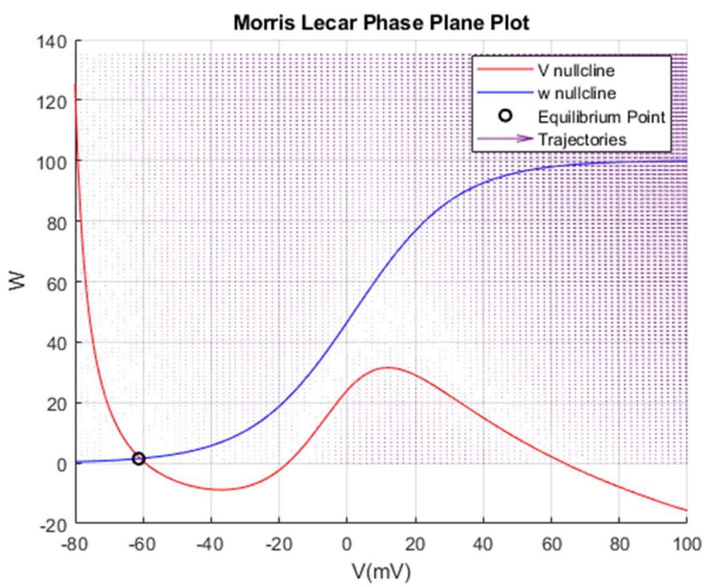
EC60007 Computational Neuroscience- Project II

Part I- Morris–Lecar model

1. Given- Conductance- μS/cm^2 and for the current to be given as μA/cm^2, the units of voltage should be mV, time should be in ms, capacitance should be μF/cm^2. The above set of units are not unique as the same set of results can be obtained scaling of different units in correspondence with change in one unit.
2. For I_{ext}=0 and given set of parameters, the equilibrium points, nullclines and quiver plot are given as:

```
The equilibrium point of MLE is located at (-6.131579e+01,1.447071e-02)
```



3. The eigen values of the Jacobian of the given system with the specified parameters are:

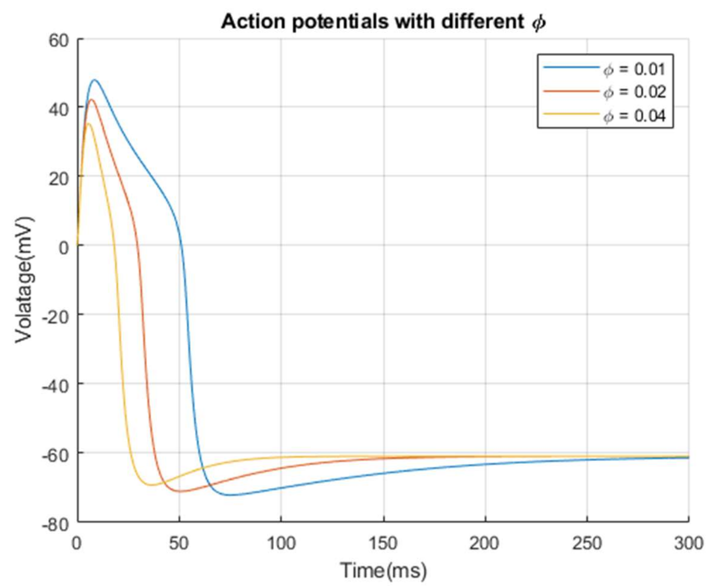
```
Question 3. The eigen values of the system are -9.616547e-02 and -3.655273e-02
```

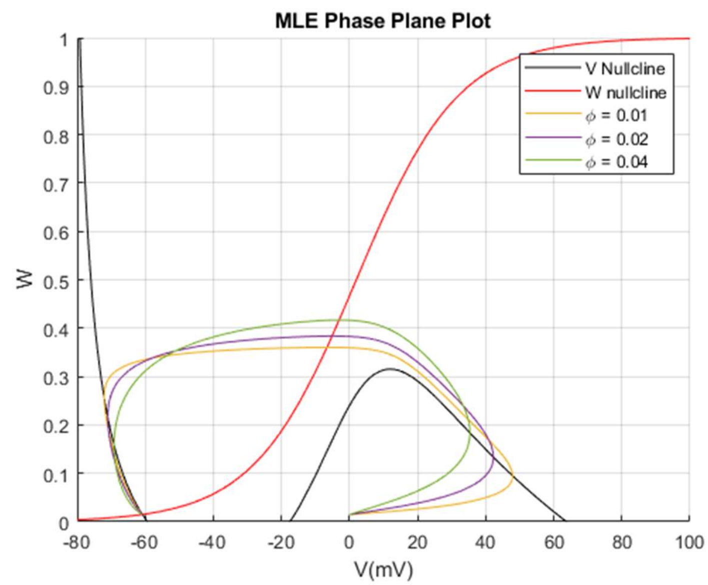
The above eigen values are determined to be stable.

4. The default numerical tolerance values built in MATLAB (AbsTol=10^{-6} and RelTol=10^{-3}) are indeed reasonable as the precision is well within the limits of the solution of the given set of equations. If the units for voltage is changed to kV in MLE, the default inbuilt tolerance would have to be modified so as to accommodate the change. It is changed as following

```
op = odeset ('RelTol',1e-3,'AbsTol',1e-12, 'refine',5, 'MaxStep', 1);
```

