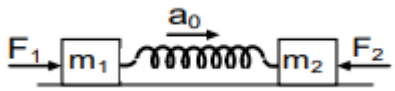
PHYSICS

SECTION-A

Question Number – 1

Two blocks m_1 and m_2 are connected with a compressed spring and placed on a smooth horizontal surface as shown in figure.



Force constant of spring is k. Under the influence of forces F_1 and F_2 at an instant blocks move with common acceleration a_0 At that instant force F_2 is suddenly withdrawn. Mark correct option.

(a) Instantaneous acceleration of m_2 is $a_0 + \frac{F_1}{m_1}$

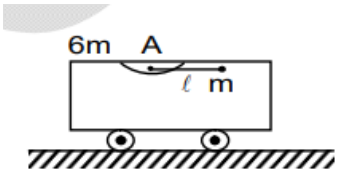
(b) Instantaneous acceleration of m_2 is $a_0 + \frac{F_2}{m_2}$

(c) Instantaneous acceleration of m_1 is $a_1 = 0$

(d) Instantaneous acceleration of m_2 is $a_2 = 0$

Question Number – 2

In the figure shown the cart of mass 6 m is initially at rest. A particle of mass m is attached to the end of the light rod which can rotate freely about A. If the rod is released form rest in a horizontal position shown, determine the velocity V_{rel} of the particle with respect to the cart when the rod is vertical. Assume friction less surface.



(a) $\sqrt{\frac{7}{3}gl}$

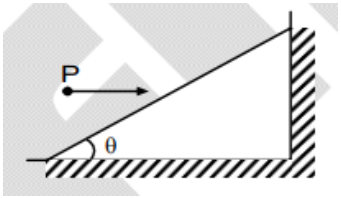
(b) $\sqrt{\frac{7}{6}gl}$

(c) $\sqrt{\frac{14}{3}gl}$

(d) $\sqrt{\frac{8}{3}gl}$

Question Number – 3

In the figure shown a particle P strikes the inclined fixed smooth plane horizontally and rebounds vertically. If the angle θ is 60°, then the coefficient of restitution is:



(a) 1/3

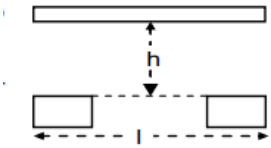
(b) $1\sqrt{3}$

(c) 1/2

(d) 1

Question Number – 4

A uniform horizontal rod of length l falls vertically from height h on two identical blocks placed symmetrically below the rod as shown in figure. The coefficients of restitution are e_1 and e_2. The maximum height through which the centre of mass of the rod will rise after bouncing off the blocks is



(a) $\frac{h}{(e_1+e_2)}$

(b) $\frac{(e_1+e_2)^2 h}{4}$

(c) $\frac{(e_1+e_2)^2 h}{2}$

(d) $\frac{4h}{(e_1^2+e_2^2)}$

Question Number – 5

The displacement of a particle is represented by the equation $y = \sin^3 \omega t$. The motion is

(a) Non-periodic

(b) Periodic but not simple harmonic

(c) Simple harmonic with period $2\pi/\omega$

(d) Simple harmonic with period π/ω

Question Number – 6

A steel ring of radius r and cross-section area A is fitted on to a wooden disc of radius R(R>r). If Young's modulus be E, then the force with which the steel ring is expanded is:

(a) $AR\frac{R}{r}$

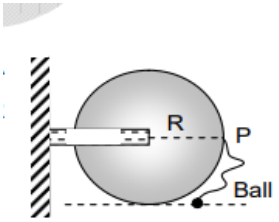
(b) $AE\left(\frac{R-r}{r}\right)$

(c) $\frac{E}{A}\left(\frac{R-r}{A}\right)$

(d) $\frac{Er}{AR}$

Question Number – 7

Figure shows a disc of mass M and radius R hinged at the centre. A small ball of mass $\frac{M}{2}$ is attached to point P with a thread of length 2R and held at rest at position shown. Now, the ball is released to fall under gravity with what angular speed the disc starts turning when the string becomes taut?



(a) $\sqrt{\frac{g}{2R}}$

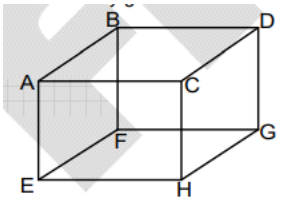
(b) $\sqrt{\frac{g}{R}}$

(c) $\sqrt{\frac{R}{g}}$

(d) $\sqrt{\frac{2g}{R}}$

Question Number – 8

1 mole of an ideal gas is contained in a cubical volume V, ABCDEFGH at 300 K in figure. One face of the cube (EFGH) is made up of a material which totally absorbs any gas molecule incident on it. At any given time



(a) The pressure on EFGH would be zero

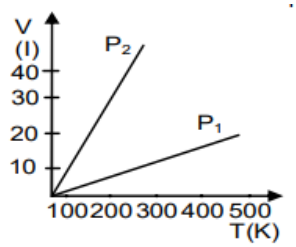
(b) The pressure on all the faces will be equal

(c) The pressure of EFGH would be double the pressure on ABCD

(d) The pressure on EFGH would be half that on ABCD

Question Number – 9

Volume versus temperature graphs for a given mass of an ideal gas are shown in figure at two different values of constant pressure. What can be inferred about relation between P_1 & P_2?



