

## **Chapter 5 - Monte Carlo Methods ( Aravind S - EE14B013 - RL project RA 2 )**

### **Monte Carlo prediction**

- In most cases we won't have complete knowledge of the system dynamics ( MDP params are not known ) but we will be able to interact with the environment. In that case, Monte Carlo methods seem suitable as we obtain an estimate of the value/action-value functions and hence policy from the different episodes ( sampled from the environment ).
- Following a policy  $\pi$ , episodes are generated. Say, we want to estimate value functions.  $V(s)$  is the average of returns observed from the state  $s$ . This is true as the expected return is the average of a large number of samples drawn according to the distribution ( remember: policy  $\pi$  is followed ).
- **Advantages over DP**
  - Optimal behaviour can be learnt just from interaction with the environment.
  - Focussed learning - value functions of a particular state can be estimated separately. This is because to estimate  $V(s)$  we need a bunch of episodes starting from state 's' and we need not compute values for other states.
  - No bootstrapping - less Markovian estimate. Practically, systems may be non-markovian. In such cases, MC gives a more reliable estimate whereas DP/TD gives a biased ( Markov nature is assumed ) estimate.
- **Variants**
  - First visit - Returns from first visit to a state is considered. True estimate of  $V(s)$ , but may not be reliable if 's' is a rare state for the policy  $\pi$ .
  - Every visit - Returns from multiple visit to a state is considered. A biased estimate, but we obtain a relatively large number of samples for a rare state 's'.

### **Monte Carlo estimation of action-values**

- The action-value function is estimated using returns obtained when a particular ( state, action ) pair is visited.
- Both first-visit and every-visit variants can be used depending on the type of episodes available.

### **Monte Carlo control ( on-policy )**

- To approximate optimal policies, we use GPI. We evaluate action-values rather than values because we don't know the dynamics of the system and hence estimation of policies from values isn't possible ( Bellman equation ).
- Action-values are estimated for an estimated  $\pi$ .  $\pi$  is computed greedily w.r.t current estimate of action-values. This is repeated until convergence ( Each (s, a) is visited infinitely often ).
- The problem is when a fixed policy  $\pi$  is followed, some ( state, action ) pairs will never occur. This problem is tackled using:
  - Exploring start s ( ES ) - Episodes are generated s.t the initial (s, a) covers all possible combinations and hence when infinite episodes are obtained, the estimates converge to true action-value functions.
  - Without ES - ES is unlikely as it can't be simulated practically. The alternate approach is to use  $\epsilon$ -soft policies ( page 109 - para 1 ). The disadvantage with this method is the optimal policy obtained is the best among all the possible  $\epsilon$ -soft policies. This may not be the overall best optimal policy.
- Policy evaluation can be implemented in an incremental fashion using cumulative sum of weights for the states encountered ( Section 5.6 ).

### **Off-policy prediction via Importance sampling ( page 112 - detailed explanation of importance sampling )**

- The problem with MC methods is - you need to obtain samples following the policy  $\pi$ . But for your estimate to be reliable you need to explore.
- This is solved using off-policy methods where you deal with 2 policies - behaviour policy ( used to generate episodes ) & target policy ( the policy which is learnt ). We generate the episodes following a policy  $\mu$  and estimate values for policy  $\pi$ .
- 2 things to note:
  - Coverage -  $\mu$  should be stochastic and hence have a non-zero probability of selecting actions which might be selected when we follow  $\pi$ .
  - Ordinary ( unbiased, higher variance [ sometimes infinite ] ) Vs Weighted ( biased, lower variance ) - Estimates from weighted importance sampling are more reliable even though the estimates don't correspond to an expectation over policy  $\pi$ . But given a large number of episodes both these estimates are close enough and hence converge to the same true optimal value.

### **Off-policy Monte Carlo control**

- As expected, Monte Carlo control is performed using GPI. Prediction step is done using importance sampling ( policy evaluation following some behaviour policy,  $\mu$  with an estimate of target policy,  $\pi$ ).
- The policy improvement step is a greedification step.  $\pi$  is estimated greedily with the current estimate of action-values.
- $\mu$  is preferably an  $\epsilon$ -soft policy to make sure sufficient exploration is done. The problem with this is learning occurs when part of the trajectory is obtained from a greedy policy. If there are lot of exploratory actions, learning is slow and hence it takes a long time converge. This situation can be avoided by using methods like temporal difference learning.

### **Return-specific importance sampling**

- Discount  $\gamma$  - measure of probability of termination. Now a flat partial return up to a horizon  $h$  ( Page 121 ) is defined. Following the derivation in page 121, we obtain a different importance sampling based estimator.
- The advantage of this is, for a discounted return setup, later rewards contribute less to the return and hence the weights in the ordinary importance sampling estimator result in higher variance in the estimate.
- Per-reward importance sampling - importance-sampling weights are simplified resulting in a low-variance estimate ( derivation in page 122 ).