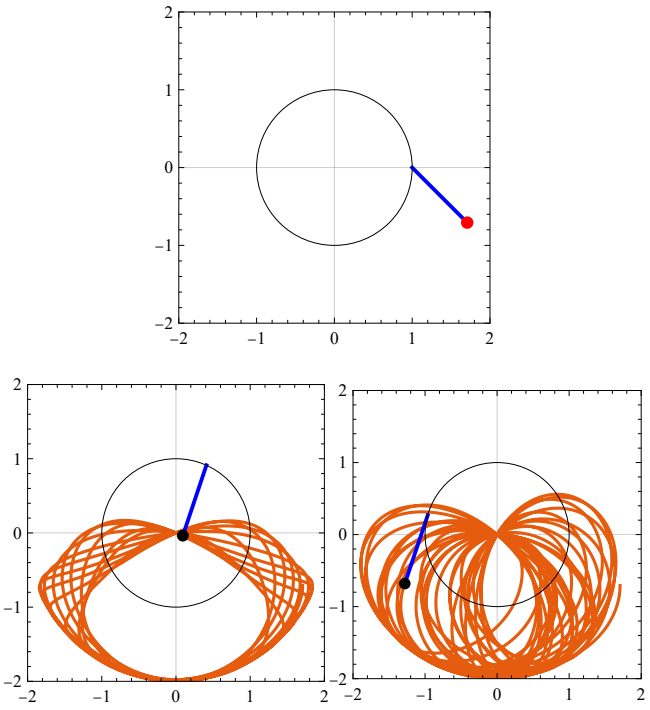
Step 0

In this project we trying to simulate trajectory of pendulum which is connected to a disk, and the disk it self has an angular momentum ω and analysis phase space of ω and the pendulum initial angel at t = 0 ($\theta_{t=0}$).
we use Lagrangian mechanics and Euler-Lagrange equation to analysis this problem.



$$\begin{cases} x[t] = & l\text{Sin}[\theta] + r\text{Cos}[\phi] \\ y[t] = & r\text{Sin}[\phi] - l\text{Cos}[\theta] \end{cases}$$

$$\phi = \omega t$$

$$\rightarrow \begin{cases} x'[t] = & l\theta' \text{ Cos}[\theta] - r\omega\text{Sin}[\omega t] \\ y'[t] = & r\omega\text{Cos}[\omega t] + l\theta' \text{ Sin}[\theta] \end{cases}$$

$$\rightarrow T = \frac{1}{2} m \left(x'^2 + y'^2\right) = \frac{1}{2} m\left[l^2 \theta'^2 + r^2 \omega^2 + 2\, r\omega l\theta' + \text{Sin}[\theta - \omega t]\right]$$

$$U = mgy = mg[r\text{Sin}[\phi] - l\text{Cos}[\theta]]$$

$$L = T - U = \frac{1}{2} m\left[l^2 \theta'^2 + a^2 \omega^2 + 2\, r\omega l\theta' + \text{Sin}[\theta - \omega t]\right] - mg[r\text{Sin}[\phi] - l\text{Cos}[\theta]]$$

$$\frac{\partial L}{\partial \theta} - \frac{d}{dt}\frac{\partial L}{\partial \theta'} = 0$$

$$\theta''[t] = \frac{-g}{l} \text{Sin}[\theta[t]] + r\, \frac{\omega^2}{l} \text{Cos}[\theta[t] - \omega t]$$

Step 1

In[ ]:=
```
Quit
```

In[1]:=
```
g := 9.8
l := 1(*Pendulum arm length*)
r := 1(*radius of disk*)
w := 6(*Angular momentum of disk*)
tmax := 50(*time range*)
(*{t=50, π/5, 1}not*)
(*{t=50, π/10, 3}Rot*)
```