5. For different values of ϕ=0.01,0.02 and 0.04, the action potential plotted is as follows

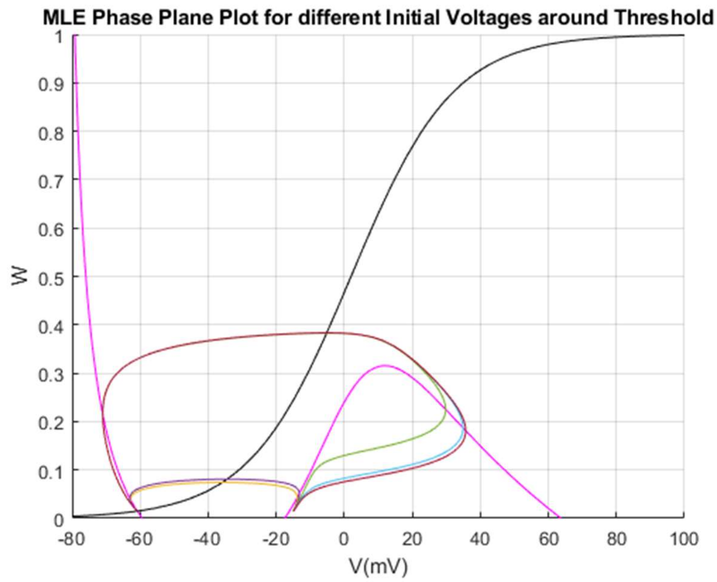


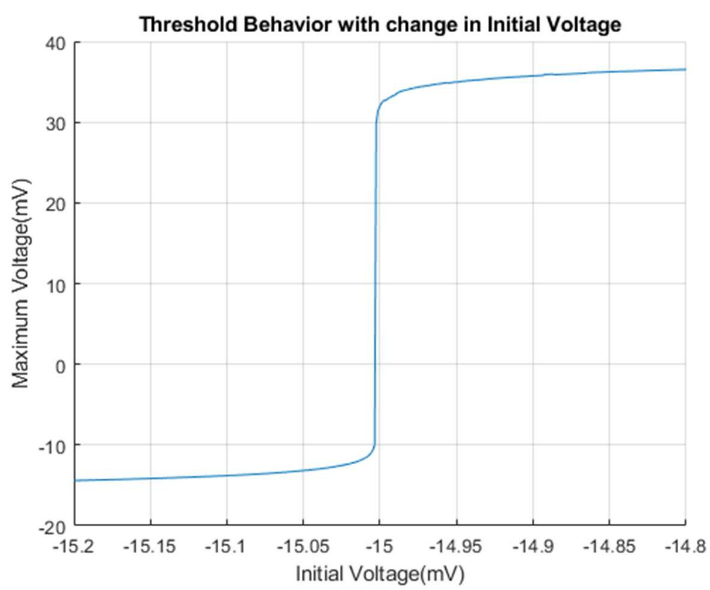


From the above graph we can observe that when the value of ϕ is increased, the occurrence of Action Potential becomes faster. The maximum value of voltage also decreases as ϕ is increased which can be attributed to the closing of K^+ channels as the time increases.

6. For understanding the behaviour of Depolarizing Threshold, the following graphs are plotted and the relevant calculations are made from the given parameters to find the threshold

```
Question 6. Threshold is (-15.002506)
```

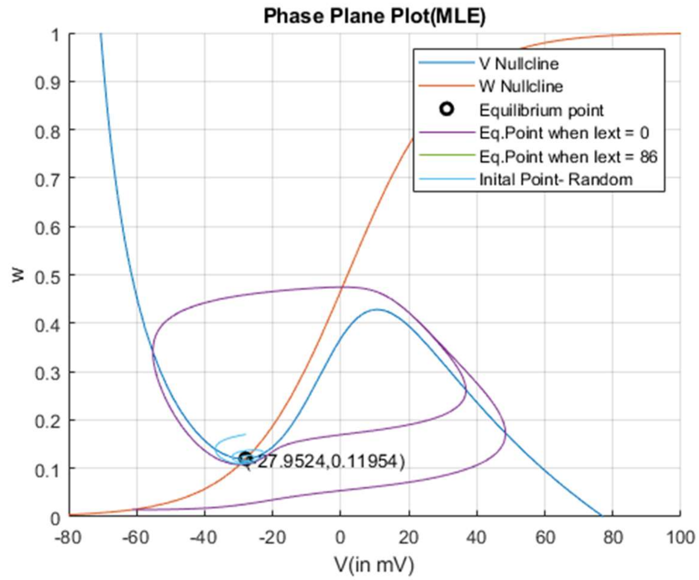




From the above calculations we observe that after -15.002506 mV, the trajectory moves far away from rest before finally returning. This can be termed as an impulse, or action potential.

7. For I_{ext}=86µA/cm^2, the phase plane plot and equilibrium point are given as:

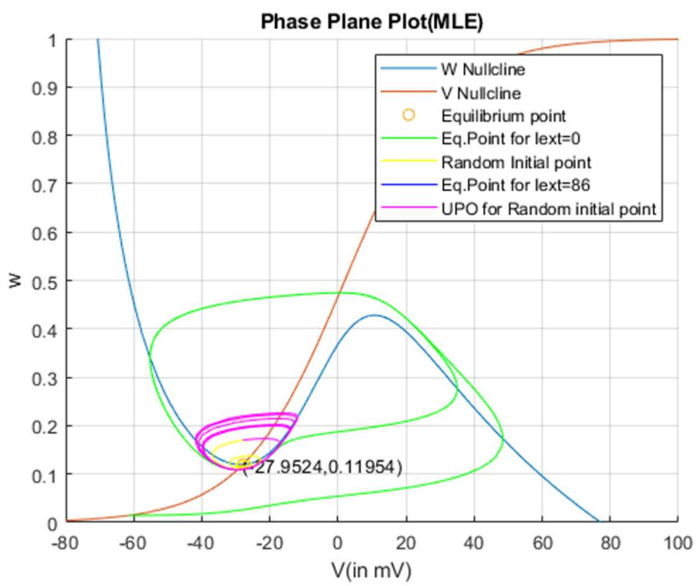
```
Question 7. The equilibrium point is located at (-2.795241e+01,1.195364e-01)
The point is stable and converging spiral
```



When an external current of 86μA/cm^2 is injected to the system, the V nullcline graph is modified (rises from the original) and a new equilibrium point is obtained which is evaluated to be stable by calculating its eigen values from its Jacobian matrix.

When the initial point is set for I_{ext}=0μA/cm^2, the trajectory of the curve is forced to be a limit cycle. As the starting point is set closer to the equilibrium point, the trajectory of the curve is shorter when compared to the earlier cases.

8. Running the model backwards in time, the same equilibrium point is obtained as before.



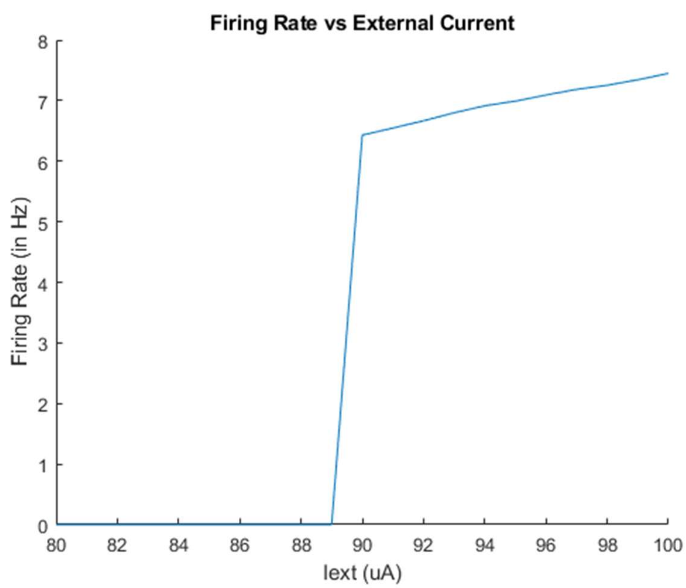
The trajectory obtained is one of Unstable Periodic Orbit (UPO). Any point outside the UPO would generate a limit cycle whereas a point inside the UPO will converge into the stable equilibrium point.