(a) $P_1 > P_2$

(b) $P_1 = P_2$

(c) $P_1 < P_2$

(d) Data is insufficient

Question Number – 10

A positively charged particle is released from rest in an uniform electric field. The electric potential energy of the charge

(a) Remains a constant because the electric field is uniform

(b) Increases because the charge moves along the electric field

(c) Decreases because the charge moves along the electric field

(d) Decreases because the charge moves opposite to the electric field

Question Number – 11

A hollow conducting sphere of inner radius R and outer radius 2R has resistivity $'\rho'$ a function of the distance 'r' from the centre of the sphere: $\rho = kr^2/R$. The inner and outer surfaces are painted with a perfectly conducting 'paint and a potential difference ΔV is applied between the two surfaces. Then as 'r' increases from R to 2R, the electric field inside the sphere

(a) Increases

(b) Decreases

(c) Remains constant

(d) Passes through a maxima

Question Number – 12

A galvanometer shows a reading of 0.65 mA. When a galvanometer is shunted with a 4Ω resistance, the deflection is reduced to 0.13 mA. If the resistance of 2Ω is further connected parallel to 4Ω resistance (ie. galvanometer is further shunted with a 2Ω wire) the new reading will be (Assume main current remains the same)

(a) 0.60 mA

(b) 0.8 mA

(c) 0.12 mA

(d) 0.05 mA

Question Number – 13

A soap bubble of radius R is blown. After heating the solution a second bubble of radius 2R is blown. The work required to blow the second bubble in comparison to that required for the first bubble is:

(a) Double

(b) Slightly less than double

(c) Slightly less than four times

(d) Slightly more than four times

Question Number – 14

In an experiment to measure the length of a rod by four different instruments, the measurement is reported as

(i) 200.0 cm

(ii) 20 cm

(iii) 20.00 cm

(iv) 0.200 cm

From the above data, we can infer that

(a) All measurements have same accuracy

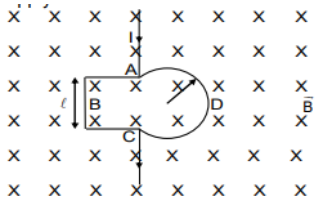
(b) A has least accuracy

(c) B has least accuracy

(d) D has least accuracy

Question Number – 15

The figure shown a uniform conducting loop ABCDA placed in a uniform magnetic field perpendicular to its plane. The part ABC is the $(3/4)^{\text{th}}$ portion of the square of side length ℓ. The part ADC is a circular arc of radius R. The points A and C are connected to a battery which supply a current l to the circuit. The magnetic force on the loop due to the field B is



XBX

(a) Zero

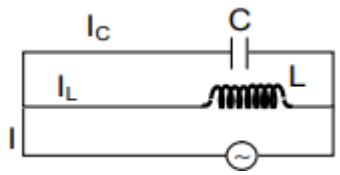
(b) $Bl\ell$

(c) 2BlR

(d) $\frac{Bl\ell R}{l+R}$

Question Number – 16

In the figure, if $l_L = 0.8A, l_C = 0.6A$, then l=? (given currents are rms values)



(a) 0.4 A

(b) 0.2 A

(c) 1.0 A

(d) 1.4 A

Question Number – 17

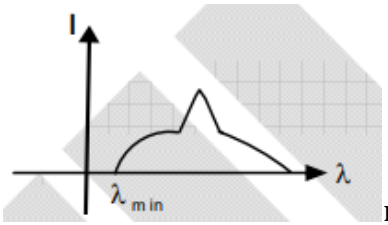
An x-ray tube has three main controls.

(i) The target material (its atomic number Z)

(ii) The filament current (l_f) and

(iii) The acceleration voltage (V)

Figure shows a typical intensity distribution against wavelength. Which of the following is incorrect?



min

(a) The limit λ_{min} is proportional to V^{-1}

(b) The sharp peak shifts to the right as Z is increased

(c) The penetrating power of X ray increases if V increased

(d) The intensity everywhere increases if filament current l_f is increased

Question Number – 18

Which of the following statement is correct in connection with hydrogen spectrum?

(a) The longest wavelength in the Balmer series is longer than the longest wavelength in Lyman series

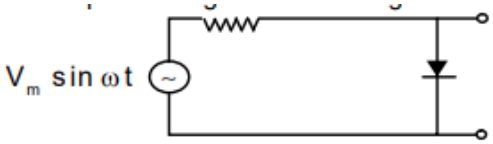
(b) The shortest wavelength in the Balmer series is shorter than the shortest wavelength in the Lyman series

(c) The longest wavelength in both Balmer and Lyman series are equal

(d) The longest wavelength in Balmer series is shorter than the longest wavelength in the Lyman series.