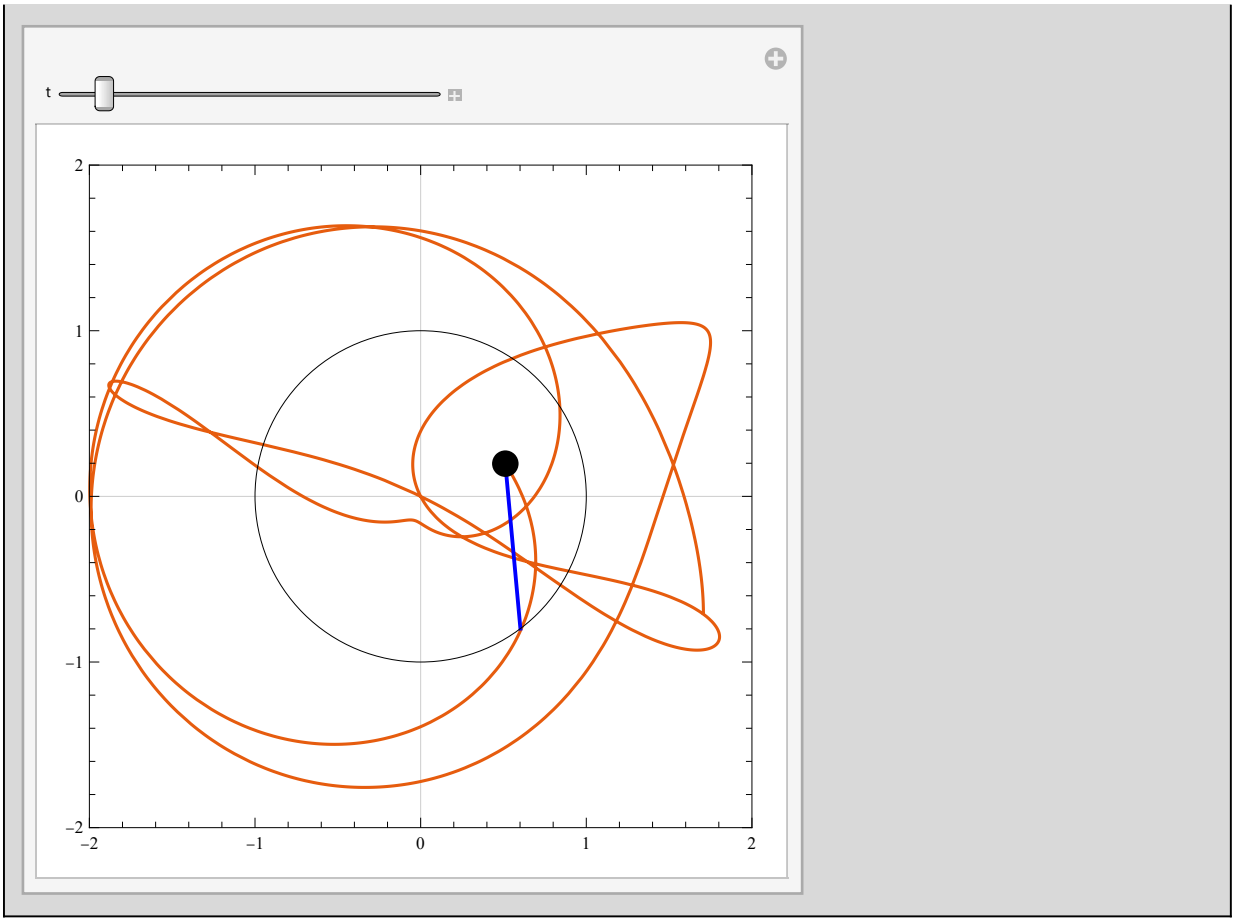
In[6]:=
```
solv =
 Flatten[NDSolve[{θ''[t] == -g/l * Sin[θ[t]] + r * w^2/l Cos[θ[t] - w * t], θ[0] == π/4, θ'[0] == 0},
   {θ[t]}, {t, 0, tmax}]]
θ[t_] := Evaluate[θ[t] /. solv]
```

Out[6]=
$\{\theta[t] \rightarrow$ InterpolatingFunction[ Domain: {{0., 50.}} Output: scalar][t]$\}$

In[8]:=
```
y1[t_] := r * Sin[w * t]
x1[t_] := r * Cos[w * t]
x2[t_] := x1[t] + l * Sin[θ[t]]
y2[t_] := y1[t] - l * Cos[θ[t]]
```

In[ ]:=
```
Manipulate[Show[ParametricPlot[{x2[t1], y2[t1]}, {t1, 0, t},
    Background → White, PlotTheme → "Scientific", PlotRange → {{-2, 2}, {-2, 2}}],
  Graphics[{Thick, Blue, Line[{{x1[t], y1[t]}, {x2[t], y2[t]}}]}],
  Graphics[{Circle[{0, 0}, r]}],
  Graphics[{PointSize[0.04], Black, Point[{x2[t], y2[t]}]}]], {t, 10^-10, tmax}]
```

