

# Application of Matrices in Assignment Problem

**Introduction:** An assignment problem is an optimization problem in which a task has to be assigned to workers to minimize the cost and maximize the total profit. Every worker is assigned exactly one task.

It makes use of column matrix in which each element shows the cost required by assigning task ‘j’ to worker ‘I’.

## Real Life Applications:

- *Job scheduling*: giving projects to employees to minimize cost & maximize productivity
- *Transportation & logistics*: Assigning delivery routes to trucks to reduce fuel usage.
- *Education*: assigning specific teachers to students to improve the mentors
- *Airline crew assignment*: minimizing the crew idle time with the help of proper shift allocation.
- *Resource management in hospitals*: assigning doctors or nurses to specific department to improve the efficiency

## Hungarian algorithm:

### Objective function

$$\text{Minimize: } Z = \sum_{i=1}^n \sum_{j=1}^n c_{ij} x_{ij}$$

### Constraints:

$$\sum_{i=1}^n x_{ij} = 1 \text{ (for each job } j\text{)}$$

$$\sum_{j=1}^n x_{ij} = 1 \text{ (for each person } i\text{)}$$

### Example:

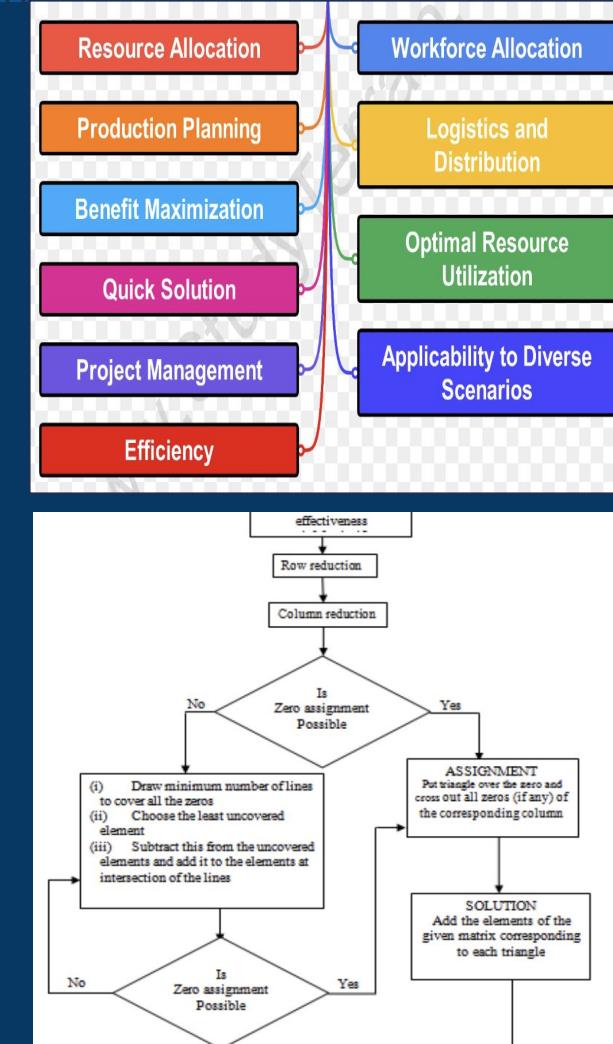
	Task 1	Task 2	Task 3
Worker A	8	15	13
Worker B	10	12	18
Worker C	14	8	11

**Objective:** Each worker should be assigned one task such that the total cost is minimum  
**Using the Hungarian algorithm given below we get the result for the minimum cost as follows:**

- A □ Task 1
- B □ Task 2
- C □ Task 3

This results in minimum cost:  $8+12+11=31$

$$\begin{array}{ccccc} \text{Step 1} & & \text{Step 2} & & \\ \left[ \begin{array}{ccc} 0 & 1 & 2 \\ 0 & 2 & 4 \\ 0 & 3 & 6 \end{array} \right] & & \left[ \begin{array}{ccc} 0 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 2 & 4 \end{array} \right] & & \\ \text{Step 3} & & \text{Step 4} & & \\ \left[ \begin{array}{ccc} 0 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 2 & 4 \end{array} \right] & & \left[ \begin{array}{ccc} 2 & 1 & 1 \\ 1 & 1 & 2 \\ 1 & 2 & 4 \end{array} \right] & & \\ \text{Step 5} & & \text{Step 6} & & \\ \left[ \begin{array}{ccc} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 3 \end{array} \right] & & \left[ \begin{array}{ccc} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 3 \end{array} \right] & & \end{array}$$



### References:

- 1) <https://onlinelibrary.wiley.com/doi/abs/10.1002/nav.3800020109>
- 2) <https://dl.acm.org/doi/10.1145/355873.355883>
- 3) <https://egyankosh.ac.in/bitstream/123456789/20790/1/Unit-5.pdf>
- 4) <https://cp-algorithms.com/graph/hungarian-algorithm.html>
- 5) [https://www.researchgate.net/figure/Flowchart-of-the-Hungarian-algorithm-fig3\\_61815026](https://www.researchgate.net/figure/Flowchart-of-the-Hungarian-algorithm-fig3_61815026)