Question Number – 19

The output of the given circuit in figure



(a) Would be zero at all times

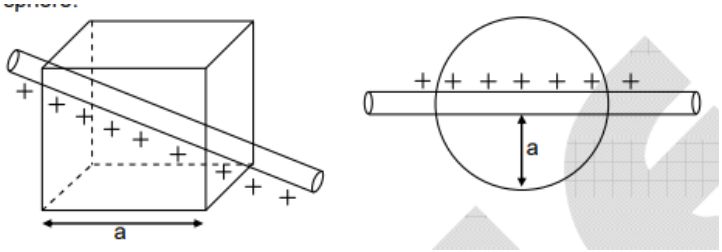
(b) Would be like a half wave rectifier with positive cycles output

(c) Would be like a half wave rectifier with negative cycles output

(d) Would be like that of a full wave rectifier

Question Number – 20

A linear charge having linear charge density λ penetrates a cube diagonally and then it penetrates a sphere diametrically as shown. What will be the ratio of flux coming cut of cube and sphere.



(a) $\frac{1}{2}$

(b) $\frac{2}{\sqrt{3}}$

(c) $\frac{\sqrt{3}}{2}$

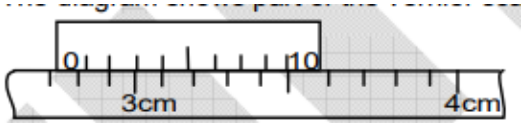
(d) $\frac{1}{1}$

SECTION-B

(Numerical Answer Type)

Question Number – 21

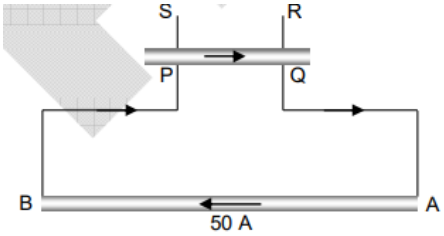
The diagram shows part of the vernier scale on a pair of callipers.



If the correct reading is "n" cm. Then the volume of "100n" is _____.

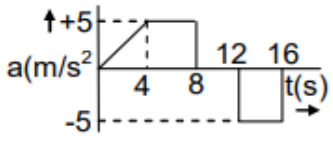
Question Number – 22

A long wire AB is placed on a table. Another wire PQ of mass 1.0 g and length 50 cm is set to slide on two rails PS and QR. A current of 50 A is passed through the wires. At what distance above AB, will the wire PQ be in equilibrium (in mm):



Question Number – 23

The acceleration of a train between two stations 2 km apart is shown in the figure. The maximum speed of the train is _____ (m/s)



↑+5+

Question Number – 24

A ray of light incident at an angle θ on a refracting face of a prism emerges from the other face normally. If the angle of the prism is 5° and the prism is made of a material of refractive index 1.5 the value of 10 θ is (in degree)

Question Number – 25

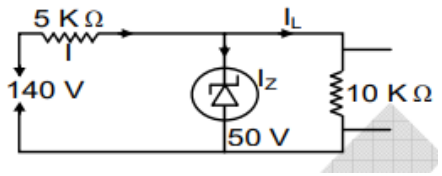
If the light of wavelength of maximum intensity emitted from surface at temperature T_1 is used to cause photoelectric emission from a metallic surface, the maximum kinetic energy of the emitted electron is 6 eV, which is 3 times the work function of the metallic surface. If light of wavelength of maximum intensity emitted from a surface at temperature $T_2 (T_2 = 2T_1)$ is used, the maximum kinetic energy of the photoelectrons emitted is

Question Number – 26

The Q-factor of a series LCR circuit with $L = 3\text{mH}, C = 2.7\ \mu\text{F}, R = \frac{10}{3}\ \Omega$, _____

Question Number – 27

The Zener current in the circuit is _____ (in mA)



Question Number – 28

One mole of ideal gas whose adiabatic exponent $\gamma = \frac{4}{3}$ undergoes process $P = 200 + \frac{1}{V}$, change in internal energy of gas when volume changes from $2m^3$ to $4m^3$ is (in Joules)

Question Number – 29

A container filled with liquid of density 10^3 kg/m^2 is kept on horizontal surface. An orifice of area 10 cm^2 is made in the wall of container at a distance 100 cm below the free surface of liquid. Find the thrust on the container at the instant when height of liquid is 50 cm above the orifice is _____ (in newton's) (Assuming container is at rest and area of orifice is very small than the area of the container) Take $g = 10$ m/s^2

Question Number – 30

An object is placed in front of a convex mirror at a distance of 50 cm. A plane mirror is introduced covering the lower half of the convex mirror. If the distance between the object and plane mirror is 30 cm, it is found that there is no parallax between the images formed by two mirrors. Radius of curvature of mirror will be: (in cm)

CHEMISTRY

SECTION-A

Question Number – 31

If we accelerate α-partide through a potential difference of V volts. The de-Broglie wave length associated with it is:

(a) $\sqrt{150.V^{-1}}$ A°

(b) $0.286.V^{-\frac{1}{2}}$A°

(c) $\frac{0.101}{\sqrt{V}}$A°

(d) $\frac{0.983}{\sqrt{V}}$A°

Question Number – 32

A certain quantity of a gas occupies 100 mL when collected over water at 15°C and 750 mm pressure. It occupies 91.9 mL in dry state at NTP. Find the aqueous vapour pressure at 15°C?