9. Analysing the Equilibrium Points for $I_{ext} = 80, 86, 90 \mu A/cm^2$, the following results were obtained

```
Question 9. For Iext = 80
The Equilibrium point is at (-2.996618e+01,1.061127e-01)
The eigen values are -0.017784+0.055717i, -0.017784-0.055717i
The point is stable and converging spiral

Question 9. For Iext = 86
The Equilibrium point is at (-2.795241e+01,1.195364e-01)
The eigen values are -0.006785+0.057427i, -0.006785-0.057427i
The point is stable and converging spiral

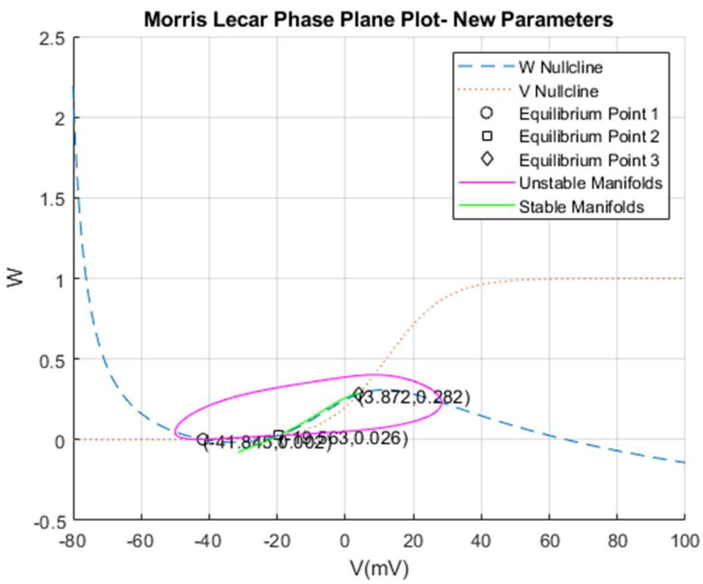
Question 9. For Iext = 90
The Equilibrium point is at (-2.659687e+01,1.293793e-01)
The eigen values are- 0.001753+0.057170i, 0.001753-0.057170i
The point is unstable and diverging spiral
```



10. For the new parameters, the equilibrium points and Phase Plane Plots are given as

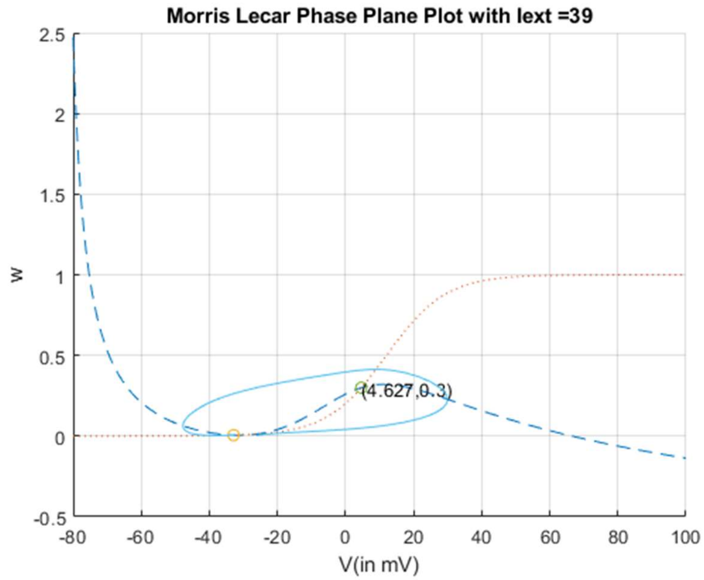
```
Question 10. Equilibrium points are:
 1. (-41.845162, 0.002047)
 2. (-19.563243, 0.025883)
 3. (3.871510, 0.282051)
```

```
For Equilibrium point 1: The eigen values are -0.071544+0.000000i, -0.156766+0.000000i
The point is stable
For Equilibrium point 2: The eigen values are 0.153619+0.000000i, -0.067328+0.000000i
The point is a saddle node
For Equilibrium point 3: The eigen values are 0.093868+0.172310i, 0.093868-0.172310i
The point is unstable and diverging spiral
```

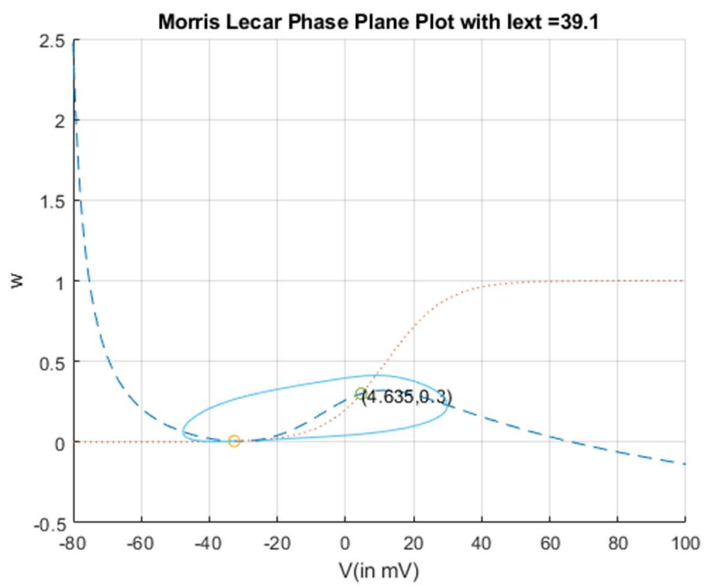


11. For current range 30-50µA/cm^2, the following were observed

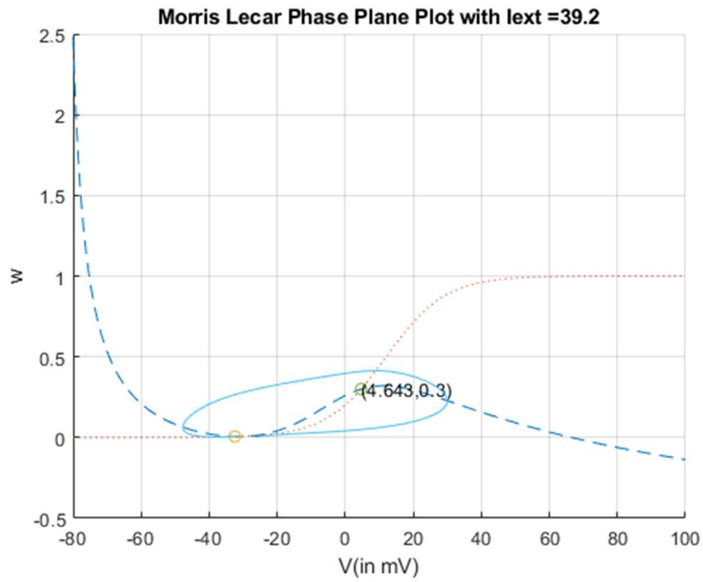
```
Question 11. For Iext = 30.000000, there are three distinct equilibrium points
Question 11. For Iext = 35.000000, there are three distinct equilibrium points
Question 11. For Iext = 39.000000, there are three distinct equilibrium points
```



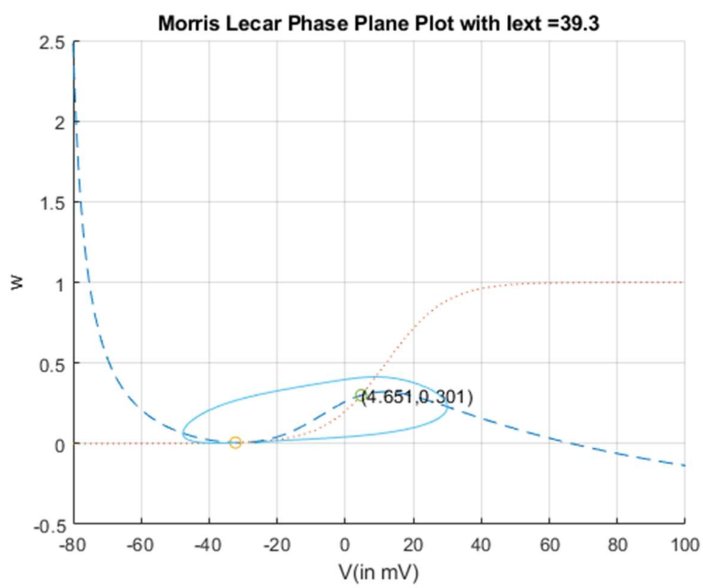
Question 11. For Iext = 39.100000, there are three distinct equilibrium points



