Qubit operations

qubit : base states 0 and 1

0 and 1 can be spin ↑ ↓

ground / state

polarization

magnetization

"Ket"

state 0 $\equiv |0\rangle$ state 1 $\equiv |1\rangle$

vector

↕

Dirac notations

matrix

$|0\rangle \equiv \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $|1\rangle \equiv \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$\bar{V} \equiv \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix}$

$|0\rangle \longrightarrow$ transform to $\langle 0|$

$\uparrow$

conjugate $|0\rangle$

Let any qubit $|q\rangle \longrightarrow |q\rangle^{*T}$ transpose

complex conjugate

$*T \equiv \dagger$ dagger

$$|q\rangle^{\dagger} = \langle q|$$

$$|0\rangle^{\dagger} \equiv \begin{bmatrix} 1 \\ 0 \end{bmatrix}^{*T} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}^{T} = \begin{bmatrix} 1 & 0 \end{bmatrix} = \langle 0|$$

$$\text{let } |\Sigma\rangle = \begin{bmatrix} i & -i \\ 0 & 1 \end{bmatrix} \quad \text{find } \langle\Sigma|$$

$$\xrightarrow[\text{conj}]{\text{com}} \begin{bmatrix} -i & i \\ 0 & 1 \end{bmatrix} \xrightarrow[\text{trans}]{} \begin{bmatrix} -i & 0 \\ i & 1 \end{bmatrix} = \langle\Sigma|$$

Qubit addition/subtraction

$$|\Sigma_1\rangle \pm |\Sigma_2\rangle = ? \quad \text{let } |\Sigma_1\rangle = |\Sigma_2\rangle = |0\rangle$$

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$|\Sigma_1\rangle + |\Sigma_2\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ 0 \end{bmatrix} = 2\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$|\Sigma_1\rangle = |1\rangle, \ |\Sigma_2\rangle = |0\rangle \quad \begin{bmatrix} 0 \\ 1 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

K

Qubit multiplication

(i) Scalar product : $|q\rangle$ and $|p\rangle$ → take conjugate

↳ $\langle p|q\rangle$

$|q\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $|p\rangle = |0\rangle \equiv \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$|0\rangle$ $\quad$ $p|$

$|0\rangle^{+}$ $+ = * \, T$ $\quad \langle 0|$

$\downarrow \begin{bmatrix} 1 & 0 \end{bmatrix}$

$$\langle p|q\rangle = \underset{1\times 2}{\begin{bmatrix} 1 & 0 \end{bmatrix}} \underset{2\times 1}{\begin{bmatrix} 1 \\ 0 \end{bmatrix}} = 1 \checkmark$$

number

find the T.T. (truth table)

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}^{+} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

$\vert q\rangle$	$\vert p\rangle$	O/P
$\vert 0\rangle$	$\vert 0\rangle$	1
$\vert 0\rangle$	$\vert 1\rangle$	?
$\vert 1\rangle$	$\vert 0\rangle$	
$\vert 1\rangle$	$\vert 1\rangle$	

H.W

$$|q\rangle^{+} \equiv \langle q|$$

$$\langle q|^{+} \equiv |q\rangle$$

$\underline{|q\rangle}$ $\underline{|p\rangle}$ $\Rightarrow$ state vector

$\langle q|$ $\langle p|$ $\Rightarrow$ conjugate state

$+ = * + T.$

vector product

scale

$\langle p | q \rangle$

$|q\rangle \langle p| \rightarrow$ different answer

non-commutative property ;

$AB \neq BA \qquad \langle p|q\rangle \neq |q\rangle\langle p|$

$$|q\rangle = |0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$|p\rangle = |0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \; ; \quad \langle p| = \langle 0| = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$|q\rangle\langle p| = \underbrace{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}_{2\times 1} \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{1\times 2} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}_{2\times 2}$$

matrix of higher order

p

H.W. find T.T. $|s\rangle \; \langle p| \; \begin{bmatrix} 1 & 0 \\ 0 & d \end{bmatrix}$

$\lvert s\rangle$	$\lvert p\rangle$	
$\lvert 0\rangle$	$\lvert 0\rangle$	?
$\lvert 0\rangle$	$\lvert 1\rangle$	
$\lvert 1\rangle$	$\lvert 0\rangle$	
$\lvert 1\rangle$	$\lvert 1\rangle$	

Tensor product: $|\varepsilon\rangle|\rho\rangle$

let $|\varepsilon\rangle = |\rho\rangle = |0\rangle$

$$|\varepsilon\rangle|\rho\rangle = \underbrace{\begin{array}{cc} \begin{bmatrix} 1 \\ 0 \end{bmatrix} & \begin{array}{c} \varepsilon_1 \\ \varepsilon_2 \end{array} \end{array}}_{|\varepsilon\rangle} \underbrace{\begin{array}{cc} \begin{bmatrix} 1 \\ 0 \end{bmatrix} & \begin{array}{c} p_1 \\ p_2 \end{array} \end{array}}_{|\rho\rangle} = \begin{array}{cc} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} & \begin{array}{c} \varepsilon_1 p_1 \\ \varepsilon_1 p_2 \\ \varepsilon_2 p_1 \\ \varepsilon_2 p_2 \end{array} \end{array}$$

4×1

HW

$|\varepsilon\rangle \quad |p\rangle \quad 0/1$

$|0\rangle \quad |0\rangle$

$\vdots \qquad\qquad ?$

$|1\rangle \quad |1\rangle$

1) Superposed state $|\psi\rangle$

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

α, β = arbitrary const.