(a) 15.7 mm

(b) 7.9 mm

(c) 736.8 mm

(d) 13.2 mm

Question Number – 33

For the reversible reaction

$N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3\ (g)$

At 500°C, the value of K_P is 1.44×10^{-5} when partial pressure is measured in atmospheres. The corresponding value of K_C with concentration in mole litre^{-1}, is

(a) $\frac{1.44\times10^{-5}}{(0.082\times500)^{-2}}$

(b) $\frac{1.44\times10^{-5}}{(8.314\times773)^{-2}}$

(c) $\frac{1.44\times10^{-5}}{(0.082\times773)^{2}}$

(d) $\frac{1.44\times10^{-5}}{(0.082\times773)^{-2}}$

Question Number – 34

Predict the order of Δ_o for the following compounds

I $[Mn(H_2O)_6]^{2+}$

II $[Mn(CN)_2(H_2O)_4]$

III $[Mn(CN)_2(H_2O)_2]^{2-}$

(a) $\Delta_o(I) < \Delta_o(II) < \Delta_o(III)$

(b) $\Delta_o(II) < \Delta_o(I) < \Delta_o(III)$

(c) $\Delta_o(III) < \Delta_o(II) < \Delta_o(I)$

(d) $\Delta_o(I) < \Delta_o(III) < \Delta_o(II)$

Question Number – 35

Match the items of List I and List II:

Reactions	**Unit of rate constant**

A.	Decomposition of N_2O on hot Pt.	(I)	$mol^{-2}L^2Time^{-2}$
B.	Hydrolysis of ester by acid	(II)	$molL^{-1}Time^{-1}$
C.	Alkaline hydrolysis of ester	(III)	$mol^{-1}LTime^{-1}$
D.	Reduction of $FeCl_3$ by $SnCl_2$	(IV)	$Time^{-1}$

(a) A-III, B-II, C-I, D-IV
(b) A-I, B-II, C-III, D-IV
(c) A-IV, B-II, C-I, D-III
(d) A-II, B-IV, C-III, D-I

Question Number – 36

The oxidations states of X, Y and Z are $+2$, $+5$ and -2 respectively. The possible formula of compound formed by them is:

(a) XYZ_2
(b) $X_3(Y_4Z)_2$
(c) $Y_2(XZ_3)_2$
(d) $X_3(YZ_4)_2$

Question Number – 37

The lP_1, lP_2, lP_3, lP_4, and lP_5, of an element are 7.1, 14.3, 34.5, 46.8 and 162.2 eV respectively. The element is likely to be

(a) Cl
(b) Na
(c) C
(d) Ba

Question Number – 38

Which option is **not** matched in correct sequence?

(a) $d^5, d^3, d^1, d^4 \longrightarrow$ dincreasing magnetic moment
(b) $MO, M_2O_3, MO_2, M_2O_5 \longrightarrow$ decreasing basic strength
(c) Sc, V, Cr, Mn $\longrightarrow$ increasing number of oxidation states
(d) $Co^{2+}, Fe^{+3}, Cr^{+3}, Sc^{+3} \longrightarrow$ increasing stability

Question Number – 39

$K_2[HgI_4]$ detects the ion/group

(a) NH_2

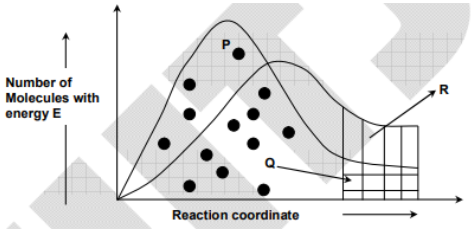
(b) NO

(c) NH_4^+

(d) Cl^-

Question Number – 40

The distribution of the number of molecules with energy E is given in the figure for two temperatures T_1 and a higher temperature T_2. The letters P, Q, R refer to the separate and differently shaded areas. The energy of activation on the energy axis



Which expression gives the fraction of the molecules present which have at least activation energy at the higher temperature T_2.

(a) $\frac{Q}{P}$

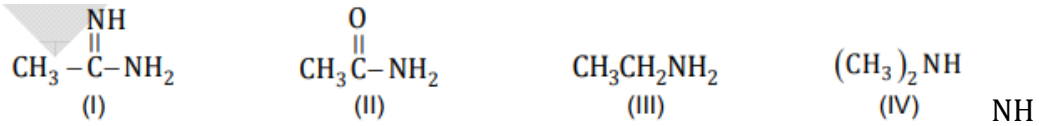
(b) $\frac{Q+R}{P+Q}$

(c) $\frac{Q+R}{P}$

(d) $\frac{Q+R}{P+Q-R}$

Question Number – 41

Arrange the following compounds in their decreasing basic strength in aq. Solution.



(a) IV > II > III > I

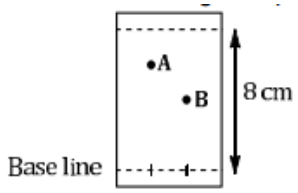
(b) II > I > IV > III

(c) I > IV > III > II

(d) IV > I > II > III

Question Number – 42

From the following TLC plate



if the R_f values of A and B are 0.75 and 0.5 respectively, then the difference in the height of A and B on TLC plate is

(a) 1 cm

(b) 1.5 cm

(c) 2 cm

(d) 2.5 cm

Question Number – 43

What is the order of reactivity of the alkenes

$(CH_3)_2C = CH_2(I), CH_3CH = CH_2$ (II) and $CH_2 = CH_2$ (III) when subject to acid-catalyzed hydration?