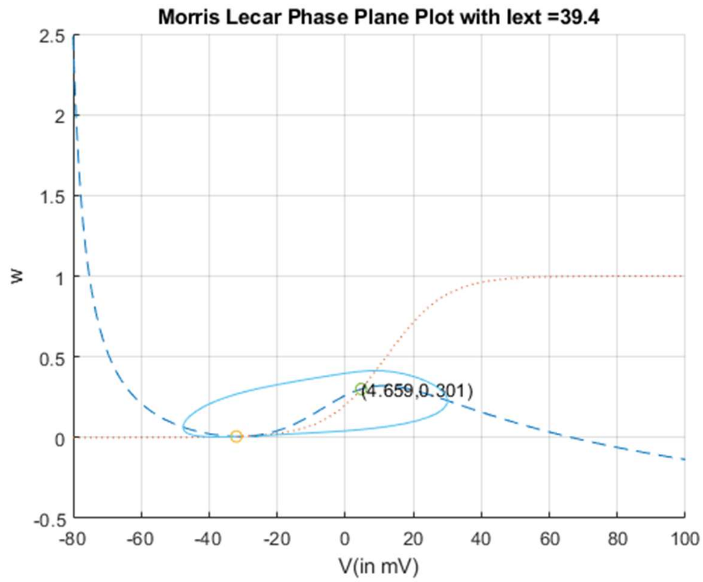
Question 11. For Iext = 39.200000, there are three distinct equilibrium points



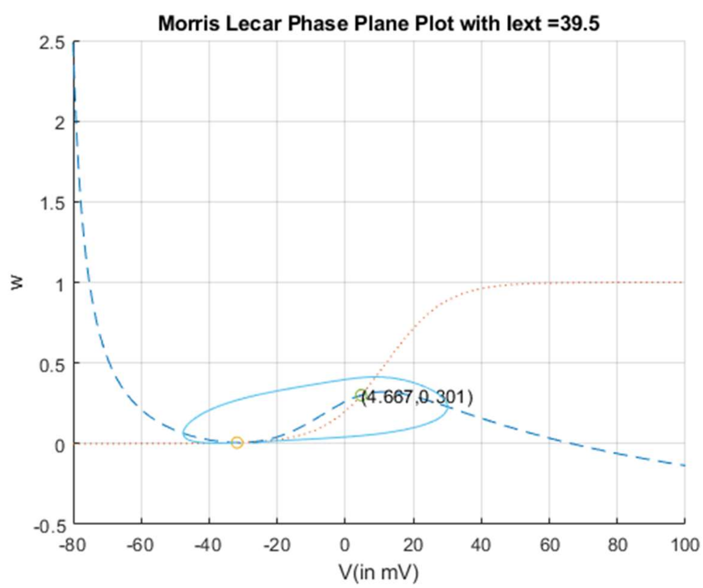
Question 11. For Iext = 39.300000, there are three distinct equilibrium points



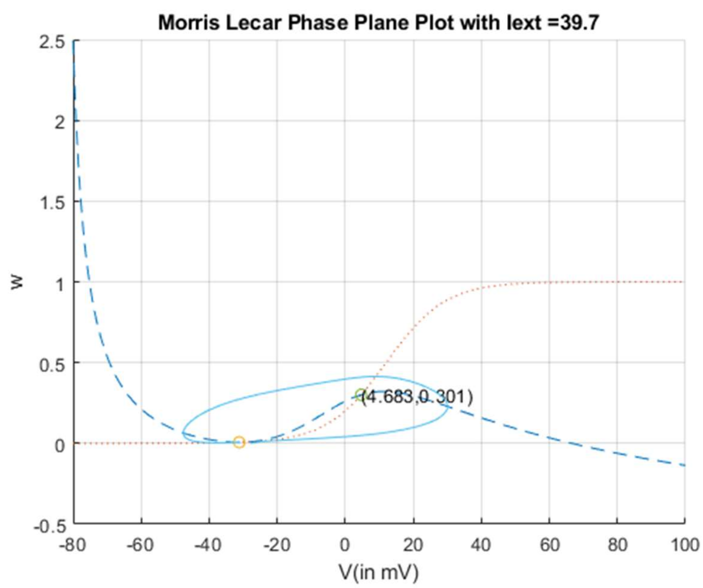
```
Question 11. For Iext = 39.400000, there are three distinct equilibrium points
```




```
Question 11. For Iext = 39.500000, there are three distinct equilibrium points
```



```
Question 11. For Iext = 39.700000, there are three distinct equilibrium points
```

