Roteiro 3a - Transformadas de Laplace

1. a) $\mathcal{L}[f(t)] = F(s) = \int_0^\infty f(t)\, e^{-st}\, dt$

$$F(s) = \int_0^\infty e^{-st}\, dt$$

$$F(s) = -\frac{1}{s} e^{-st}\Big|_0^\infty = \frac{1}{s}$$

b) $f(t) = t\,u(t)$

$$\mathcal{L}[f(t)] = F(s) = \int_0^\infty t\, e^{-st}\, dt$$

$\int u\,dv = u.v - \int v\,du$

$u = t \qquad dv = e^{-st}$

$du = 1dt$

$v = -\frac{1}{s} e^{-st}$

$$= \frac{-t}{s} e^{-st}\Big|_0^\infty - \int_0^\infty -\frac{1}{s} e^{-st}\, dt$$

$$= \frac{-t}{s} e^{-st} - \frac{1}{s^2} e^{-st}\Big|_0^\infty$$

$$= \frac{e^{-st}}{s^2}(-st - 1)\Big|_0^\infty$$

$$= \frac{-(st+1)}{s^2 e^{st}}\Big|_0^\infty = \cancel{\frac{-(st+1)}{s^2 e^{st}}\Big|_{t\to\infty}}^{\,0} - \left(\frac{-(st+1)}{s^2 e^{st}}\right)\Big|_{t\to 0}$$

$$F(s) = \frac{1}{s^2}$$

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c) $F(s) = \int_0^{\infty} \operatorname{sen} \omega t \, e^{-st} \, dt = \frac{e^{-st}}{s^2+\omega^2}(-s \operatorname{sen} \omega t - \omega \cos \omega t)\Big|_0^{\infty}$

$F(s) = \frac{\omega}{s^2+\omega^2}$

d) $F(s) = \int_0^{\infty} \cos \omega t \, e^{-st} \, dt = \frac{e^{-st}}{s^2+\omega^2}(-s \cos \omega t + \omega \operatorname{sen} \omega t)\Big|_0^{\infty}$

$F(s) = \frac{s}{s^2+\omega^2}$

2. a) $e^{-at} \operatorname{sen} \omega t \; u(t)$

$F(s) = \mathcal{L}\{e^{-at} \operatorname{sen} \omega t \; u(t)\} = \frac{\omega}{(s+a)^2 + \omega^2}$

b) $e^{-at} \cos \omega t \; u(t)$

$F(s) = \mathcal{L}\{e^{-at} \cos \omega t \; u(t)\} = \frac{(s+a)}{(s+a)^2 + \omega^2}$

c) $t^3 \, u(t)$

$F(s) = \mathcal{L}\{t^3 \, u(t)\} = \frac{3!}{s^4} = \frac{6}{s^4}$

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8. $\frac{d^3y}{dt^3} + 3\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + y = \frac{d^3x}{dt^3} + 4\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 8x$

$s^3Y(s) + 3s^2Y(s) + 5sY(s) + Y(s) = s^3X(s) + 4s^2X(s) + 6sX(s) + 8X(s)$

$Y(s)(s^3+3s^2+5s+1) = X(s)(s^3+4s^2+6s+8)$

$$\frac{Y(s)}{X(s)} = \frac{s^3+4s^2+6s+8}{s^3+3s^2+5s+1}$$

9. a) $\frac{X(s)}{F(s)} = \frac{7}{s^2+5s+10}$

$X(s)(s^2+5s+10) = F(s)7$

$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 10x = 7f$

b) $\frac{X(s)}{F(s)} = \frac{15}{(s+10)(s+11)} \Rightarrow (s^2+21s+110)X(s) = 15F(s)$

Aplicando a transformada inversa de Laplace

$\frac{d^2x}{dt^2} + 21\frac{dx}{dt} + 110x = 15f$

c) $\frac{X(s)}{F(s)} = \frac{s+3}{s^3+11s^2+12s+18} \Rightarrow (s^3+11s^2+12s+18)X(s) = (s+3)F(s)$

Aplicando a transmormada inversa de Laplace

$\frac{d^3x}{dt^3} + 11\frac{d^2x}{dt^2} + 12\frac{dx}{dt} + 18x = \frac{df}{dt} + 3f$

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10. $$\frac{C(s)}{R(s)} = \frac{s^5 + 2s^4 + 4s^3 + s^2 + 4}{s^6 + 7s^5 + 3s^4 + 2s^3 + s^2 + 5}$$

$$\Rightarrow C(s)(s^6 + 7s^5 + 3s^4 + 2s^3 + s^2 + 5) = R(s)(s^5 + 2s^4 + 4s^3 + s^2 + 4)$$

Aplicando a transformada inversa de Laplace

$$\frac{d^6c}{dt^6} + \frac{7d^5c}{dt^5} + \frac{3d^4c}{dt^4} + \frac{2d^3c}{dt^3} + \frac{d^2c}{dt^2} + 5c = \frac{d^5r}{dt^5} + \frac{2d^4r}{dt^4} + \frac{4d^3r}{dt^3} + \frac{d^2r}{dt^2} + 4r$$

11. $$\frac{C(s)}{R(s)} = \frac{s^4 + 3s^3 + 2s^2 + s + 1}{s^5 + 4s^4 + 3s^3 + 2s^2 + 3s + 2}$$

$$\Rightarrow C(s)(s^5 + 4s^4 + 3s^3 + 2s^2 + 3s + 2) = R(s)(s^4 + 3s^3 + 2s^2 + s + 1)$$

Aplicando a transformada inversa de Laplace

$$\frac{d^5c}{dt^5} + \frac{4d^4c}{dt^4} + \frac{3d^3c}{dt^3} + \frac{2d^2c}{dt^2} + \frac{3dc}{dt} + 2c = \frac{d^4r}{dt^4} + \frac{3d^3r}{dt^3} + \frac{2d^2r}{dt^2} + \frac{dr}{dt} + 1r$$

$$\frac{d^5c}{dt^5} + \frac{4d^4c}{dt^4} + \frac{3d^3c}{dt^3} + \frac{2d^2c}{dt^2} + \frac{3dc}{dt} + 2c = \quad u(t)(54 + 36t + 9t^2 + 3t^3)$$

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