(a) I > II > III

(b) I > III > II

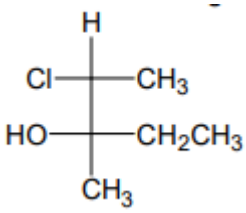
(c) III > II > I

(d) II > I > III

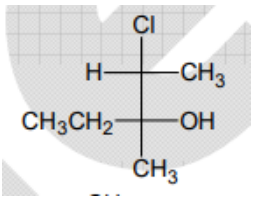
Question Number – 44

Which of the following structure are superimposable?

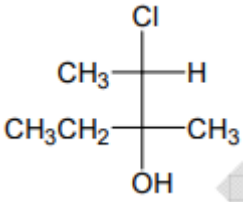
(a)

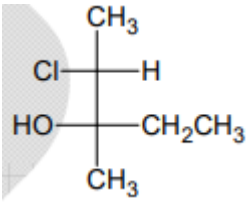


(b)



(c)



(d) 

(a) 1 and 2

(b) 2 and 3

(c) 1 and 4

(d) 1 and 3

Question Number – 45

In Williamson synthesis of mixed ether having a primary alcohol and a tertiary alkyl group if tertiary halide is used then:

(a) Rate of reaction will be slow due to slow cleavage of carbon-halogen bond

(b) Alkene will be the main product

(c) Simple ether will form instead of mixed ether

(d) Expected mixed ether will be formed

Question Number – 46

When HCl gas is passed through propene in the presence of benzoyl peroxide it gives

(a) n-propyl chloride

(b) 2-chloropropane

(c) allyl chloride

(d) no reaction

Question Number – 47

At 25°C, pK_b of NH_2 and pK_a of COOH are x and y respectively. Then, pl of the glycine is

(a) $\frac{x+y}{2}$

(b) $\frac{14+x+y}{2}$

(c) $\frac{14+x-y}{2}$

(d) $\frac{14-x+y}{2}$

Question Number – 48

The formation constant of complex HgI_4^{2-} at 25°C is

Given: $E°_{(Hg^{2+}/Hg)} = 0.85$ V and $E°_{(HgI_4^{2-}/Hg,I^-)} = -0.05$ V.

(a) $1.0 \times 10^{-26.66}$

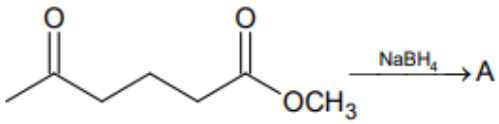
(b) 1.0×10^{60}

(c) $1.0 \times 10^{26.66}$

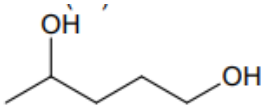
(d) 1.0×10^{30}

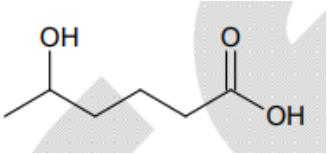
Question Number – 49

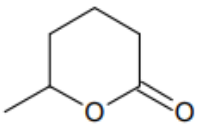
Consider the following reaction:

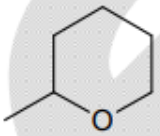


The product (A) is:

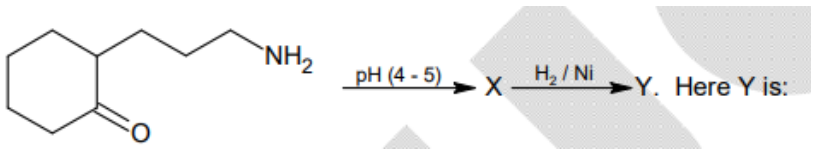
(a) 

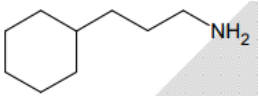
(b) 

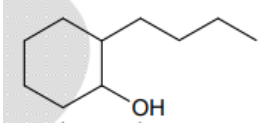
(c) 

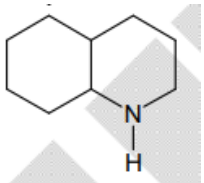
(d) 

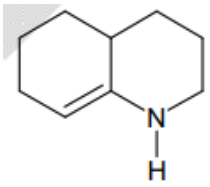
Question Number – 50



(a) 

(b) 

(c) 

(d) 

SECTION-B

Question Number – 51

Find out the change in pH of water (at 25°C) when a drop of 1 M HCl is dipped in 1 litre H_2O. (Given 100 drop = 1 mL)

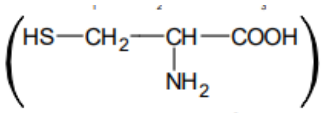
Question Number – 52

Given Reaction	Energy Change (in kJ)
$Li(s) \rightarrow Li(g)$	161
$Li(g) \rightarrow Li^+(g)$	520
$\frac{1}{2}F_2(g) \rightarrow F(g)$	77
$F(g) + e^- \rightarrow F^-(g)$	(Electron gain enthalpy)
$Li^+(g) + F^-(g) \rightarrow Li\,F(s)$	-1047
$Li(s) + \frac{1}{2}F_2(g) \rightarrow LiF(s)$	-617

Based on data provided the value of electron gain enthalpy (in kJ mol^{-1}) of fluorine is (neglecting sign):

Question Number – 53

The pK_{a_1}, pK_{a_2} and pK_{a_3} values for the amino acid cysteine are respectively 1.8, 8.0, 10.0.