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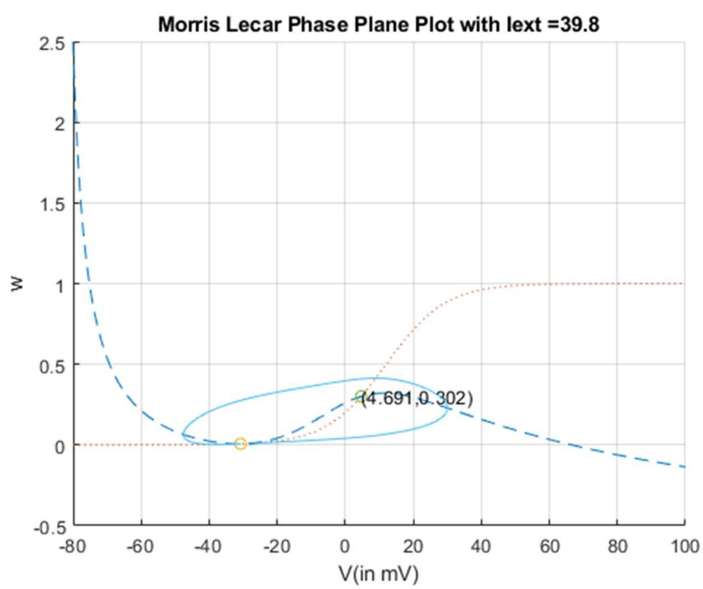
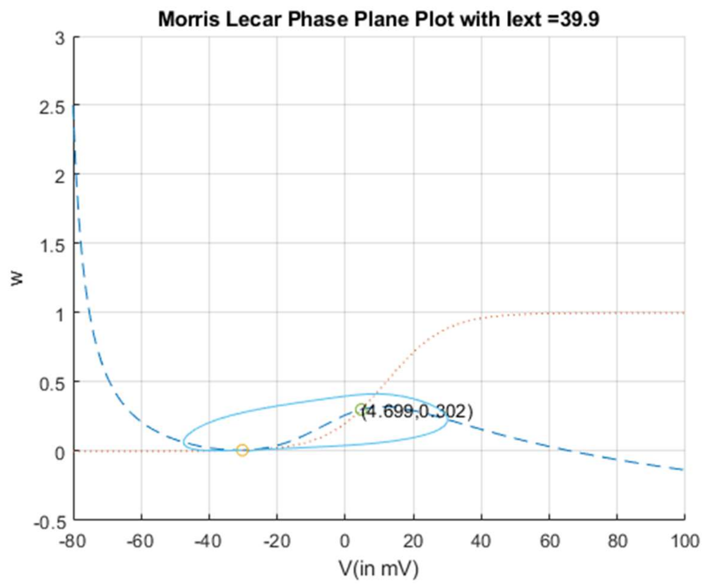
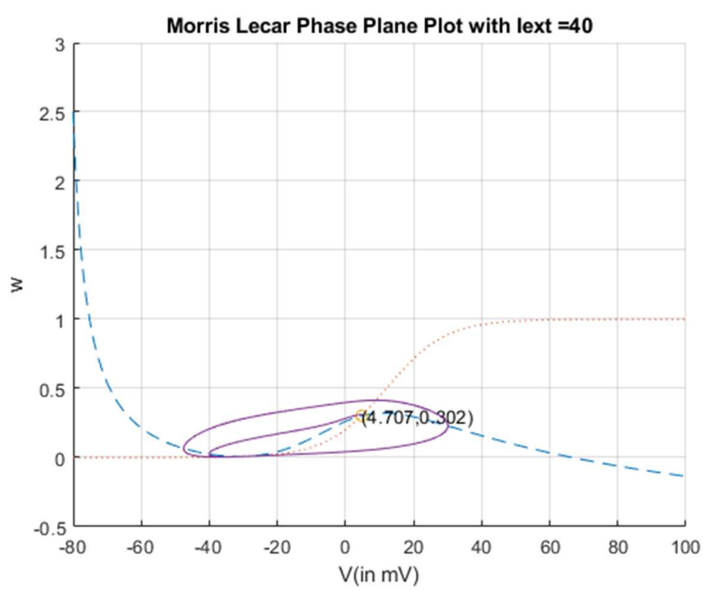
Question 11. For Iext = 39.800000, there are three distinct equilibrium points
```



Question 11. For Iext = 39.900000, there are three distinct equilibrium points



Question 11. For Iext = 40.000000, there is only one distinct equilibrium point



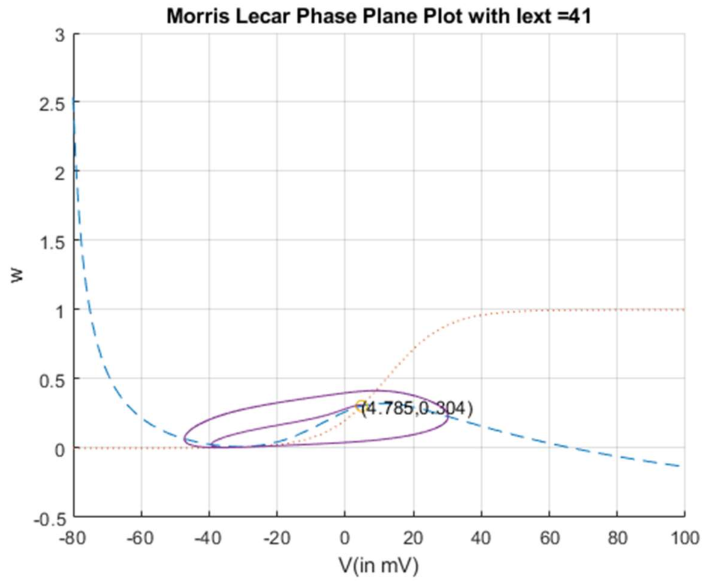
Question 11. For Iext = 41.000000, there is only one distinct equilibrium point

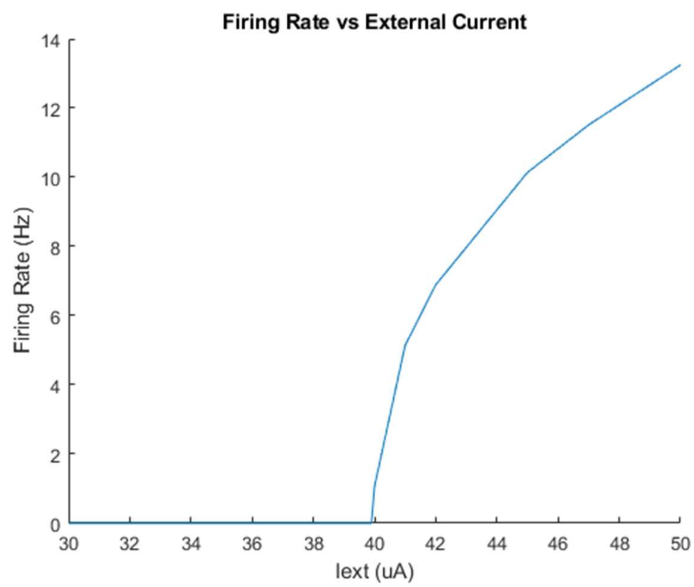
Question 11. For Iext = 42.000000, there is only one distinct equilibrium point

Question 11. For Iext = 45.000000, there is only one distinct equilibrium point

Question 11. For Iext = 47.000000, there is only one distinct equilibrium point

Question 11. For Iext = 50.000000, there is only one distinct equilibrium point





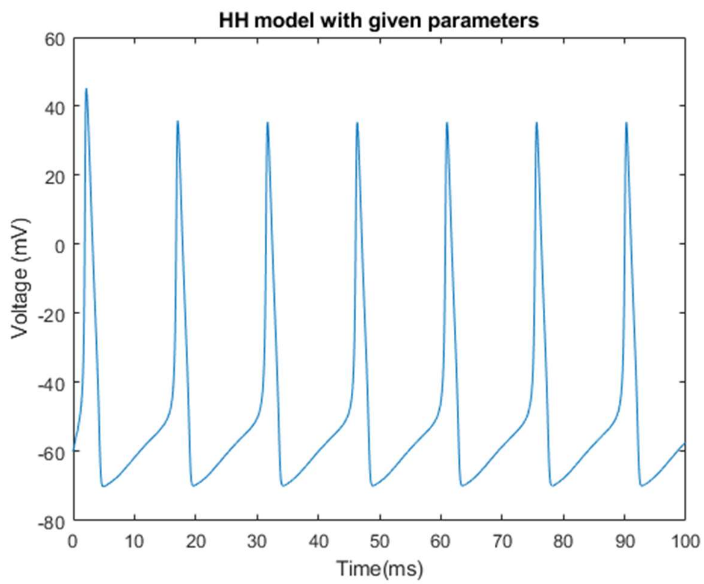
The number of equilibrium points of the given system reduces after I_{ext}= 39.9 μA/cm^2, which is indicative of the system reaching its threshold voltage.

PART II - Hodgkin Huxley Equations

12. Refer the attached m-file for the Hodgkin Huxley Model simulation.

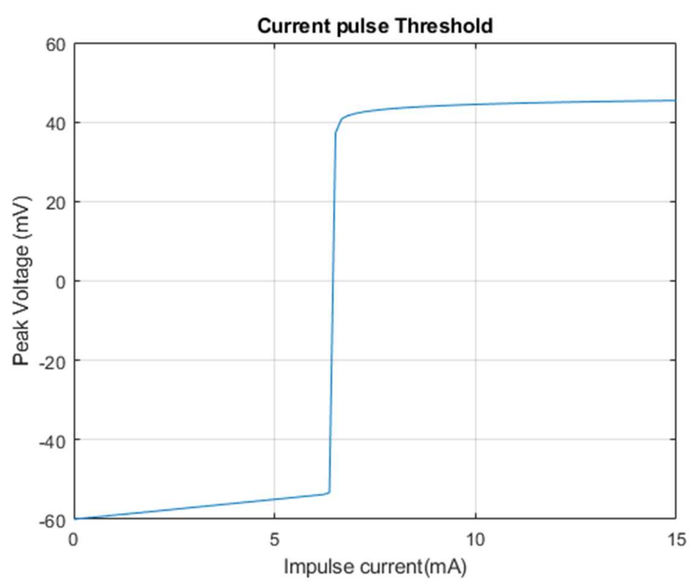
13. The value of E_L is found by fixing the values of V=-60mV and $\frac{dm}{dt}, \frac{dn}{dt}, \frac{dh}{dt}$ =0 and the obtained values are plugged in for V=-49mV to find the value of E_L.