Out[ ]=



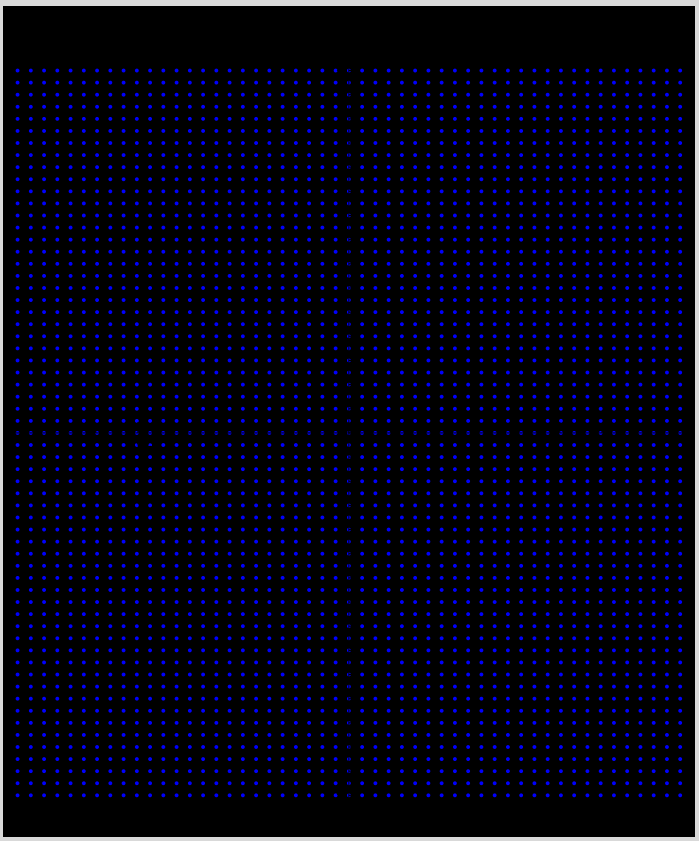
Step 2

In[ ]:=
```
Quit
```

In[•]:=
```
g := 9.8
l := 1(*Pendulum arm length*)
r := 1(*radius of disk*)
tmax := 25(*time range*)
ωstart := -3
ωend := 3
ωstep := 0.1
θstart := -π/4
θend := π/4
θstep := π/100
```

In[•]:=
```
xy = Table[{θ0, ω}, {θ0, θstart, θend, θstep}, {ω, ωstart, ωend, ωstep}];



lenθ := (θend - θstart)/θstep + 1
lenω := (ωend - ωstart)/ωstep + 1



PHS := {}
Do[{
  Do[{
    data = xy[[i]][[j]]
    , AppendTo[PHS, data]}
   , {j, 1, lenω}]}
 , {i, 1, lenθ}]


ListPlot[PHS, AspectRatio → Full,
 PlotStyle → {Blue, PointSize[Small]},
 Background → Black,
 Axes → True,
 PlotLabel → "Phase space",
 PlotRange → All]
```

Out[ ]=



In[ ]:=
```
PHS;
```

In[ ]:=
```
Length[PHS]
```

Out[ ]=
```
3111
```

Step 3

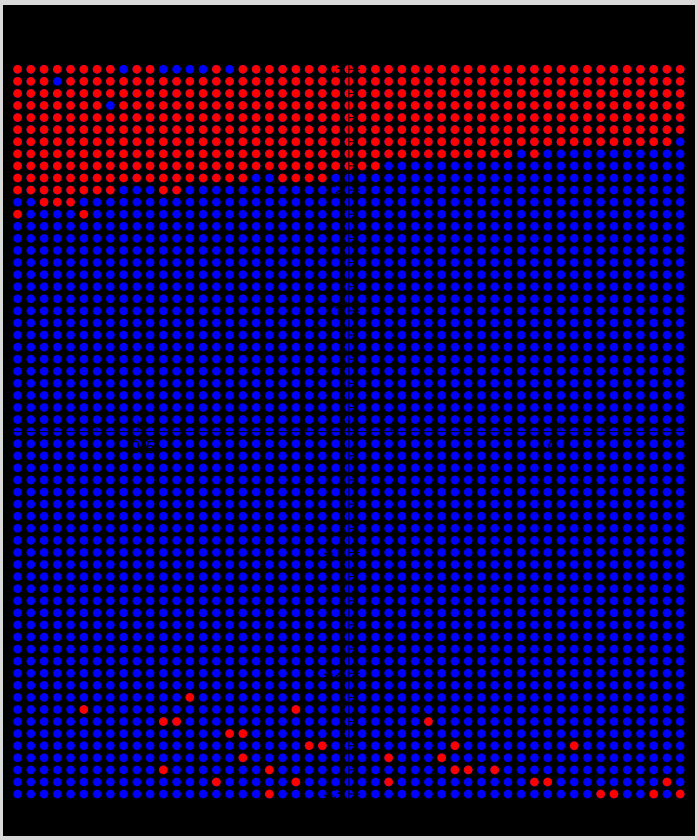
In[•]:=
```
firstRdata := {}
Do[{
  Clear[θ],
  theta = PHS[[α]][[1]];

  ω = PHS[[α]][[2]];

  eqf1 = θ[t] /. Flatten[NDSolve[{θ''[t] + g/l * Sin[θ[t]] - r * ω^2/l Cos[θ[t] - ω * t] == 0,
      θ[0] == theta, θ'[0] == 0}, θ[t], {t, 0, tmax}]];
  Do[{
    If[(eqf1 /. t → t1) > π, {AppendTo[firstRdata, {theta, ω}], Break[]}, Null]
   }, {t1, 0, tmax}]

 }, {α, 1, Length@PHS}]
```

In[•]:=
```
Length[firstRdata]
```

Out[•]=
```
483
```

In[•]:=
```
Show[ListPlot[PHS, AspectRatio → Full,
  PlotStyle → {Blue, PointSize[Medium]},
  Axes → True,
  PlotLabel → "Phase space",
  PlotRange → All],
 ListPlot[firstRdata, PlotStyle → {Red, PointSize[Medium]}], Background → Black]
```