The isoelectric point of cysteine amino acid is

Question Number – 54

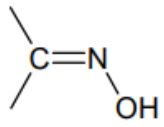
If formic acid, acetic acid, propanoic acid and benzoic acid is mixed with Phosphorus and Bromine then the number of products (including stereoisomer) formed are

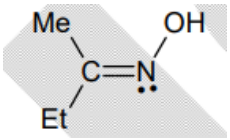
Question Number – 55

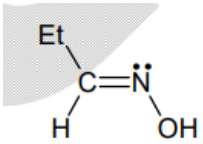
The number of 3d electrons occupied in t_{2g} orbitals of hydrated Cr^{3+} ion (octahedral).

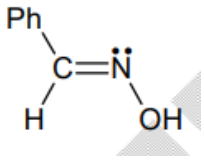
Question Number – 56

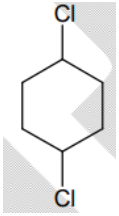
How many of the following can show geometrical isomerism?

(I) 

(II) 

(III) 

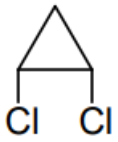
(IV) 

(V) 

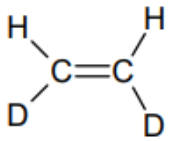
(VI) $CHCl = C = CHCl$

(VII) $Ph - N = N - Ph$

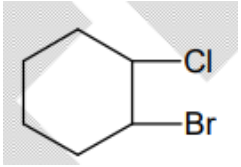
(VIII) $MeCH = N - NH_2$

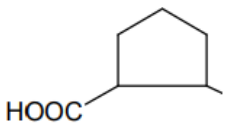
(IX) 

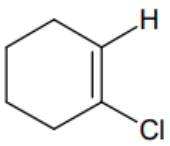
(X) $CHCl = C = C = CHCl$

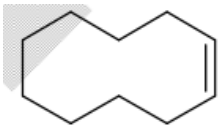
(XI) 

(XII) $CH_3 - C \equiv C - CH_2Cl$

(XIII) 

(XIV) 

(XV) 

(XVI) 

Question Number – 57

For the strong electrolytes NaOH, NaCl and $BaCl_2$ the molar ionic conductivities at infinite dilution are 250, 125 and 300 mho cm^2 mol^{-1} respectively. The molar conductivity of $Ba(OH)_2$ at infinite dilution (mho cm^2 mol^{-1}) is

Question Number – 58

Two moles of a gas at 8.21 bar and 300K are expand at constant temperature up to 2.73 bar against a constant pressure of 1 bar. How much work is done by the gas in atm-litre? (neglect the sign)

Question Number – 59

30 ml of 0.2 M NaOH is added with 50 ml 0.2 M CH_3COOH solution. The extra volume of 0.2 M NaOH required to make the pH of the solution 5.00 is $\frac{10}{x}$ ml. The value of x is: (The ionization constant of $CH_3COOH = 2 \times 10^{-5}$)

Question Number – 60

The combustion of sodium in excess air yields a higher oxide. What is the oxidation state of the oxygen in the product? Neglect the negative sign

MATHS

SECTION-A

Question Number – 61

Let $y = \tan^{-1}\left(\frac{4x}{1+5x^2}\right) + \tan^{-1}\left(\frac{2+3x}{3-2x}\right)$, where $x \in \left(0, \frac{2}{3}\right)$. If $\frac{dy}{dx} = \frac{\alpha}{1+25x^2}$, then the value of α is equal to:

(a) 3
(b) 4
(c) 5
(d) 6

Question Number – 62

If the value of $\lim_{n\to\infty}\sum_{k=0}^{n}\frac{{}^{n}C_{k}}{n^{k(k+3)}}$ equals L. Then [L] is equal to [Note: Where [k] denotes greatest integer function less than or equal to k]

(a) 0

(b) 1

(c) 2

(d) 3

Question Number – 63

Let f(x) and g(x) are functions defined in the real domain and co-domain, such that $\sqrt{1-f^2(x)}=g(x)$, then which of the following statements are necessarily true?

(a) If g(x) is periodic with period 1, then f(x) is periodic with period half.

(b) If $f'(c)=-f(c)=0.5$, then $\frac{g'(c)}{g(c)}=\frac{1}{3}$

(c) If g(x) is an even function, then f(x) is odd.

(d) If g(x) is continuous function then f(x) is also continuous in their respective domains.