```
Question 13. EL = -4.940108e+01
```



14. The stability and threshold of the model is given as

```
Question 14. Threshold: 6.515152 mA
```



```
Question 14. Iext =0

Equilibrium Point V=-60.000000 n=0.317677 m=0.052932 h=0.596121

The eigen values are -4.675345+0.000000i, -0.202718+0.383061i, -0.202718-0.383061i,
-0.120659+0.000000i

Stable
```

15. Behaviour of the equilibrium points for steady current injections from 8-12 $\mu A/cm^2$

```
Question 15. Iext = 8

Equilibrium Point V=-55.355128 n=0.390607 m=0.090048 h=0.430515

The eigen values are -4.690140+0.000000i, -0.034549+0.566792i, -0.034549-0.566792i,
-0.135013+0.000000i

Stable


Question 15. Iext = 9

Equilibrium Point V=-54.952404 n=0.397027 m=0.094133 h=0.416502

The eigen values are -4.730588+0.000000i, -0.014865+0.578298i, -0.014865-0.578298i,
-0.136960+0.000000i

Stable


Question 15. Iext = 10

Equilibrium Point V=-54.572150 n=0.403092 m=0.098131 h=0.403419

The eigen values are -4.774093+0.000000i, 0.004122+0.588328i, 0.004122-0.588328i, -
0.138902+0.000000i

Stability cannot be determined as 4-D plot is required


Question 15. Iext = 11

Equilibrium Point V=-54.211777 n=0.408841 m=0.102052 h=0.391169

The eigen values are -4.819956+0.000000i, 0.022390+0.597090i, 0.022390-0.597090i, -
0.140836+0.000000i

Stability cannot be determined as 4-D plot is required

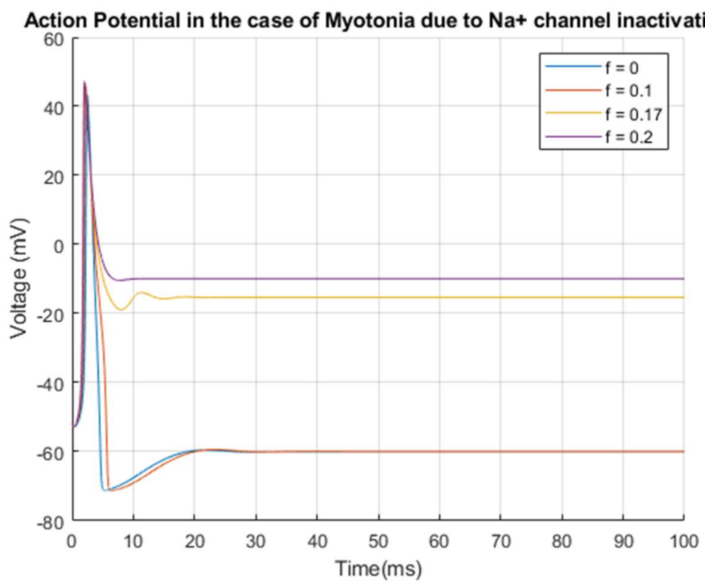

Question 15. Iext = 12

Equilibrium Point V=-53.869127 n=0.414306 m=0.105899 h=0.379667

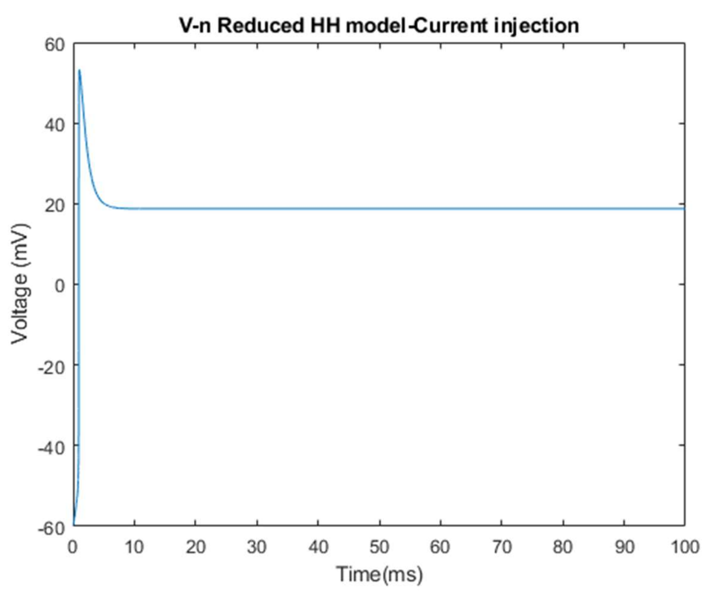
The eigen values are -4.867630+0.000000i, 0.039926+0.604761i, 0.039926-0.604761i, -
0.142760+0.000000i