2) probability of computation $|\psi\rangle$ can be found in state $|0\rangle$ after computation?

$$P(|\psi\rangle = |0\rangle) = |\langle 0|\psi\rangle|^2$$

$$\left|\langle 0|(\alpha|0\rangle + \beta|1\rangle)\right|^2 = \left|\alpha\underbrace{\langle 0|0\rangle}_{1} + \beta\underbrace{\langle 0|1\rangle}_{0}\right|^2$$

$$= |\alpha|^2$$

prob $|\psi\rangle$ in $|1\rangle \longleftrightarrow |\beta|^2$

total prob $= |\alpha|^2 + |\beta|^2 = 1$

in special case $\alpha = \beta$

$$2|\alpha|^2 = 1 \implies \alpha = \pm \frac{1}{\sqrt{2}}$$

let $\alpha = \frac{1}{\sqrt{2}}$

$|0\rangle$ and $|1\rangle \longrightarrow$ combine

$\longrightarrow \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \equiv |+\rangle \rightarrow$ special superposed state

$\longrightarrow \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \equiv |-\rangle \rightarrow$

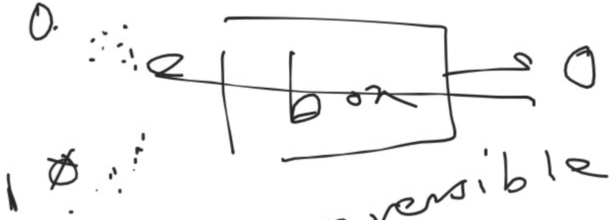
logic gates

AND, OR, NAND, NOR, XOR, NOT

b	o/p
1	0
0	1

bit flip

		AND
0	0	0
0	1	0
1	0	0
1	1	1



non-reversible

Quantum logic gates

Pauli-X gate σ_x : Quantum analogue of NOT gate

$$\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\sigma_x |0\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \longrightarrow |1\rangle$$

$$\sigma_x |1\rangle \longrightarrow |0\rangle$$

$|+\rangle$

$$\sigma_x |+\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$|+\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right) = \frac{1}{\sqrt{2}}\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}\right)$$

$$|+\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\sigma_x |+\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} \cdot \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$|+\rangle$$

$$|+\rangle = ?$$

σ_z : Pauli-Z gate

$$\sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\sigma_z |+\rangle = ? \quad \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = ?$$

$$\longrightarrow |-\rangle$$

$$|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) = \frac{1}{\sqrt{2}}\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\sigma_z |0\rangle = ?$$

$$\sigma_z |1\rangle = ?$$

Imp : How do you produce a superposed state?

'Hadamard' $H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

check $H \equiv \frac{1}{\sqrt{2}} \left(|+\rangle + |-\rangle \right)$

$$H|0\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = |+\rangle$$

superposed state

$H|+\rangle = ?$

$$\underline{\underline{|0\rangle}} \longrightarrow \left[\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)\right]$$

$|1\rangle$ excited stat

$|0\rangle$ - ground

physical meaning ?

$$H|+\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix} \cdot \frac{1}{\sqrt{2}}\begin{bmatrix}1\\ 1\end{bmatrix}$$

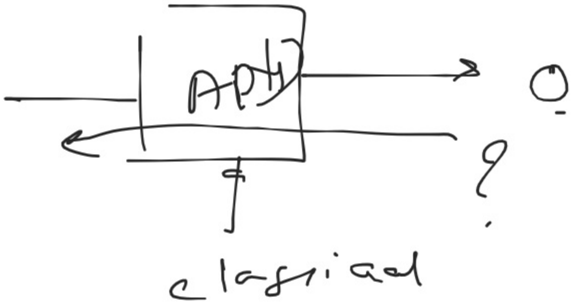
$$= \frac{1}{2}\begin{bmatrix}2\\ 0\end{bmatrix} = \frac{2}{2}\begin{bmatrix}1\\ 0\end{bmatrix}$$

$$= \begin{bmatrix}1\\ 0\end{bmatrix}$$

$$\downarrow$$

$$|0\rangle$$

$$H|1\rangle \to |-\rangle$$



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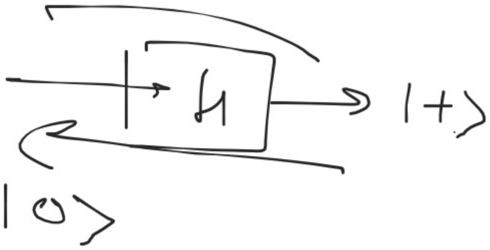

C-NOT gate

2-qubit gate

$|q_1 q_2\rangle$

q_1 → control bit

q_2 → data bit

q_1	q_2	O/P
0	0	0
0	1	1
1	0	1
1	1	0

→ XOR

operation : if $q_1 = 1$, flip data bit

if $q_1 = 0$, don't change data bit

Quantum Ckt

a combination of different Quantum logic gates to perform a computational task