cart spring pendulum problem

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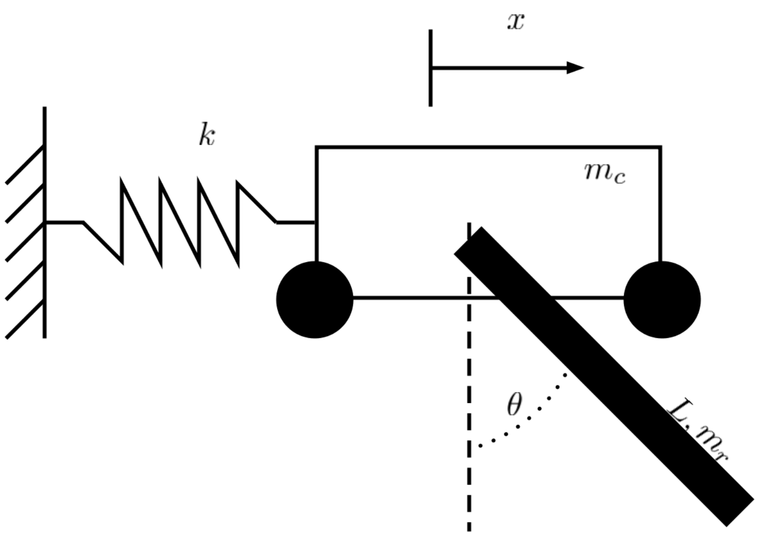
1 picture



Also note that:
$k = 10N/m$
$L = 0.5m$
$m_c = 1kg$
$m_r = 0.25kg$

2 solve for EOM

$$Y_G = \frac{L}{2}$$

$$I_G = \frac{1}{3}mL^2$$

$$r_G = (x + \frac{L}{2}s\theta)\vec{i} + (-\frac{1}{2}Lc\theta)\vec{j}$$

$$\dot{r}_G = (\dot{x} + \frac{L}{2}c\theta\dot{\theta})\vec{i} + (\frac{L}{2}s\theta\dot{\theta})\vec{j}$$

note that :
$T_{total} = T_{cart} + T_{rod}$
so we first find T_{rod}

$$T_{rod} = \frac{1}{2}m\dot{r}\cdot\dot{r} + \frac{1}{2}I_G\dot{\theta}^2$$

$$\dot{r}\cdot\dot{r} = (\dot{x} + \frac{L}{2}c\theta\dot{\theta})(\dot{x} + \frac{L}{2}c\theta\dot{\theta}) + (\frac{L}{2}s\theta\dot{\theta})(\frac{L}{2}s\theta\dot{\theta}) = \dot{x}^2 + Lc\theta\dot{x}\dot{\theta} + \frac{L^2}{4}c^2\theta\dot{\theta}^2 + \frac{L^2}{4}s^2\theta\dot{\theta}^2$$

$$\dot{r}\cdot\dot{r} = \dot{x}^2 + Lc\theta\dot{x}\dot{\theta} + \frac{L^2}{4}\dot{\theta}^2$$

$$\frac{1}{2}I_G\dot{\theta}^2 = \frac{1}{2}(\frac{1}{3}m_rL^2)\dot{\theta}^2 = \frac{1}{6}m_rL^2\dot{\theta}^2$$

$$T_{rod} = \frac{1}{2}m_r(\dot{x}^2 + Lc\theta\dot{x}\dot{\theta} + \frac{L^2}{4}\dot{\theta}^2 + \frac{L^2}{3}\dot{\theta}^2)$$

$$T_{cart} = \frac{1}{2}m_c\dot{x}^2$$

So we can solve for T_{total}

$$T_{total} = T_{cart} + T_{rod} = \frac{1}{2}m_c\dot{x}^2 + \frac{1}{2}m_r(\dot{x}^2 + Lc\theta\dot{x}\dot{\theta} + \frac{7L^2}{12}\dot{\theta}^2)$$

Now we have to solve for V

$$V = \frac{1}{2}kx^2 + \frac{1}{2}m_rgL(1 - c\theta)$$

by lagrange's eqn, we have:

$$\frac{d}{dt}(\frac{dT}{d\dot{q}_k}) - \frac{dT}{dq_k} + \frac{dV}{dq_k} = 0$$

So, we need expressions for $\frac{dT}{dx}$, $\frac{dT}{d\dot{x}}$, $\frac{dV}{dx}$, $\frac{dT}{d\theta}$, $\frac{dT}{d\dot{\theta}}$, $\frac{dV}{d\theta}$ to construct our two EOM

$$\frac{dT}{dx} = 0$$

$$\frac{dT}{d\dot{x}} = (m_c + m_r)\dot{x} + \frac{1}{2}m_rLc\theta\dot{\theta}$$

$$\frac{dV}{dx} = kx$$

$$\frac{dT}{d\theta} = -\frac{1}{2}m_rL\dot{\theta}\dot{x}s\theta$$

$$\frac{dT}{d\dot{\theta}} = \frac{1}{2}m_rL\dot{x}c\theta + \frac{7}{12}m_rL^2\dot{\theta}$$

$$\frac{dV}{d\theta} = \frac{1}{2}m_rgLs\theta$$

EOM1:

$$\frac{d}{dt}(\frac{dT}{d\dot{x}}) - \frac{dT}{dx} + \frac{dV}{dx} = 0$$

$$\frac{d}{dt}((m_c + m_r)\dot{x} + \frac{1}{2}m_r Lc\theta\dot{\theta}) + kx = 0$$

$$\boxed{(m_c + m_r)\ddot{x} + (\frac{1}{2}m_r Lc\theta)\ddot{\theta} - \frac{1}{2}m_r Ls\theta\dot{\theta}^2}$$

EOM2:

$$\frac{d}{dt}(\frac{dT}{d\dot{\theta}}) - \frac{dT}{d\theta} + \frac{dV}{d\theta} = 0$$

$$\frac{d}{dt}(\frac{1}{2}m_r L\dot{x}c\theta + \frac{7}{12}m_r L^2\dot{\theta}) + \frac{1}{2}m_r L\dot{\theta}\dot{x}s\theta + \frac{1}{2}m_r gLs\theta = 0$$

$$\frac{1}{2}m_r L\ddot{x}c\theta - \frac{1}{2}m_r L\dot{x}\dot{\theta}s\theta + \frac{7}{12}m_r L^2\ddot{\theta} + \frac{1}{2}m_r L\dot{\theta}\dot{x}s\theta + \frac{1}{2}m_r gLs\theta$$

$$\frac{1}{2}m_r L\ddot{x}c\theta + \frac{7}{12}m_r L^2\ddot{\theta} + \frac{1}{2}m_r gLs\theta = 0$$

$$\boxed{\ddot{x}c\theta + \frac{7}{6}L\ddot{\theta} + gs\theta = 0}$$