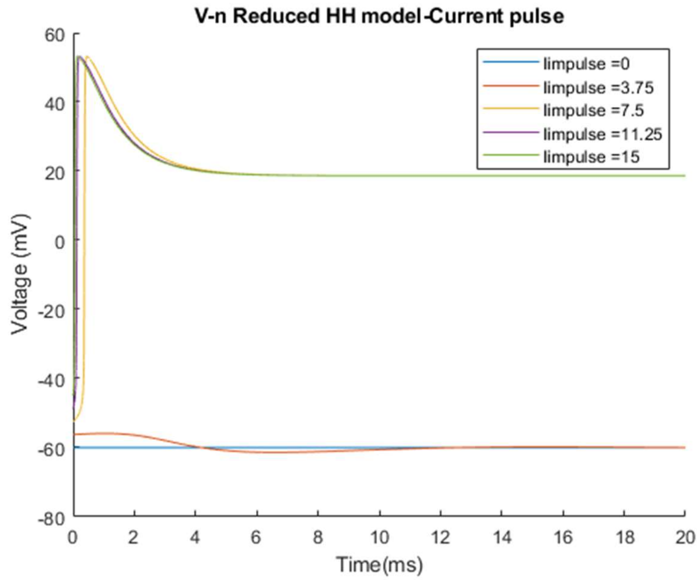
Stability cannot be determined as 4-D plot is required
```

16. In the case of Myotonia caused due to Na^+ channel inactivation, action potentials are fired for f=0 and 0.1 but aren't for the other cases. This is due to the loss of inactivating behaviour of Na^+ channels which are essential to the cell returning to its resting state. A loss of such behaviour leads to the inability to fire action potentials in neurons which is illustrated below:



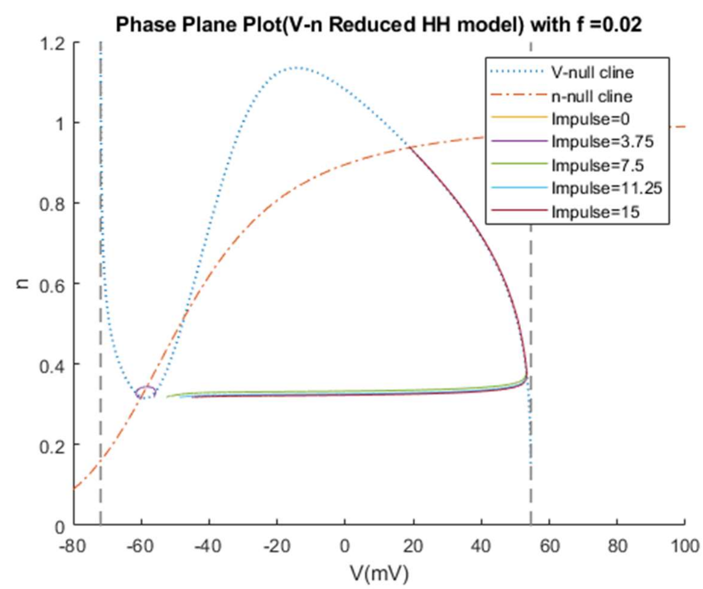
17. V-n reduced Hodgkin Huxley model behaves as following

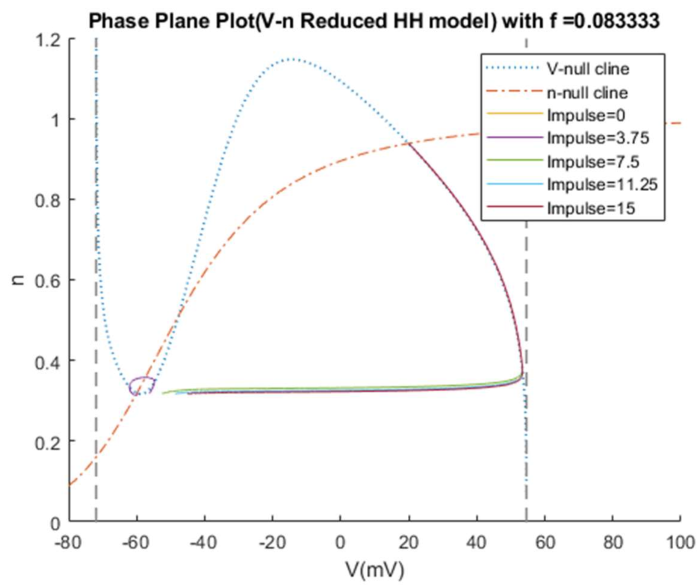


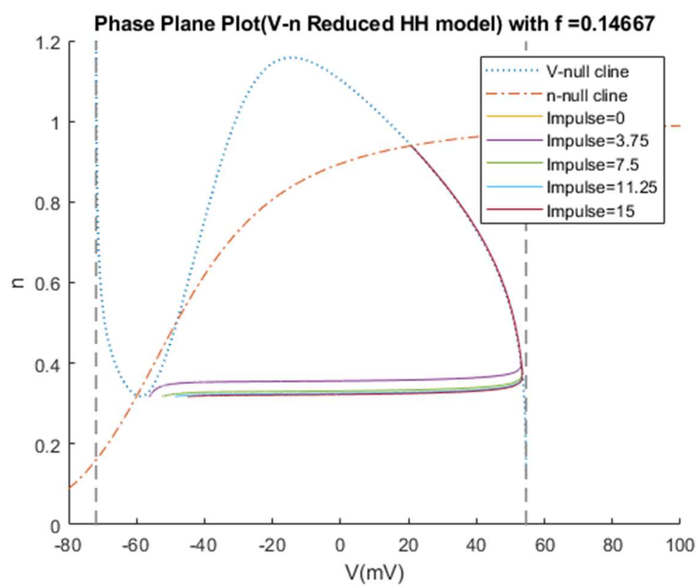


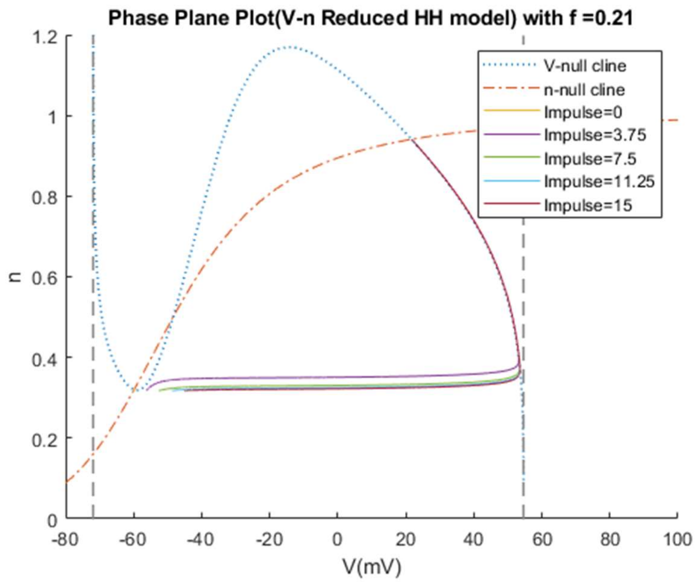
The behaviour of the V-n reduced similar to the original system barring some differences in the shape of the impulse which might be caused due to replacing m with $\mathbf{m}_{\infty}$.

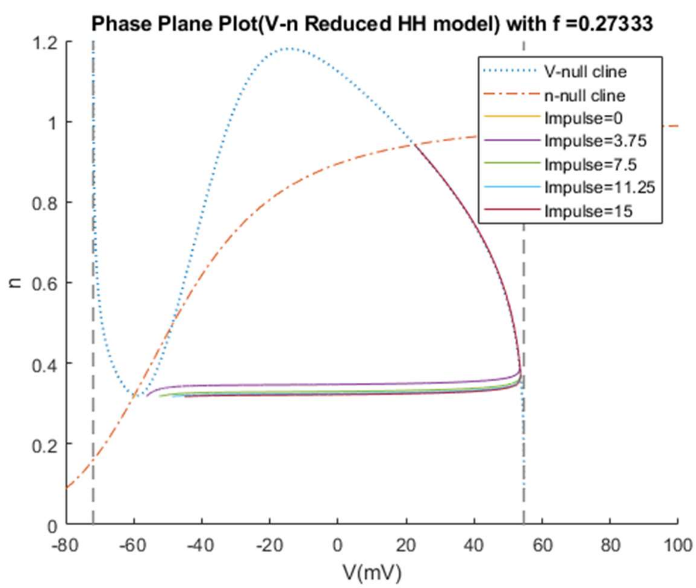
18. Phase Plane analysis of V-n reduced HH model is as follows

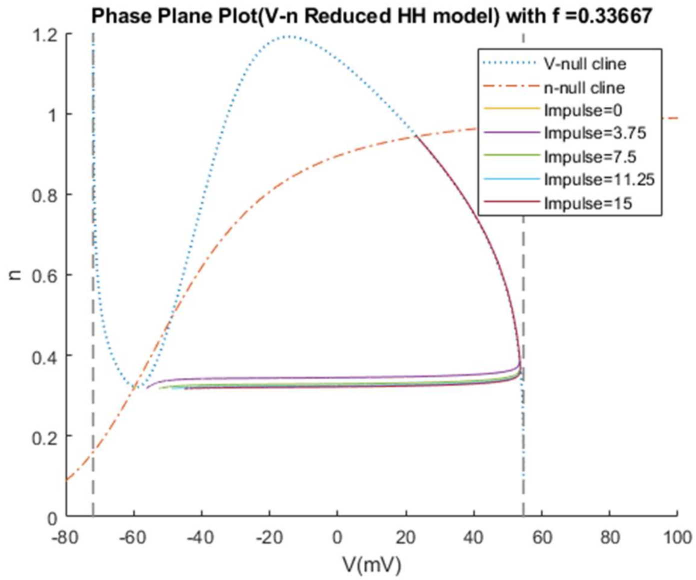


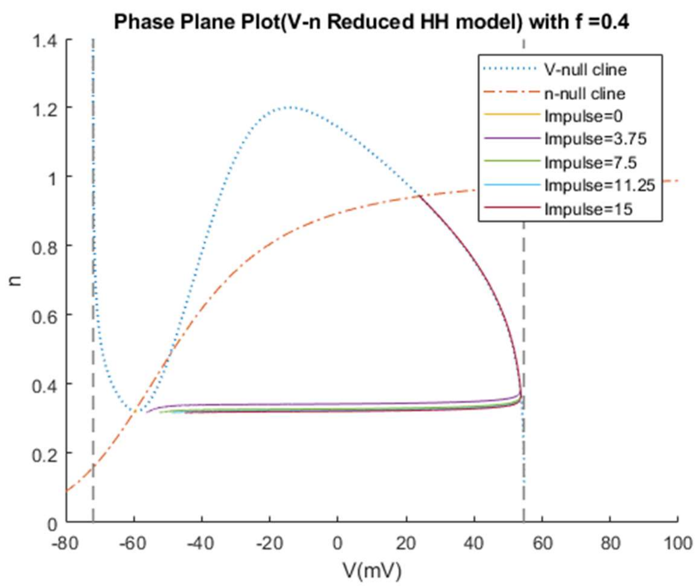

Phase Plane Plot(V-n Reduced HH model) with f =0.083333
V-null cline
n-null cline
Impulse=0
Impulse=3.75
Impulse=7.5
Impulse=11.25
Impulse=15
n
V(mV)


Phase Plane Plot(V-n Reduced HH model) with f =0.14667
V-null cline
n-null cline
Impulse=0
Impulse=3.75
Impulse=7.5
Impulse=11.25