Out[•]=

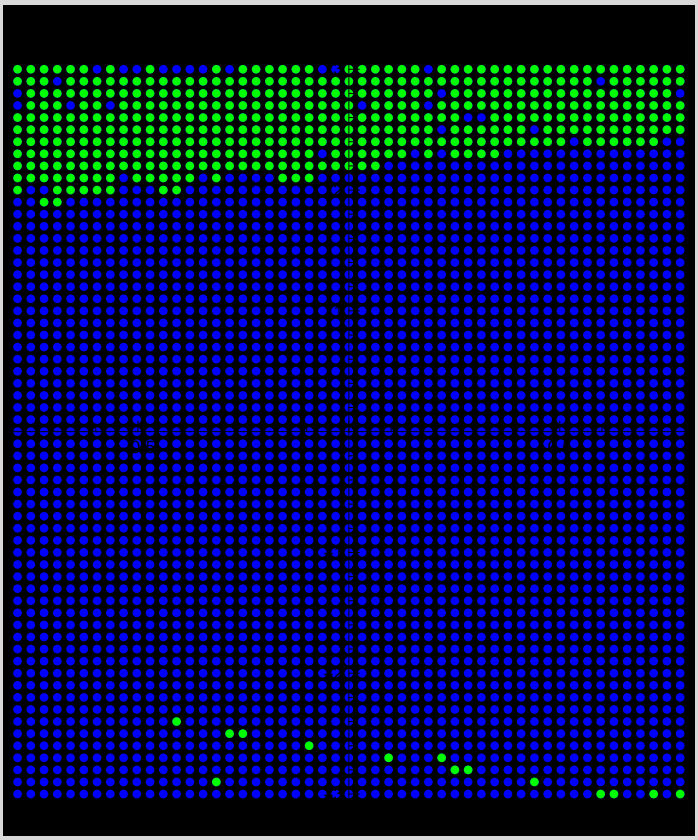


In[ ]:=
```
secondRdata := {}
Do[{
  Clear[θ],
  theta = PHS[[α]][[1]];

  ω = PHS[[α]][[2]];

  eqf1 = θ[t] /. Flatten[NDSolve[{θ''[t] + g/l * Sin[θ[t]] - r * ω^2/l Cos[θ[t] - ω * t] == 0,
       θ[0] == theta, θ'[0] == 0}, θ[t], {t, 0, tmax}]];
  Do[{
    If[(eqf1 /. t → t1) > 3 * π, {AppendTo[secondRdata, {theta, ω}], Break[]}, Null]
   }, {t1, 0, tmax}]

 }, {α, 1, Length@PHS}]
```

In[ ]:=
```
Length[secondRdata];
```

In[•]:=
```
Show[ListPlot[PHS, AspectRatio → Full,
  PlotStyle → {Blue, PointSize[Medium]},
  Axes → True,
  PlotLabel → "Phase space",
  PlotRange → All],
 ListPlot[secondRdata, PlotStyle → {Green, PointSize[Medium]}], Background → Black]
```