Question Number – 64

The value of $\lim_{x\to\infty}\frac{e^{x}\left(\left(2^{x^n}\right)^{1/e^x}-\left(3^{x^n}\right)^{1/e^x}\right)}{x^n}$ where n is positive integer, is:

(a) ln 2-ln 3

(b) ln 3-ln 2

(c) 0

(d) None of these

Question Number – 65

Let f be an invertible function form R → R satisfying the equation

$f^3(x)-(x^3+2)f^2(x)+(2x^3+1)f(x)-x^3=0$. Then the value of $f'(8)\times(f^{-1})'(8)$, is:

(a) 12

(b) 16

(c) 20

(d) 32

Question Number – 66

If $\int\frac{3\tan\left(x-\frac{\pi}{4}\right)}{\cos^2 x\sqrt{\tan^3 x+\tan^2 x+\tan x}}dx=k\tan^{-1}\left(\sqrt{\tan x+1+\cot x}\right)+C$, then the value of k is: [where C is constant of integration]

(a) 2

(b) 3

(c) 6

(d) 8

Question Number – 67

The solution of the differential equation $e^{-x}(y+1)dy + (\cos^2 x - \sin 2x)y\,dx = 0$ subjected to condition that $y = 1$ when $x = 0$, is:

(a) $(y+1) + e^x \cos^2 x = 2$

(b) $y + \ln y = e^x \cos^2 x$

(c) $\ln(y+1) + e^x \cos^2 x = 1$

(d) $y + \ln y + e^x \cos^2 x = 2$

Question Number – 68

Let f be a function defined by $y = f(x)$ where $x = 2t - |t|$ and $y = t^2 + t|t|$ for $t \in R$, then:

(a) f(x) is both continuous and differentiable at $x = 0$

(b) f(x) is non-differentiable at $x = 0$

(c) f(x) is discontinuous at $x = 0$

(d) f(x) is neither continuous nor differentiable at $x = 0$

Question Number – 69

If $32\tan^8\theta = 2\cos^2\alpha - 3\cos\alpha$ and $3\cos 2\theta = 1$, then the most general value of α are:

(a) $n\pi - \frac{\pi}{3}$

(b) $2n\pi \pm \frac{2\pi}{3}$

(c) $n\pi + \frac{\pi}{3}$

(d) $n\pi \pm \frac{\pi}{3}$

(where $n \in l$)

Question Number – 70

The difference between the radii of the largest and the smallest circles, which have their centres on the circumference of the circle $x^2 + 2x + y^2 + 4y = 4$, and pass through the point (3, 4) is equal to:

(a) 4

(b) 6

(c) 8

(d) 10

Question Number – 71

If a line with direction ratio 2:1:1 intersects the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-2}{1}=\frac{y+1}{2}=\frac{z+2}{3}$ at A and B then $\left|\overrightarrow{AB}\right|$ is

(a) $2\sqrt{3}$

(b) $2\sqrt{5}$

(c) $2\sqrt{6}$

(d) $2\sqrt{7}$

Question Number – 72

Let $\vec{a},\vec{b}$ and $\vec{c}$ be three vectors such that $|\vec{a}|=2\left|\vec{b}\right|=4|\vec{c}|$ and $2\vec{a}-3\vec{b}+6\vec{c}=\vec{0}$. If θ is the angle between $\vec{a}$ and $\vec{b}$, then cos θ equals:

(a) $\frac{2}{3}$

(b) $\frac{1}{3}$

(c) $\frac{3}{5}$

(d) $\frac{4}{5}$

Question Number – 73

Let P and Q are points on the parabola $y^2=4ax$ with vertex O, such that OP is perpendicular to OQ and have length r_1 and r_2 respectively, then the value of $\frac{r_1^{4/3}r_2^{4/3}}{r_1^{2/3}+r_2^{2/3}}$ is

(a) $16a^2$

(b) a^2

(c) 4a

(d) None of these

Question Number – 74

Consider the ellipse $\frac{x^2}{f(k^2+2k+5)}+\frac{y^2}{f(k+11)}=1$, where f(x) is a positive decreasing function, then the value of k for which major axis coincides with x-axis is:

(a) $k\in(-7,-5)$

(b) $k\in(-5,-3)$

(c) $k \in (-3, 2)$

(d) None of these

Question Number – 75

Let $A_1, A_2, A_3 \ldots, A_n$ are the vertices of a polygon of n sides. If the number of pentagons that can be constructed by joining these vertices such that none of the side of the polygon is also the side of the pentagon is 36, then the value of n is equal to:

(a) 8

(b) 10

(c) 11

(d) 12

Question Number – 76

Let $(1 + 3x + 2x^2)^6 = \sum_{k=0}^{12} a_k x^k$

(a) The coefficient of x^{12} equals 2^{12}

(b) The coefficient of x^{11} equals $9(2)^8$

(c) $\frac{\sum_{k=0}^{6} a_{2k}}{a_{12}} = \frac{3^6}{2}$

(d) $\frac{\sum_{k=1}^{6} a_{2k-1}}{a_{12}} = 2(3)^6$

Question Number – 77

If α, β, γ are the roots of the equation $x^3 - px + 1 = 0, p \in R$ and A, B, C three matrices defined as:

$$A = \begin{bmatrix} \alpha^3 + 1 & -2 & 3 \\ 2 & \beta^3 + 1 & 1 \\ -3 & -1 & \gamma^3 + 1 \end{bmatrix}, B = \begin{bmatrix} \beta^3 - 2 & 3 & -1 \\ -3 & \gamma^3 - 2 & -2 \\ 1 & 2 & \alpha^3 - 2 \end{bmatrix} \text{ and } C = \begin{bmatrix} \gamma^3 + 5 & -1 & -2 \\ 1 & \alpha^3 + 5 & 1 \\ 2 & -1 & \beta^3 + 5 \end{bmatrix}.$$