Impulse=15
n
V(mV)

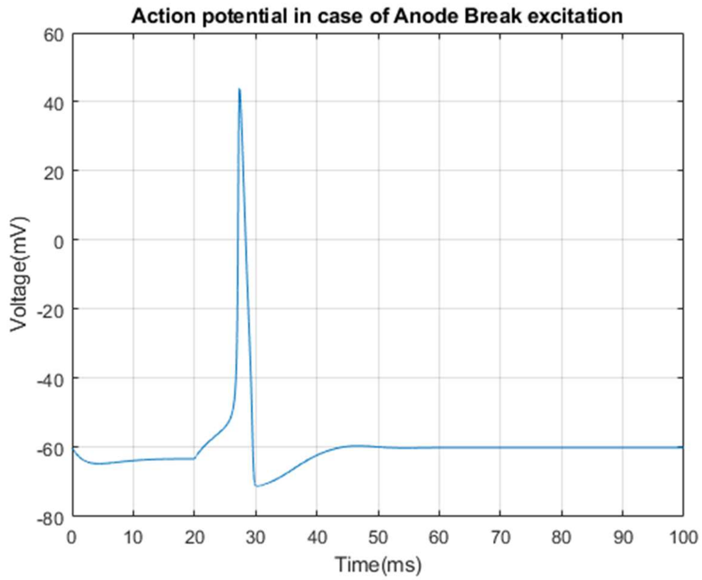

Phase Plane Plot(V-n Reduced HH model) with f =0.21
V-null cline
n-null cline
Impulse=0
Impulse=3.75
Impulse=7.5
Impulse=11.25
Impulse=15
n
V(mV)


Phase Plane Plot(V-n Reduced HH model) with f =0.27333
V-null cline
n-null cline
Impulse=0
Impulse=3.75
Impulse=7.5
Impulse=11.25
Impulse=15
n
V(mV)

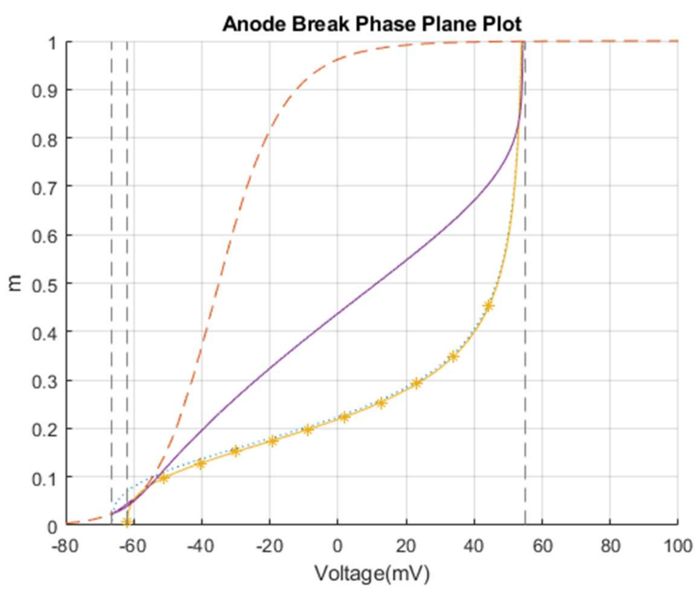




19. Anode break excitation behavior is observed due to decrease in resting potassium conductance and relieving sodium channel inactivation due to hyperpolarizing current and when the hyperpolarizing potential is removed, the open sodium channels lead to the influx of Na^+ ions into the cell which causes the overshooting of the membrane potential beyond resting state which causes further opening of Na^+ channels resulting in spike production.



20. Phase Plane analysis of Anode break excitation




```
Question 20. Case 1-Equilibrium Point V=-54.329239 m=0.100760
Question 20. Case 2-Equilibrium Point V=54.348821 m=0.999221
```

The initial stable equilibrium point near the resting membrane potential changes its behavior due to the hyperpolarizing current and becomes an unstable equilibrium point. This leads to the production of an action potential which follows the yellow trajectory in the above figure to reach the remaining stable point in the system.