Out[•]=

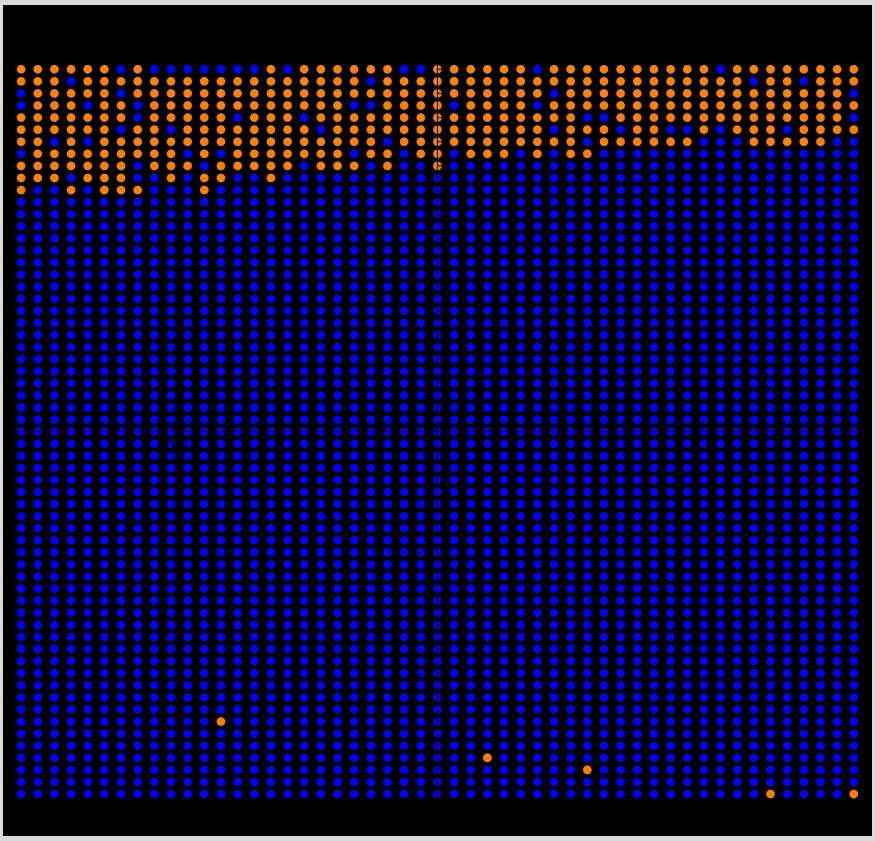


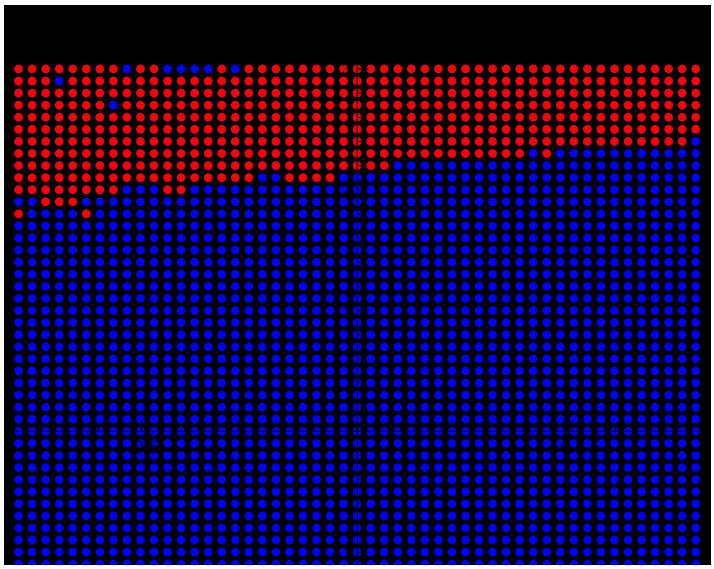
In[ ]:=
```
thirdRdata := {}
Do[{
  Clear[θ],
  theta = PHS[[α]][[1]];

  ω = PHS[[α]][[2]];

  eqf1 = θ[t] /. Flatten[NDSolve[{θ''[t] + g/l * Sin[θ[t]] - r * ω^2/l Cos[θ[t] - ω * t] == 0,
       θ[0] == theta, θ'[0] == 0}, θ[t], {t, 0, tmax}]];
  Do[{
    If[(eqf1 /. t → t1) > 5 * π, {AppendTo[thirdRdata, {theta, ω}], Break[]}, Null]
   }, {t1, 0, tmax}]

 }, {α, 1, Length@PHS}]
```