Then the value of det. (A+B+C) equals:

(a) 0

(b) 1

(c) 5

(d) 6

Question Number – 78

If A and B are square matrices of order 3 such that det. $A = -2$ and det. $B = 1$, then

det. $\left(A^{-1}\text{adj}(B^{-1}).\text{adj}(2A^{-1})\right)$ is equal to:

(a) 8

(b) -8

(c) 1

(d) -1

Question Number – 79

z_1 and z_2 are two distinct points in an argand plane. If $a|z_1| = b|z_2|$, (where $a, b \in R$) then the point $\frac{az_1}{bz_2} + \frac{bz_2}{az_1}$ is a point on the:

(a) Line segment $[-2, 2]$ of the real axis

(b) Line segment $[-2, 2]$ of the imaginary axis

(c) Unit circle $|z| = 1$

(d) $z = \tan^{-1} 2$

Question Number – 80

A square OABC is formed by line pairs xy=0 and $xy + 1 = x + y$ where 'O' is the origin. A circle with centre C_1 inside the square is drawn to touch the line pair $xy = 0$ and another circle with centre C_2 and radius twice that of C_1, is drawn to touch the circle C_1 and the other line pair. The radius of the circle with centre C_1 is:

(a) $\frac{\sqrt{2}}{\sqrt{3}(\sqrt{2}+1)}$

(b) $\frac{2\sqrt{2}}{3(\sqrt{2}+1)}$

(c) $\frac{\sqrt{2}}{3(\sqrt{2}+1)}$

(d) $\frac{\sqrt{2}+1}{3\sqrt{2}}$

Question Number – 81

Let E and M be 3×3 matrices satisfying the system of equations

$EM^T = (EM)^T = 20I$

And $(E + M)^T = 17(E - M)^T$

where I denote identity matrix of order 3.

If $E^2 + M^2 = \frac{a}{b}I$ (where a and b are co-prime), then find the value of $(a + b)$.

Question Number – 82

Let f(x) be monotonically strictly increasing function in [3, 5] such that $\int_3^5 f^2(x)dx = 9, f(1) = 3;\ f(4) = 5$. Find the value of $2\int_1^4 x\left(5 - f^{-1}(x)\right)dx$.

Question Number – 83

Let S_k be the area bounded by the curve $y = x^2(1-x)^k$ and the lines $x = 0$, $y = 0$ and $x = 1$. If $\lim_{n\to\infty} \sum_{k=1}^{n} S_k$ is equal to $\frac{p}{q}$, where p and q are co-prime positive integers, find (p+q).

Question Number – 84

Let $\vec{k}, \vec{l}, \vec{m}, \vec{n}$ are four distinct unit's vectors in space such that $\vec{k}.\vec{l} = \vec{l}.\vec{m} = \vec{m}.\vec{k} = \vec{n}.\vec{l} = \vec{n}.\vec{m} = \frac{-1}{11}$. The value of $\vec{k}.\vec{n}$ can be expressed as $\frac{-A}{B}$, where A, B are co-prime positive integer. Find the value of A+ B.

Question Number – 85

If eccentricity of conjugate hyperbola of the given hyperbola:

$$\left|\sqrt{(x-1)^2+(y-2)^2} - \sqrt{(x-5)^2+(y-5)^2}\right| = 3$$

is e', then the value of 8e' is:

Question Number – 86

Let (a, b) be the outcome of throwing a pair of fair dice. If the probability for which $\lim_{x\to 0} \frac{\ln((\cos x)^a)}{x^b}$ exist and is finite can be expressed as $\frac{p}{q}$, where p and q are co- prime positive integers, then find the value of (p + q).

Question Number – 87

Let $z_1, z_2 \in C$ and satisfy the equation $|z+1|^2 + |z-1|^2 + 2|z| = 6$. If maximum value of $(2|z_1 - 2| + |2z_2 + 1|)$ is equal to λ, then find the value of 4λ.

Question Number – 88

Consider $f(x) = x^4 + ax^3 + bx^2 + cx + d$. If straight line $y = 3x + 2$ is tangent to the curve y=f(x) at P(1, f(1)) which again intersects f(x) at Q (2,8) and $f''(1) = 0$, then find the value of f(3).

Question Number – 89

Consider a family of n children. Let two events A and B are defined as follows:

A: is the event that the family has both boys and girls

B: is the event that the family has at most one girl

If the vents A and B are independent, then find the value of n.

[Note: Probability that a randomly selected child is a boy or girl is same]

Question Number – 90

If the function $f(x) = 2x^3 - (8 - a)x^2 + \left(a^2 + \frac{16}{9}\right)x - 12$ has local minima at some $x \in R^-$, then find the number of integers in the range of a.