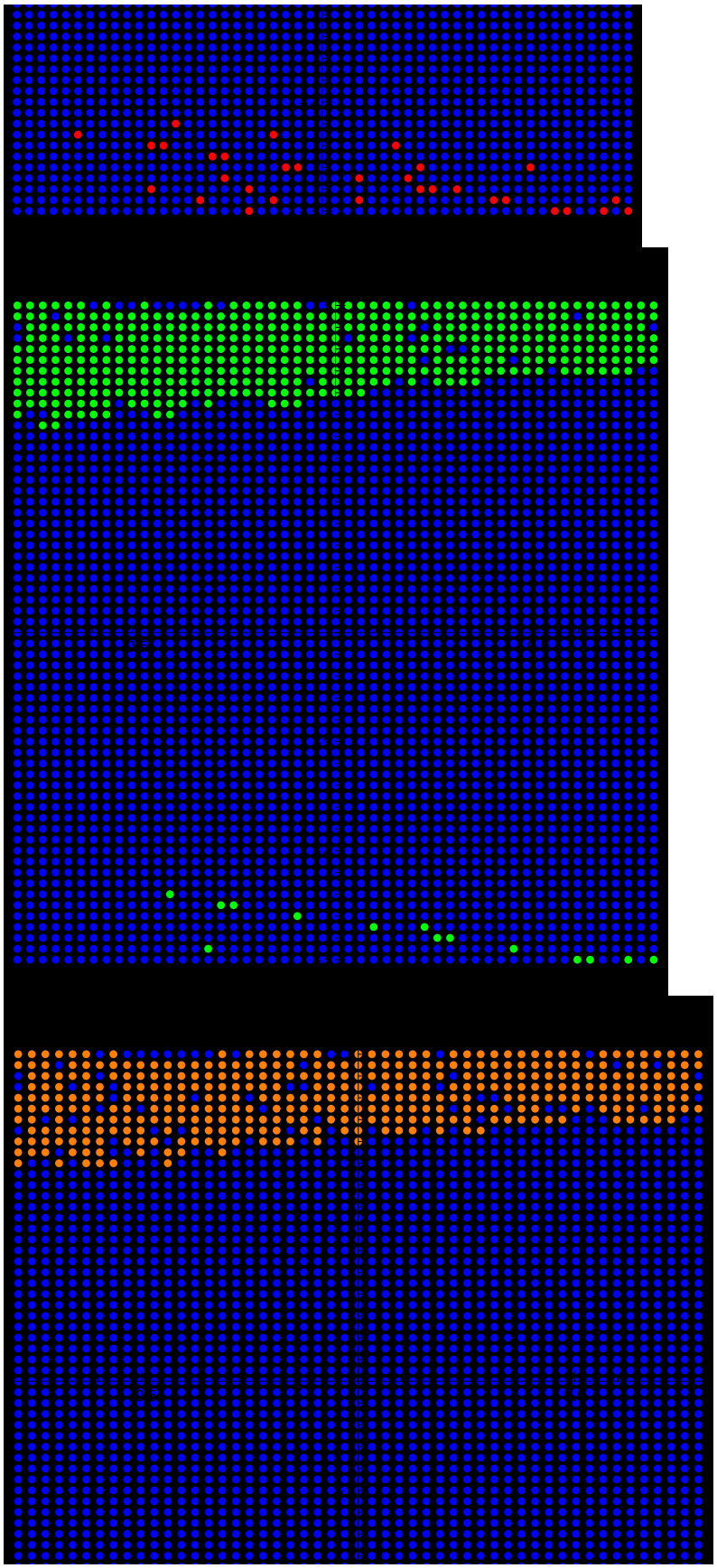
In[ ]:=
```
Length[thirdRdata];
```

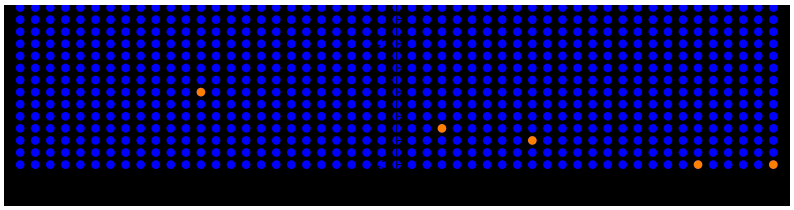
In[•]:=
```
Show[ListPlot[PHS, AspectRatio → Full,
  PlotStyle → {Blue, PointSize[Medium]},
  Axes → True,
  PlotLabel → "Phase space",
  PlotRange → All],
 ListPlot[thirdRdata, PlotStyle → {Orange, PointSize[Medium]}], Background → Black]
```

Out[•]=



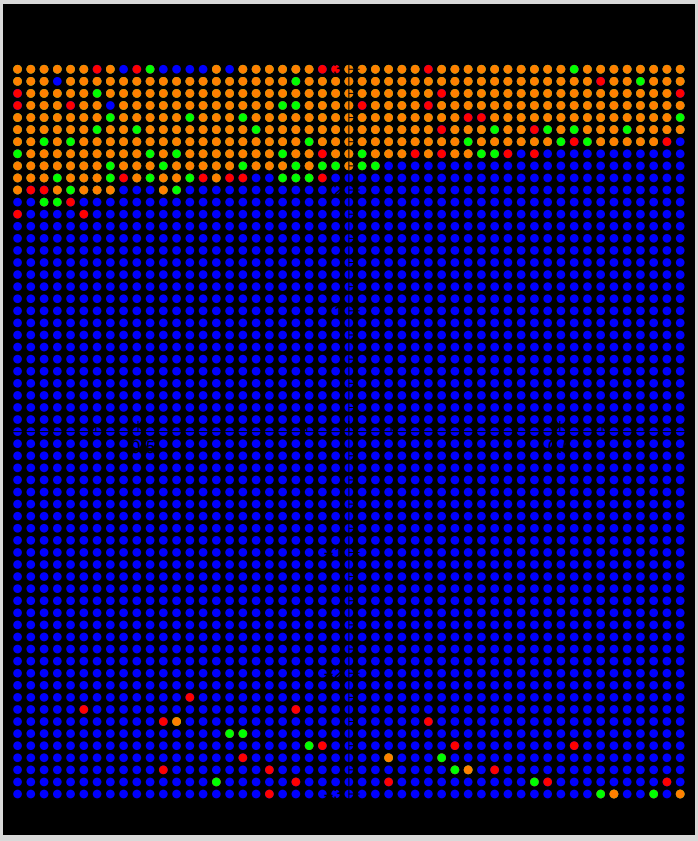


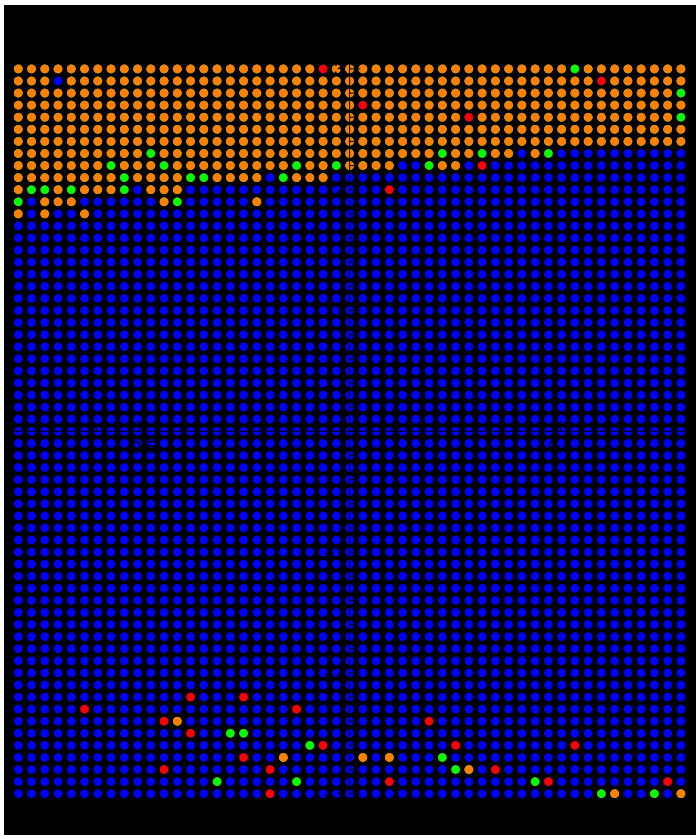




In[•]:=
```
Show[
 ListPlot[PHS, AspectRatio → Full,
  PlotStyle → {Blue, PointSize[Medium]},
  Axes → True,
  PlotLabel → "Phase space",
  PlotRange → All],
 ListPlot[firstRdata, PlotStyle → {Red, PointSize[Medium]}],

 ListPlot[secondRdata, PlotStyle → {Green, PointSize[Medium]}],

 ListPlot[thirdRdata, PlotStyle → {Orange, PointSize[Medium]}]
 , Background → Black]
```

Out[•]=





Step 4

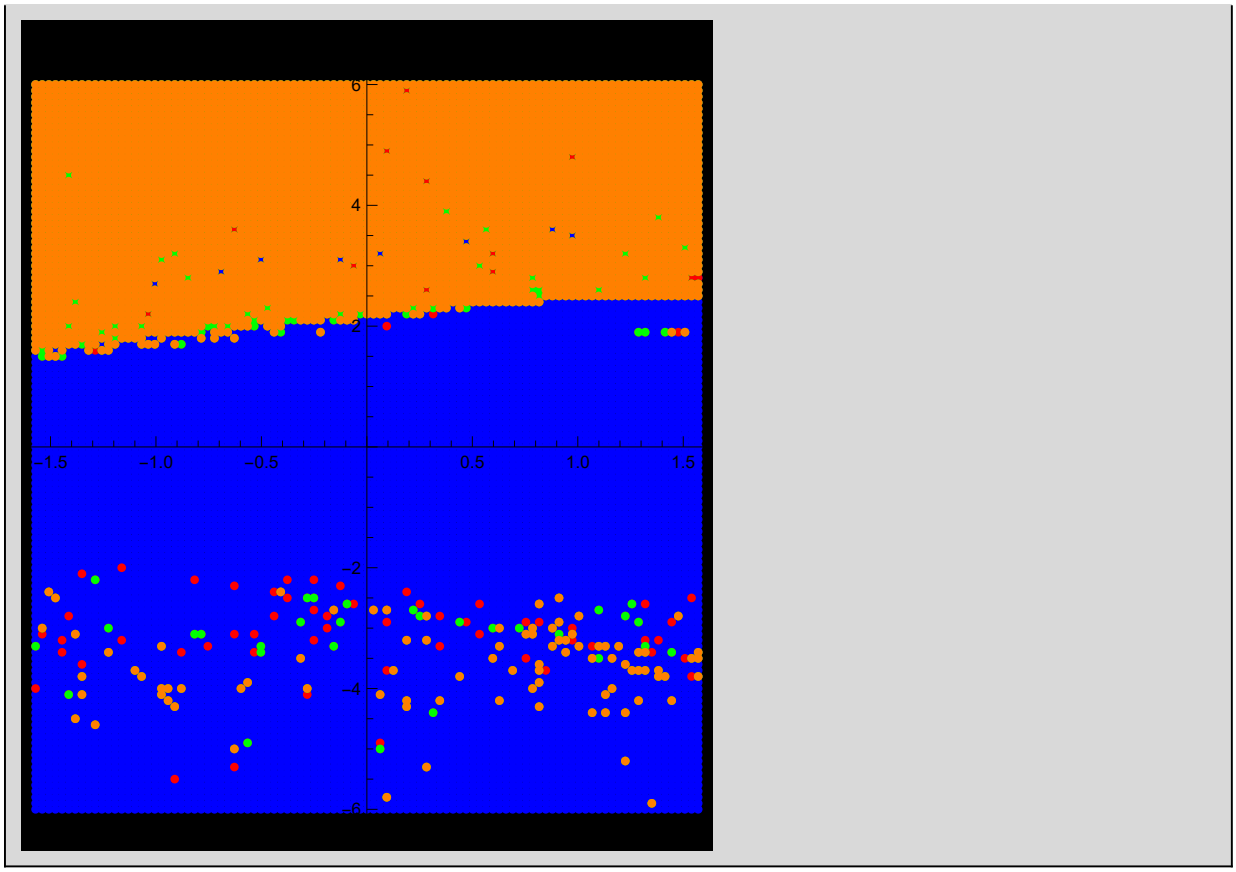
In[ ]:=
```
Quit
```

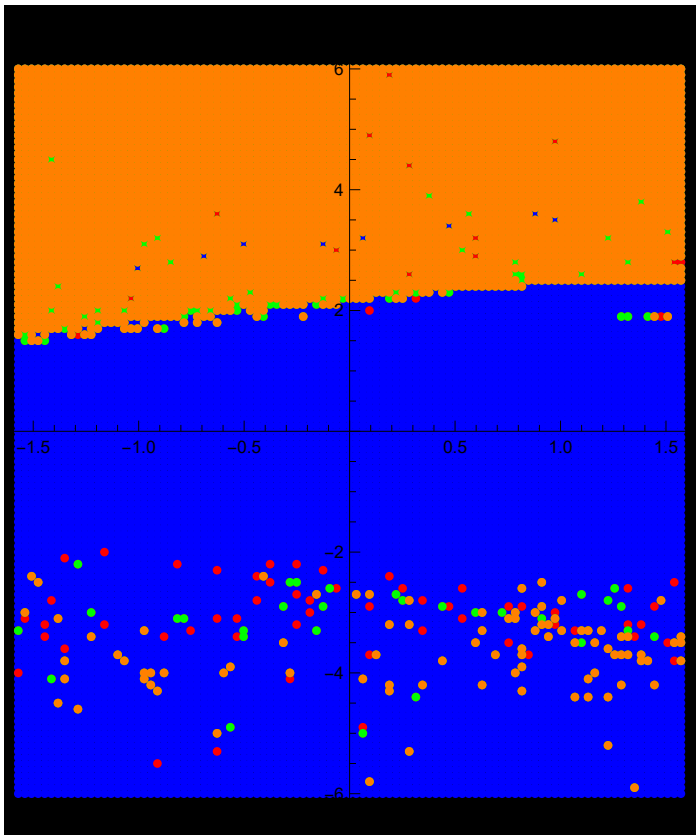
In[ ]:=
```
g := 9.8
l := 1(*Pendulum arm length*)
r := 1(*radius of disk*)
tmax := 50(*time range*)
ωstart := -6
ωend := 6
ωstep := 0.1
θstart := -π/2
θend := π/2
θstep := π/100

firstRdata := {}
secondRdata := {}
thirdRdata := {}
```

```
xy = Table[{θ0, ω}, {θ0, θstart, θend, θstep}, {ω, ωstart, ωend, ωstep}];

lenθ := (θend - θstart)/θstep + 1
lenω := (ωend - ωstart)/ωstep + 1

PHS := {}
Do[{
  Do[{
    data = xy[[i]][[j]]
    , AppendTo[PHS, data]}
   , {j, 1, lenω}]}
 , {i, 1, lenθ}]

firstRdata := {}
Do[{
  Clear[θ],
  theta = PHS[[α]][[1]];

  ω = PHS[[α]][[2]];

  eqf1 = θ[t] /. Flatten[NDSolve[{θ''[t] + g/l * Sin[θ[t]] - r * ω^2/l Cos[θ[t] - ω * t] == 0,
       θ[0] == theta, θ'[0] == 0}, θ[t], {t, 0, tmax}]];
  Do[{
    If[(eqf1 /. t → t1) > π, {AppendTo[firstRdata, {theta, ω}], Break[]}, Null]
   }, {t1, 0, tmax}]

 }, {α, 1, Length@PHS}]

Do[{
  Clear[θ],
  theta = PHS[[α]][[1]];

  ω = PHS[[α]][[2]];

  eqf1 = θ[t] /. Flatten[NDSolve[{θ''[t] + g/l * Sin[θ[t]] - r * ω^2/l Cos[θ[t] - ω * t] == 0,
       θ[0] == theta, θ'[0] == 0}, θ[t], {t, 0, tmax}]];
  Do[{
    If[(eqf1 /. t → t1) > 3 * π, {AppendTo[secondRdata, {theta, ω}], Break[]}, Null]
   }, {t1, 0, tmax}]
```

```
 }, {α, 1, Length@PHS}]

Do[{
  Clear[θ],
  theta = PHS[[α]][[1]];

  ω = PHS[[α]][[2]];

  eqf1 = θ[t] /. Flatten[NDSolve[{θ''[t] + g/l * Sin[θ[t]] - r * ω^2/l Cos[θ[t] - ω * t] == 0,
       θ[0] == theta, θ'[0] == 0}, θ[t], {t, 0, tmax}]];
  Do[{
    If[(eqf1 /. t → t1) > 5 * π, {AppendTo[thirdRdata, {theta, ω}], Break[]}, Null]
   }, {t1, 0, tmax}]

 }, {α, 1, Length@PHS}]

Show[
 ListPlot[PHS, AspectRatio → Full,
  PlotStyle → {Blue, PointSize[Medium]},
  Axes → True,
  PlotLabel → "Phase space",
  PlotRange → All],
 ListPlot[firstRdata, PlotStyle → {Red, PointSize[Medium]}],

 ListPlot[secondRdata, PlotStyle → {Green, PointSize[Medium]}],

 ListPlot[thirdRdata, PlotStyle → {Orange, PointSize[Medium]}]
 , Background → Black]
```

Out[ ]=




6
4
2
−2
−4
−6
−1.5
−1.0
−0.5
0.5
1.0
1.5